INTEGRATION INTRODUCTION

STANDARD INTEGRATION PRACTICE

Question 1

Integrate the following expressions with respect to x.

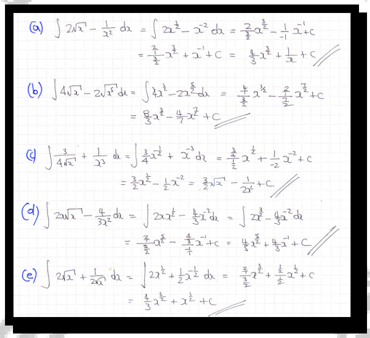
a) $\displaystyle\int 2\sqrt{x} - \frac{1}{x^2}\,dx$ $\quad\boxed{\frac{4}{3}x^{\frac{3}{2}} + \frac{1}{x} + C}$

b) $\displaystyle\int 4\sqrt{x} - 2\sqrt{x^5}\,dx$ $\quad\boxed{\frac{8}{3}x^{\frac{3}{2}} - \frac{4}{7}x^{\frac{7}{2}} + C}$

c) $\displaystyle\int \frac{3}{4\sqrt{x}} + \frac{1}{x^3}\,dx$ $\quad\boxed{\frac{3}{2}\sqrt{x} - \frac{1}{2x^2} + C}$

d) $\displaystyle\int 2x\sqrt{x} - \frac{4}{3x^2}\,dx$ $\quad\boxed{\frac{4}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{-1} + C}$

e) $\displaystyle\int 2\sqrt{x} + \frac{1}{2\sqrt{x}}\,dx$ $\quad\boxed{\frac{4}{3}x^{\frac{3}{2}} + \sqrt{x} + C}$



(a) $\int 2\sqrt{x} - \frac{1}{x^2}\,dx = \int 2x^{\frac{1}{2}} - x^{-2}\,dx = \frac{2}{\frac{3}{2}}x^{\frac{3}{2}} - \frac{1}{-1}x^{-1} + C$
$= \frac{\frac{2}{1}}{\frac{3}{2}}x^{\frac{3}{2}} + x^{-1} + C = \frac{4}{3}x^{\frac{3}{2}} + \frac{1}{x} + C$

(b) $\int 4\sqrt{x} - 2\sqrt{x^5}\,dx = \int 4x^{\frac{1}{2}} - 2x^{\frac{5}{2}}\,dx = \frac{\frac{4}{1}}{\frac{3}{2}}x^{\frac{3}{2}} - \frac{\frac{2}{1}}{\frac{7}{2}}x^{\frac{7}{2}} + C$
$= \frac{8}{3}x^{\frac{3}{2}} - \frac{4}{7}x^{\frac{7}{2}} + C$

(c) $\int \frac{3}{4\sqrt{x}} + \frac{1}{x^3}\,dx = \int \frac{3}{4}x^{-\frac{1}{2}} + x^{-3}\,dx = \frac{\frac{3}{4}}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{1}{-2}x^{-2} + C$
$= \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-2} = \frac{3}{2}\sqrt{x} - \frac{1}{2x^2} + C$

(d) $\int 2x\sqrt{x} - \frac{4}{3x^2}\,dx = \int 2xx^{\frac{1}{2}} - \frac{4}{3}x^{-2}\,dx = \int 2x^{\frac{3}{2}} - \frac{4}{3}x^{-2}\,dx$
$= \frac{\frac{2}{1}}{\frac{5}{2}}x^{\frac{5}{2}} - \frac{\frac{4}{3}}{-1}x^{-1} + C = \frac{4}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{-1} + C$

(e) $\int 2\sqrt{x} + \frac{1}{2\sqrt{x}}\,dx = \int 2x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\,dx = \frac{\frac{2}{1}}{\frac{3}{2}}x^{\frac{3}{2}} + \frac{\frac{1}{2}}{\frac{1}{2}}x^{\frac{1}{2}} + C$
$= \frac{4}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + C$

Question 2

Integrate the following expressions with respect to x.

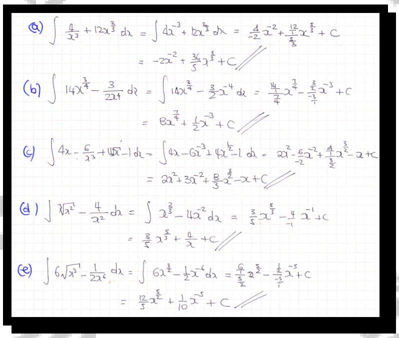
a) $\displaystyle\int \frac{4}{x^3}+12x^{\frac{2}{3}}\ dx$ $\boxed{-2x^{-2}+\frac{36}{5}x^{\frac{5}{3}}+C}$

b) $\displaystyle\int 14x^{\frac{3}{4}}-\frac{3}{2x^4}\ dx$ $\boxed{8x^{\frac{7}{4}}+\frac{1}{2}x^{-3}+C}$

c) $\displaystyle\int 4x-\frac{6}{x^3}+4\sqrt{x}-1\ dx$ $\boxed{2x^2-3x^{-2}+\frac{8}{3}x^{\frac{3}{2}}-x+C}$

d) $\displaystyle\int \sqrt[3]{x^2}-\frac{4}{x^2}\ dx$ $\boxed{\frac{3}{5}x^{\frac{5}{3}}+\frac{4}{x}+C}$

e) $\displaystyle\int 6\sqrt{x^3}-\frac{1}{2x^6}\ dx$ $\boxed{\frac{12}{5}x^{\frac{5}{2}}+\frac{1}{10}x^{-5}+C}$



Question 3

Integrate the following expressions with respect to x.

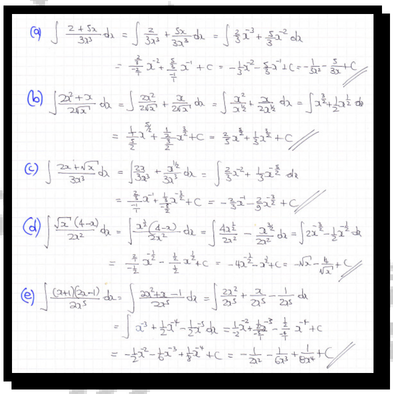
a) $\displaystyle\int \frac{2+5x}{3x^3}\,dx$ $\qquad$ $\boxed{-\frac{1}{3x^2}-\frac{5}{3x}+C}$

b) $\displaystyle\int \frac{2x^2+x}{2\sqrt{x}}\,dx$ $\qquad$ $\boxed{\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{3}x^{\frac{3}{2}}+C}$

c) $\displaystyle\int \frac{2x+\sqrt{x}}{3x^3}\,dx$ $\qquad$ $\boxed{-\frac{2}{3}x^{-1}-\frac{2}{9}x^{-\frac{3}{2}}+C}$

d) $\displaystyle\int \frac{\sqrt{x}(4-x)}{2x^2}\,dx$ $\qquad$ $\boxed{-\sqrt{x}-\frac{4}{\sqrt{x}}+C}$

e) $\displaystyle\int \frac{(x+1)(2x-1)}{2x^5}\,dx$ $\qquad$ $\boxed{-\frac{1}{2x^2}-\frac{1}{6x^3}+\frac{1}{8x^4}+C}$



Question 4

Integrate the following expressions with respect to x.

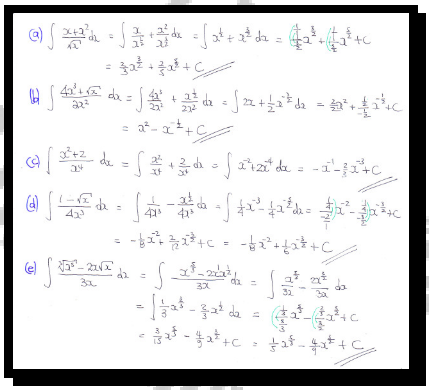
a) $\displaystyle\int \frac{x+x^2}{\sqrt{x}}\,dx$ $\qquad\boxed{\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{5}x^{\frac{5}{2}}+C}$

b) $\displaystyle\int \frac{4x^3+\sqrt{x}}{2x^2}\,dx$ $\qquad\boxed{x^2-x^{-\frac{1}{2}}+C}$

c) $\displaystyle\int \frac{x^2+2}{x^4}\,dx$ $\qquad\boxed{-x^{-1}-\frac{2}{3}x^{-3}+C}$

d) $\displaystyle\int \frac{1-\sqrt{x}}{4x^3}\,dx$ $\qquad\boxed{-\frac{1}{8}x^{-2}+\frac{1}{6}x^{-\frac{3}{2}}+C}$

e) $\displaystyle\int \frac{\sqrt[3]{x^5}-2x\sqrt{x}}{3x}\,dx$ $\qquad\boxed{\frac{1}{5}x^{\frac{5}{3}}-\frac{4}{9}x^{\frac{3}{2}}+C}$



Question 5

Integrate the following expressions with respect to x.

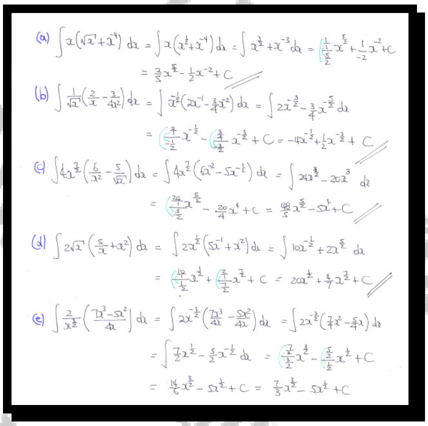
a) $\displaystyle\int x\left(\sqrt{x}+x^{-4}\right)\,dx$ $\qquad \boxed{\frac{2}{5}x^{\frac{5}{2}}-\frac{1}{2}x^{-2}+C}$

b) $\displaystyle\int \frac{1}{\sqrt{x}}\left(\frac{2}{x}-\frac{3}{4x^2}\right)dx$ $\qquad \boxed{-4x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}+C}$

c) $\displaystyle\int 4x^{\frac{7}{2}}\left(\frac{6}{x^2}-\frac{5}{\sqrt{x}}\right)dx$ $\qquad \boxed{\frac{48}{5}x^{\frac{5}{2}}-5x^4+C}$

d) $\displaystyle\int 2\sqrt{x}\left(\frac{5}{x}+x^2\right)dx$ $\qquad \boxed{20x^{\frac{1}{2}}+\frac{4}{7}x^{\frac{7}{2}}+C}$

e) $\displaystyle\int \frac{2}{x^{\frac{3}{2}}}\left(\frac{7x^3-5x^2}{4x}\right)dx$ $\qquad \boxed{\frac{7}{3}x^{\frac{3}{2}}-5x^{\frac{1}{2}}+C}$


(a) $\int x(\sqrt{x}+x^{-4})\,dx = \int x(x^{\frac{1}{2}}+x^{-4})\,dx = \int x^{\frac{3}{2}}+x^{-3}\,dx = \frac{1}{\frac{5}{2}}x^{\frac{5}{2}}+\frac{1}{-2}x^{-2}+C$
$= \frac{2}{5}x^{\frac{5}{2}}-\frac{1}{2}x^{-2}+C$

(b) $\int \frac{1}{\sqrt{x}}\left(\frac{2}{x}-\frac{3}{4x^2}\right)dx = \int x^{-\frac{1}{2}}(2x^{-1}-\frac{3}{4}x^{-2})\,dx = \int 2x^{-\frac{3}{2}}-\frac{3}{4}x^{-\frac{5}{2}}\,dx$
$= \frac{2}{-\frac{1}{2}}x^{-\frac{1}{2}}-\frac{\frac{3}{4}}{-\frac{3}{2}}x^{-\frac{3}{2}}+C = -4x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}+C$

(c) $\int 4x^{\frac{7}{2}}\left(\frac{6}{x^2}-\frac{5}{\sqrt{x}}\right)dx = \int 4x^{\frac{7}{2}}(6x^{-2}-5x^{-\frac{1}{2}})\,dx = \int 24x^{\frac{3}{2}}-20x^3\,dx$
$= \frac{24}{\frac{5}{2}}x^{\frac{5}{2}}-\frac{20}{4}x^4+C = \frac{48}{5}x^{\frac{5}{2}}-5x^4+C$

(d) $\int 2\sqrt{x}\left(\frac{5}{x}+x^2\right)dx = \int 2x^{\frac{1}{2}}(5x^{-1}+x^2)\,dx = \int 10x^{-\frac{1}{2}}+2x^{\frac{5}{2}}\,dx$
$= \frac{10}{\frac{1}{2}}x^{\frac{1}{2}}+\frac{2}{\frac{7}{2}}x^{\frac{7}{2}}+C = 20x^{\frac{1}{2}}+\frac{4}{7}x^{\frac{7}{2}}+C$

(e) $\int \frac{2}{x^{\frac{3}{2}}}\left(\frac{7x^3-5x^2}{4x}\right)dx = \int 2x^{-\frac{3}{2}}\left(\frac{7x^3}{4x}-\frac{5x^2}{4x}\right)dx = \int 2x^{-\frac{3}{2}}(\frac{7}{4}x^2-\frac{5}{4}x)\,dx$
$= \int \frac{7}{2}x^{\frac{1}{2}}-\frac{5}{2}x^{-\frac{1}{2}}\,dx = \frac{\frac{7}{2}}{\frac{3}{2}}x^{\frac{3}{2}}-\frac{\frac{5}{2}}{\frac{1}{2}}x^{\frac{1}{2}}+C$
$= \frac{14}{6}x^{\frac{3}{2}}-5x^{\frac{1}{2}}+C = \frac{7}{3}x^{\frac{3}{2}}-5x^{\frac{1}{2}}+C$

Question 6

Integrate the following expressions with respect to x.

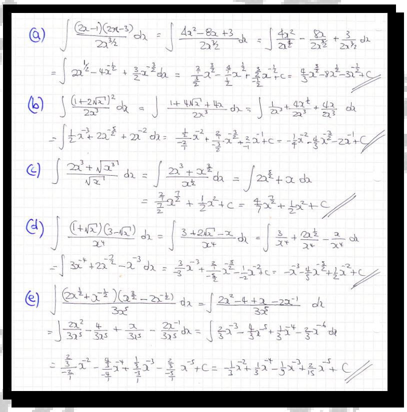
a) $\displaystyle\int \frac{(2x-1)(2x-3)}{2x^{\frac{3}{2}}}\,dx$ $\qquad$ $\boxed{\frac{4}{3}x^{\frac{3}{2}} - 8x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} + C}$

b) $\displaystyle\int \frac{\left(1+2\sqrt{x}\right)^2}{2x^3}\,dx$ $\qquad$ $\boxed{-\frac{1}{4}x^{-2} - \frac{4}{3}x^{-\frac{3}{2}} - 2x^{-1} + C}$

c) $\displaystyle\int \frac{2x^3+\sqrt{x^3}}{\sqrt{x}}\,dx$ $\qquad$ $\boxed{\frac{4}{7}x^{\frac{7}{2}} + \frac{1}{2}x^2 + C}$

d) $\displaystyle\int \frac{\left(1+\sqrt{x}\right)\left(3-\sqrt{x}\right)}{x^4}\,dx$ $\qquad$ $\boxed{-x^{-3} - \frac{4}{5}x^{-\frac{5}{2}} + \frac{1}{2}x^{-2} + C}$

e) $\displaystyle\int \frac{\left(2x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)\left(x^{\frac{3}{2}}-2x^{-\frac{1}{2}}\right)}{3x^5}\,dx$ $\qquad$ $\boxed{-\frac{1}{3}x^{-2} + \frac{1}{3}x^{-4} - \frac{1}{9}x^{-3} + \frac{2}{15}x^{-5} + C}$



Question 7

Evaluate the following integrals.

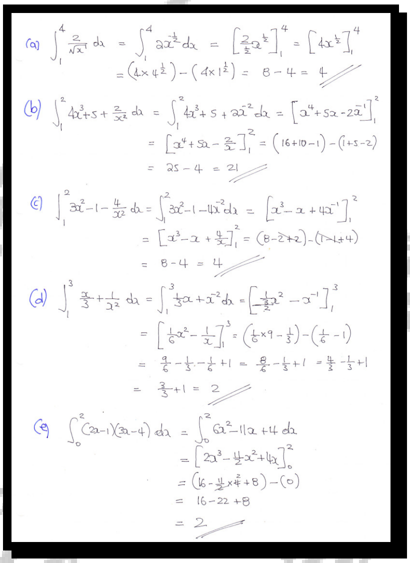
a) $\displaystyle\int_1^4 \frac{2}{\sqrt{x}}\ dx$ $\boxed{4}$

b) $\displaystyle\int_1^2 4x^3+5+\frac{2}{x^2}\ dx$ $\boxed{21}$

c) $\displaystyle\int_1^2 3x^2-1-\frac{4}{x^2}\ dx$ $\boxed{4}$

d) $\displaystyle\int_1^3 \frac{x}{3}+\frac{1}{x^2}\ dx$ $\boxed{2}$

e) $\displaystyle\int_0^2 (2x-1)(3x-4)\ dx$ $\boxed{2}$



(a) $\int_1^4 \frac{2}{\sqrt{x}}\,dx = \int_1^4 2x^{-\frac{1}{2}}\,dx = \left[\frac{2}{\frac{1}{2}}x^{\frac{1}{2}}\right]_1^4 = \left[4x^{\frac{1}{2}}\right]_1^4$
$= \left(4\times 4^{\frac{1}{2}}\right) - \left(4\times 1^{\frac{1}{2}}\right) = 8-4 = 4$

(b) $\int_1^2 4x^3+5+\frac{2}{x^2}\,dx = \int_1^2 4x^3+5+2x^{-2}\,dx = \left[x^4+5x-2x^{-1}\right]_1^2$
$= \left[x^4+5x-\frac{2}{x}\right]_1^2 = (16+10-1)-(1+5-2)$
$= 25-4 = 21$

(c) $\int_1^2 3x^2-1-\frac{4}{x^2}\,dx = \int_1^2 3x^2-1-4x^{-2}\,dx = \left[x^3-x+4x^{-1}\right]_1^2$
$= \left[x^3-x+\frac{4}{x}\right]_1^2 = (8-2+2)-(1-1+4)$
$= 8-4 = 4$

(d) $\int_1^3 \frac{x}{3}+\frac{1}{x^2}\,dx = \int_1^3 \frac{1}{3}x+x^{-2}\,dx = \left[\frac{\frac{1}{3}}{2}x^2 - x^{-1}\right]_1^3$
$= \left[\frac{1}{6}x^2-\frac{1}{x}\right]_1^3 = \left(\frac{1}{6}\times 9-\frac{1}{3}\right)-\left(\frac{1}{6}-1\right)$
$= \frac{9}{6}-\frac{1}{3}-\frac{1}{6}+1 = \frac{8}{6}-\frac{1}{3}+1 = \frac{4}{3}-\frac{1}{3}+1$
$= \frac{3}{3}+1 = 2$

(e) $\int_0^2 (2x-1)(3x-4)\,dx = \int_0^2 6x^2-11x+4\,dx$
$= \left[2x^3-\frac{11}{2}x^2+4x\right]_0^2$
$= \left(16-\frac{11}{2}\times 4+8\right)-(0)$
$= 16-22+8$
$= 2$

Question 8

Evaluate the following integrals.

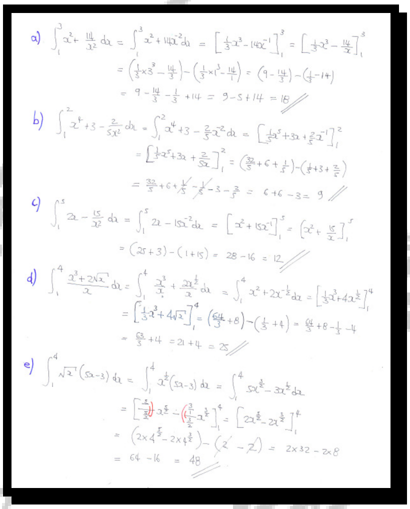
a) $\displaystyle\int_1^3 x^2 + \frac{14}{x^2}\ dx$ $\boxed{18}$

b) $\displaystyle\int_1^2 x^4 + 3 - \frac{2}{5x^2}\ dx$ $\boxed{9}$

c) $\displaystyle\int_1^5 2x - \frac{15}{x^2}\ dx$ $\boxed{12}$

d) $\displaystyle\int_1^4 \frac{x^3 + 2\sqrt{x}}{x}\ dx$ $\boxed{25}$

e) $\displaystyle\int_1^4 \sqrt{x}\,(5x-3)\,dx$ $\boxed{48}$



Question 9

Evaluate the following integrals.

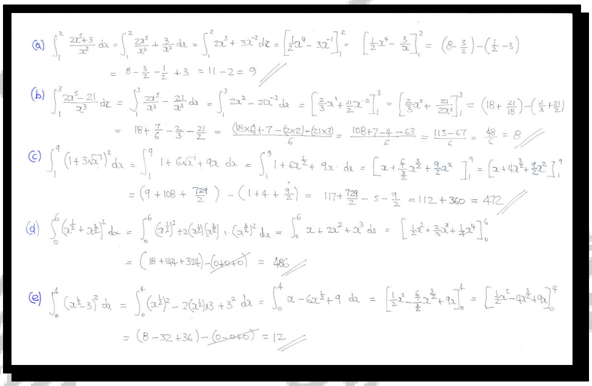
a) $\displaystyle\int_1^2 \frac{2x^5+3}{x^2}\,dx$ $\boxed{9}$

b) $\displaystyle\int_1^3 \frac{2x^5-21}{x^3}\,dx$ $\boxed{8}$

c) $\displaystyle\int_1^9 \left(1+3\sqrt{x}\right)^2\,dx$ $\boxed{472}$

d) $\displaystyle\int_0^6 \left(x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)^2\,dx$ $\boxed{486}$

e) $\displaystyle\int_0^4 \left(x^{\frac{1}{2}}-3\right)^2\,dx$ $\boxed{12}$



(a) $\int_1^2 \frac{2x^5+3}{x^2}\,dx = \int_1^2 \frac{2x^5}{x^2}+\frac{3}{x^2}\,dx = \int_1^2 2x^3+3x^{-2}\,dx = \left[\frac{1}{2}x^4-3x^{-1}\right]_1^2 = \left[\frac{1}{2}x^4-\frac{3}{x}\right]_1^2 = \left(8-\frac{3}{2}\right)-\left(\frac{1}{2}-3\right)$

$= 8-\frac{3}{2}-\frac{1}{2}+3 = 11-2 = 9$

(b) $\int_1^3 \frac{2x^5-21}{x^3}\,dx = \int_1^3 \frac{2x^5}{x^3}-\frac{21}{x^3}\,dx = \int_1^3 2x^2-21x^{-3}\,dx = \left[\frac{2}{3}x^3+\frac{21}{2}x^{-2}\right]_1^3 = \left[\frac{2}{3}x^3+\frac{21}{2x^2}\right]_1^3 = \left(18+\frac{21}{18}\right)-\left(\frac{2}{3}+\frac{21}{2}\right)$

$= 18+\frac{7}{6}-\frac{2}{3}-\frac{21}{2} = \frac{(18\times6)+7-(2\times2)-(21\times3)}{6} = \frac{108+7-4-63}{6} = \frac{115-67}{6} = \frac{48}{6} = 8$

(c) $\int_1^9 (1+3\sqrt{x})^2\,dx = \int_1^9 1+6\sqrt{x}+9x\,dx = \int_1^9 1+6x^{\frac{1}{2}}+9x\,dx = \left[x+\frac{6}{\frac{3}{2}}x^{\frac{3}{2}}+\frac{9}{2}x^2\right]_1^9 = \left[x+4x^{\frac{3}{2}}+\frac{9}{2}x^2\right]_1^9$

$= \left(9+108+\frac{729}{2}\right)-\left(1+4+\frac{9}{2}\right) = 117+\frac{729}{2}-5-\frac{9}{2} = 112+360 = 472$

(d) $\int_0^6 \left(x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)^2 dx = \int_0^6 \left(x^{\frac{1}{2}}\right)^2+2\left(x^{\frac{1}{2}}\right)\left(x^{\frac{3}{2}}\right)+\left(x^{\frac{3}{2}}\right)^2 dx = \int_0^6 x+2x^2+x^3\,dx = \left[\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4\right]_0^6$

$= (18+144+324)-(0+0+0) = 486$

(e) $\int_0^4 \left(x^{\frac{1}{2}}-3\right)^2 dx = \int_0^4 \left(x^{\frac{1}{2}}\right)^2-2\left(x^{\frac{1}{2}}\right)\times3+3^2\,dx = \int_0^4 x-6x^{\frac{1}{2}}+9\,dx = \left[\frac{1}{2}x^2-\frac{6}{\frac{3}{2}}x^{\frac{3}{2}}+9x\right]_0^4 = \left[\frac{1}{2}x^2-4x^{\frac{3}{2}}+9x\right]_0^4$

$= (8-32+36)-(0-0+0) = 12$

Question 10

Evaluate the following integrals.

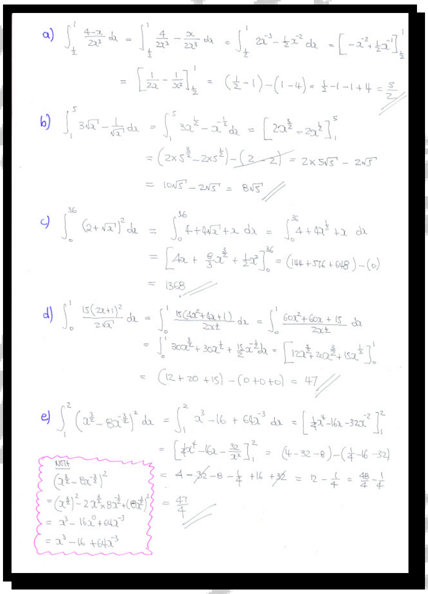
a) $\displaystyle\int_{\frac{1}{2}}^{1} \frac{4-x}{2x^3}\,dx$ $\boxed{\frac{5}{2}}$

b) $\displaystyle\int_{1}^{5} 3\sqrt{x}-\frac{1}{\sqrt{x}}\,dx$ $\boxed{8\sqrt{5}}$

c) $\displaystyle\int_{0}^{36} \left(2+\sqrt{x}\right)^2\,dx$ $\boxed{1368}$

d) $\displaystyle\int_{0}^{1} \frac{15(2x+1)^2}{2\sqrt{x}}\,dx$ $\boxed{47}$

e) $\displaystyle\int_{1}^{2} \left(x^{\frac{3}{2}}-8x^{-\frac{3}{2}}\right)^2\,dx$ $\boxed{\frac{47}{4}}$



VARIOUS INTEGRATION QUESTIONS

Question 1

$$y = 2x^2 - \frac{6}{x^3} + 8x^3, \ x \neq 0.$$

Find an expression for

$$\int y \, dx .$$

$$\boxed{\frac{2}{3}x^3 + 3x^{-2} + 2x^4 + C}$$

$$\int y \, dx = \int 2x^2 - 6x^{-3} + 8x^3 \, dx = \frac{2}{3}x^3 - \frac{6}{-2}x^{-2} + \frac{8}{4}x^4 + C$$
$$= \frac{2}{3}x^3 + 3x^{-2} + 2x^4 + C$$

Question 2

$$y = 3x^2 - 6\sqrt{x} - \frac{1}{x^2} + 4, \ x > 0.$$

Find an expression for

$$\int y \, dx .$$

$$\boxed{4x + x^3 - 4x^{\frac{3}{2}} + x^{-1} + C}$$

$$\int y \, dx = \int 3x^2 - 6x^{\frac{1}{2}} - x^{-2} + 4 \, dx = \frac{3}{3}x^3 - \frac{6}{\frac{3}{2}}x^{\frac{3}{2}} - \frac{1}{-1}x^{-1} + 4x + C$$
$$= x^3 - 4x^{\frac{3}{2}} + x^{-1} + 4x + C$$

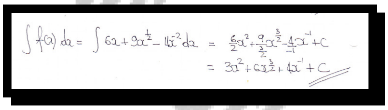
Question 3

$$f(x) = 6x + 9\sqrt{x} - \frac{4}{x^2}, \ x > 0.$$

Find an expression for

$$\int f(x)\, dx.$$

$$\boxed{3x^2 + 6x^{\frac{3}{2}} + 4x^{-1} + C}$$



$$\int f(x)\,dx = \int 6x + 9x^{\frac{1}{2}} - 4x^{-2}\,dx = \frac{6}{2}x^2 + \frac{9}{\frac{3}{2}}x^{\frac{3}{2}} - \frac{4}{-1}x^{-1} + C$$
$$= 3x^2 + 6x^{\frac{3}{2}} + 4x^{-1} + C$$

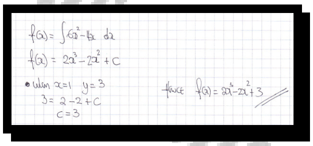
Question 4

The point $P(1,3)$ lies on the curve with equation $y = f(x)$, whose gradient function is given by

$$f'(x) = 6x^2 - 4x, \ x \in \mathbb{R}.$$

Find an equation for $f(x)$.

$$\boxed{f(x) = 2x^3 - 2x^2 + 3}$$



$f(x) = \int 6x^2 - 4x \, dx$

$f(x) = 2x^3 - 2x^2 + C$

• when $x=1$ $y=3$

$3 = 2 - 2 + C$

$C = 3$

thus $f(x) = 2x^3 - 2x^2 + 3$

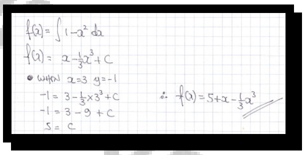
Question 5

The point $P(3,-1)$ lies on the curve with equation $y = f(x)$, whose gradient function is given by

$$f'(x) = 1 - x^2, \; x \in \mathbb{R}.$$

Find an equation for $f(x)$.

$$\boxed{f(x) = 5 + x - \frac{1}{3}x^3}$$

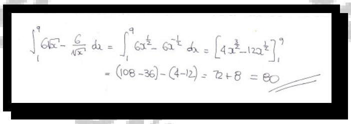


Question 6

By showing clear workings, find the value of

$$\int_1^9 6\sqrt{x} - \frac{6}{\sqrt{x}} \, dx.$$

$$\boxed{80}$$



Question 7

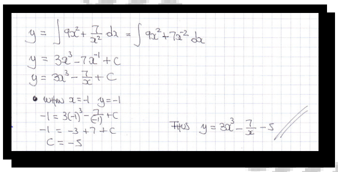
The curve C with equation $y = f(x)$ has gradient function

$$\frac{dy}{dx} = 9x^2 + \frac{7}{x^2}, \quad x \neq 0.$$

The point $A(-1,-1)$ lies on C.

Find an equation for C.

$$\boxed{y = 3x^3 - \frac{7}{x} - 5}$$



$y = \int 9x^2 + \frac{7}{x^2}\,dx = \int 9x^2 + 7x^{-2}\,dx$

$y = 3x^3 - 7x^{-1} + C$

$y = 3x^3 - \frac{7}{x} + C$

• when $x = -1$ $y = -1$

$-1 = 3(-1)^3 - \frac{7}{(-1)} + C$

$-1 = -3 + 7 + C$

$C = -5$

Thus $y = 3x^3 - \frac{7}{x} - 5$

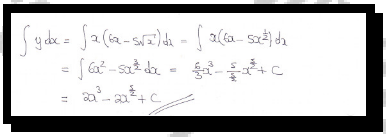
Question 8

$$y = x\left(6x - 5\sqrt{x}\right), \quad x \geq 0.$$

By showing all steps in the workings, find an expression for

$$\int y\,dx.$$

$$\boxed{2x^3 - 2x^{\frac{5}{2}} + C}$$



$\int y\,dx = \int x(6x - 5\sqrt{x})\,dx = \int x(6x - 5x^{\frac{1}{2}})\,dx$

$= \int 6x^2 - 5x^{\frac{3}{2}}\,dx = \frac{6}{3}x^3 - \frac{5}{\frac{5}{2}}x^{\frac{5}{2}} + C$

$= 2x^3 - 2x^{\frac{5}{2}} + C$

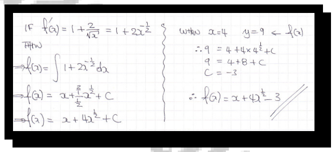
Question 9

The point $P(4,9)$ lies on the curve with equation $y = f(x)$, whose gradient function is given by

$$f'(x) = 1 + \frac{2}{\sqrt{x}}, \quad x > 0.$$

Find an equation for $f(x)$.

$$\boxed{f(x) = x + 4\sqrt{x} - 3}$$

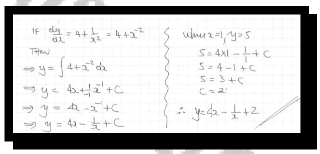


Question 10

$$\frac{dy}{dx} = 4 + \frac{1}{x^2}, \quad x \neq 0.$$

Given that $y = 5$ when $x = 1$, express y in terms of x.

$$\boxed{y = 4x - \frac{1}{x} + 2}$$

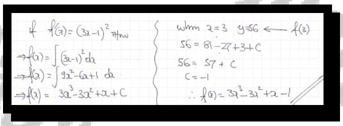


Question 11

$$f'(x) = (3x-1)^2 .$$

Given that $f(3) = 56$, find an expression for $f(x)$.

$$\boxed{f(x) = 3x^3 - 3x^2 + x - 1}$$



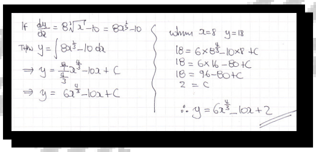
Question 12

The point $P(8,18)$ lies on the curve C, whose gradient function is given by

$$\frac{dy}{dx} = 8\sqrt[3]{x} - 10, \quad x \geq 0 .$$

Find an equation for C.

$$\boxed{y = 6x^{\frac{4}{3}} - 10x + 2}$$



Question 13

$$f(x) = \frac{5\sqrt{x}\left(3x^2 - 2\right)}{x}, \quad x > 0.$$

Show clearly that

$$\int f(x)\,dx = P\sqrt{x} + Qx^{\frac{5}{2}} + C,$$

where P and Q are integers to be found, and C is an arbitrary constant.

$\boxed{P = -20}$, $\boxed{Q = 6}$

$$\int f(x)\,dx = \int \frac{5\sqrt{x}\left(3x^2-2\right)}{x}\,dx = \int \frac{5x^{\frac{1}{2}}\left(3x^2-2\right)}{x}\,dx$$

$$= \int \frac{15x^{\frac{5}{2}} - 10x^{\frac{1}{2}}}{x}\,dx = \int \frac{15x^{\frac{5}{2}}}{x} - \frac{10x^{\frac{1}{2}}}{x}\,dx$$

$$= \int 15x^{\frac{3}{2}} - 10x^{-\frac{1}{2}}\,dx = \frac{15}{\frac{5}{2}}x^{\frac{5}{2}} - \frac{10}{\frac{1}{2}}x^{\frac{1}{2}} + C$$

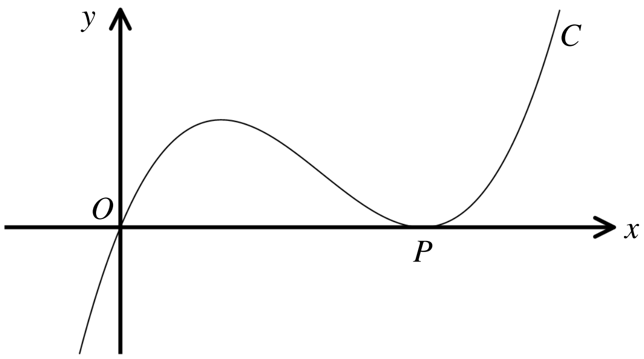
$$= 6x^{\frac{5}{2}} - 20x^{\frac{1}{2}} + C$$

$$= 6x^{\frac{5}{2}} - 20\sqrt{x} + C$$

It $P = -20$

$Q = 6$

Question 14



The figure above shows the cubic curve C which meets the coordinates axes at the origin O and at the point P.

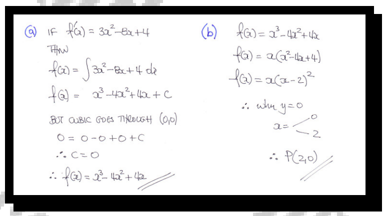
The gradient function of C is given by

$$f'(x) = 3x^2 - 8x + 4 .$$

a) Find an equation for C.

b) Determine the coordinates of P.

$\boxed{f(x) = x^3 - 4x^2 + 4x}$, $\boxed{P(2,0)}$



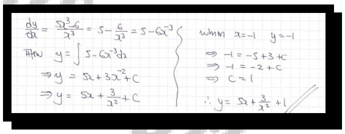
Question 15

The point $P(-1,-1)$ lies on the curve C, whose gradient function is given by

$$\frac{dy}{dx} = \frac{5x^3 - 6}{x^3}, \ x \neq 0.$$

Find an equation for C.

$$\boxed{y = 5x + \frac{3}{x^2} + 1}$$



$\frac{dy}{dx} = \frac{5x^3-6}{x^3} = 5 - \frac{6}{x^3} = 5 - 6x^{-3}$

Then $y = \int 5 - 6x^{-3}\,dx$

$\Rightarrow y = 5x + 3x^{-2} + C$

$\Rightarrow y = 5x + \frac{3}{x^2} + C$

when $x=-1$ $y=-1$

$\Rightarrow -1 = -5 + 3 + C$

$\Rightarrow -1 = -2 + C$

$\Rightarrow C = 1$

$\therefore y = 5x + \frac{3}{x^2} + 1$

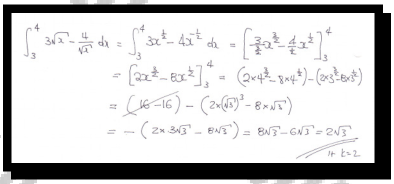
Question 16

Show clearly that

$$\int_3^4 3\sqrt{x} - \frac{4}{\sqrt{x}} \ dx = k\sqrt{3},$$

where k is an integer to be found.

$$\boxed{k = 2}$$



$\int_3^4 3\sqrt{x} - \frac{4}{\sqrt{x}}\,dx = \int_3^4 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}\,dx = \left[\frac{3}{\frac{3}{2}}x^{\frac{3}{2}} - \frac{4}{\frac{1}{2}}x^{\frac{1}{2}}\right]_3^4$

$= \left[2x^{\frac{3}{2}} - 8x^{\frac{1}{2}}\right]_3^4 = \left(2\times4^{\frac{3}{2}} - 8\times4^{\frac{1}{2}}\right) - \left(2\times3^{\frac{3}{2}} - 8\times3^{\frac{1}{2}}\right)$

$= (16 - 16) - \left(2\times(\sqrt{3})^3 - 8\times\sqrt{3}\right)$

$= -\left(2\times3\sqrt{3} - 8\sqrt{3}\right) = 8\sqrt{3} - 6\sqrt{3} = 2\sqrt{3}$

i.e. $k=2$

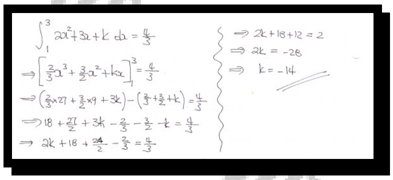
Question 17

$$f(x) = 2x^2 + 3x + k \text{, where } k \text{ is a constant.}$$

Find the value of k, given that

$$\int_1^3 f(x)\ dx = \frac{4}{3}.$$

$\boxed{k = -14}$



Question 18

The cubic equation C passes through the origin O and its gradient function is

$$\frac{dy}{dx} = 6x^2 - 6x - 20.$$

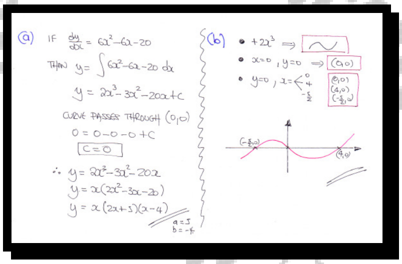
a) Show clearly that the equation of C can be written as

$$y = x(2x+a)(x+b),$$

where a and b are constants.

b) Sketch the graph of C, indicating clearly the coordinates of the points where the graph meets the coordinate axes.

$\boxed{a=5}$, $\boxed{b=-4}$



Question 19

The gradient of every point on the curve C, with equation $y = f(x)$, satisfies

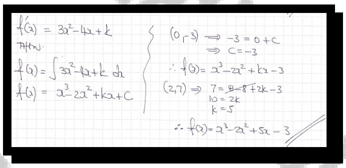
$$f'(x) = 3x^2 - 4x + k \, ,$$

where k is a constant.

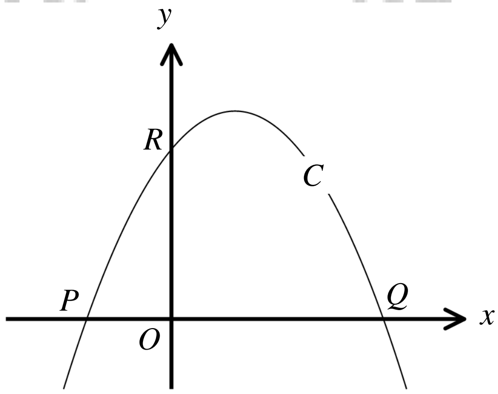
The points $P(0,-3)$ and $Q(2,7)$ both lie on C.

Find an equation for C.

$$\boxed{y = x^3 - 2x^2 + 5x - 3}$$



Question 20



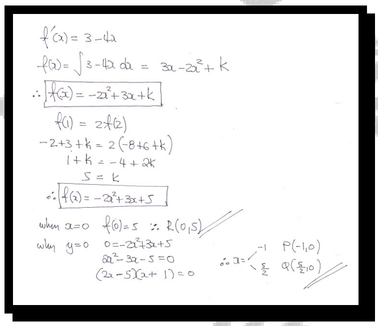
The figure above shows the curve C which meets the coordinates axes at the points P, Q and R.

Given the gradient function of C is given by

$$f'(x) = 3 - 4x,$$

and that $f(1) = 2f(2)$, determine the coordinates of P, Q and R.

$\boxed{P(-1,0)}$, $\boxed{P\left(\frac{5}{2},0\right)}$, $\boxed{R(0,5)}$



$f'(x) = 3 - 4x$

$f(x) = \int 3 - 4x \, dx = 3x - 2x^2 + k$

$\therefore \boxed{f(x) = -2x^2 + 3x + k}$

$f(1) = 2f(2)$

$-2 + 3 + k = 2(-8 + 6 + k)$

$1 + k = -4 + 2k$

$5 = k$

$\therefore \boxed{f(x) = -2x^2 + 3x + 5}$

when $x=0$ $f(0) = 5$ $\therefore R(0,5)$

when $y=0$ $0 = -2x^2 + 3x + 5$

$2x^2 - 3x - 5 = 0$

$(2x-5)(x+1) = 0$

$\therefore x = -1$, $P(-1,0)$; $x = \frac{5}{2}$, $Q(\frac{5}{2},0)$

Question 21

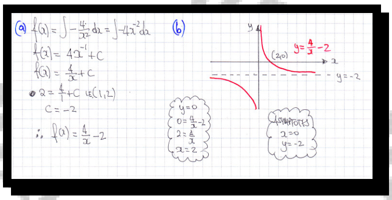
The curve C with equation $y = f(x)$ satisfies

$$f'(x) = -\frac{4}{x^2}, \quad x \neq 0.$$

a) Given that $f(1) = 2$, find an expression for $f(x)$.

b) Sketch the graph of $f(x)$, indicating clearly the asymptotes of the curve and the coordinates of any points where the curve crosses the coordinate axes.

$\boxed{f(x) = \frac{4}{x} - 2}$, $\boxed{(2,0)}$



Question 22

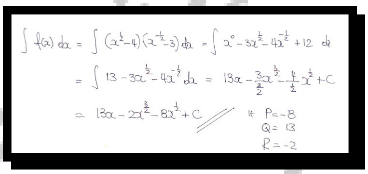
$$f(x) = \left(x^{\frac{1}{2}} - 4\right)\left(x^{-\frac{1}{2}} - 3\right), \quad x > 0.$$

Show clearly that

$$\int f(x)\,dx = P\sqrt{x} + Qx + Rx^{\frac{3}{2}} + C,$$

where P, Q and R are integers to be found, and C is an arbitrary constant.

$\boxed{P = -8}$, $\boxed{Q = 13}$, $\boxed{R = -2}$

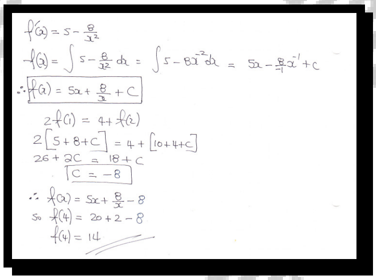


Question 23

$$f'(x) = 5 - \frac{8}{x^2}, \ x \neq 0.$$

Find the value of $f(4)$, given that $2f(1) = 4 + f(2)$.

$\boxed{f(4) = 14}$


$f'(x) = 5 - \frac{8}{x^2}$

$f(x) = \int 5 - \frac{8}{x^2} \, dx = \int 5 - 8x^{-2} \, dx = 5x - \frac{8}{-1}x^{-1} + c$

$\therefore \boxed{f(x) = 5x + \frac{8}{x} + C}$

$2f(1) = 4 + f(2)$

$2[5 + 8 + C] = 4 + [10 + 4 + C]$

$26 + 2C = 18 + C$

$\boxed{C = -8}$

$\therefore f(x) = 5x + \frac{8}{x} - 8$

so $f(4) = 20 + 2 - 8$

$f(4) = 14$

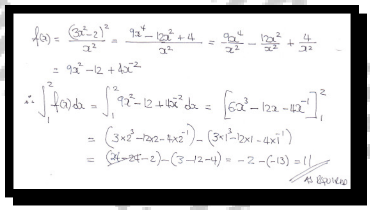
Question 24

$$f(x) = \frac{\left(3x^2 - 2\right)^2}{x^2}, \ x \neq 0.$$

Show clearly that

$$\int_1^2 f(x) \ dx = 11.$$

$\boxed{\text{proof}}$


$f(x) = \frac{(3x^2-2)^2}{x^2} = \frac{9x^4 - 12x^2 + 4}{x^2} = \frac{9x^4}{x^2} - \frac{12x^2}{x^2} + \frac{4}{x^2}$

$= 9x^2 - 12 + 4x^{-2}$

$\therefore \int_1^2 f(x) \, dx = \int_1^2 9x^2 - 12 + 4x^{-2} \, dx = \left[6x^3 - 12x - 4x^{-1}\right]_1^2$

$= (3 \times 2^3 - 12 \times 2 - 4 \times 2^{-1}) - (3 \times 1^3 - 12 \times 1 - 4 \times 1^{-1})$

$= (24 - 24 - 2) - (3 - 12 - 4) = -2 - (-13) = 11$

AS REQUIRED

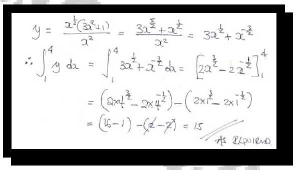
Question 25

$$y = \frac{x^{\frac{1}{2}}(3x^2+1)}{x^2}, \ \ x > 0.$$

Show clearly that

$$\int_1^4 y \ \ dx = 15.$$

proof



$$y = \frac{x^{\frac{1}{2}}(3x^2+1)}{x^2} = \frac{3x^{\frac{5}{2}} + x^{\frac{1}{2}}}{x^2} = 3x^{\frac{1}{2}} + x^{-\frac{3}{2}}$$

$$\therefore \int_1^4 y \, dx = \int_1^4 3x^{\frac{1}{2}} + x^{-\frac{3}{2}} \, dx = \left[2x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}\right]_1^4$$

$$= \left(2\times4^{\frac{3}{2}} - 2\times4^{-\frac{1}{2}}\right) - \left(2\times1^{\frac{3}{2}} - 2\times1^{-\frac{1}{2}}\right)$$

$$= (16-1) - (2-2) = 15$$

AS REQUIRED

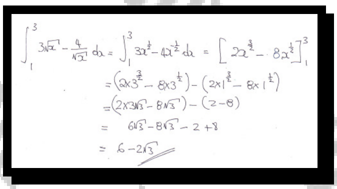
Question 26

Find the exact value of

$$\int_1^3 3\sqrt{x} - \frac{4}{\sqrt{x}} \, dx,$$

giving the answer in the form $p + q\sqrt{3}$, where p and q are integers.

$6 - 2\sqrt{3}$



$$\int_1^3 3\sqrt{x} - \frac{4}{\sqrt{x}} \, dx = \int_1^3 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} \, dx = \left[2x^{\frac{3}{2}} - 8x^{\frac{1}{2}}\right]_1^3$$

$$= \left(2\times3^{\frac{3}{2}} - 8\times3^{\frac{1}{2}}\right) - \left(2\times1^{\frac{3}{2}} - 8\times1^{\frac{1}{2}}\right)$$

$$= \left(2\times3\sqrt{3} - 8\sqrt{3}\right) - (2-8)$$

$$= 6\sqrt{3} - 8\sqrt{3} - 2 + 8$$

$$= 6 - 2\sqrt{3}$$

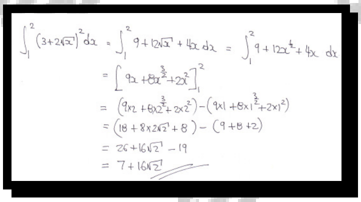
Question 27

Find the exact value of

$$\int_1^2 \left(3+2\sqrt{x}\right)^2 \, dx,$$

giving the answer in the form $a+b\sqrt{2}$, where a and b are integers.

$\boxed{7+16\sqrt{2}}$



$$\int_1^2 (3+2\sqrt{x})^2 dx = \int_1^2 9+12\sqrt{x}+4x \, dx = \int_1^2 9+12x^{\frac{1}{2}}+4x \, dx$$
$$= \left[9x+8x^{\frac{3}{2}}+2x^2\right]_1^2$$
$$= \left(9\times2+8\times2^{\frac{3}{2}}+2\times2^2\right)-\left(9\times1+8\times1^{\frac{3}{2}}+2\times1^2\right)$$
$$= \left(18+8\times2\sqrt{2}+8\right)-\left(9+8+2\right)$$
$$= 26+16\sqrt{2}-19$$
$$= 7+16\sqrt{2}$$

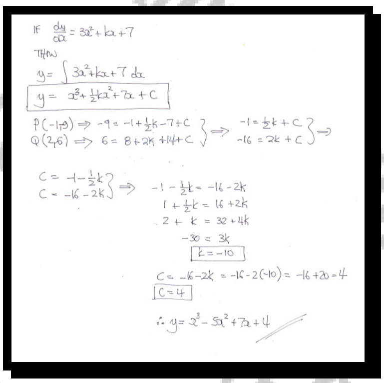
Question 28

A cubic curve passes through the points $P(-1,-9)$ and $Q(2,6)$ and its gradient function is given by

$$\frac{dy}{dx} = 3x^2+kx+7, \text{ where } k \text{ is a constant.}$$

Find an equation for this cubic curve.

$\boxed{y = x^3-5x^2+7x+4}$



IF $\frac{dy}{dx} = 3x^2+kx+7$

THEN

$y = \int 3x^2+kx+7 \, dx$

$\boxed{y = x^3+\frac{1}{2}kx^2+7x+C}$

$P(-1,-9) \Rightarrow -9 = -1+\frac{1}{2}k-7+C$, $Q(2,6) \Rightarrow 6 = 8+2k+14+C$ $\Rightarrow$ $-1 = \frac{1}{2}k+C$, $-16 = 2k+C$ $\Rightarrow$

$C = -1-\frac{1}{2}k$, $C = -16-2k$ $\Rightarrow$ $-1-\frac{1}{2}k = -16-2k$

$1+\frac{1}{2}k = 16+2k$

$2+k = 32+4k$

$-30 = 3k$

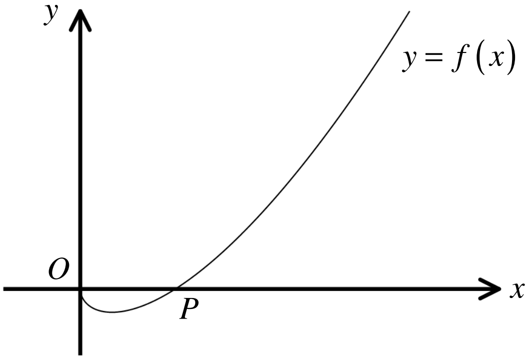
$\boxed{k=-10}$

$C = -16-2k = -16-2(-10) = -16+20 = 4$

$\boxed{C=4}$

$\therefore y = x^3-5x^2+7x+4$

Question 29



The figure above shows a curve with equation $y = f(x)$ which meets the x axis at the origin O and at the point P.

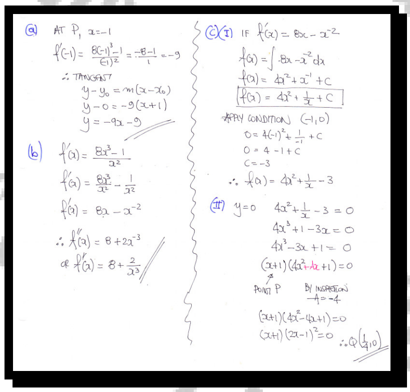
The gradient function of the curve is given by

$$f'(x) = \frac{12x-1}{\sqrt{x}}, \quad x > 0.$$

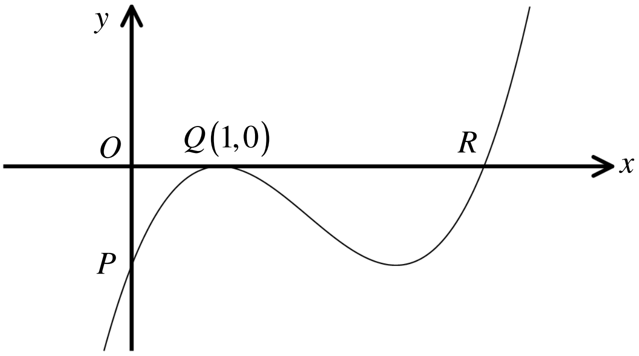
a) Find an equation of the curve.

b) Determine the coordinates of P.

$\boxed{f(x) = 8x^{\frac{3}{2}} - 2\sqrt{x}}$, $\boxed{P\left(\frac{1}{4}, 0\right)}$



Question 30



The figure above shows the graph of a cubic curve, which touches the x axis at the point $Q(1,0)$.

a) Determine an equation for the cubic curve , given its gradient is given by

$$\frac{dy}{dx} = 3x^2 - 12x + 9.$$

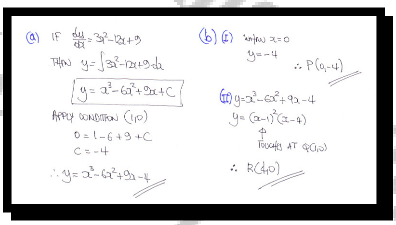
The cubic curve crosses the x axis and the y axis at the points R and P, respectively.

b) Determine the coordinates …

i. … of the point P.

ii. … of the point R.

$\boxed{y = x^3 - 6x^2 + 9x - 4}$, $\boxed{P(0,-4)}$, $\boxed{R(4,0)}$



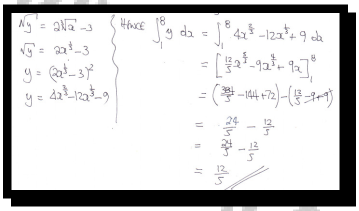
Question 31

$$\sqrt{y} = 2\sqrt[3]{x} - 3, \ \ x > 0.$$

Show clearly that

$$\int_1^8 y \ \ dx = \frac{12}{5}.$$

proof



Question 32

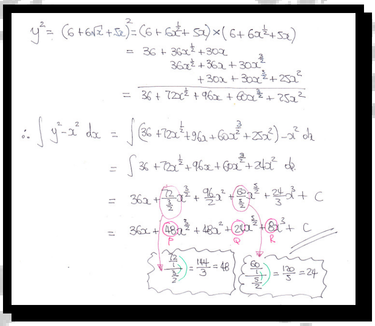
$$y = 6 + 6\sqrt{x} + 5x, \ \ x \geq 0.$$

Show clearly that

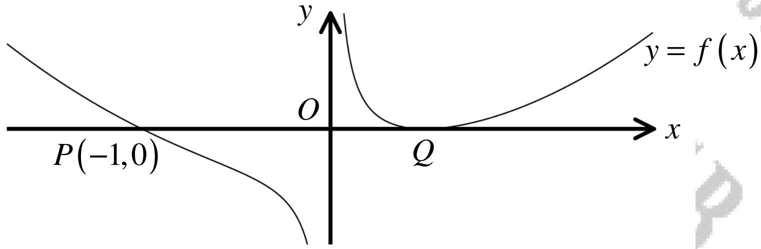
$$\int \left(y^2 - x^2 \right) \, dx = 36x + Px^{\frac{3}{2}} + 48x^2\sqrt{x} + Qx^{\frac{5}{2}} + Rx^3 + C,$$

where P, Q and R are constants to be found, and C is an arbitrary constant.

$P = 48$, $Q = 24$, $R = 8$



Question 33



The figure above shows a curve with equation $y = f(x)$.

The curve meets the x axis at the points $P(-1,0)$ and Q, and its gradient function is given by

$$f'(x) = \frac{8x^3 - 1}{x^2}, \quad x \neq 0.$$

a) Find an equation of the tangent to the curve at P.

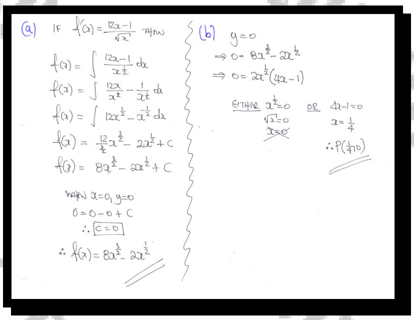
b) Find an expression for $f''(x)$.

c) Determine …

i. … an equation of the curve.

ii. … the coordinates of Q.

$\boxed{y = -9x - 9}$, $\boxed{f''(x) = 8 + 2x^{-3}}$, $\boxed{y = 4x^2 + \frac{1}{x} - 3}$, $\boxed{Q\left(\frac{1}{2}, 0\right)}$



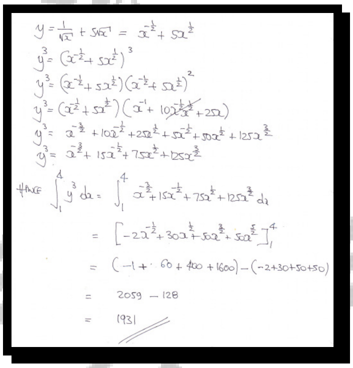
Question 34

$$y = \frac{1}{\sqrt{x}} + 5\sqrt{x}, \quad x > 0.$$

Show clearly that

$$\int_1^4 y^3 \, dx = 1931.$$

proof


$y = \frac{1}{\sqrt{x}} + 5\sqrt{x} = x^{-\frac{1}{2}} + 5x^{\frac{1}{2}}$

$y^3 = (x^{-\frac{1}{2}} + 5x^{\frac{1}{2}})^3$

$y^3 = (x^{-\frac{1}{2}} + 5x^{\frac{1}{2}})(x^{-\frac{1}{2}} + 5x^{\frac{1}{2}})^2$

$y^3 = (x^{-\frac{1}{2}} + 5x^{\frac{1}{2}})(x^{-1} + 10x^{-\frac{1}{2}}x^{\frac{1}{2}} + 25x)$

$y^3 = x^{-\frac{3}{2}} + 10x^{-\frac{1}{2}} + 25x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} + 50x^{\frac{1}{2}} + 125x^{\frac{3}{2}}$

$y^3 = x^{-\frac{3}{2}} + 15x^{-\frac{1}{2}} + 75x^{\frac{1}{2}} + 125x^{\frac{3}{2}}$

HENCE $\int_1^4 y^3 \, dx = \int_1^4 x^{-\frac{3}{2}} + 15x^{-\frac{1}{2}} + 75x^{\frac{1}{2}} + 125x^{\frac{3}{2}} \, dx$

$= \left[-2x^{-\frac{1}{2}} + 30x^{\frac{1}{2}} + 50x^{\frac{3}{2}} + 50x^{\frac{5}{2}}\right]_1^4$

$= (-1 + 60 + 400 + 1600) - (-2 + 30 + 50 + 50)$

$= 2059 - 128$

$= 1931$

Question 35

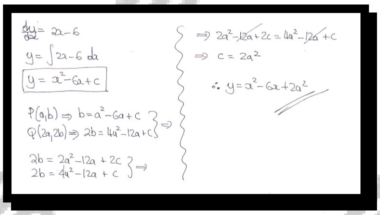
A quadratic curve C passes through the points $P(a,b)$ and $Q(2a,2b)$, where a and b are constants.

The gradient at any given point on C is given by

$$\frac{dy}{dx} = 2x - 6.$$

Find an equation for C, in terms of a.

$$\boxed{y = x^2 - 6x + 2a^2}$$



$\frac{dy}{dx} = 2x - 6$

$y = \int 2x - 6 \, dx$

$\boxed{y = x^2 - 6x + c}$

$P(a,b) \Rightarrow b = a^2 - 6a + c$

$Q(2a,2b) \Rightarrow 2b = 4a^2 - 12a + c$

$2b = 2a^2 - 12a + 2c$

$2b = 4a^2 - 12a + c$

$\Rightarrow 2a^2 - 12a + 2c = 4a^2 - 12a + c$

$\Rightarrow c = 2a^2$

$\therefore y = x^2 - 6x + 2a^2$