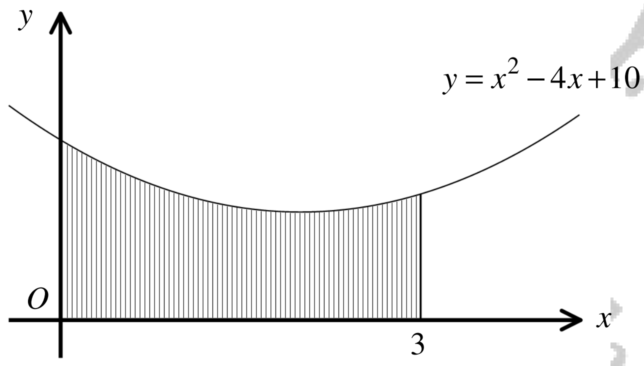
INTEGRATION

FINDING AREAS

Question 1 **(**)**

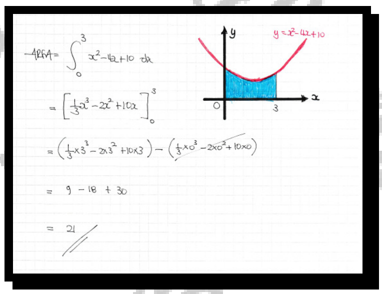


The figure above shows the curve with equation

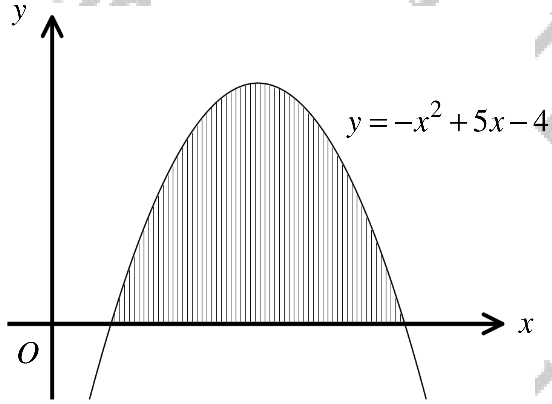
$$y = x^2 - 4x + 10, \ x \in \mathbb{R}.$$

Find the area of the region, bounded by the curve the coordinate axes and the straight line with equation $x = 3$.

$\boxed{\text{area} = 21}$



Question 2 **(**)**



The figure above shows the curve with equation

$$y = -x^2 + 5x - 4 .$$

a) Find the coordinates of the points where the curve crosses the x axis.

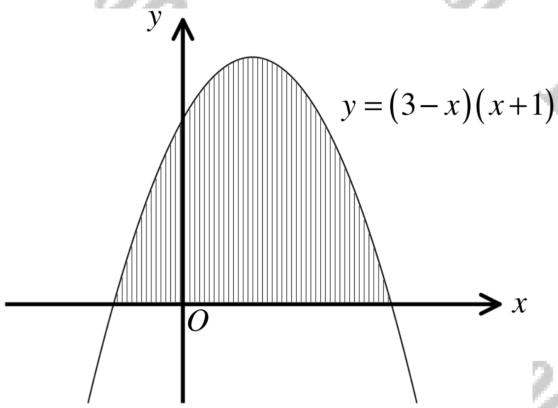
b) Determine the exact area of the shaded region.

$\boxed{(1,0),\ (4,0)}$, $\boxed{\text{area} = \frac{9}{2}}$

(a) $y = -x^2 + 5x - 4$
$\Rightarrow 0 = -x^2 + 5x - 4$
$\Rightarrow x^2 - 5x + 4 = 0$
$\Rightarrow (x-4)(x-1) = 0$
$\Rightarrow x = 1, 4$
$\therefore (1,0)$ & $(4,0)$

(b) $A = \int_1^4 -x^2 + 5x - 4 \, dx = \left[-\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x\right]_1^4$
$= \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x\right]_4^1$
$= \left(\frac{1}{3} - \frac{5}{2} + 4\right) - \left(\frac{64}{3} - 40 + 16\right)$
$= \frac{1}{3} - \frac{5}{2} + 4 - \frac{64}{3} + 40 - 16$
$= 28 - \frac{5}{2} - \frac{61}{3} = 28 - \frac{5}{2} - 21 = 7 - \frac{5}{2}$
$= \frac{14}{2} - \frac{5}{2} = \frac{9}{2}$

Question 3 (**)

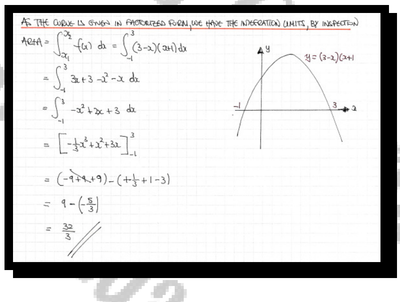


The figure above shows the curve with equation

$$y=(3-x)(x+1),\ x\in\mathbb{R}.$$

Find the exact area of the region, bounded by the curve and the x axis, shown shaded in the figure above.

$$\boxed{\text{area}=\frac{32}{3}}$$



AS THE CURVE IS GIVEN IN FACTORIZED FORM, WE HAVE THE INTEGRATION LIMITS, BY INSPECTION

AREA $= \int_{x_1}^{x_2} f(x)\,dx = \int_{-1}^{3} (3-x)(x+1)\,dx$

$= \int_{-1}^{3} 3x+3-x^2-x\,dx$

$= \int_{-1}^{3} -x^2+2x+3\,dx$

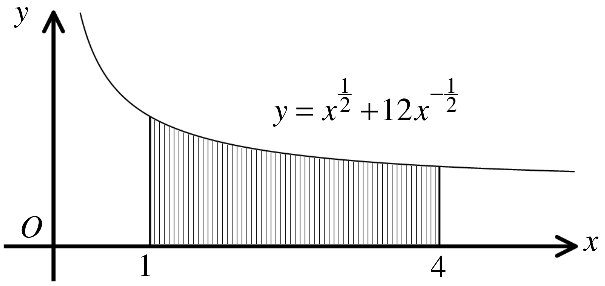
$= \left[-\frac{1}{3}x^3+x^2+3x\right]_{-1}^{3}$

$= (-9\cancel{+9}+9)-(+\frac{1}{3}+1-3)$

$= 9-\left(-\frac{5}{3}\right)$

$= \frac{32}{3}$

Question 4 (**)



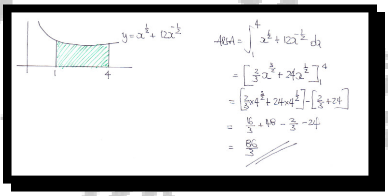
The figure above shows the curve with equation

$$y = x^{\frac{1}{2}} + 12x^{-\frac{1}{2}}, \ x \in \mathbb{R}, \ x > 0 .$$

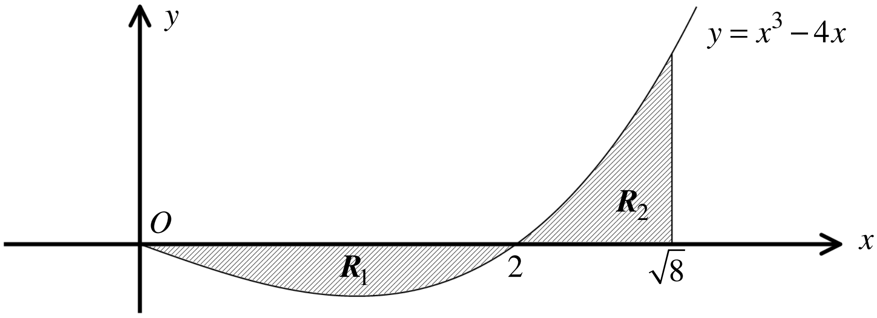
The region bounded by the curve, the x axis and the straight lines with equations $x = 1$ and $x = 4$, is shown shaded in the figure.

Find the exact area of the shaded region.

$\boxed{\text{area} = \frac{86}{3}}$



Question 5 (**)



The figure above shows the cubic curve with equation

$$y = x^3 - 4x, \ x \geq 0.$$

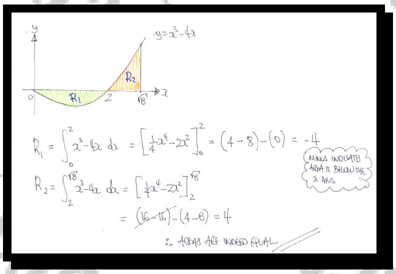
The curve meets the x axis at the origin O and at the point where $x = 2$.

The finite region R_1 is bounded by the curve and the x axis, for $0 \leq x \leq 2$.

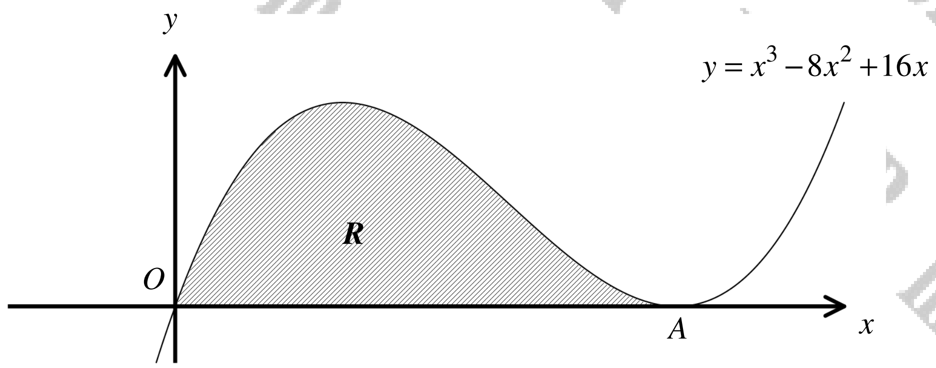
The region R_2 is bounded by the curve and the x axis, for $2 \leq x \leq \sqrt{8}$.

Show that the area of R_1 is equal to the area of R_2.

$\boxed{\text{proof}}$



Question 6 (**)



The figure above shows the cubic curve with equation

$$y = x^3 - 8x^2 + 16x, \; x \in \mathbb{R}.$$

The curve meets the x axis at the origin O and at the point A.

a) Show clearly that $x = 4$ at A.

The finite region R is bounded by the curve and the x axis.

b) Find the exact area of R.

$\boxed{\frac{64}{3}}$

(a) $y = x^3 - 8x^2 + 16x$

$0 = x^3 - 8x^2 + 16x$

$0 = x(x^2 - 8x + 16)$

$0 = x(x-4)^2$

$\therefore x = 0 \leftarrow$ ORIGIN, $\; 4 \leftarrow A$

(b) AREA $= \int_0^4 x^3 - 8x^2 + 16x \, dx$

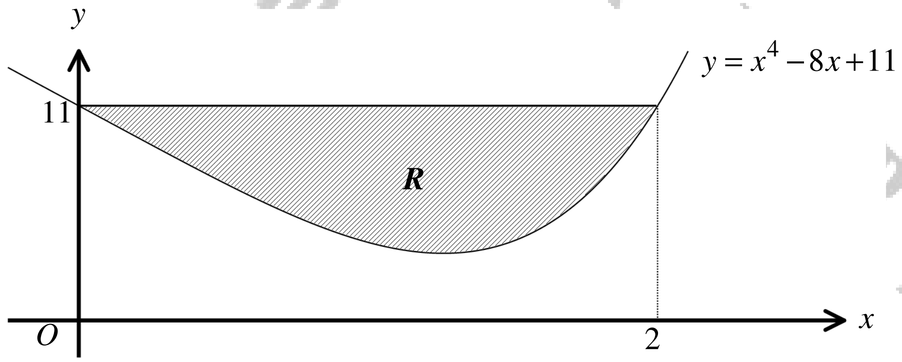
$= \left[\frac{1}{4}x^4 - \frac{8}{3}x^3 + 8x^2\right]_0^4$

$= \left(\frac{1}{4} \times 4^4 - \frac{8}{3} \times 4^3 + 8 \times 4^2\right) - (0)$

$= 64 - \frac{512}{3} + 128$

$= \frac{64}{3}$

Question 7 (**)



The figure above shows the curve with equation

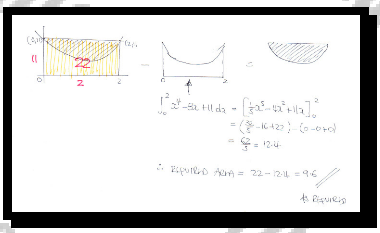
$$y = x^4 - 8x + 11, \ x \in \mathbb{R}.$$

The point with coordinates $(2,11)$ lies on the curve.

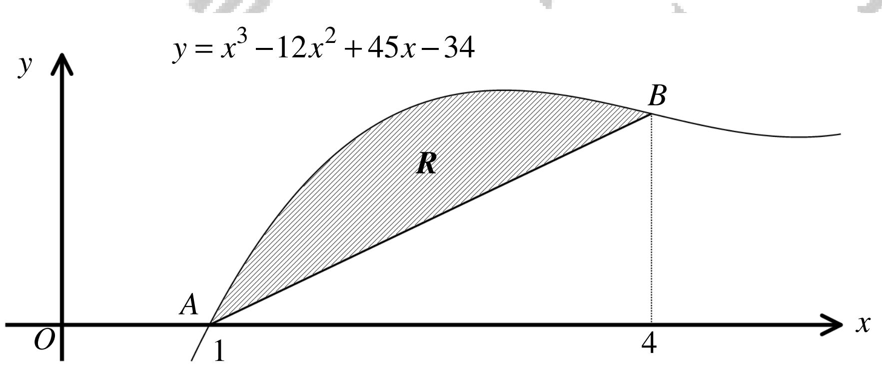
The finite region R is bounded by the curve and the straight line with equation $y = 11$.

Show that the area of R, shown shaded in the figure, is 9.6 square units.

$\boxed{\text{proof}}$



Question 8 (**)



The figure above shows the curve with equation

$$y = x^3 - 12x^2 + 45x - 34 .$$

The points A and B lie on the curve, where $x = 1$ and $x = 4$, respectively.

The finite region R is bounded by the curve and the straight line segment AB.

Show that the area of R, shown shaded in the figure, is exactly $\frac{81}{4}$.

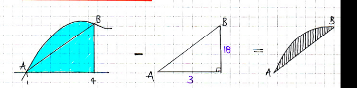
$\boxed{\text{proof}}$

START BY COMPUTING THE y CO-ORDS OF A & B

$y(1) = 1 - 12 + 45 - 34 = 0$ IE $A(1,0)$

$y(4) = 64 - 192 + 180 - 34 = 18$ IE $B(4,18)$

LOOKING AT THE DIAGRAM BELOW



$\int_1^4 x^3 - 12x^2 + 45x - 34 \, dx$

$\frac{1}{2} \times 3 \times 18 = 27$

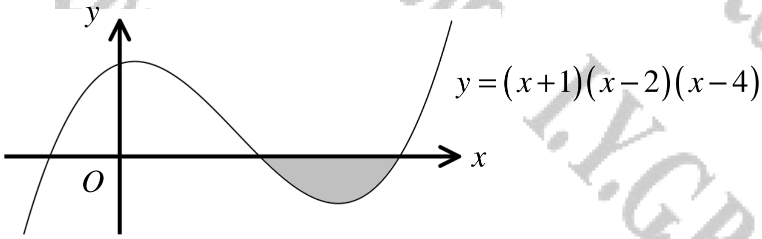
$= \left[\frac{1}{4}x^4 - 4x^3 + \frac{45}{2}x^2 - 34x \right]_1^4$

$= (64 - 256 + 360 + 136) - (\frac{1}{4} - 4 + \frac{45}{2} - 34)$

$= \frac{189}{4} = 47.25$

$\therefore$ REQUIRED AREA $= 47.25 - 27 = 20.25 = \frac{81}{4}$ // AS REQUIRED

Question 9 (**+)



The figure above shows the curve with equation

$$y=(x+1)(x-2)(x-4),\ x\in\mathbb{R}.$$

a) Write the equation of the curve in the form

$$y=x^3+ax^2+bx+c,$$

where a, b and c are constants.

b) Find the exact area the shaded region.

$\boxed{y=x^3-5x^2+2x+8}$, $\boxed{\text{area}=\frac{16}{3}}$

(a) $y=(x+1)(x-2)(x-4)$

$y=(x+1)(x^2-2x-4x+8)$

$y=(x+1)(x^2-6x+8)$

$y=x^3-6x^2+8x$
$\quad\ \ x^2-6x+8$

$y=x^3-5x^2+2x+8$

(b) AREA $=\int_2^4 x^3-5x^2+2x+8\,dx$

$=\left[\frac{1}{4}x^4-\frac{5}{3}x^3+x^2+8x\right]_2^4$

$=\left(64-\frac{5}{3}\times 64+16+32\right)-\left(4-\frac{40}{3}+4+16\right)$

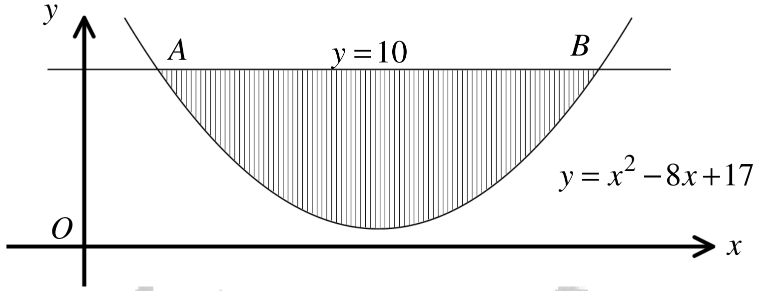
$=64-\frac{320}{3}\cancel{+16}+32-4+\frac{40}{3}-4\cancel{-16}$

$=88-\frac{280}{3}$

$=\frac{264}{3}-\frac{280}{3}$

$=-\frac{16}{3}$ $\quad\therefore$ AREA $\frac{16}{3}$

Question 10 (**+)



The figure above shows a curve and a straight line with respective equations

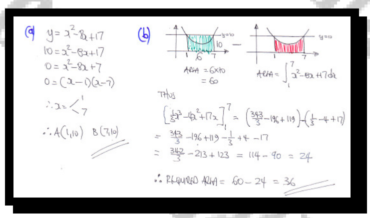
$$y = x^2 - 8x + 17 \quad \text{and} \quad y = 10 .$$

The points A and B are the points of intersection between the straight line and the quadratic curve.

a) Find the coordinates of A and B.

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{A(1,10),\ B(7,10)}$, $\boxed{\text{area} = 36}$



(a) $y = x^2 - 8x + 17$
$10 = x^2 - 8x + 17$
$0 = x^2 - 8x + 7$
$0 = (x-1)(x-7)$
$\therefore x = 1, 7$
$\therefore A(1,10)\ B(7,10)$

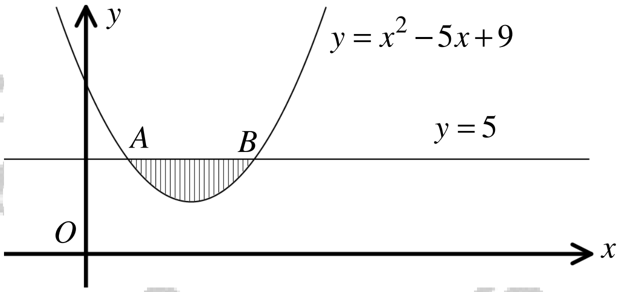
(b) AREA $= 6 \times 10 = 60$ − AREA $= \int_1^7 x^2 - 8x + 17\,dx$

THUS

$\left[\frac{1}{3}x^3 - 4x^2 + 17x\right]_1^7 = \left(\frac{343}{3} - 196 + 119\right) - \left(\frac{1}{3} - 4 + 17\right)$
$= \frac{343}{3} - 196 + 119 - \frac{1}{3} + 4 - 17$
$= \frac{342}{3} - 213 + 123 = 114 - 90 = 24$

$\therefore$ REQUIRED AREA $= 60 - 24 = 36$

Question 11 (**+)



The figure above shows a quadratic curve and a straight line with respective equations

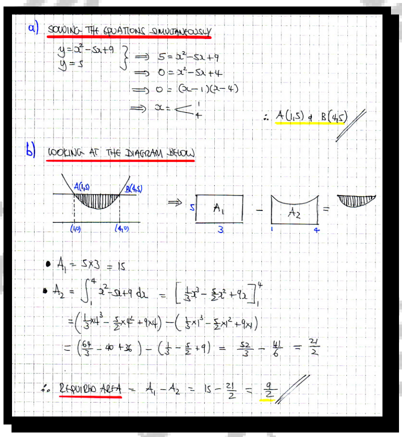
$$y = x^2 - 5x + 9 \quad \text{and} \quad y = 5.$$

The points A and B are the points of intersection between the straight line and the quadratic curve.

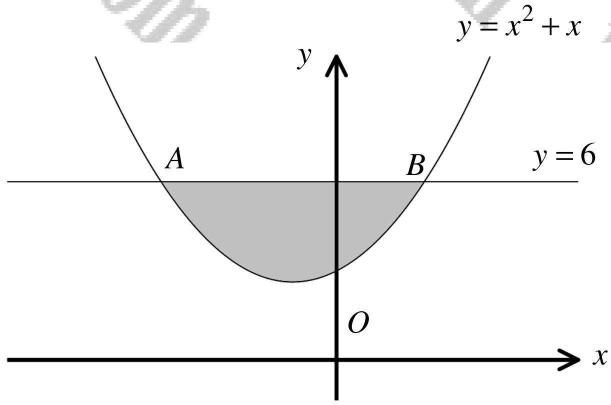
a) Find the coordinates of A and B.

c) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{A(1,5), B(4,5)}$, $\boxed{\text{area} = \frac{9}{2}}$



Question 12 (**+)



The figure above shows a quadratic curve and a straight line with respective equations

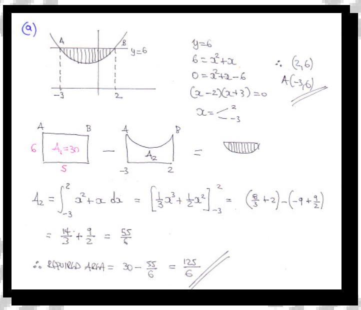
$$y = x^2 + x \quad \text{and} \quad y = 6.$$

The points A and B are the points of intersection between the straight line and the quadratic curve.

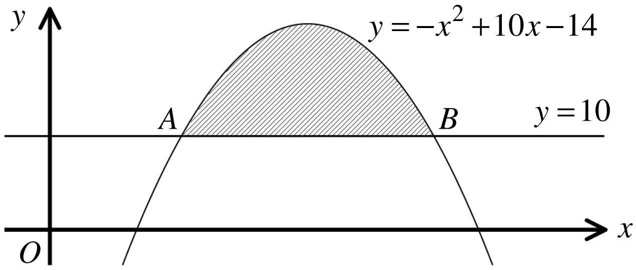
a) Find the coordinates of A and B.

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{A(-3,6),\ B(2,6)}$, $\boxed{\text{area} = \frac{125}{6}}$



Question 13 (**+)



The figure above shows a quadratic curve and a straight line with respective equations

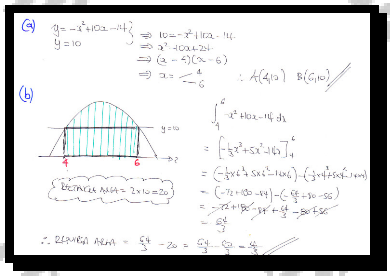
$$y=-x^2+10x-14 \quad \text{and} \quad y=10.$$

The points A and B are the points of intersection between the straight line and the quadratic curve.

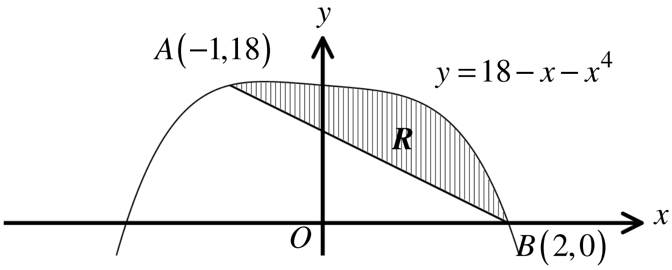
a) Find the coordinates of A and B.

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{A(4,10), B(6,10)}$, $\boxed{\text{area}=\frac{4}{3}}$



Question 14 (**+)



The figure above shows the curve C with equation

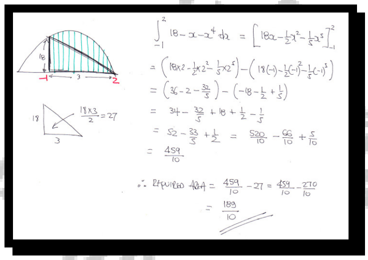
$$y = 18 - x - x^4 .$$

The curve crosses the x axis at $B(2,0)$ and the point $A(-1,18)$ lies on C.

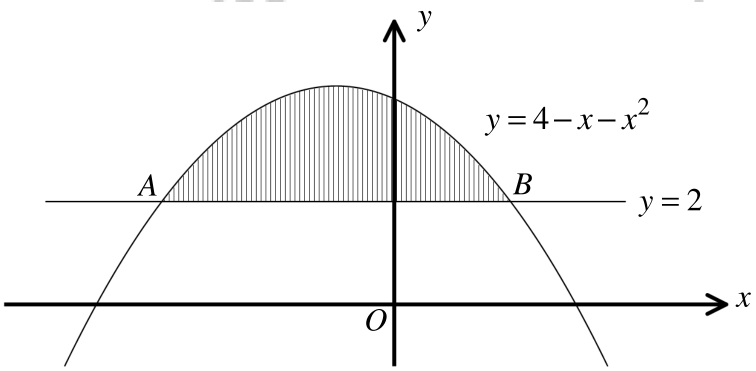
The shaded region R is bounded by the curve and the straight line segment AB.

Find the area of the shaded region.

$\boxed{\text{area} = 18.9}$



Question 15 (**+)



The figure above shows a quadratic curve and a straight line with respective equations

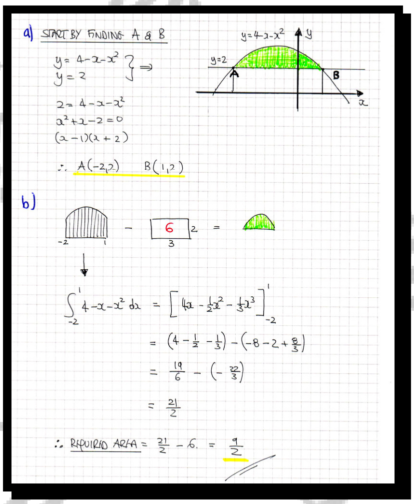
$$y = 4 - x - x^2 \quad \text{and} \quad y = 2 .$$

The points A and B are the points of intersection between the quadratic curve and the straight line.

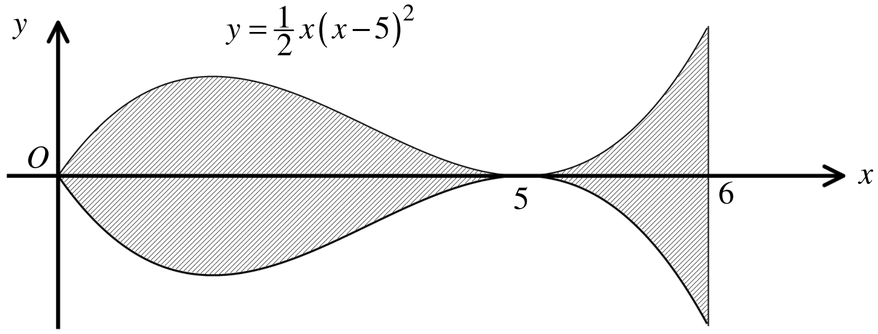
a) Find the coordinates of A and B.

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{A(-2,2),\ B(1,2)}$, $\boxed{\text{area} = \frac{9}{2}}$



Question 16 (**+)



A fish logo is generated by the curve C with equation

$$y = \frac{1}{2}x(x-5)^2, \ 0 \le x \le 6,$$

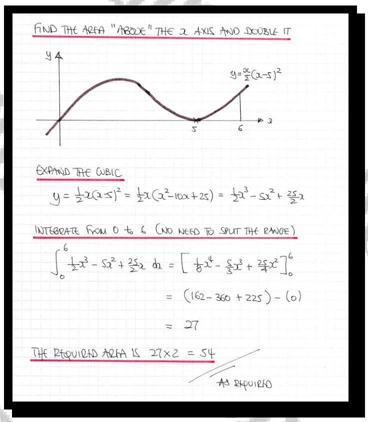
and its reflection in the x axis.

The curve C meets the x axis at the origin O and at the point $(5,0)$.

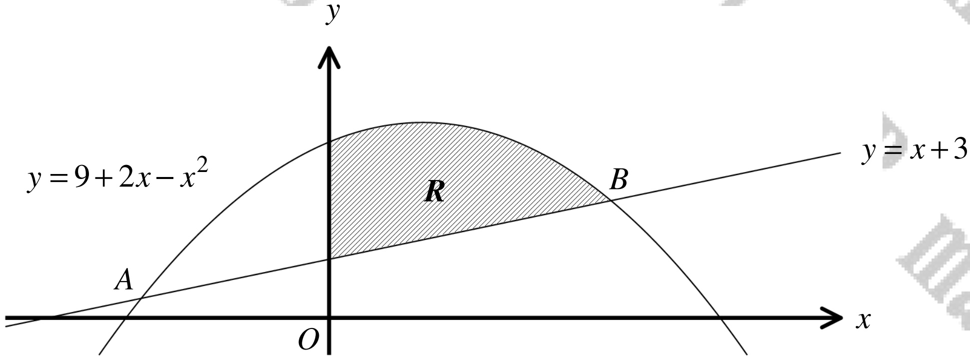
The finite region R is bounded by C, its reflection in the x axis and the straight line with equation $x = 6$.

Show that the area of R, shown shaded in the figure, is 54 square units.

$\boxed{\text{proof}}$



Question 17 (**+)



The figure above shows the graph of the curve C with equation

$$y = 9 + 2x - x^2,$$

and the straight line L with equation

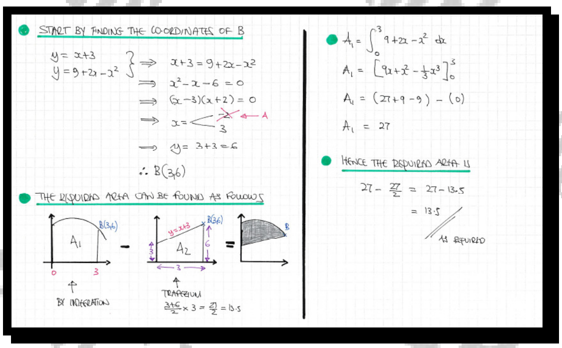
$$y = x + 3.$$

The curve meets the straight line at the points A and B.

The finite region R, shown shaded in the figure, is bounded by the curve C, the straight line L and the coordinate axes.

Show that the area of R is 13.5 square units.

proof



START BY FINDING THE CO-ORDINATES OF B

$$\left.\begin{array}{l} y = x+3 \\ y = 9+2x-x^2 \end{array}\right\} \Rightarrow x+3 = 9+2x-x^2$$
$$\Rightarrow x^2 - x - 6 = 0$$
$$\Rightarrow (x-3)(x+2) = 0$$
$$\Rightarrow x = \cancel{-2} \leftarrow A, \quad 3$$
$$\Rightarrow y = 3+3 = 6$$
$$\therefore B(3,6)$$

THE REQUIRED AREA CAN BE FOUND AS FOLLOWS

A_1 − A_2 =

B(3,6) 0 3 BY INTEGRATION

$y=x+3$ B(3,6) 3 6 3 TRAPEZIUM $\frac{3+6}{2} \times 3 = \frac{27}{2} = 13.5$

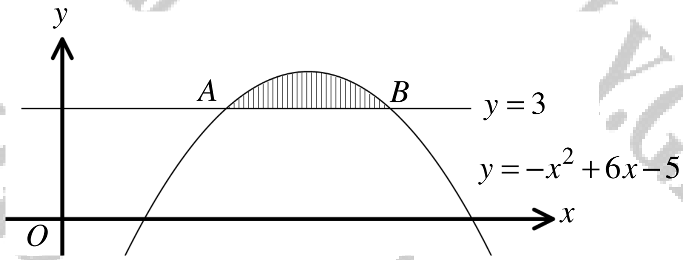
$$A_1 = \int_0^3 9+2x-x^2 \, dx$$
$$A_1 = \left[9x + x^2 - \tfrac{1}{3}x^3\right]_0^3$$
$$A_1 = (27+9-9) - (0)$$
$$A_1 = 27$$

HENCE THE REQUIRED AREA IS

$$27 - \frac{27}{2} = 27 - 13.5$$
$$= 13.5$$

AS REQUIRED

Question 18 (**+)



The figure above shows a quadratic curve and a straight line with respective equations

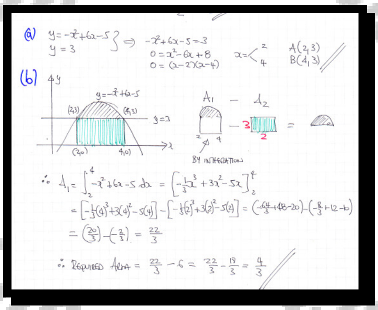
$$y = -x^2 + 6x - 5 \quad \text{and} \quad y = 3 .$$

The points A and B are the points of intersection between the quadratic curve and the straight line.

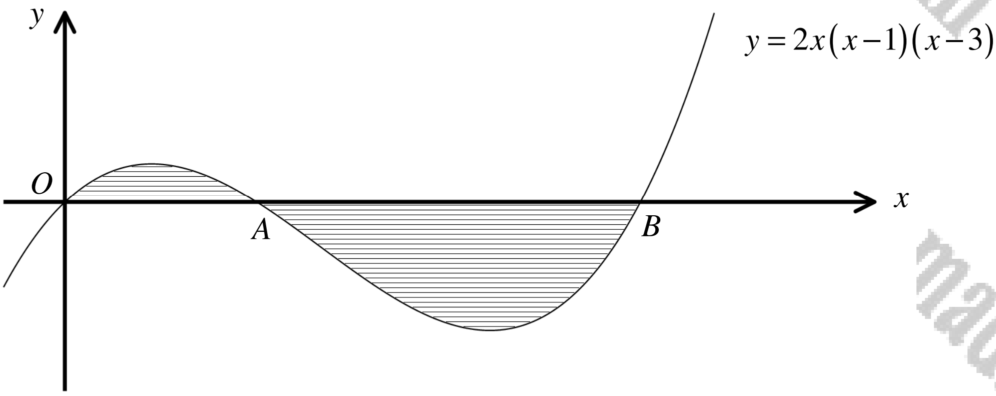
a) Find the coordinates of A and B.

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{A(2,3),\ B(4,3)}$, $\boxed{\text{area} = \frac{4}{3}}$



Question 19 (***)



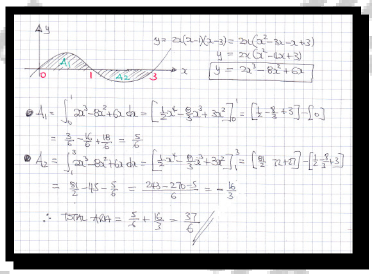
The figure above shows part of the curve with equation

$$y = 2x(x-1)(x-3), \ x \in \mathbb{R}.$$

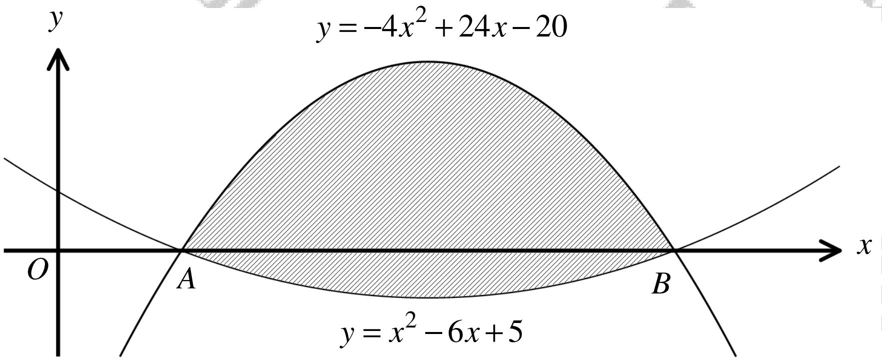
The curve meets the x axis at the origin and at the points A and B.

Determine the exact area of the finite region bounded by the curve and the x axis, shown shaded in the figure above.

$\boxed{\text{area} = \frac{37}{6}}$



Question 20 (***)



The figure above shows the graph of the curves with equations

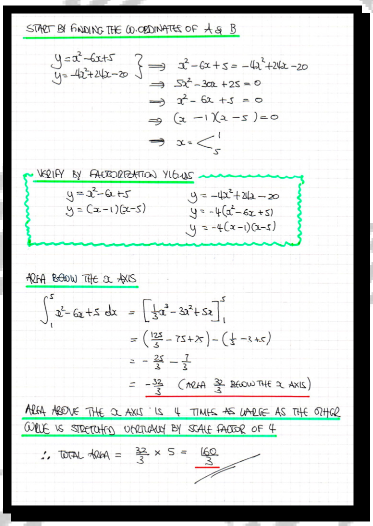
$$y = -4x^2 + 24x - 20 \quad \text{and} \quad y = x^2 - 6x + 5.$$

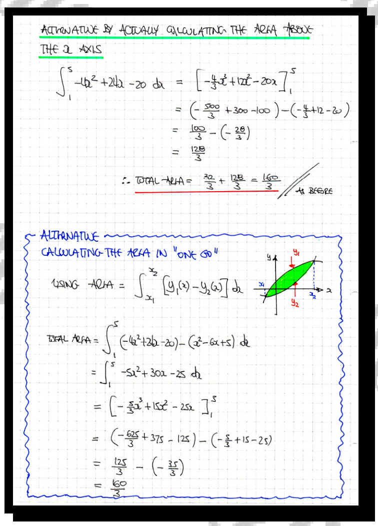
The two curves intersect each other at the points A and B.

The finite region R bounded by the two curves is shown shaded in the figure.

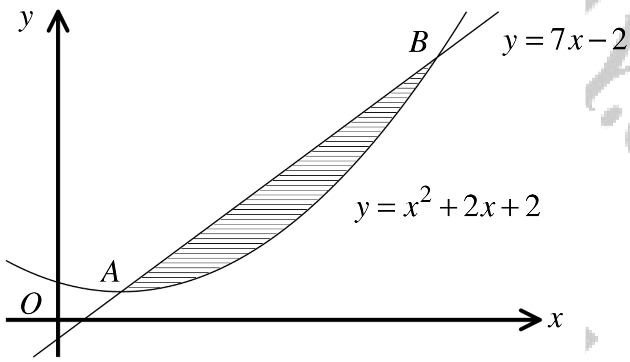
Find the exact area of R.

$\boxed{\frac{160}{3}}$





Question 21 (***)



The figure above shows a quadratic curve and a straight line with respective equations

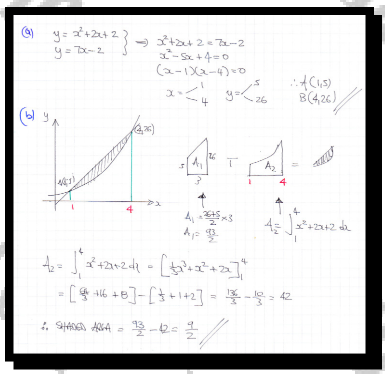
$$y = x^2 + 2x + 2 \quad \text{and} \quad y = 7x - 2 .$$

The points A and B are the points of intersection between the quadratic curve and the straight line.

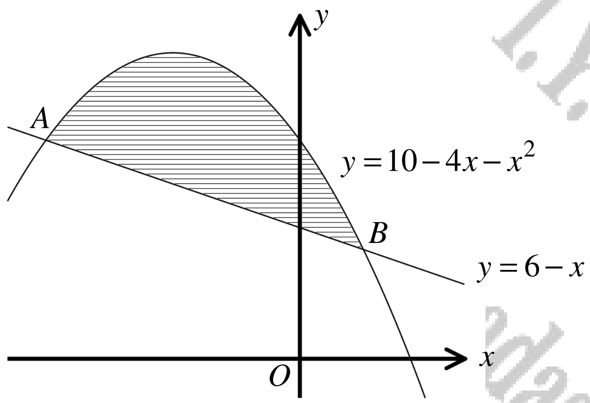
a) Find the coordinates of A and B.

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{A(1,5),\ B(4,26)}$, $\boxed{\text{area} = \frac{9}{2}}$



Question 22 (***)



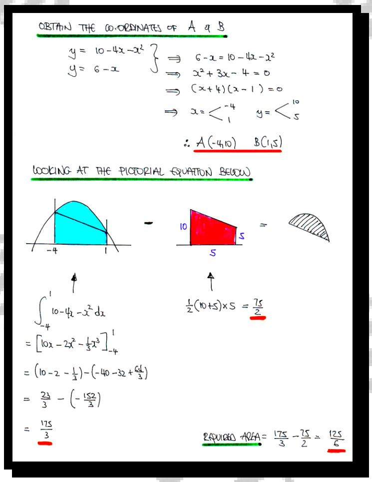
The figure above shows a quadratic curve and a straight line with respective equations

$$y = 10 - 4x - x^2 \qquad \text{and} \qquad y = 6 - x.$$

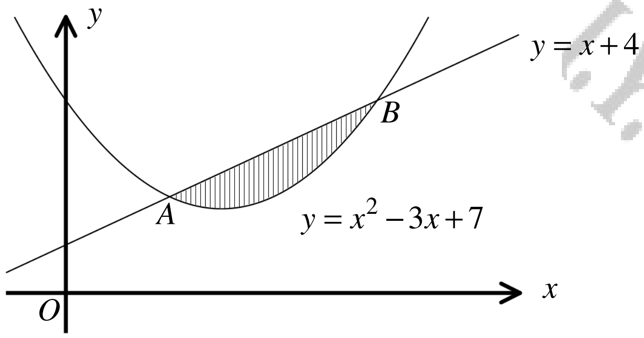
The points A and B, are the points of intersection between the quadratic curve and the straight line.

Calculate the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$$\boxed{\text{area} = \frac{125}{6}}$$



Question 23 (***)



The figure above shows a curve C and a straight line L with respective equations

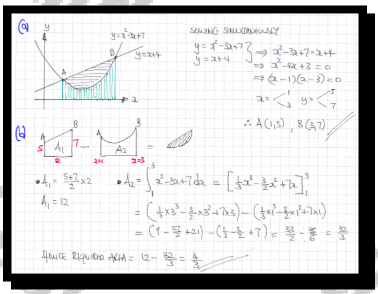
$$y = x^2 - 3x + 7 \quad \text{and} \quad y = x + 4\,.$$

The curve and the straight line meet at the points A and B.

a) Find the coordinates of A and B.

b) Find the exact area of the region bounded by C and L, shown shaded in the figure above.

$\boxed{A(1,5), B(3,7)}$, $\boxed{\text{area} = \frac{4}{3}}$



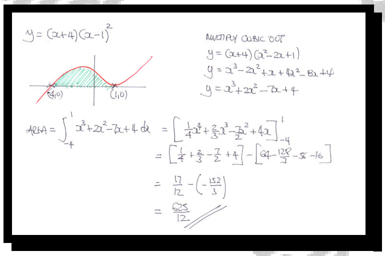
Question 24 (***)

A cubic curve C has equation

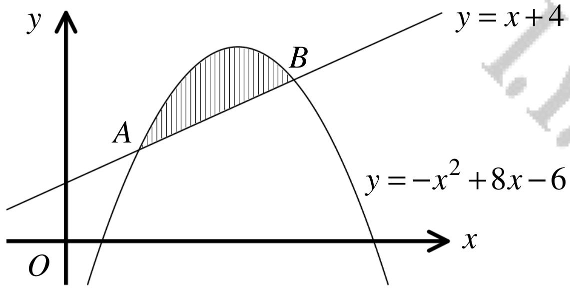
$$y = (x+4)(x-1)^2, \ x \in \mathbb{R}.$$

Sketch the graph of C and hence find the exact area of the finite region bounded by C and the x axis.

$$\boxed{\text{area} = \frac{625}{12}}$$



Question 25 (***)



The figure above shows a quadratic curve and a straight line with respective equations

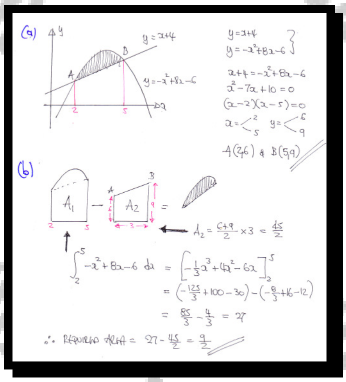
$$y = -x^2 + 8x - 6 \quad \text{and} \quad y = x + 4.$$

The points A and B are the points of intersection between the quadratic curve and the straight line.

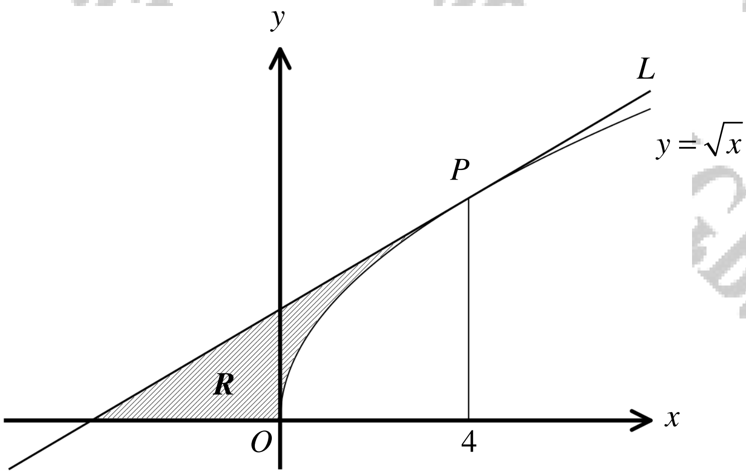
a) Find the coordinates of A and B.

b) Determine the exact area of the finite region bounded by the quadratic curve and the straight line, shown shaded in the above figure.

$\boxed{A(2,6), B(5,9)}$, $\boxed{\text{area} = \frac{9}{2}}$



Question 26 (***)



The figure above shows the graph of the curve C with equation

$$y=\sqrt{x}\,,\ x \geq 0\,.$$

The point P lies on C where $x=4$.

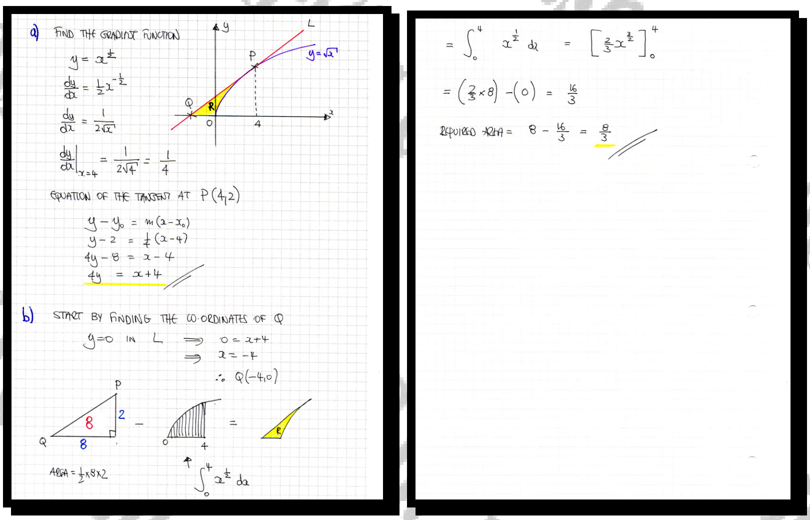
The straight line L is the tangent to C at P.

a) Find an equation of L.

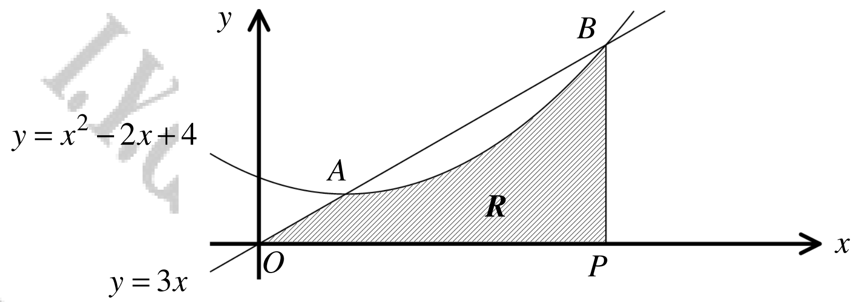
The finite region R, shown shaded in the figure, is bounded by C, L and the x axis.

b) Find the exact area of R.

$\boxed{y=\frac{1}{4}x+1}$, $\boxed{\frac{8}{3}}$



Question 27 (***)



The figure above shows the graph of the curve C with equation

$$y = x^2 - 2x + 4, \ x \in \mathbb{R}$$

intersected by the straight line L with equation

$$y = 3x, \ x \in \mathbb{R}.$$

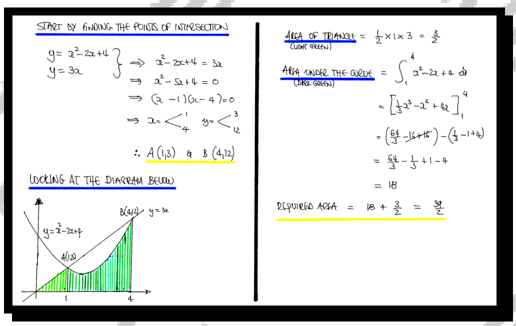
The curve meets the straight line at the points A and B.

The point P is located on the x axis so that the straight line segment BP is parallel to the y axis.

The finite region R is bounded by C, L, BP and the x axis.

Show that the area of R, shown shaded in the figure, is $\frac{39}{2}$.

$\boxed{\text{proof}}$



START BY FINDING THE POINTS OF INTERSECTION

$$\left.\begin{array}{l} y = x^2 - 2x + 4 \\ y = 3x \end{array}\right\} \Rightarrow x^2 - 2x + 4 = 3x$$

$$\Rightarrow x^2 - 5x + 4 = 0$$

$$\Rightarrow (x-1)(x-4) = 0$$

$$\Rightarrow x = \begin{cases} 1 \\ 4 \end{cases} \quad y = \begin{cases} 3 \\ 12 \end{cases}$$

$\therefore A(1,3)$ & $B(4,12)$

LOOKING AT THE DIAGRAM BELOW

AREA OF TRIANGLE (LIGHT GREEN) $= \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$

AREA UNDER THE CURVE (DARK GREEN) $= \int_1^4 x^2 - 2x + 4 \, dx$

$$= \left[\tfrac{1}{3}x^3 - x^2 + 4x\right]_1^4$$

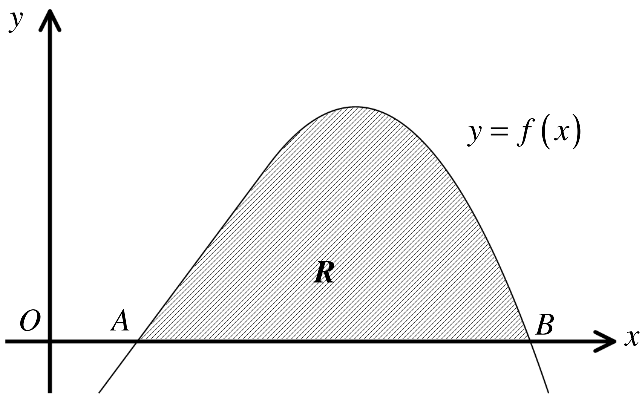
$$= \left(\tfrac{64}{3} \cancel{- 16 + 16}\right) - \left(\tfrac{1}{3} - 1 + 4\right)$$

$$= \tfrac{64}{3} - \tfrac{1}{3} + 1 - 4$$

$$= 18$$

REQUIRED AREA $= 18 + \frac{3}{2} = \frac{39}{2}$

Question 28 **(***)**



The figure above shows the graph of the curve with equation

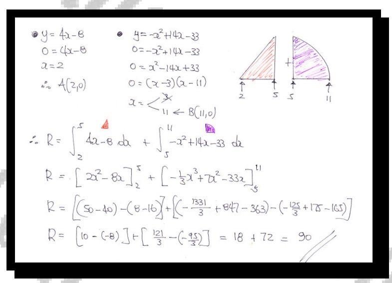
$$f(x) = \begin{cases} 4x - 8 & x < 5 \\ -x^2 + 14x - 33 & x \geq 5 \end{cases}.$$

The curve meets the x axis at the points A and B.

The finite region R, shown shaded in the figure above, is bounded by the curve and the x axis.

Find the area of R.

$\boxed{90}$

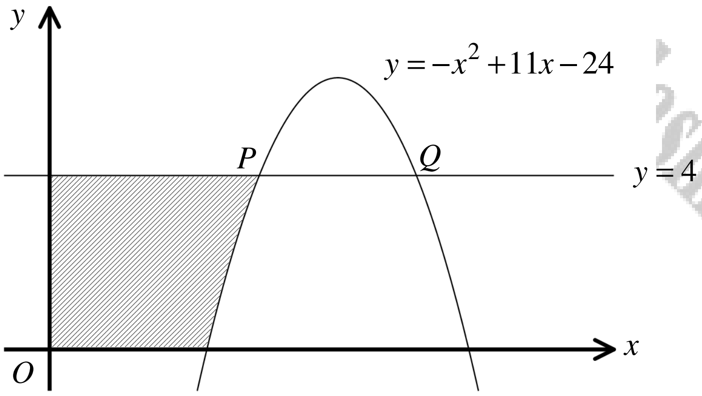


Question 29 (***+)

The diagram below shows a parabola and a straight line with respective equations

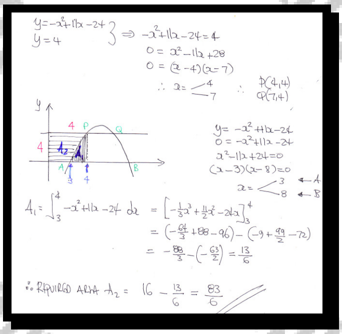
$$y = -x^2 + 11x - 24 \quad \text{and} \quad y = 4 .$$

The points P and Q are the intersections between the parabola and the straight line.



Find the exact area of the shaded region, bounded by the curve, the coordinate axes and the straight line with equation $y = 4$.

$$\boxed{\text{area} = \frac{83}{6}}$$



$y=-x^2+11x-24$, $y=4$ $\Rightarrow -x^2+11x-24=4$

$0 = x^2-11x+28$

$0 = (x-4)(x-7)$

$\therefore x = 4, 7 \quad \therefore P(4,4), \; Q(7,4)$

$y = -x^2+11x-24$

$0 = -x^2+11x-24$

$x^2-11x+24=0$

$(x-3)(x-8)=0$

$x = 3 \leftarrow A$, $x = 8 \leftarrow B$

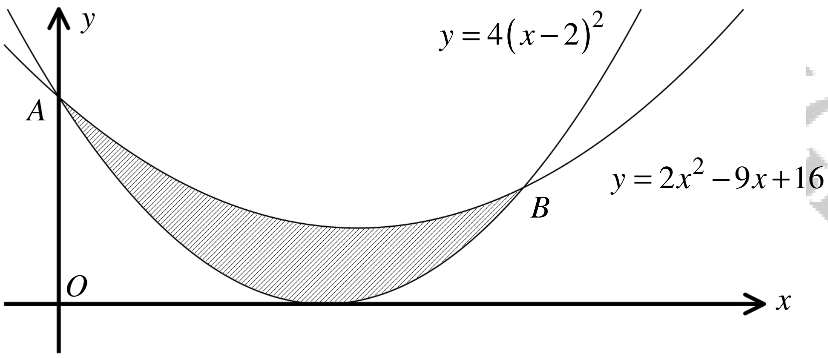
$A_1 = \int_3^4 -x^2+11x-24 \, dx = \left[-\frac{1}{3}x^3+\frac{11}{2}x^2-24x\right]_3^4$

$= \left(-\frac{64}{3}+88-96\right)-\left(-9+\frac{99}{2}-72\right)$

$= -\frac{88}{3}-\left(-\frac{63}{2}\right) = \frac{13}{6}$

$\therefore$ REQUIRED AREA $A_2 = 16 - \frac{13}{6} = \frac{83}{6}$

Question 30 (***+)



The figure above shows the graph of the curves with equations

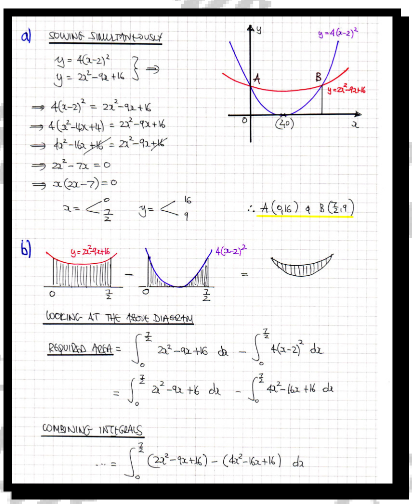
$$y=4(x-2)^2 \quad \text{and} \quad y=2x^2-9x+16.$$

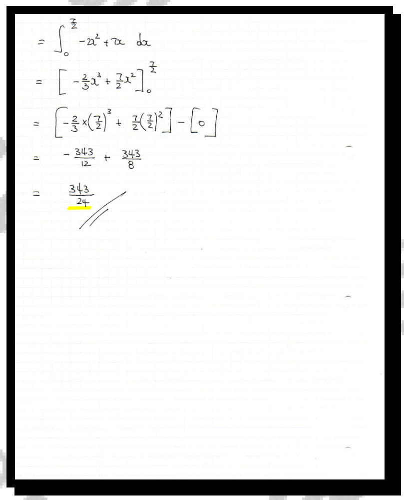
The curves meet each other at the points A and B.

a) Determine the coordinates of A and B.

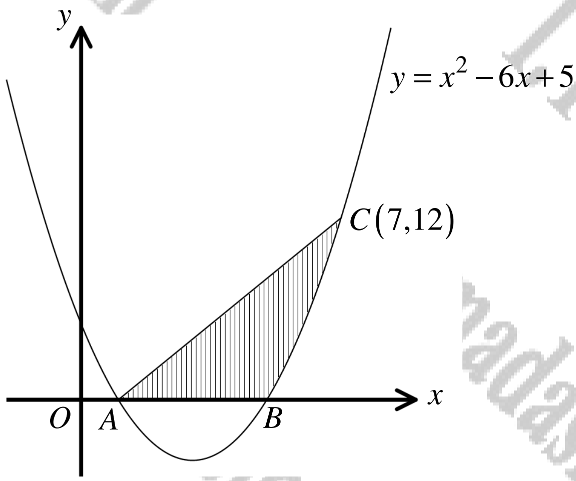
b) Find the exact area of the finite region bounded by the two curves, shown shaded in the above figure.

$\boxed{A(0,16)\ \ B\left(\frac{7}{2},9\right)}$, $\boxed{\frac{343}{24}}$





Question 31 (***+)



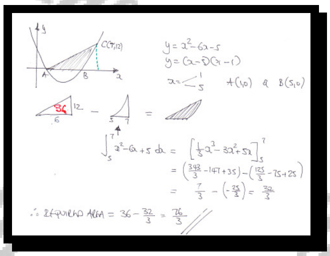
The diagram above shows the curve with equation

$$y = x^2 - 6x + 5.$$

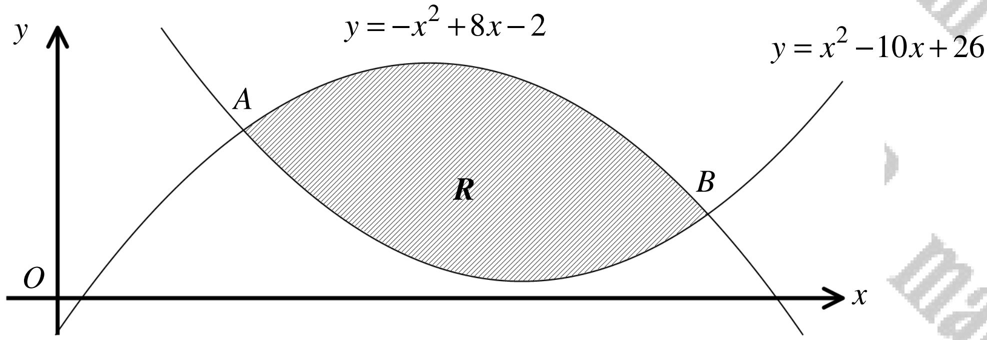
The point $C(7,12)$ lies on the curve while A and B are the points of intersection of the curve and the x axis.

Find the exact area of the shaded region, bounded by the curve, the straight line segment AC and the x axis.

$\boxed{\text{area} = \frac{76}{3}}$



Question 32 (***+)



The figure above shows the graphs of the curves with equations

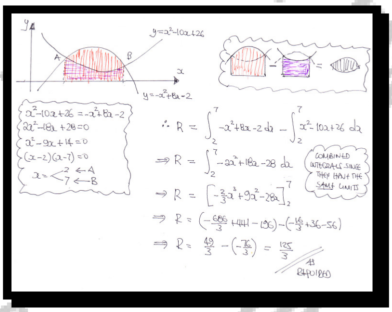
$$y = -x^2 + 8x - 2 \quad \text{and} \quad y = x^2 - 10x + 26.$$

The two curves intersect each other at the points A and B.

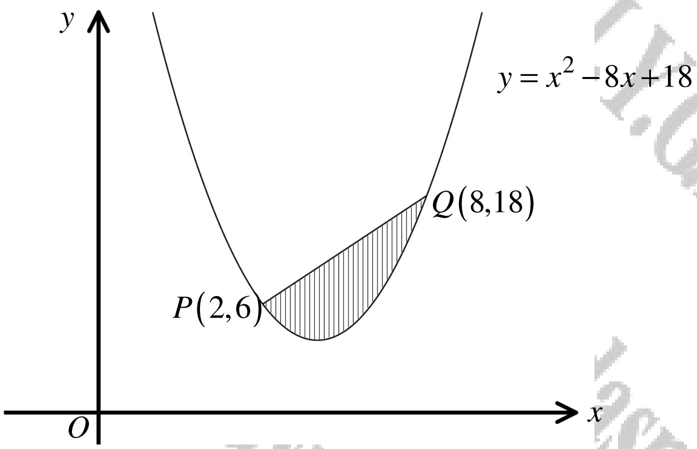
The finite region R bounded by the two curves is shown shaded in the figure above.

Show that the area of R is exactly $\frac{125}{3}$.

proof



Question 33 (***+)



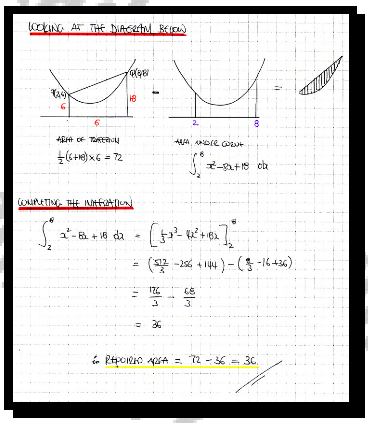
The figure above shows the parabola with equation

$$y = x^2 - 8x + 18, \ x \in \mathbb{R}.$$

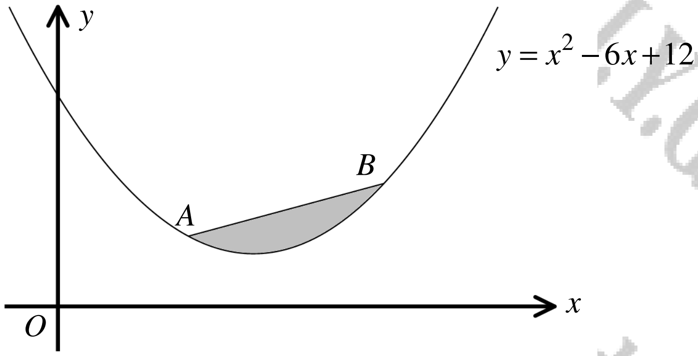
The points $P(3,3)$ and $Q(6,6)$ both lie on the parabola.

Find the exact of the shaded region, bounded by the curve and the straight line segment between P and Q.

$\boxed{\text{area} = 36}$



Question 34 (***+)



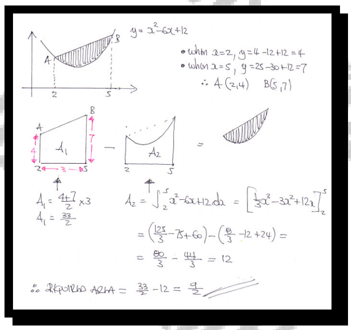
The figure above shows the curve C whose equation is

$$y = x^2 - 6x + 12 .$$

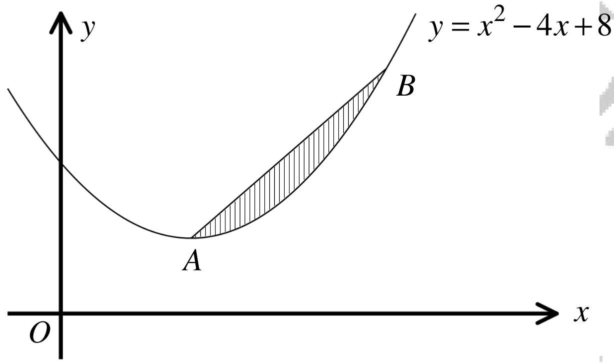
The points A and B both lie on C and have x coordinates 2 and 5 respectively.

Calculate the exact area of the shaded region, bounded by C and the straight line segment AB.

$\boxed{\text{area} = \frac{9}{2}}$



Question 35 (***+)



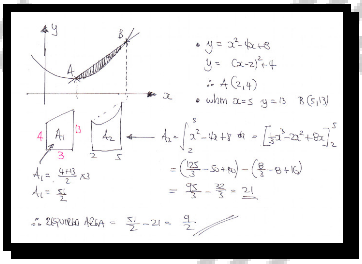
The figure above shows the curve C whose equation is

$$y = x^2 - 4x + 8.$$

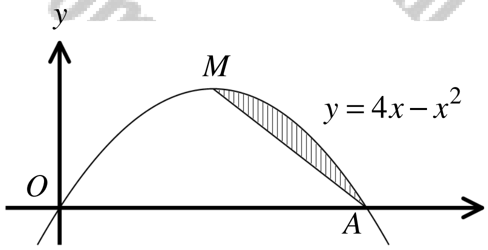
The point A is the minimum point of C and B is a point on C where $x = 5$.

Calculate the exact area of the shaded region, bounded by C and the straight line segment AB.

$$\boxed{\text{area} = \frac{9}{2}}$$



Question 36 (***+)



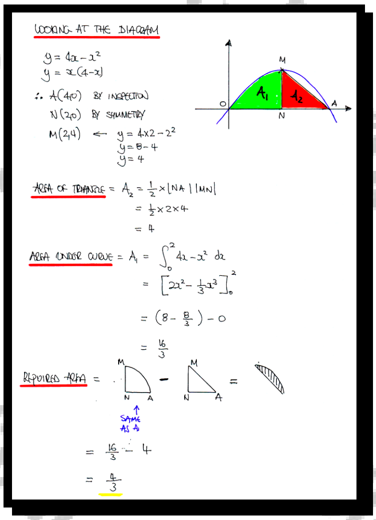
The figure above shows the curve with equation

$$y = 4x - x^2, \ x \in \mathbb{R}.$$

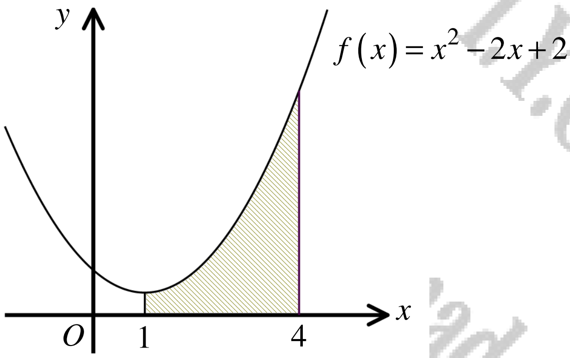
The point M is the maximum point of the curve and the point A is one of the x intercepts of the curve.

Find the exact area of the shaded region, bounded by the curve and the straight line segment joining A and M.

$\boxed{\text{area} = \frac{4}{3}}$



Question 37 (***+)



The curve C has equation

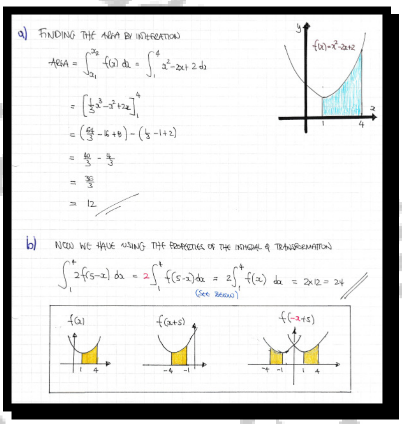
$$f(x)=x^2-2x+2, \quad x \in \mathbb{R}.$$

a) Find the area of the finite region bounded by C, the x axis and the straight lines with equations $x=1$ and $x=4$, shown shaded in the figure above.

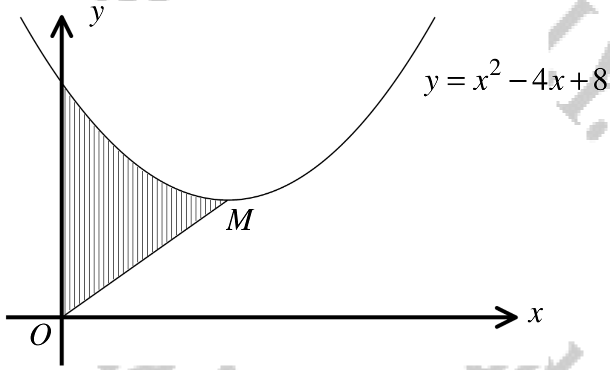
b) Hence evaluate

$$\int_1^4 2f(5-x)\,dx.$$

$\boxed{12}$, $\boxed{24}$



Question 38 (***+)



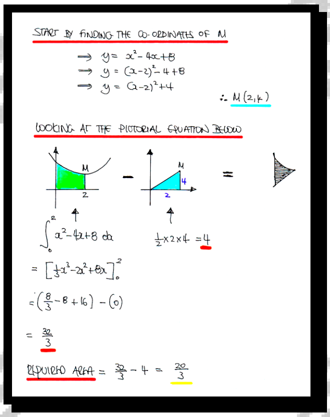
The figure above shows the curve with equation

$$y = x^2 - 4x + 8, \ x \in \mathbb{R}.$$

The point M is the minimum point of the curve.

Find the area of the shaded region, bounded by the curve, the y axis and the straight line segment from O to M.

$$\boxed{\text{area} = \frac{20}{3}}$$



START BY FINDING THE CO-ORDINATES OF M

$\Rightarrow y = x^2 - 4x + 8$
$\Rightarrow y = (x-2)^2 - 4 + 8$
$\Rightarrow y = (x-2)^2 + 4$

$\therefore M(2,4)$

LOOKING AT THE PICTORIAL EQUATION BELOW

$\int_0^2 x^2 - 4x + 8 \, dx$

$= \left[\frac{1}{3}x^3 - 2x^2 + 8x\right]_0^2$

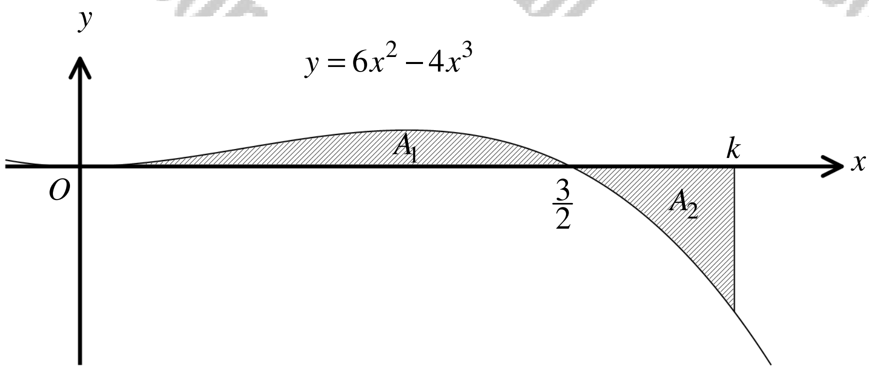
$= \left(\frac{8}{3} - 8 + 16\right) - (0)$

$= \frac{32}{3}$

$\frac{1}{2} \times 2 \times 4 = 4$

REQUIRED AREA $= \frac{32}{3} - 4 = \frac{20}{3}$

Question 39 (***+)



The figure above shows the graph of the curve with equation

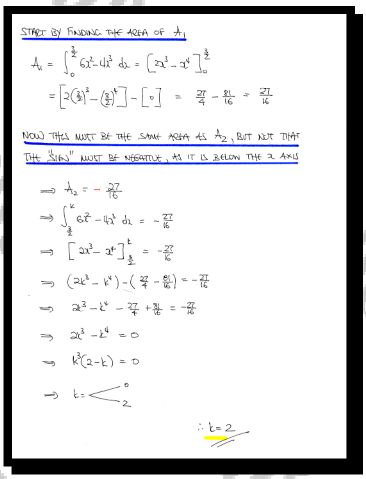
$$y = 6x^2 - 4x^3, \ x \in \mathbb{R}.$$

The curve meets x axis at the origin O and at the point $\left(\frac{3}{2}, 0\right)$.

The point $(k, 0)$, $k > \frac{3}{2}$ is such so that, the area A_1 of the region between the curve and the x axis for which $0 \le x \le \frac{3}{2}$, **is equal** to the area A_2 of the region between the curve and the x axis for which $\frac{3}{2} \le x \le k$.

Determine the value of k.

$\boxed{k = 2}$



START BY FINDING THE AREA OF A_1

$$A_1 = \int_0^{\frac{3}{2}} 6x^2 - 4x^3 \, dx = \left[2x^3 - x^4\right]_0^{\frac{3}{2}}$$

$$= \left[2\left(\tfrac{3}{2}\right)^3 - \left(\tfrac{3}{2}\right)^4\right] - \left[0\right] = \frac{27}{4} - \frac{81}{16} = \frac{27}{16}$$

NOW THIS MUST BE THE SAME AREA AS A_2, BUT NOT THAT THE "SIGN" MUST BE NEGATIVE, AS IT IS BELOW THE x AXIS

$\Rightarrow A_2 = -\frac{27}{16}$

$\Rightarrow \int_{\frac{3}{2}}^{k} 6x^2 - 4x^3 \, dx = -\frac{27}{16}$

$\Rightarrow \left[2x^3 - x^4\right]_{\frac{3}{2}}^{k} = -\frac{27}{16}$

$\Rightarrow (2k^3 - k^4) - \left(\frac{27}{4} - \frac{81}{16}\right) = -\frac{27}{16}$

$\Rightarrow 2k^3 - k^4 - \frac{27}{4} + \frac{81}{16} = -\frac{27}{16}$

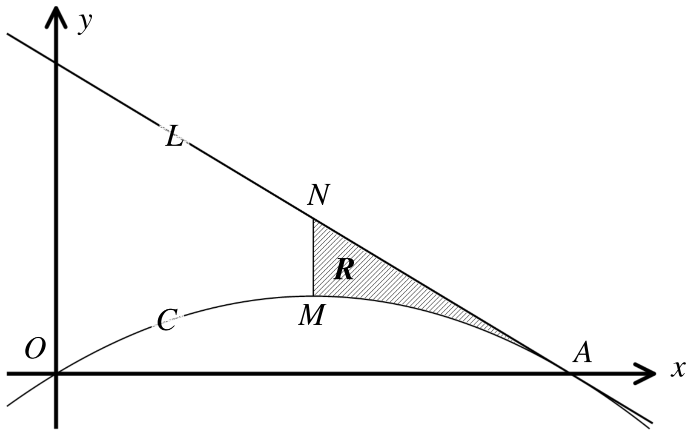
$\Rightarrow 2k^3 - k^4 = 0$

$\Rightarrow k^3(2 - k) = 0$

$\Rightarrow k = \begin{cases} 0 \\ 2 \end{cases}$

$\therefore k = 2$

Question 40 (***+)



The figure above shows the graph of the curve C with equation

$$y = 6x - x^2,\ x \in \mathbb{R}.$$

The curve meets the x axis at the origin O and at the point A. The straight line L is the tangent to C at A.

a) Find an equation of L.

The point M is the maximum point of C. The point N lies on L so that MN is parallel to the y axis. The finite region R, shown shaded in the figure above, is bounded by C, L and the straight line segment MN.

b) Determine the area of R.

$\boxed{y = 36 - 6x}$, $\boxed{9}$

a) START BY FINDING THE CO-ORDINATES OF A

$y = 6x - x^2$
$y = x(6-x)$
$\therefore A(6,0)$

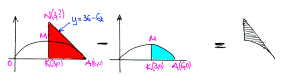
FIND THE GRADIENT AT A

$y = 6x - x^2$
$\frac{dy}{dx} = 6 - 2x$
$\left.\frac{dy}{dx}\right|_{x=6} = 6 - 2\times 6 = -6$ ← TANGENT GRADIENT

EQUATION OF TANGENT

$y - y_0 = m(x - x_0)$
$y - 0 = -6(x-6)$
$y = -6x + 36$
$y = 36 - 6x$

b)



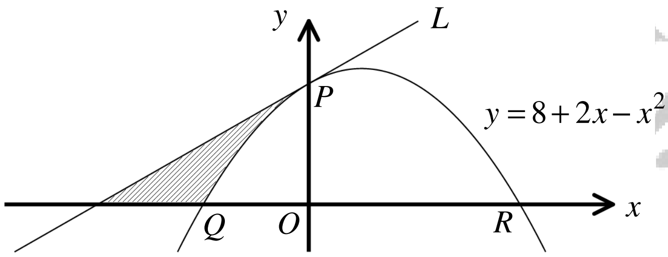
$y = 36 - 6\times 3$
$y = 18$
$\therefore N(3,18)$

AREA $= \frac{1}{2}|KN||KA|$
$= \frac{1}{2}\times 18\times 3 = 27$

$\int_3^6 6x - x^2\,dx$
$= \left[3x^2 - \frac{1}{3}x^3\right]_3^6$
$= (108-72) - (27-9)$
$= 18$

$= 9$

Question 41 (***+)



The figure above shows the graph of the curve C with equation

$$y = 8 + 2x - x^2 .$$

The curve meets the y axis at the point P, and the x axis at the points Q and R.

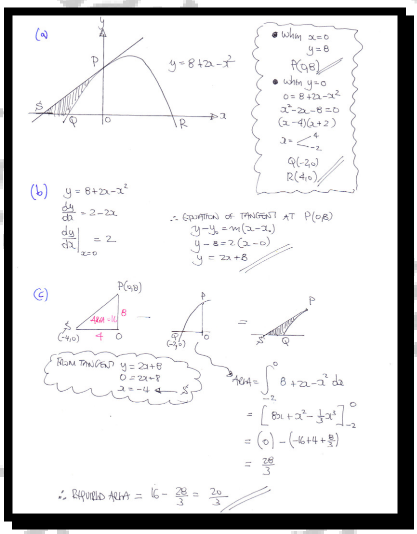
a) Determine the coordinates of P, Q and R.

The straight line L is the tangent to C at P.

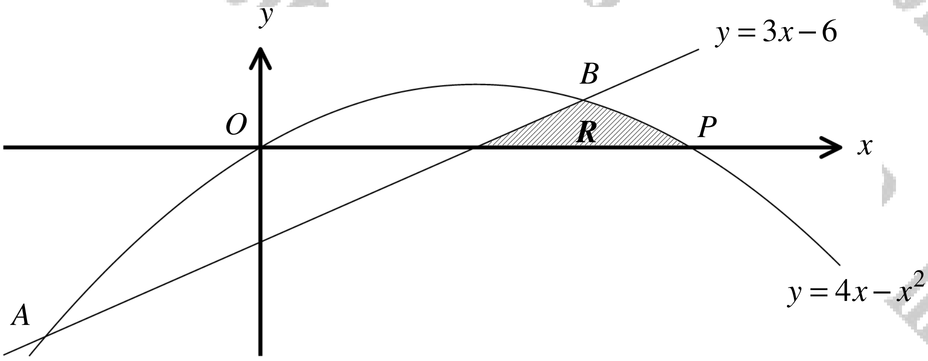
b) Find an equation of L.

c) Show that the area of the finite region bounded by C, L and the x axis is $\frac{20}{3}$.

$\boxed{P(0,8)}$, $\boxed{Q(-2,0)}$, $\boxed{R(4,0)}$, $\boxed{y = 2x - 8}$



Question 42 (****)



The figure above shows the graph of the curve C with equation

$$y = 4x - x^2, \ x \in \mathbb{R},$$

intersected by the straight line L with equation

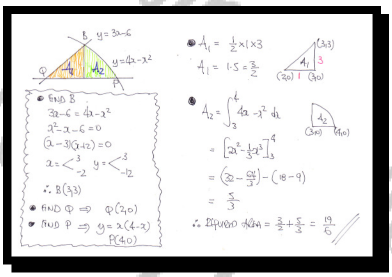
$$y = 3x - 6, \ x \in \mathbb{R}.$$

As shown in the above figure, C meets L at the points A and B, and crosses the x axis at the origin O and at the point P.

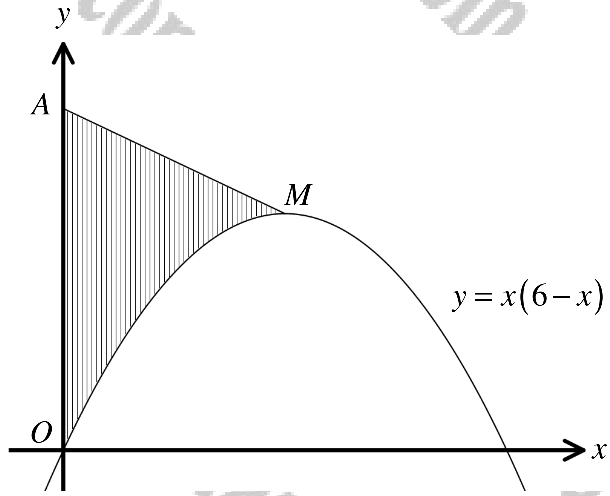
The finite region R is bounded by C, L and the x axis.

Show that the area of R, shown shaded in the figure, is $\frac{19}{6}$.

proof



Question 43 (****)



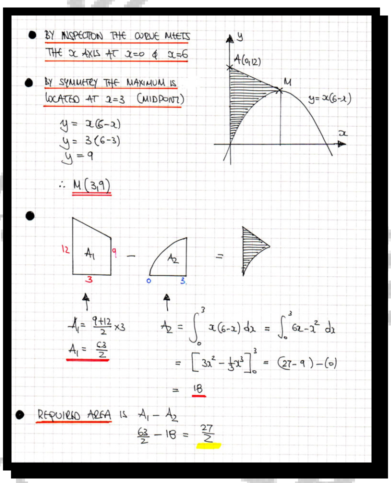
The figure above shows the curve C with equation

$$y = x(6-x), \ x \in \mathbb{R}.$$

The point M is the maximum point of C and the point A has coordinates $(0,12)$.

Find the exact area of the shaded region, bounded by the curve, the y axis and the straight line segment from A to M.

$\boxed{\text{area} = \frac{27}{2}}$



- BY INSPECTION THE CURVE MEETS THE x AXIS AT $x=0$ & $x=6$
- BY SYMMETRY THE MAXIMUM IS LOCATED AT $x=3$ (MIDPOINT)

$y = x(6-x)$
$y = 3(6-3)$
$y = 9$

$\therefore M(3,9)$

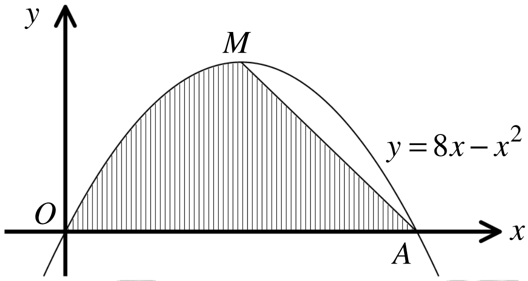
$A_1 = \frac{9+12}{2} \times 3$
$A_1 = \frac{63}{2}$

$A_2 = \int_0^3 x(6-x)\,dx = \int_0^3 6x - x^2\,dx$
$= \left[3x^2 - \frac{1}{3}x^3\right]_0^3 = (27-9)-(0)$
$= 18$

- REQUIRED AREA IS $A_1 - A_2$

$\frac{63}{2} - 18 = \frac{27}{2}$

Question 44 (****)



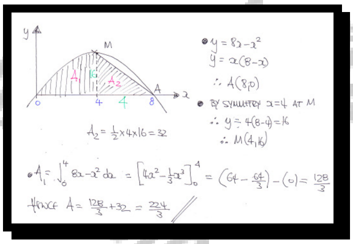
The figure above shows the quadratic curve with equation

$$y = 8x - x^2, \ x \in \mathbb{R}.$$

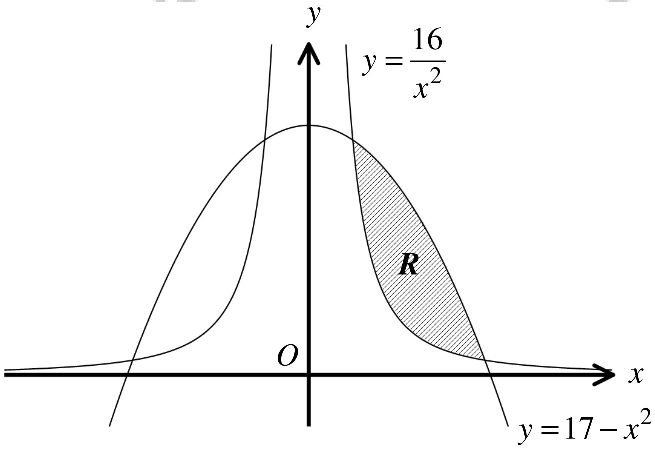
The point M is the maximum point of the curve and the point A is one of the curve's x intercepts.

Find the exact area of the shaded region, bounded by the curve, the x axis and the straight line segment from A to M.

$\boxed{\text{area} = \frac{224}{3}}$



Question 45 (****)



The figure above shows the graphs of the curves with equations

$$y = \frac{16}{x^2} \qquad \text{and} \qquad y = 17 - x^2 .$$

The finite region R, shown shaded in the figure above, is bounded by the two curves in the first quadrant.

Find the area of R.

$\boxed{18}$

START BY FINDING THE INTERSECTIONS OF THE TWO CURVES

$$\left.\begin{array}{l} y = \frac{16}{x^2} \\ y = 17 - x^2 \end{array}\right\} \Rightarrow \frac{16}{x^2} = 17 - x^2$$

$$\Rightarrow x^2 - 17 + \frac{16}{x^2} = 0$$

$$\Rightarrow x^4 - 17x^2 + 16 = 0$$

$$\Rightarrow (x^2 - 1)(x^2 - 16) = 0$$

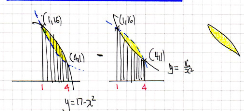
$$\Rightarrow x^2 = \begin{cases} 1 \\ 16 \end{cases}$$

$$\Rightarrow x = \begin{cases} +1 \\ +4 \end{cases} \quad \text{(FIRST QUADRANT)}$$

$$\Rightarrow y = \begin{cases} 16 \\ 1 \end{cases}$$

$$\therefore \ (1,16) \ \& \ (4,1)$$

LOOKING AT THE DIAGRAM BELOW



$$\text{AREA} = \int_1^4 17 - x^2 \, dx - \int_1^4 \frac{16}{x^2} \, dx = \int_1^4 17 - x^2 - \frac{16}{x^2} \, dx$$

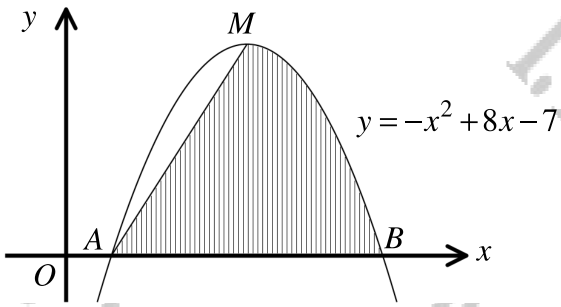
$$\text{AREA} = \left[17x - \frac{1}{3}x^3 + \frac{16}{x} \right]_1^4$$

$$= \left(68 - \frac{64}{3} + 4\right) - \left(17 - \frac{1}{3} + 16\right)$$

$$= \frac{152}{3} - \frac{98}{3}$$

$$= 18$$

Question 46 (****)



The figure above shows the quadratic curve with equation

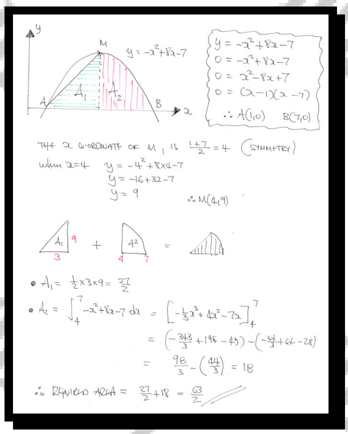
$$y = -x^2 + 8x - 7 .$$

The point M is the maximum point of the curve.

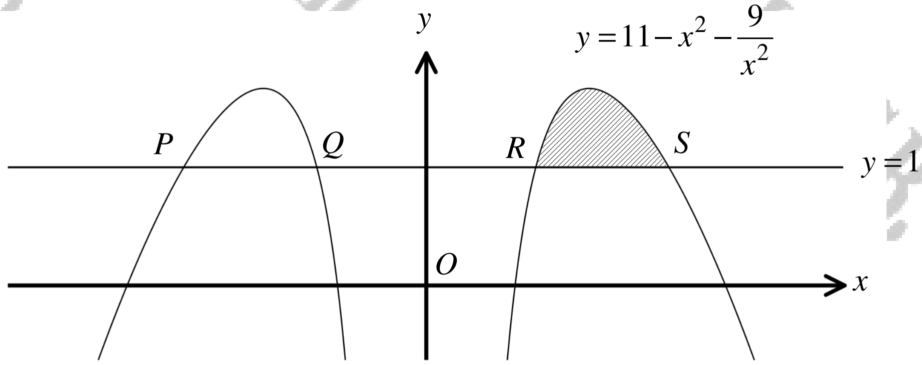
The points A and B are the points where the curve meets the x axis.

Calculate the area of the shaded region bounded by the curve, the x axis and the straight line segment from A to M.

$\boxed{\text{area} = \frac{63}{2}}$



Question 47 (****)



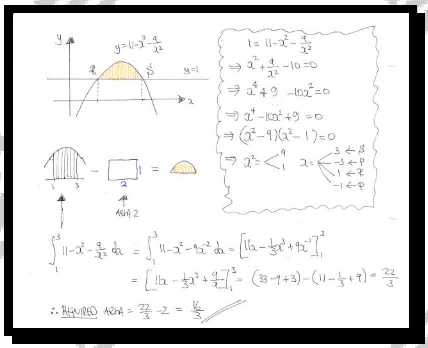
The figure above shows the curve C with equation

$$y = 11 - x^2 - \frac{9}{x^2}, \ x \neq 0 .$$

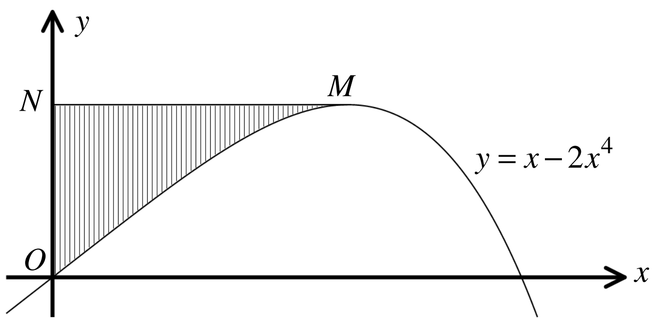
The line with equation $y = 1$ meets C at the points P, Q, R and S, where R and S have positive x coordinates, as shown in the figure.

Find the area of the finite region bounded by C and the line segment RS.

$\boxed{\frac{16}{3}}$



Question 48 (****)



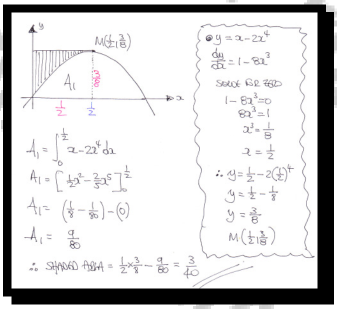
The diagram below shows the quartic curve with equation

$$y = x - 2x^4, \; x \in \mathbb{R}.$$

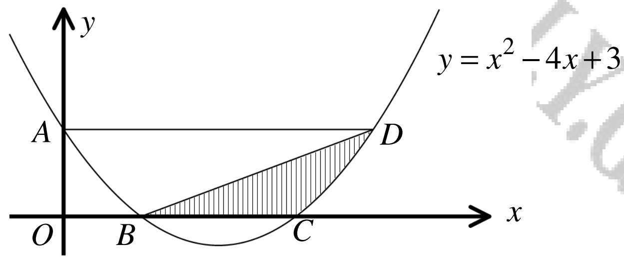
The point M is the maximum point on the curve and the point N lies on the y axis so that the straight line segment MN is parallel to the x axis.

Find the exact area of the shaded region, bounded by the curve, the y axis and the straight line segment from M to N.

$$\boxed{\text{area} = \frac{3}{40}}$$



Question 49 (****)



The figure above shows a quadratic curve with equation

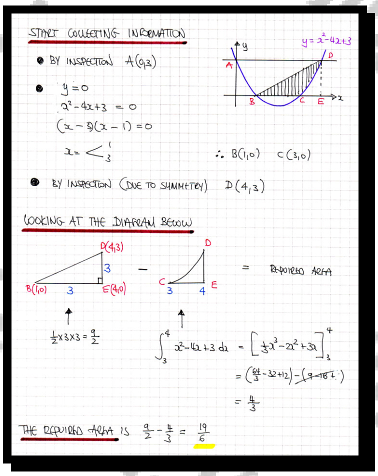
$$y = x^2 - 4x + 3.$$

The points A, B and C are the points where the curve meets the coordinate axes.

The point D lies on the curve so that AD is parallel to the x axis.

Calculate the exact area of the shaded region, bounded by the curve, the x axis and the straight line segment BD.

$\boxed{\text{area} = \frac{19}{6}}$



START COLLECTING INFORMATION

- BY INSPECTION $A(0,3)$
- $y = 0$
  $x^2 - 4x + 3 = 0$
  $(x-3)(x-1) = 0$
  $x = \begin{cases} 1 \\ 3 \end{cases}$
  $\therefore B(1,0) \quad C(3,0)$
- BY INSPECTION (DUE TO SYMMETRY) $D(4,3)$

LOOKING AT THE DIAGRAM BELOW

$-$ $=$ REQUIRED AREA

$\frac{1}{2} \times 3 \times 3 = \frac{9}{2}$

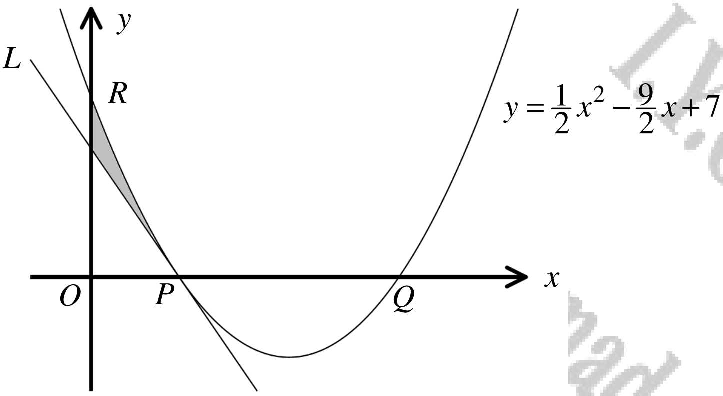
$\int_3^4 x^2 - 4x + 3 \, dx = \left[\frac{1}{3}x^3 - 2x^2 + 3x\right]_3^4$

$= \left(\frac{64}{3} - 32 + 12\right) - \cancel{(9 - 18 + 9)}$

$= \frac{4}{3}$

THE REQUIRED AREA IS $\frac{9}{2} - \frac{4}{3} = \frac{19}{6}$

Question 50 (****)



The diagram above shows the quadratic curve C with equation

$$y = \frac{1}{2}x^2 - \frac{9}{2}x + 7.$$

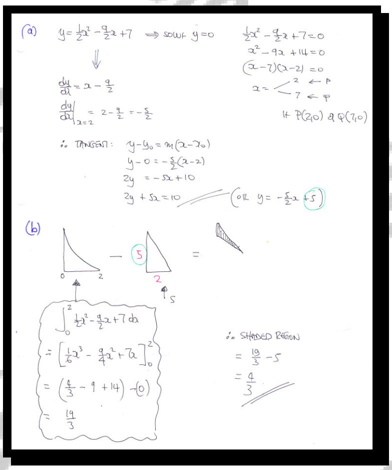
The curve crosses the x axis at the points P and Q, and the y axis at the point R.

The line L is the tangent to C at the point P.

a) Find an equation of L.

b) Find the exact area of the shaded region bounded by the tangent at P, the curve and the y axis.

$\boxed{2y + 5x = 10}$, $\boxed{\text{area} = \frac{4}{3}}$



(a) $y = \frac{1}{2}x^2 - \frac{9}{2}x + 7 \Rightarrow$ SOLVE $y = 0$

$\frac{1}{2}x^2 - \frac{9}{2}x + 7 = 0$
$x^2 - 9x + 14 = 0$
$(x-7)(x-2) = 0$
$x = 2 \leftarrow P$, $7 \leftarrow Q$
$\therefore P(2,0)$ & $Q(7,0)$

$\frac{dy}{dx} = x - \frac{9}{2}$
$\left.\frac{dy}{dx}\right|_{x=2} = 2 - \frac{9}{2} = -\frac{5}{2}$

$\therefore$ TANGENT: $y - y_0 = m(x - x_0)$
$y - 0 = -\frac{5}{2}(x-2)$
$2y = -5x + 10$
$2y + 5x = 10$ (OR $y = -\frac{5}{2}x + 5$)

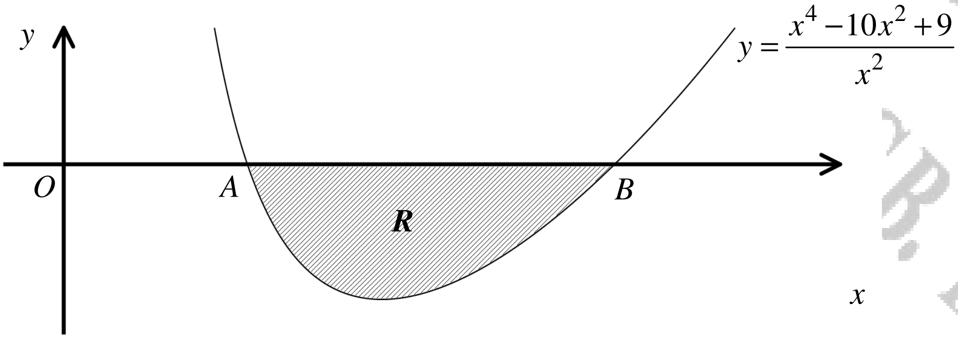
(b) $-$ 5 $=$

0 2 2 5

$\int_0^2 \frac{1}{2}x^2 - \frac{9}{2}x + 7 \, dx$
$= \left[\frac{1}{6}x^3 - \frac{9}{4}x^2 + 7x\right]_0^2$
$= \left(\frac{4}{3} - 9 + 14\right) - (0)$
$= \frac{19}{3}$

$\therefore$ SHADED REGION
$= \frac{19}{3} - 5$
$= \frac{4}{3}$

Question 51 (****)



The figure above shows the graph of the curve with equation

$$y = \frac{x^4 - 10x^2 + 9}{x^2}, \ x > 0 .$$

The curve meets the x axis at the points A and B.

The finite region R, shown shaded in the figure above, is bounded by the curve and the x axis.

Find the exact area of R.

$\boxed{\dfrac{16}{3}}$

$y = 0$

$\dfrac{x^4 - 10x^2 + 9}{x^2} = 0$

$x^4 - 10x^2 + 9 = 0$

$(x^2 - 9)(x^2 - 1) = 0$

$x^2 = \begin{cases} 9 \\ 1 \end{cases}$

$x = 3, \cancel{-3}, \cancel{-1}, 1$

$\therefore A(1,0) \quad B(3,0)$

$R = \int_1^3 \dfrac{x^4 - 10x^2 + 9}{x^2} \, dx = \int_1^3 \dfrac{x^4}{x^2} - \dfrac{10x^2}{x^2} + \dfrac{9}{x^2} \, dx$

$= \int_1^3 x^2 - 10 + 9x^{-2} \, dx = \left[\tfrac{1}{3}x^3 - 10x - 9x^{-1} \right]_1^3$

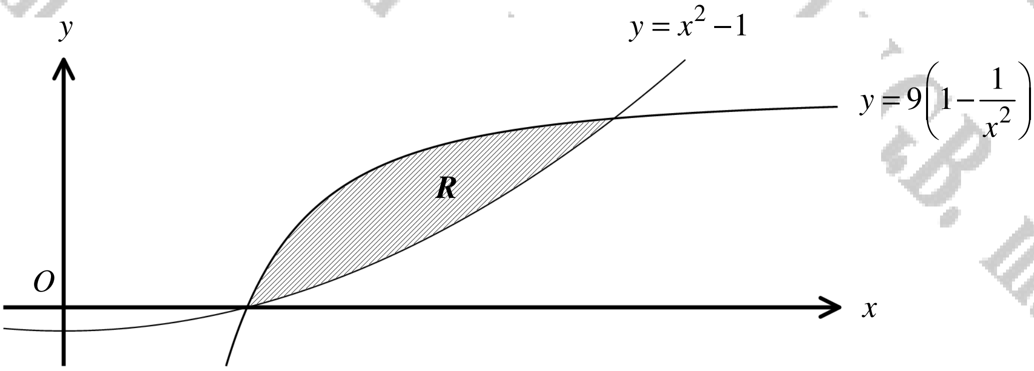
$= \left[\tfrac{1}{3}x^3 - 10x - \dfrac{9}{x} \right]_1^3$

$= (9 - 30 - 3) - (\tfrac{1}{3} - 10 - 9)$

$= -\dfrac{16}{3}$

$\therefore$ AREA IS $\dfrac{16}{3}$ // MINUS INDICATES AREA IS ENTIRELY BELOW THE x AXIS

Question 52 (****)



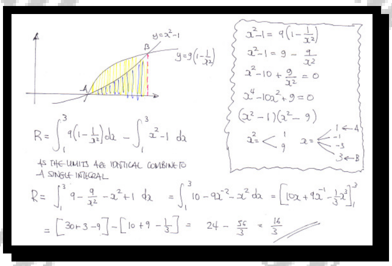
The figure above shows the graphs of the curves with equations

$$y = x^2 - 1 \quad \text{and} \quad y = 9\left(1 - \frac{1}{x^2}\right).$$

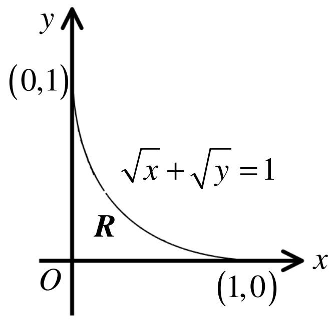
The finite region R is bounded by the two curves in the 1st quadrant, and is shown shaded in the figure above.

Determine the exact area of R.

$\boxed{\frac{16}{3}}$



Question 53 **(****)**



The figure above shows the curve with equation

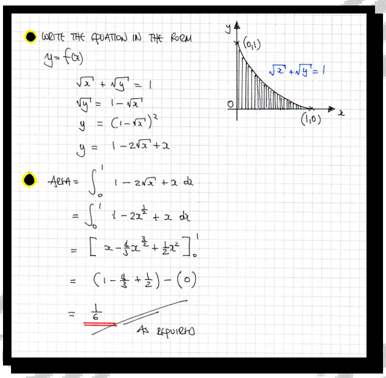
$$\sqrt{x}+\sqrt{y}=1\ ,\ x\in\mathbb{R},\ 0\le x\le 1.$$

The curve meets the coordinate axes at the points $(1,0)$ and $(0,1)$.

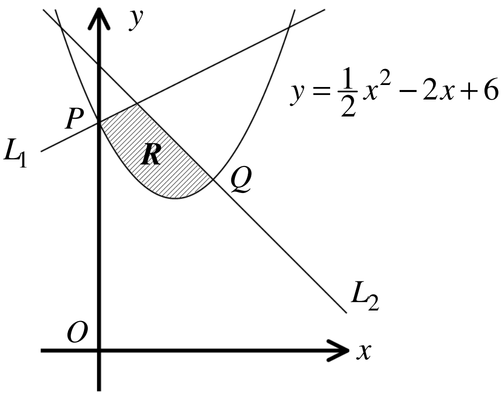
The finite region R is bounded by the curve and the coordinate axes.

Show that the area of R is $\frac{1}{6}$.

proof



Question 54 (****)



The figure above shows the graph of the curve C with equation

$$y=\frac{1}{2}x^2-2x+6.$$

The point P is the point where C meets the y axis so that the straight line L_1 is the normal to C at P.

a) Find an equation for L_1.

The point $Q\left(3,\frac{9}{2}\right)$ lies on C and the straight line L_2 is the normal to C at Q.

The finite region R, shown shaded in the figure above, is bounded by L_1, L_2 and C.

b) Find the area of R.

$\boxed{y=\frac{1}{2}x+6}$, $\boxed{\frac{15}{4}}$

a) BY INSPECTION $P(0,6)$

$y=\frac{1}{2}x^2-2x+6$

$\frac{dy}{dx}=x-2$

$\left.\frac{dy}{dx}\right|_{x=0}=-2$

NORMAL GRADIENT $=+\frac{1}{2}$

EQUATION OF L_1 MUST BE

$y=\frac{1}{2}x+6$

(USING $y=mx+c$)

b) $\left.\frac{dy}{dx}\right|_{x=3}=1 \Rightarrow$ NORMAL GRADIENT MUST BE -1

$\Rightarrow y-y_0=m(x-x_0)$

$\Rightarrow y-\frac{9}{2}=-1(x-3)$

$\Rightarrow 2y-9=-2(x-3)$

$\Rightarrow 2y-9=-2x+6$

$\Rightarrow 2x+2y=15$

FIND THE INTERSECTION BETWEEN L_1 & L_2

$L_1: \; y=\frac{1}{2}x+6$
$L_2: \; 2x+2y=15$

$\Rightarrow 2x+2\left(\frac{1}{2}x+6\right)=15$

$\Rightarrow 2x+x+12=15$

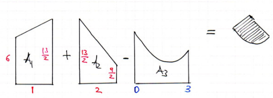
$\Rightarrow 3x=3$

$\Rightarrow x=1$

$\Rightarrow y=6\frac{1}{2}=\frac{13}{2}$

$\therefore \left(1,\frac{13}{2}\right)$

LOOKING AT THE DIAGRAM BELOW



• $A_1=\frac{\frac{13}{2}+6}{2}\times 1=\frac{25}{4}$

• $A_2=\frac{\frac{13}{2}+\frac{9}{2}}{2}\times 2=11$

• $A_3=\int_0^3 \frac{1}{2}x^2-2x+6 \, dx=\left[\frac{1}{6}x^3-x^2+6x\right]_0^3$

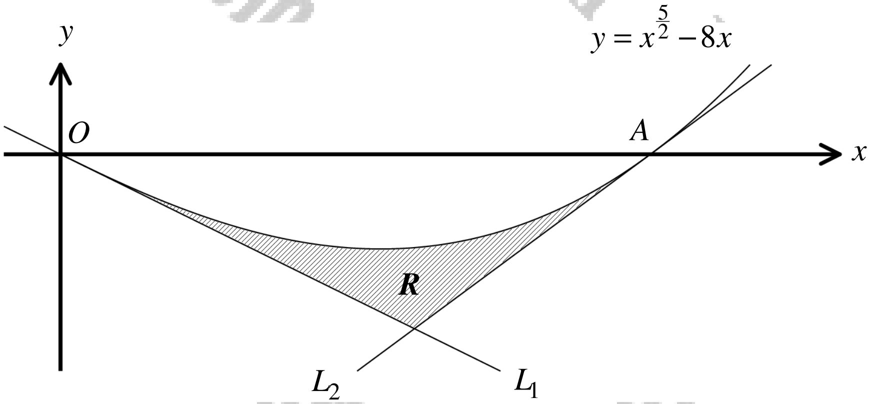
$=\left(\frac{9}{2}-9+18\right)-(0)=\frac{27}{2}$

REQUIRED AREA $=A_1+A_2-A_3$

$=\frac{25}{4}+11-\frac{27}{2}$

$=\frac{15}{4}$

Question 55 (****+)



The figure above shows the graph of the curve C with equation

$$y = x^{\frac{5}{2}} - 8x \, , \; x \geq 0 \, .$$

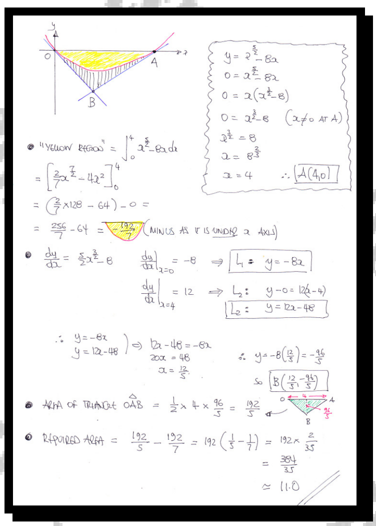
The curve meets the x axis at the origin O and at the point A.

The tangent to C at O is denoted by L_1 and the tangent to C at A is denoted by L_2.

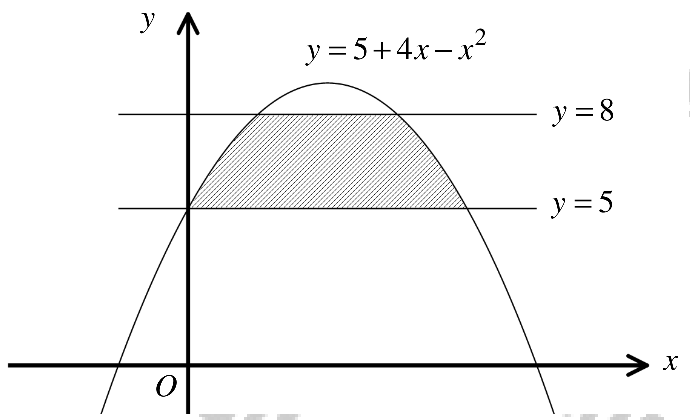
The finite region R, shown shaded in the figure above, is bounded by C, L_1 and L_2.

Determine the area of R.

$$\boxed{\frac{384}{35} \approx 11.0}$$



Question 56 (****+)



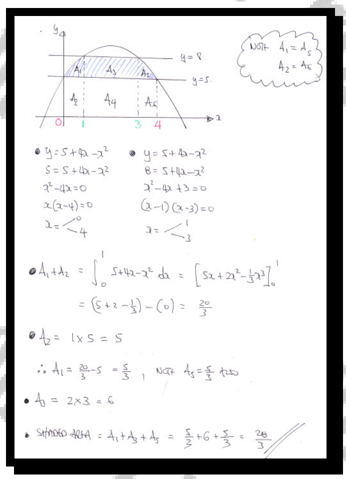
The figure above shows the curve C with equation

$$y = 5 + 4x - x^2,$$

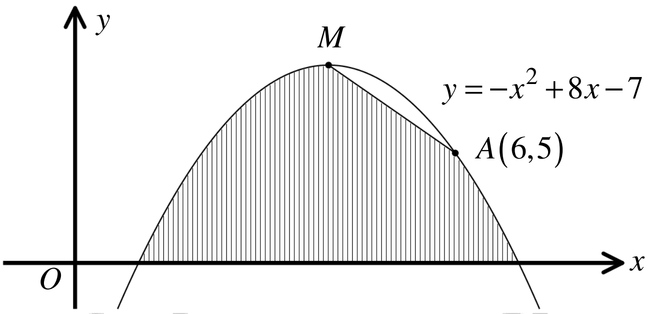
intersected by the horizontal straight lines with equations $y = 5$ and $y = 8$.

Calculate the exact area of the shaded region, bounded by C and the two straight lines.

$\boxed{\text{area} = \frac{28}{3}}$



Question 57 (****+)



The figure above shows the quadratic curve with equation

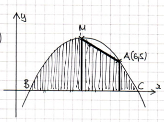
$$y=-x^2+8x-7.$$

The point M is the maximum point of the curve and A is another point on the curve whose coordinates are $(6,5)$.

Find the exact area of the shaded region, bounded by the curve, the x axis and the straight line segment from A to M.

$$\boxed{\text{area} = \frac{104}{3}}$$

FIND THE CO-ORDS OF M, BY COMPLETING THE SQUARE (OR CALCULUS)



$\Rightarrow y=-x^2+8x-7$

$\Rightarrow -y=x^2-8x+7$

$\Rightarrow -y=(x-4)^2-16+7$

$\Rightarrow -y=(x-4)^2-9$

$\Rightarrow y=9-(x-4)^2$

$\therefore M(4,9)$

ALSO THE CO-ORDINATES OF B & C ARE NEEDED

$\Rightarrow y=0$

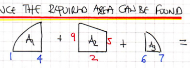
$\Rightarrow -x^2+8x-7=0$

$\Rightarrow x^2-8x+7=0$

$\Rightarrow (x-1)(x-7)=0$

$\Rightarrow x = \begin{cases}1\\7\end{cases}$ $\therefore B(1,0)$ $C(7,0)$

HENCE THE REQUIRED AREA CAN BE FOUND



$A_1 + A_2 + A_3 =$

$A_1=\int_1^4 -x^2+8x-7\,dx$ $\quad A_2=\frac{9+5}{2}\times 2$ $\quad A_3=\int_6^7 -x^2+8x-7\,dx$

$A_2=14$

$A_1=\int_1^4 -x^2+8x-7\,dx = \left[-\frac{1}{3}x^3+4x^2-7x\right]_1^4$

$=\left(-\frac{64}{3}+64-28\right)-\left(-\frac{1}{3}+4-7\right)$

$=\frac{44}{3}-\left(-\frac{10}{3}\right)$

$=18$

$A_3=\int_6^7 -x^2+8x-7\,dx = \left[-\frac{1}{3}x^3+4x^2-7x\right]_6^7$

$=\left(-\frac{343}{3}+196-49\right)-\left(-72+144-42\right)$

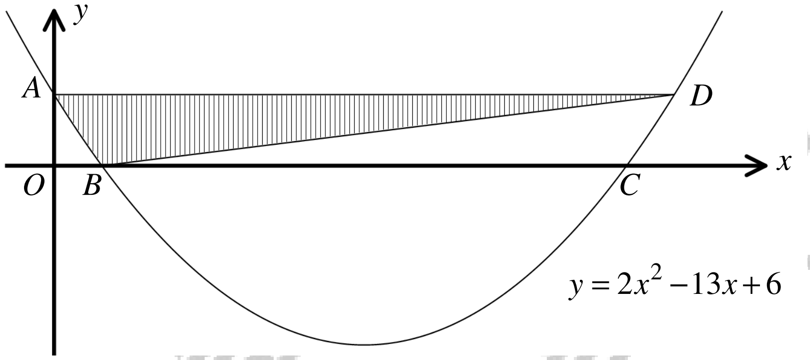
$=\frac{98}{3}-30$

$=\frac{8}{3}$

THE AREA OF THE SHADED REGION IS $A_1+A_2+A_3$

$18+14+\frac{8}{3}=\frac{104}{3}$

Question 58 (****+)



The figure above shows the curve with equation

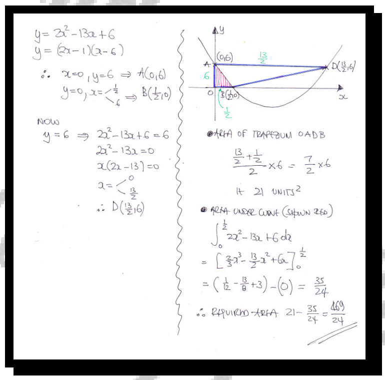
$$y = 2x^2 - 13x + 6 .$$

The points A, B and C are the points where the curve meets the coordinate axes.

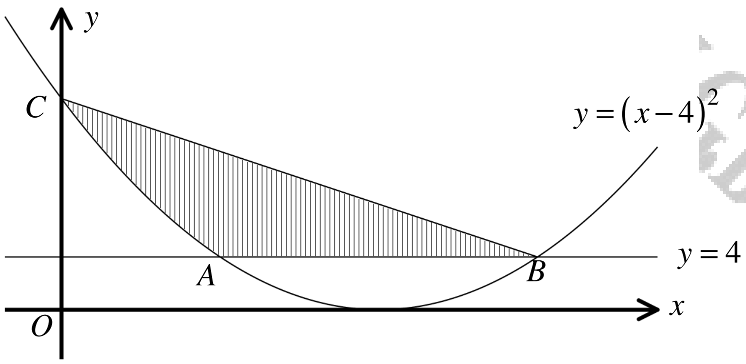
The point D is such so that the straight line segment AD is parallel to the x axis.

Find the exact area of the shaded region, bounded by the curve and the straight line segments BD and AD.

$\boxed{\text{area} = \frac{469}{24}}$



Question 59 **(****+)**



The diagram above shows the curve with equation

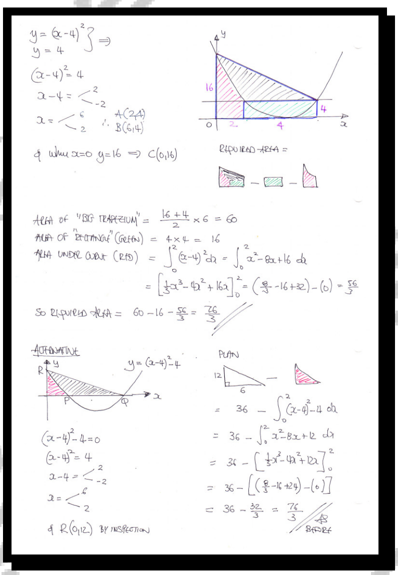
$$y=(x-4)^2,\ x\in\mathbb{R},$$

intersected by the straight line with equation $y=4$, at the points A and B.

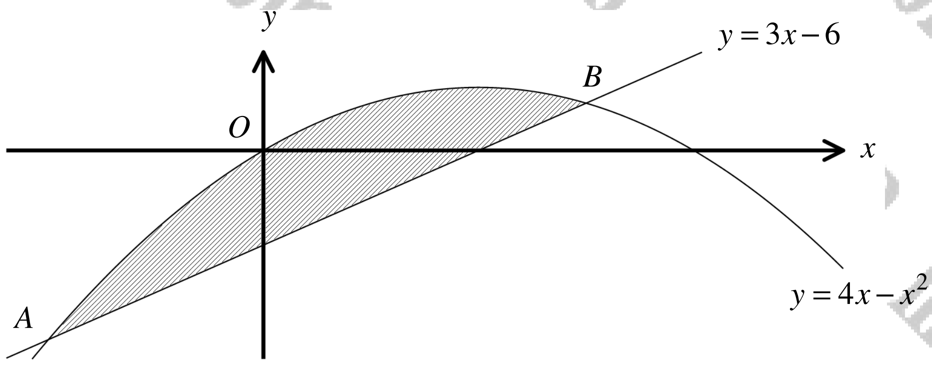
The curve meets the y axis at the point C.

Calculate the exact area of the shaded region, bounded by the curve and the straight line segments AB and BC.

$\boxed{\text{area} = \frac{76}{3}}$



Question 60 (*****)



The figure above shows the graph of the curve C with equation

$$y = 4x - x^2,$$

intersected by the straight line L with equation

$$y = 3x - 6.$$

The finite region R is bounded by C and L.

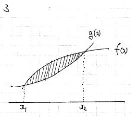
Show that the area of R, shown shaded in the above figure, is $\frac{125}{6}$.

proof

OBTAIN THE x CO-ORDINATES OF A & B

$$\left.\begin{array}{l} y = 3x-6 \\ y = 4x - x^2 \end{array}\right\} \Rightarrow 3x - 6 = 4x - x^2$$
$$\Rightarrow x^2 - x - 6 = 0$$
$$\Rightarrow (x-3)(x+2) = 0$$
$$\Rightarrow x = \genfrac{}{}{0pt}{}{-2}{3}$$

THE REQUIRED AREA IS GIVEN BY



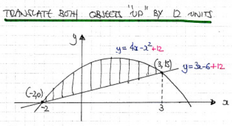
$$\int_{x_1}^{x_2} f(x) - g(x)\, dx$$
$$= \int_{-2}^{3} (4x - x^2) - (3x - 6)\, dx$$
$$= \int_{-2}^{3} 4x - x^2 - 3x + 6\, dx$$
$$= \int_{-2}^{3} 6 + x - x^2\, dx$$
$$= \left[6x + \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_{-2}^{3}$$
$$= \left(18 + \tfrac{9}{2} - 9\right) - \left(-12 + 2 + \tfrac{8}{3}\right)$$
$$= \tfrac{27}{2} - \left(-\tfrac{22}{3}\right)$$
$$= \frac{125}{6}$$

AS REQUIRED

ALTERNATIVE APPROACH BY TRANSFORMATIONS – FIND THE x CO-ORDINATES OF A & B AS BEFORE.

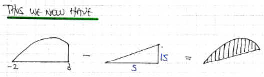
$x = \genfrac{}{}{0pt}{}{-2}{3}$ USING $y = 3x - 6$ $y = \genfrac{}{}{0pt}{}{-12}{3}$

TRANSLATE BOTH OBJECTS "UP" BY 12 UNITS



$(-2,-12)$ & $(3,3)$
$\downarrow \qquad \downarrow$
$(-2,0)$ & $(3,15)$

THUS WE NOW HAVE



$\frac{1}{2} \times 5 \times 15 = \frac{75}{2}$

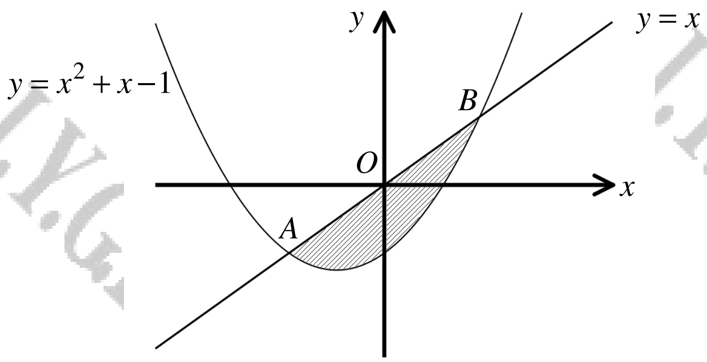
$$\int_{-2}^{3} 4x - x^2 + 12\, dx = \left[2x^2 - \tfrac{1}{3}x^3 + 12x\right]_{-2}^{3} = (18 - 9 + 36) - \left(8 + \tfrac{8}{3} - 24\right)$$
$$= 45 - \left(-\tfrac{40}{3}\right)$$
$$= \frac{175}{3}$$

HENCE THE REQUIRED AREA IS GIVEN BY

$$\frac{175}{3} - \frac{75}{2} = \frac{125}{6}$$

AS BEFORE

Question 61 (*****)



The figure above shows the graph of the curve C with equation

$$y = x^2 + x - 1,$$

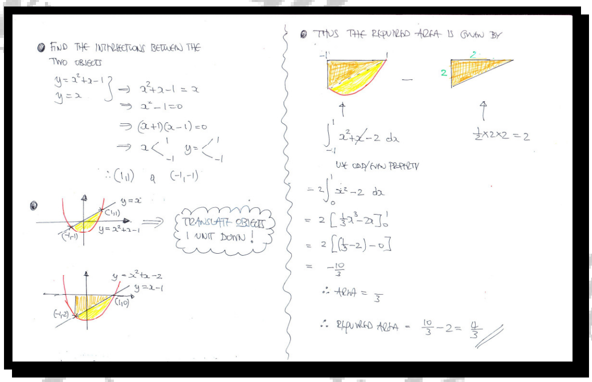
intersected by the straight line L with equation

$$y = x.$$

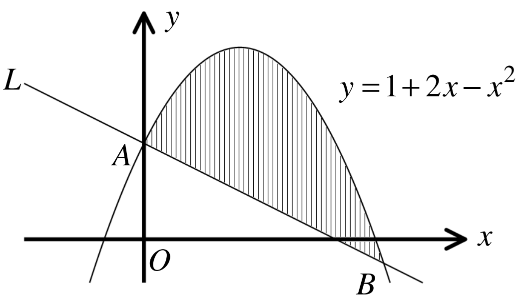
The points A and B, are the points of intersection between C and L, as shown in the above figure. The finite region R is bounded by C and L.

Show that the area of R, shown shaded in the above figure, is $\frac{4}{3}$.

$\boxed{\text{proof}}$



Question 62 (*****)



The diagram above shows part of the curve C, with equation

$$y = 1 + 2x - x^2.$$

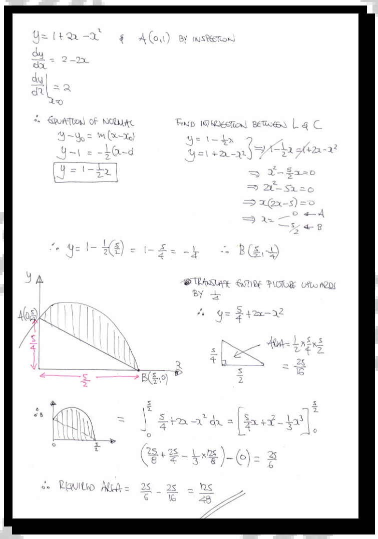
The curve crosses the y axis at the point A.

The straight line L is the normal to C at A.

The point B is a point of intersection between C and A.

Find the exact area of the finite region, bounded by C and L.

$\boxed{\text{area} = \frac{125}{48}}$



$y = 1 + 2x - x^2$ & $A(0,1)$ BY INSPECTION

$\frac{dy}{dx} = 2 - 2x$

$\left.\frac{dy}{dx}\right|_{x=0} = 2$

∴ EQUATION OF NORMAL

$y - y_0 = m(x - x_0)$

$y - 1 = -\frac{1}{2}(x - 0)$

$\boxed{y = 1 - \frac{1}{2}x}$

FIND INTERSECTION BETWEEN L & C

$\left.\begin{array}{l} y = 1 - \frac{1}{2}x \\ y = 1 + 2x - x^2 \end{array}\right\} \Rightarrow \cancel{1} - \frac{1}{2}x = \cancel{1} + 2x - x^2$

$\Rightarrow x^2 - \frac{5}{2}x = 0$

$\Rightarrow 2x^2 - 5x = 0$

$\Rightarrow x(2x - 5) = 0$

$\Rightarrow x = 0 \leftarrow A$, $x = \frac{5}{2} \leftarrow B$

∴ $y = 1 - \frac{1}{2}\left(\frac{5}{2}\right) = 1 - \frac{5}{4} = -\frac{1}{4}$ ∴ $B\left(\frac{5}{2}, -\frac{1}{4}\right)$

$A\left(0, \frac{5}{4}\right)$, $\frac{5}{4}$, $\frac{5}{2}$, $B\left(\frac{5}{2}, 0\right)$

TRANSLATE ENTIRE PICTURE UPWARDS BY $\frac{1}{4}$

∴ $y = \frac{5}{4} + 2x - x^2$

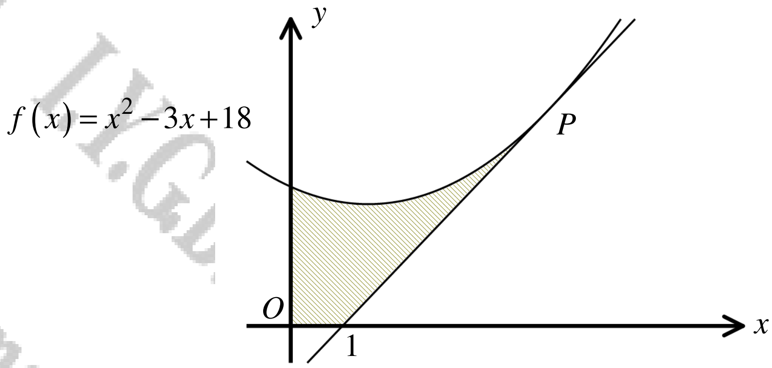
AREA $= \frac{1}{2} \times \frac{5}{4} \times \frac{5}{2} = \frac{25}{16}$

∴ $= \int_0^{\frac{5}{2}} \frac{5}{4} + 2x - x^2 \, dx = \left[\frac{5}{4}x + x^2 - \frac{1}{3}x^3\right]_0^{\frac{5}{2}}$

$\left(\frac{25}{8} + \frac{25}{4} - \frac{1}{3} \times \frac{125}{8}\right) - (0) = \frac{25}{6}$

∴ REQUIRED AREA $= \frac{25}{6} - \frac{25}{16} = \frac{125}{48}$

Question 63 (*****)



A quadratic curve has equation

$$y = x^2 - 3x + 18, \quad x \in \mathbb{R}.$$

The tangent to the curve at the point P meets the x axis at the point with coordinates $(1,0)$, as shown in the figure above.

Find the area of the finite region bounded by the curve, the coordinates axes and the tangent to the curve at P, shown shaded in figure above.

$$\boxed{\text{area} = \frac{229}{6}}$$