IYGB - MP2 PAPER U - QUESTION 1

USING APPROXIMATIONS FOR "SMALL ANGLES"

$$\sin\theta \simeq \theta$$

$$\cos\theta \simeq 1 - \frac{\theta^2}{2}$$

$$\cos\left(\tfrac{1}{2}\theta\right) \simeq 1 - \frac{\left(\frac{1}{2}\theta\right)^2}{2}$$

$$\simeq 1 - \tfrac{1}{8}\theta^2$$

HENCE WE HAVE

$$\Longrightarrow \frac{\cos\frac{1}{2}x}{1+\sin x} = 0.925$$

$$\Longrightarrow \frac{1 - \frac{1}{8}x^2}{1+x} = 0.925$$

$$\Longrightarrow \frac{8 - x^2}{8 + 8x} = 0.925$$

$$\Longrightarrow 7.4 + 7.4x = 8 - x^2$$

$$\Longrightarrow x^2 + 7.4x - 0.6 = 0$$

QUADRATIC FORMULA OR COMPLETING THE SQUARE

$$\Longrightarrow (x+3.7)^2 - 3.7^2 - 0.6 = 0$$

$$\Longrightarrow (x+3.7)^2 = 14.29$$

$$\Longrightarrow x + 3.7 = \begin{cases} 0.0802\ldots \\ -3.78021\ldots \end{cases}$$

$$\Longrightarrow x = \begin{cases} \\ \cancel{7.4802}\ldots \end{cases}$$

$$\therefore\ x \simeq 0.08$$

IYGB - MP2 PAPER U - QUESTION 2

PROCEED AS FOLLOWS

$\Longrightarrow\ 2^x + 4^x + 8^x + 16^x + \dots = 1$

$\Longrightarrow\ 2^x + (2^2)^x + (2^3)^x + (2^4)^x + \dots = 1$

$\Longrightarrow\ 2^x + (2^x)^2 + (2^x)^3 + (2^x)^4 + \dots = 1$

THIS IS A CONVERGENT G.P WITH $a = 2^x$ & $r = 2^x$

$\Longrightarrow\ \dfrac{2^x}{1-2^x} = 1$ $\qquad \left\{ S_\infty = \dfrac{a}{1-r} \right\}$

$\Longrightarrow\ 2^x = 1 - 2^x$

$\Longrightarrow\ 2 \times 2^x = 1$

$\Longrightarrow\ 2^x = \dfrac{1}{2}$

$\Longrightarrow\ 2^x = 2^{-1}$

$\Longrightarrow\ x = -1$

IYGB - MP2 PAPER U - QUESTION 3

a) SOLVING THE STANDARD EQUATION FOR $0 \leq x \leq 2\pi$

$$2 + \sec\left(x - \frac{\pi}{3}\right) = 0$$

$$\sec\left(x - \frac{\pi}{3}\right) = -2$$

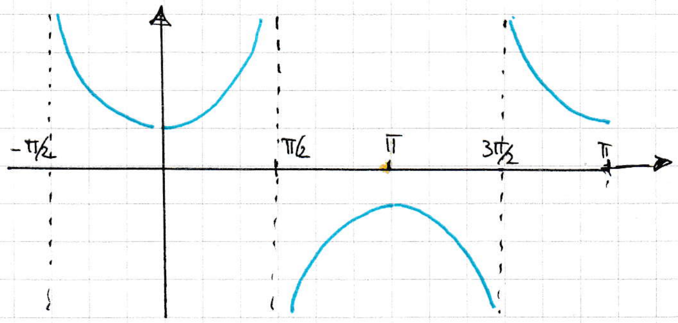
$$\cos\left(x - \frac{\pi}{3}\right) = -\frac{1}{2}$$

$$\begin{cases} x - \pi/3 = 2\pi/3 \pm 2n\pi \\ x - \pi/3 = 4\pi/3 \pm 2n\pi \end{cases} \qquad n = 0,1,2,3,\ldots$$

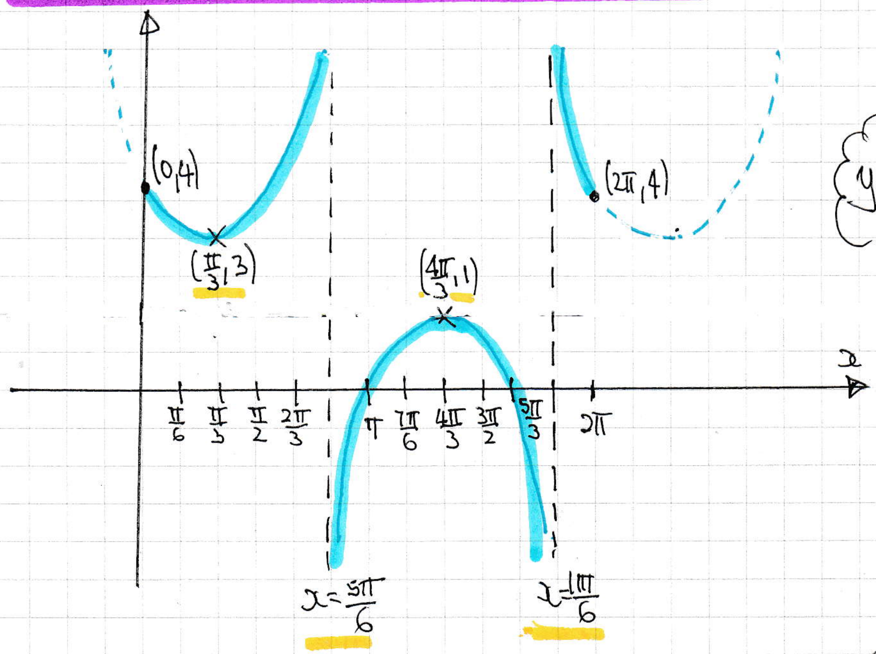
$$\begin{cases} x = \pi \pm 2n\pi \\ x = \frac{5\pi}{3} \pm 2n\pi \end{cases}$$

$$\therefore \; x = \pi, \; \frac{5\pi}{3}$$

b) STARTING WITH THE GRAPH OF $\sec x$



TRANSLATING BY $\pi/3$ TO THE "RIGHT" & BY 2 "UPWARDS"



$$y = f(x) = 2 + \sec\left(x - \frac{\pi}{3}\right)$$

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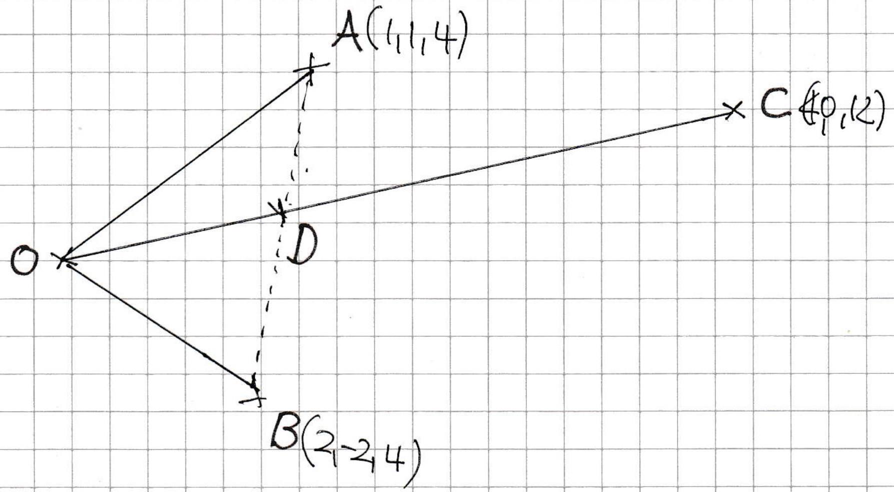
DRAWING A DIAGRAM



DETERMINE THE POSITION VECTOR OF D

$$\vec{OD} = \frac{1}{3}\vec{OC} = \frac{1}{3}(4,0,12) = \left(\frac{4}{3},0,4\right) \quad \text{i.e. } D\left(\frac{4}{3},0,4\right)$$

DETERMINE THE VECTORS $\vec{AD}$ & $\vec{DB}$

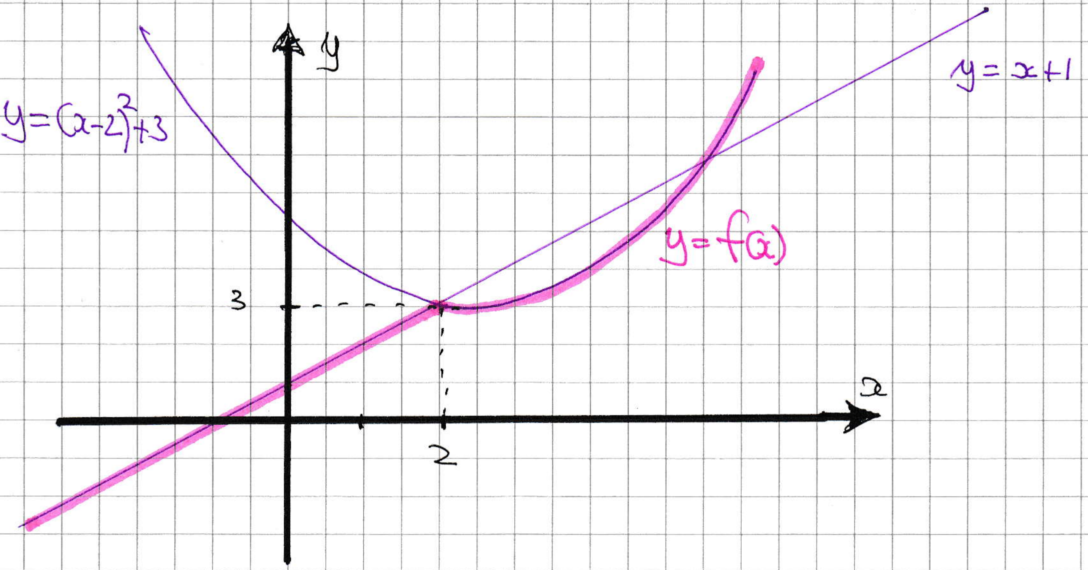
$$\vec{AD} = \underline{d} - \underline{a} = \left(\frac{4}{3},0,4\right) - (1,1,4) = \left(\frac{1}{3},-1,0\right) = 1\left(\frac{1}{3},-1,0\right)$$

$$\vec{DB} = \underline{b} - \underline{d} = (2,-2,4) - \left(\frac{4}{3},0,4\right) = \left(\frac{2}{3},-2,0\right) = 2\left(\frac{1}{3},-1,0\right)$$

AS $\vec{AD}$ IS IN THE SAME DIRECTION AS $\vec{DB}$, AND SHARE A POINT A, D & B ARE COLLINEAR, SO D LIES ON THE LINE AB

IYGB – MP2 PAPER U – QUESTION 5

a) SKETCHING EACH SECTION SEPARATELY NOTING THAT BOTH SECTIONS, AGREE AT $x=2$



b) TREATING EACH SECTION SEPARATELY

- $f_1(x) = x+1$

$y = x+1$

$x = y-1$

$f_1^{-1}(x) = x-1$

- $f_2(x) = (x-2)^2+3$

$y = (x-2)^2+3$

$y-3 = (x-2)^2$

$x-2 = \pm\sqrt{y-3}$

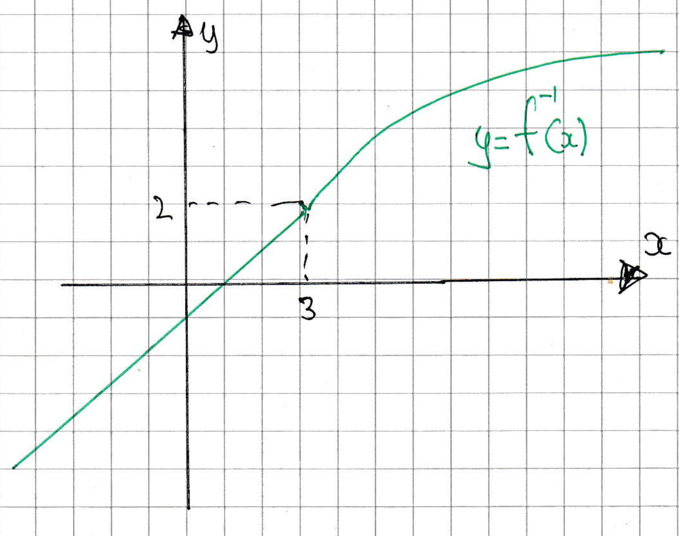
$(x>2 \Rightarrow$ RHS IS POSITIVE$)$

$x-2 = \sqrt{y-3}$

$x = 2+\sqrt{y-3}$

$f_2^{-1}(x) = 2+\sqrt{x-3}$

$$\therefore f^{-1}(x) = \begin{cases} x-1 & x \leqslant 3 \\ 2+\sqrt{x-3} & x>3 \end{cases}$$



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- START BY REARRANGING THE EQUATION

$$4\sin\frac{\theta}{2} + \sqrt{3} = 0$$
$$4\sin\frac{\theta}{2} = -\sqrt{3}$$
$$\sin\frac{\theta}{2} = -\frac{\sqrt{3}}{4}$$

- WE NEED NO SOLUTION OVER AN ACTUAL RANGE, IF WE SUITABLY MANIPULATE THE FUNCTION

$$\Rightarrow f(\theta) = 4\cos\theta - 3\sin\frac{\theta}{2}$$
$$\Rightarrow f(\theta) = 4\left(1 - 2\sin^2\frac{\theta}{2}\right) - 3\sin\frac{\theta}{2}$$
$$\Rightarrow f(\theta) = 4 - 8\sin^2\frac{\theta}{2} - 3\sin\frac{\theta}{2}$$

$$\boxed{\begin{aligned} \cos 2A &\equiv 1 - 2\sin^2 A \\ \cos 2\left(\tfrac{A}{2}\right) &\equiv 1 - 2\sin^2\left(\tfrac{A}{2}\right) \\ \cos A &\equiv 1 - 2\sin^2\tfrac{A}{2} \end{aligned}}$$

- EVALUATING THE ABOVE AT $\sin\frac{\theta}{2} = -\frac{\sqrt{3}}{4}$

$$\Rightarrow f(\theta)\Big|_{\sin\frac{\theta}{2} = -\frac{\sqrt{3}}{4}} = 4 - 8\left(-\frac{\sqrt{3}}{4}\right)^2 - 3\left(-\frac{\sqrt{3}}{4}\right)$$
$$= 4 - 8\left(\frac{3}{16}\right) + \frac{3\sqrt{3}}{4}$$
$$= 4 - \frac{3}{2} + \frac{3\sqrt{3}}{4}$$
$$= \frac{5}{2} + \frac{3\sqrt{3}}{4}$$
$$= \frac{10}{4} + \frac{3\sqrt{3}}{4}$$
$$= \frac{1}{4}\left(10 + 3\sqrt{3}\right)$$

IYGB - MP2 PAPER U - QUESTION 7

- REWRITE THE EQUATION IN INDEX FORM & DIFFERENTIATE BY USING THE PRODUCT RULE

$$\Rightarrow y = x(\ln x)^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = 1 \times (\ln x)^{\frac{1}{2}} + \cancel{x} \times \frac{1}{2}(\ln x)^{-\frac{1}{2}} \times \frac{1}{\cancel{x}}$$

$$\Rightarrow \frac{dy}{dx} = (\ln x)^{\frac{1}{2}} + \frac{1}{2}(\ln x)^{-\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\ln x} + \frac{1}{2\sqrt{\ln x}}$$

- NOW WE REQUIRE GRADIENT OF $\frac{3}{2}$

$$\Rightarrow \sqrt{\ln x} + \frac{1}{2\sqrt{\ln x}} = \frac{3}{2}$$

$$\Rightarrow a + \frac{1}{2a} = \frac{3}{2} \qquad \text{WHERE } a = \sqrt{\ln x}$$

$$\Rightarrow 2a^2 + 1 = 3a$$

$$\Rightarrow 2a^2 - 3a + 1 = 0$$

$$\Rightarrow (2a-1)(a-1) = 0$$

$$\Rightarrow a = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$$

$$\Rightarrow \sqrt{\ln x} = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$$

$$\Rightarrow \ln x = \begin{cases} 1 \\ \frac{1}{4} \end{cases}$$

IYGB - MP2 PAPER U - QUESTION 7

$$\Longrightarrow x = \begin{cases} e^{1} = e \\ e^{\frac{1}{4}} \end{cases}$$

$$\Longrightarrow y = \begin{cases} e \times 1 = e \\ e^{\frac{1}{4}} \times \frac{1}{2} = \frac{1}{2}e^{\frac{1}{4}} \end{cases}$$

- REQUIRED POINTS ARE (e, e) & $\left(e^{\frac{1}{4}}, \frac{1}{2}e^{\frac{1}{4}}\right)$

IYGB – MP2 PAPER U – QUESTION 8

ARGUE AS FOLLOWS – LET a & b BE ODD POSITIVE INTEGERS

$a+b$ IS A MULTIPLE OF 4, SO $a+b = 4m$, $m \in \mathbb{N}$

SUPPOSE NOW $a-b$ IS A MULTIPLE OF 4

$a-b = 4n$, $n \in \mathbb{N}$

ADDING THE EQUATIONS

$$\left.\begin{array}{l} a+b = 4m \\ a-b = 4n \end{array}\right\} \Longrightarrow \quad 2a = 4(m+n)$$

$$a = 2(m+n)$$

$\therefore$ a MUST BE EVEN

THIS IS A CONTRADICTION THAT a IS ODD

$\therefore$ $a-b$ CANNOT BE A MULTIPLE OF 4 //

IYGB - MP2 PAPER U - QUESTION 9

a) REWRITE THE EQUATION IN FUNCTION FORM

$$\boxed{x^2 = \frac{2}{\sqrt{x}} + \frac{3}{x^2}}$$

$$x^2 - \frac{2}{\sqrt{x}} + \frac{3}{x^2} = 0$$

$$f(x) = x^2 - \frac{2}{\sqrt{x}} - \frac{3}{x^2}$$

- $f(1) = 1 - 2 - 3 = -4 < 0$
- $f(2) = 4 - \sqrt{2} - \frac{3}{4} = 1.8357\ldots > 0$

AS $f(x)$ IS CONTINOUS ON $(1,2)$, AND CHANGES SIGN ON $(1,2)$ THERE EXISTS AT LEAST A VALUE α IN $(1,2)$ SO THAT $f(\alpha)=0$

b)
$$\boxed{\begin{aligned} f(x) &= x^2 - 2x^{-\frac{1}{2}} - 3x^{-2} \\ f'(x) &= 2x + x^{-\frac{3}{2}} + 6x^{-3} \end{aligned}}$$

NEWTON RAPHSON STATES

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^2 - 2x_n^{-\frac{1}{2}} - 3x_n^{-2}}{2x_n + x_n^{-\frac{3}{2}} + 6x_n^{-3}}$$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^5 - 2x_n^{\frac{5}{2}} - 3x_n}{2x_n^4 + x_n^{\frac{3}{2}} + 6}$$

$$\Rightarrow x_{n+1} = \frac{2x_n^5 + x_n^{\frac{5}{2}} + 6x_n - x_n^5 + 2x_n^{\frac{5}{2}} + 3x_n}{2x_n^4 + x_n^{\frac{3}{2}} + 6}$$

$$\Rightarrow x_{n+1} = \frac{x_n^5 + 3x_n^{\frac{5}{2}} + 9x_n}{2x_n^4 + x_n^{\frac{3}{2}} + 6}$$

AS REQUIRED

IYGB — MP2 PAPER U — QUESTION 9

c) USING ANY VALUE IN THE INTERVAL AND THE RECURRENCE OF PART (b)

$$\boxed{x_{n+1} = \frac{x_n^5 + 3x_n^{\frac{5}{2}} + 9x_n}{2x_n^4 + x_n^{\frac{3}{2}} + 6}}$$

- $x_1 = 1.5$
- $x_2 = 1.634594485\ldots$
- $x_3 = 1.637565406\ldots$
- $x_4 = 1.637566228\ldots$
- $x_5 = 1.637566228\ldots$

$\therefore$ REQUIRED ROOT IS 1.63756623

CORRECT TO 8 d.p

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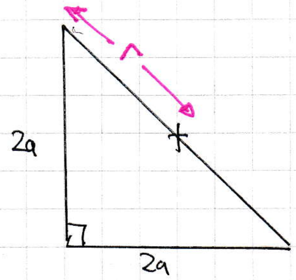
EVIDENTLY THE AREA OF THE SQUARE ABCD IS $4a^2$

THE RADIUS OF THE CIRCUMSCRIBING CIRCLE IS



$$\frac{1}{2}\sqrt{(2a)^2+(2a)^2} = \frac{1}{2}\sqrt{8a^2}$$
$$= \frac{1}{2}\times 2\sqrt{2}a$$
$$= \sqrt{2}a$$

THE AREA BETWEEN THE SIDE OF THE SQUARE AND A LUNE IS

$$\frac{1}{4}\left[\pi(\sqrt{2}a)^2 - 4a^2\right] = \frac{1}{4}\left[\pi(2a^2) - 4a^2\right] = \frac{1}{2}\left[\pi a^2 - 2a^2\right]$$
$$= \frac{1}{2}(\pi-2)a^2$$

THE AREA OF EACH LUNE IS

$$\underbrace{\frac{1}{2}\pi a^2}_{\text{SEMICIRCLE}} - \frac{1}{2}(\pi-2)a^2 = \cancel{\frac{1}{2}\pi a^2} - \cancel{\frac{1}{2}\pi a^2} + a^2 = a^2$$

$\therefore$ AREA OF 4 LUNES IS $4a^2$ WHICH IS THE SAME AREA AS THAT OF THE SQUARE

IYGB - MP2 PAPER U - QUESTION 11

PROCEED AS FOLLOWS

$$\frac{A+Bx}{(2-x)^3} = (A+Bx)(2-x)^{-3} = (A+Bx)\times 2^{-3}\left(1-\tfrac{1}{2}x\right)^{-3}$$

$$= \tfrac{1}{8}(A+Bx)\left[1 + \frac{-3}{1}\left(-\tfrac{1}{2}x\right)^1 + \frac{-3(-4)}{1\times 2}\left(-\tfrac{1}{2}x\right)^2 + \frac{-3(-4)(-5)}{1\times 2\times 3}\left(-\tfrac{1}{2}x\right)^3 + O(x^4)\right]$$

$$= \tfrac{1}{8}(A+Bx)\left[1 + \tfrac{3}{2}x + \tfrac{3}{2}x^2 + \tfrac{5}{4}x^3 + O(x^4)\right]$$

$$= (A+Bx)\left[\tfrac{1}{8} + \tfrac{3}{16}x + \tfrac{3}{16}x^2 + \tfrac{5}{32}x^3 + O(x^4)\right]$$

$$= \tfrac{1}{8}A + \tfrac{3}{16}Ax + \tfrac{3}{16}Ax^2 + \tfrac{5}{32}Ax^3 + O(x^4)$$
$$\qquad + \tfrac{1}{8}Bx + \tfrac{3}{16}Bx^2 + \tfrac{3}{16}Bx^3 + O(x^4)$$

$$\underset{\underset{\frac{1}{4}}{\uparrow}}{\tfrac{1}{8}A} + \underset{\underset{0}{\uparrow}}{\left(\tfrac{3}{16}A+\tfrac{1}{8}B\right)}x + \underset{\underset{C}{\uparrow}}{\left(\tfrac{3}{16}A+\tfrac{3}{16}B\right)}x^2 + \underset{\underset{D}{\uparrow}}{\left(\tfrac{5}{32}A+\tfrac{3}{16}B\right)}x^3 + O(x^4)$$

COMPARING COEFFICIENTS

- $\tfrac{1}{8}A = \tfrac{1}{4}$

  $A = 2$

- $\tfrac{3}{16}A + \tfrac{1}{8}B = 0$

  $3A + 2B = 0$

  $6 + 2B = 0$

  $B = -3$

- $C = \tfrac{3}{16}A + \tfrac{3}{16}B$

  $C = \tfrac{3}{16}(A+B)$

  $C = \tfrac{3}{16}(-1)$

  $C = -\tfrac{3}{16}$

- $D = \tfrac{5}{32}A + \tfrac{3}{16}B$

  $D = \tfrac{5}{16} - \tfrac{9}{16}$

  $D = -\tfrac{1}{4}$

IYGB – MP2 PAPER U – QUESTION 12

a) LOOKING AT THE FIRST SUMMATION WE REQUIRE THE FIRST 3 TERMS

$$\therefore \boxed{t_{n+1} = at_n + 3n + 2}$$

- $t_1 = -2$
- $t_2 = at_1 + 3\times1 + 2 = a(-2) + 3 + 2 = 5 - 2a$
- $t_3 = at_2 + 3\times2 + 2 = a(5-2a) + 6 + 2 = 8 + 5a - 2a^2$

NOW WE HAVE

$$\sum_{r=1}^{3}(t_r + r^3) = (t_1 + 1^3) + (t_2 + 2^3) + (t_3 + 3^3)$$
$$= t_1 + 1 + t_2 + 8 + t_3 + 27$$
$$= t_1 + t_2 + t_3 + 36$$
$$= -2 + (5 - 2a) + (8 + 5a - 2a^2) + 36$$
$$= -2a^2 + 3a + 47$$

FINALLY WE HAVE

$$\Rightarrow \sum_{r=1}^{3}(t_r + r^3) = 12$$
$$\Rightarrow -2a^2 + 3a + 47 = 12$$
$$\Rightarrow 0 = 2a^2 - 3a - 35$$
$$\Rightarrow (a-5)(2a+7) = 0$$
$$a = \begin{cases} 5 \\ -\frac{7}{2} \end{cases}$$

IYGB - MP2 PAPER U - QUESTION 12

b) WE PROCEED AS FOLLOWS (THE VALUE OF k IS IRRELEVANT)

$$\sum_{r=8}^{31}\left[t_{r+1} - a t_r\right] = \sum_{r=8}^{31}\left[(\cancel{at_r} + 3r + 2) - \cancel{at_r}\right]$$

$$= \sum_{r=8}^{31}(3r+2)$$

$$= 26 + 29 + 32 + 35 + \dots + 95$$

THIS IS AN ARITHMETIC PROGRESSION WITH $(31-7) = 24$ TERMS

$$\Rightarrow S_n = \frac{n}{2}\left[a + L\right]$$

$$\Rightarrow S_{24} = \frac{24}{2}\left[26 + 95\right]$$

$$\Rightarrow S_{24} = 12 \times 121$$

$$\Rightarrow S_{24} = \begin{array}{r}1210\\242\end{array}$$

$$\Rightarrow S_{24} = 1452$$

IYGB - MP2 PAPER U - QUESTION 13

FORMING A DIFFERENTIAL EQUATION

x = COFFEE TEMPERATURE (°C)
t = TIME (MINUTES)

$t=0$, $x=80$
$t=10$, $x=40$

$$\frac{dx}{dt} = -k(x-10)^2$$

RATE, DECREASING, PROPORTIONAL, "DIFFERENCE ... SQUARED"

SOLVING BY SEPARATION OF VARIABLES

$$\Rightarrow \frac{dx}{dt} = -k(x-10)^2$$

$$\Rightarrow dx = -k(x-10)^2\,dt$$

$$\Rightarrow \frac{1}{(x-10)^2}\,dx = -k\,dt$$

$$\Rightarrow \int (x-10)^{-2}\,dx = \int -k\,dt$$

$$\Rightarrow -(x-10)^{-1} = -kt + C$$

$$\Rightarrow \frac{1}{x-10} = At + B$$

APPLY THE CONDITIONS GIVEN

$t=0$, $x=80$ $\Rightarrow \frac{1}{70} = B$

$$\Rightarrow \frac{1}{x-10} = At + \frac{1}{70}$$

$t=10$, $x=40$ $\Rightarrow \frac{1}{30} = 10A + \frac{1}{70}$

IYGB – MP2 PAPER U – QUESTION 13

$$\Rightarrow A = \frac{1}{525}$$

$$\Rightarrow \frac{1}{x-10} = \frac{1}{525}t + \frac{1}{70}$$

$\times 1050$

$$\Rightarrow \frac{1050}{x-10} = 2t + 15$$

$$\Rightarrow \frac{1050}{2t+15} = x - 10$$

$$\Rightarrow x = \frac{1050}{2t+15} + 10$$

$$\Rightarrow x = \frac{1050 + 10(2t+15)}{2t+15}$$

$$\Rightarrow x = \frac{20t + 1200}{2t+15}$$

AS REQUIRED

FINALLY WHEN $x = 20$

$$\frac{1050}{x-10} = 2t + 15 \quad \text{(FROM EARLIER)}$$

$$\frac{1050}{20-10} = 2t + 15$$

$$105 = 2t + 15$$

$$90 = 2t$$

$$t = 45$$

IYGB - MP2 PAPER U - QUESTION 14

a)
$$\frac{1}{u^2+5u+6} \equiv \frac{1}{(u+2)(u+3)} \equiv \frac{A}{u+2} + \frac{B}{u+3}$$

$$1 \equiv A(u+3) + B(u+2)$$

If $u=-3 \implies 1 = -B$

If $u=-2 \implies 1 = A$

$$\therefore \frac{1}{u^2+5u+6} = \frac{1}{u+2} - \frac{1}{u+3}$$

b)
$$\int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}} \frac{1}{(\sin x + 2\cos x)(\sin x + 3\cos x)}\, dx$$

DIVIDE "TOP & BOTTOM" OF THE INTEGRAND BY $\cos^2 x$

$$= \int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}} \frac{\frac{1}{\cos^2 x}}{\frac{(\sin x+2\cos x)}{\cos x}\,\frac{(\sin x+3\cos x)}{\cos x}}\, dx$$

$$= \int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}} \frac{\sec^2 x}{(\tan x+2)(\tan x+3)}\, dx$$

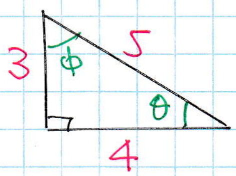
c) USING THE SUBSTITUTION GIVEN.

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

THE LIMITS BECOME



$\arccos\frac{3}{5} = \phi = \arctan\frac{4}{3}$

$\arcsin\frac{3}{5} = \theta = \arctan\frac{3}{4}$

- $x = \arccos\frac{3}{5} \longmapsto u = \frac{4}{3}$
- $x = \arcsin\frac{3}{5} \longmapsto u = \frac{3}{4}$

IYGB - MP2 PAPER U - QUESTION 14

HENCE WE NOW HAVE

$$\int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}} \frac{-\sec^2 x}{(\tan x+2)(\tan x+3)}\,dx = \int_{3/4}^{4/3} \frac{\cancel{\sec^2 x}}{(u+2)(u+3)}\,\frac{du}{\cancel{\sec^2 x}}$$

$$= \int_{\frac{3}{4}}^{\frac{4}{3}} \frac{1}{(u+2)(u+3)}\,du = \ldots \text{part (a)} \ldots = \int_{\frac{3}{4}}^{\frac{4}{3}} \frac{1}{u+2} - \frac{1}{u+3}\,du$$

$$= \Big[\ln|u+2| - \ln|u+3|\Big]_{\frac{3}{4}}^{\frac{4}{3}} = \left(\ln\frac{10}{3} - \ln\frac{13}{3}\right) - \left(\ln\frac{11}{4} - \ln\frac{15}{4}\right)$$

$$= \ln\frac{10/3}{13/3} - \ln\frac{11/4}{15/4} = \ln\frac{10}{13} - \ln\frac{11}{15}$$

$$= \ln\frac{10/13}{11/15} = \ln\frac{150}{143}$$

IYGB - SPECIAL PAPER U - QUESTION 15

- START BY FINDING THE EQUATION OF THE TANGENT AT THE POINT WHERE $t=p$ I.E $P(p^2, p^2-p)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t-1}{2t}$$

$$\left.\frac{dy}{dx}\right|_p = \frac{2p-1}{2p}$$

- EQUATION OF TANGENT IS GIVEN BY

$$y-(p^2-p) = \frac{2p-1}{2p}(x-p^2)$$

- THE TANGENT PASSES THROUGH $(4, \frac{3}{2})$

$$\frac{3}{2} - p^2+p = \frac{2p-1}{2p}(4-p^2)$$

$$3p - 2p^3+2p^2 = (2p-1)(4-p^2)$$

$$3p - \cancel{2p^3}+2p^2 = 8p - \cancel{2p^3} - 4 + p^2$$

$$p^2-5p+4=0$$

$$(p-1)(p-4)=0$$

$$p = \begin{cases} 1 \\ 4 \end{cases}$$

- HENCE WE OBTAIN

$$p=1 \qquad P(1,0)$$

$$p=4 \qquad P(16,12)$$

IYGB - MP2 PAPER U - QUESTION 16

- DIFFERENTIATE THE EXPRESSION W.R.T x, USING THE FACT $\frac{d}{dx}\left[a^{f(x)}\right] = a^{f(x)} \times \ln a \times f'(x)$

$$y = 2^{3e^{2x}} \implies \frac{dy}{dx} = 2^{3e^{2x}} \times \ln 2 \times 6e^{2x}$$

$$\implies \frac{dy}{dx} = y \ln 2 \times 2 \times (3e^{2x})$$

NOW WE NOTE THAT

$$\ln y = \ln 2^{3e^{2x}}$$

$$\ln y = (3e^{2x})(\ln 2)$$

$$\implies \frac{dy}{dx} = 2y\ln y$$

- ALTERNATIVE BY "TAKING LOGS" FIRST FOLLOWED BY IMPLICIT DIFFERENTIATION

$$\implies y = 2^{3e^{2x}}$$

$$\implies \ln y = \ln 2^{3e^{2x}}$$

$$\implies \ln y = (\ln 2)(3e^{2x})$$

DIFFERENTIATE WITH RESPECT TO x

$$\implies \frac{1}{y}\frac{dy}{dx} = (\ln 2) \times 6e^{2x}$$

$$\implies \frac{dy}{dx} = y \times (\ln 2) \times 6e^{2x}$$

$$\implies \frac{dy}{dx} = 2y \times (\ln 2)(3e^{2x})$$

$$\implies \frac{dy}{dx} = 2y\ln y$$