IDENTITIES

Question 1 (**)

Use the identity

$$(a\pm b)^3\equiv a^3\pm 3a^2b+3ab^2\pm b^3,$$

to simplify $(2-3x)^3$.

$$\boxed{(2-3x)^3=8-36x+54x^2-27x^3}$$

Question 2 (**)

$$f(x)=(2x-1)^3-(x+2)^2.$$

Use algebraic identities to simplify $f(x)$.

$$\boxed{f(x)=8x^3-13x^2+2x-5}$$

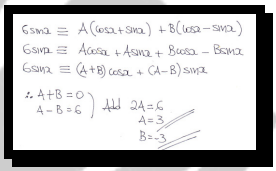
Question 3 (**)

It is given that for some constants A and B

$$6\sin x \equiv A(\cos x + \sin x) + B(\cos x - \sin x).$$

Find the value of A and the value of B.

$$\boxed{A = 3}, \quad \boxed{B = -3},$$



Question 4 (**)

Use the identity

$$(a \pm b)^3 \equiv a^3 \pm 3a^2 b + 3ab^2 \pm b^3,$$

to simplify $\left(2 - \sqrt{2}\right)^3$.

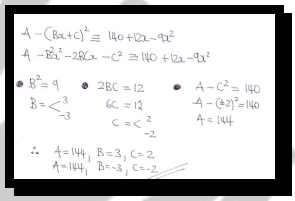
$$\boxed{\left(2 - \sqrt{2}\right)^3 = 20 - 14\sqrt{2}}$$

Question 5 (**)

$$A-\left(Bx+C\right)^2\equiv 140+12x-9x^2\ ,\ x\in\mathbb{R}\ .$$

Find the values of each of the constants A, B and C in the above identity.

$$\boxed{A=144,\ B=\pm 3,\ C=\pm 2}$$



Question 6 (+)**

$$f\left(x\right)=\left(x^2+\frac{2}{x}\right)^3\ ,\ x\in\mathbb{R}\ .$$

Use algebraic identities to simplify $f\left(x\right)$.

$$\boxed{f\left(x\right)=x^6+6x^3+12+\frac{8}{x^3}}$$

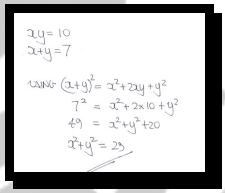
Question 7 (+)**

It is given that

$$x+y=7 \quad \text{and} \quad xy=10 .$$

Use a suitable algebraic identity to find the value of x^2+y^2.

$$\boxed{x^2+y^2=29}$$



Question 8 (***)

Determine the value of each of the constants P, Q and R in the identity

$$P(x+1)^2+Qx(x+1)+Rx\equiv 1 .$$

$$\boxed{P=1}, \ \boxed{Q=-1}, \ \boxed{R=-1}$$

Question 9 (***)

Factorize fully

$$2x-2x^4 .$$

$$\boxed{2x(1-x)(1+x+x^2)}$$

Question 10 (***)

Determine the value of each of the constants p and q in the identity

$$(2x+p)\left(6x^2-7x+2\right)\equiv\left(4x^2-1\right)(3x+q).$$

$\boxed{p=1}$, $\boxed{q=-2}$

Question 11 (***)

$$f(x)=9x^2-1,\ x\in\mathbb{R}.$$

a) Express $f(x)$ as a product of two linear factors.

b) Hence express 899 as a product of two prime factors.

$\boxed{f(x)=(3x+1)(3x-1)}$, $\boxed{899=29\times31}$

Question 12 (***)

Determine the value of each of the constants a, b and c in the identity

$$4(x+a)^2+b(x+3)+c\equiv 4x^2-21x\,.$$

$$\boxed{a=-2}\,,\ \boxed{b=-5}\,,\ \boxed{c=-1}$$

Question 13 (***)

Determine the value of each of the constants P, Q and R in the identity

$$P\left(x^2+1\right)+(x+1)(Qx+R)+x\equiv 1\,.$$

$$\boxed{P=1}\,,\ \boxed{Q=-1}\,,\ \boxed{R=0}$$

Question 14 (***)

Determine the value of each of the constants A, B and C in the identity

$$(Ax+B)(x-1)+C\left(x^2+1\right)+x+3\equiv 0.$$

$$\boxed{A=2}, \boxed{B=1}, \boxed{C=-2}$$

Question 15 (***)

$$f(x)=x^4-27x,\ x\in\mathbb{R}.$$

Express $f(x)$ as a product of three factors.

$$\boxed{f(x)=x(x-3)\left(x^2+x+9\right)}$$

Question 16 (***)

Determine the value of each of the constants A, B and C in the identity

$$(A-25)x+8(Bx-1)+A\equiv C+3(2x-1)(x+1).$$

$$\boxed{A=1},\ \boxed{B=3},\ \boxed{C=-1}$$

Question 17 (***)

Determine the value of each of the constants A, B and C in the identity

$$x^3\equiv A+B(x-1)^2+(x+2)(x+C)^2.$$

$$\boxed{A=1},\ \boxed{B=3},\ \boxed{C=-1}$$

Question 18 (***)

Determine the value of each of the constants A, B, C and D in the identity

$$(x+3)\left(Ax^2+Bx+C\right)\equiv 2x^3+2x^2+Dx+9.$$

$$\boxed{A=2},\ \boxed{B=-4},\ \boxed{C=3},\ \boxed{D=-9}$$

Question 19 (*+)**

Determine the value of each of the constants a, b, c and d in the identity

$$a(x-2)^3+b(x+4)^2\equiv 3x^3+cx^2+dx+8.$$

$$\boxed{a=3},\ \boxed{b=2},\ \boxed{c=-16},\ \boxed{d=52}$$

Question 20 (*+)**

Determine the value of each of the constants A, B, C and D in the identity

$$\left(x^2+A\right)(x+B)+(x+C)(x+D)\equiv x^3-3x^2+4x.$$

$\boxed{A=3}$, $\boxed{B=-2}$, $\boxed{C=2}$, $\boxed{D=-3}$

Question 21 (*+)**

Determine the value of each of the constants p and q in the identity

$$\left(x^2+p\right)^2\equiv x\left(x^2+q\right)\left(x-\frac{6}{x}\right).$$

$\boxed{p=q=-6}$

Question 22 (*+)**

Determine the value of each of the constants a, b, c and d in the identity

$$a(x+b)^3+c\equiv 4x^3-24x^2+48x+d\,.$$

$$\boxed{a=4},\ \boxed{b=-2},\ \boxed{c=29},\ \boxed{d=-3}$$

Question 23 (*+)**

Determine the value of each of the constants A, B, C and D in the identity

$$(x+A)(x+2)\left(x^2+B\right)\equiv Cx^4+D\,.$$

$$\boxed{A=-2},\ \boxed{B=4},\ \boxed{C=1},\ \boxed{D=-16}$$

Question 24 (****)

$$f(x)=x^3+3x^2-4,\ x\in\mathbb{R}.$$

Use algebraic identities to express $f(x)$ as a product of three linear factors.

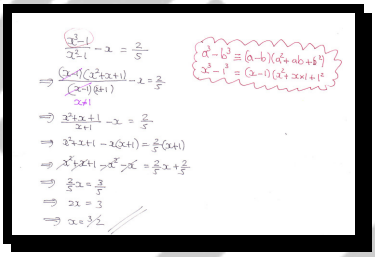
$$\boxed{f(x)=(x-1)(x+2)^2}$$

Question 25 (**+)**

Solve the following equation

$$\frac{x^3-1}{x^2-1}-x=\frac{2}{5},\ x\neq\pm1.$$

$$\boxed{x=\frac{3}{2}}$$



Question 26 (**+)**

$$f(x)=x^3-3x^2-3x+1,\ x\in\mathbb{R}.$$

Express $f(x)$ as a product of **three linear** factors

$$\boxed{f(x)=(x+1)\left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right)}$$