INDICES

Exam Questions

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Question 1 (**)

a) Evaluate the following indicial expression, giving the final answer as an exact simplified fraction.

$$4^{\frac{3}{2}} + 4^{-\frac{1}{2}}.$$

b) Simplify fully the following expression

$$\frac{12y^{-5}}{3y^{-2}}.$$

$\boxed{}$, $\boxed{\frac{17}{2}}$, $\boxed{4y^{-3} = \frac{4}{y^3}}$

a) $4^{\frac{3}{2}} + 4^{-\frac{1}{2}} = \left(\sqrt{4}\right)^3 + \frac{1}{\sqrt{4}} = 2^3 + \frac{1}{2} = 8 + \frac{1}{2} = \frac{17}{2}$ (or 8.5)

b) $\frac{12y^{-5}}{3y^{-2}} = 4y^{-5-(-2)} = 4y^{-3}$ // or $\frac{4}{y^3}$

Question 2 (**)

a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

i. $2^{-4}+8^{-1}$.

ii. $\left(\frac{81}{16}\right)^{\frac{3}{4}}$.

b) Simplify fully the following expression

$$\frac{\left(4xy^2\right)^2}{(2x)^3}.$$

$\boxed{}$, $\boxed{\frac{3}{16}}$, $\boxed{\frac{27}{8}}$, $\boxed{2y^4x^{-1}=\frac{2y^4}{x}}$

(a) (I) $2^{-4}+8^{-1}=\frac{1}{2^4}+\frac{1}{8}=\frac{1}{16}+\frac{1}{8}=\frac{1}{16}+\frac{2}{16}=\frac{3}{16}$

(II) $\left(\frac{81}{16}\right)^{\frac{3}{4}}=\left(\sqrt[4]{\frac{81}{16}}\right)^3=\left(\frac{3}{2}\right)^3=\frac{27}{8}$

(b) $\frac{(4xy^2)^2}{(2x)^3}=\frac{16x^2y^4}{8x^3}=\frac{2y^4}{x}$ (or $2x^{-1}y^4$)

Question 3 (**+)

a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

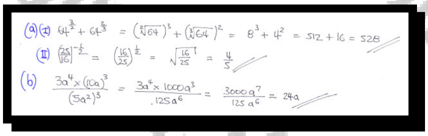
i. $64^{\frac{3}{2}} + 64^{\frac{2}{3}}$.

ii. $\left(\frac{25}{16}\right)^{-\frac{1}{2}}$.

b) Simplify fully the following expression

$$\frac{3a^4 \times (10a)^3}{\left(5a^2\right)^3}.$$

$\boxed{528}$, $\boxed{\frac{4}{5}}$, $\boxed{24a}$



Question 4 (**+)

a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

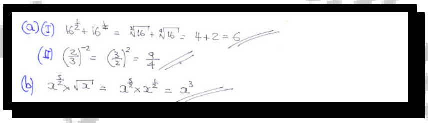
i. $16^{\frac{1}{2}} + 16^{\frac{1}{4}}$.

ii. $\left(\frac{2}{3}\right)^{-2}$.

b) Simplify fully the following expression

$$x^{\frac{5}{2}} \times \sqrt{x}\,.$$

$\boxed{6}$, $\boxed{\frac{9}{4}}$, $\boxed{x^3}$


(a)(i) $16^{\frac{1}{2}} + 16^{\frac{1}{4}} = \sqrt{16} + \sqrt[4]{16} = 4 + 2 = 6$
(ii) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$
(b) $x^{\frac{5}{2}} \times \sqrt{x} = x^{\frac{5}{2}} \times x^{\frac{1}{2}} = x^3$

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Question 5 **(**+)**

a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

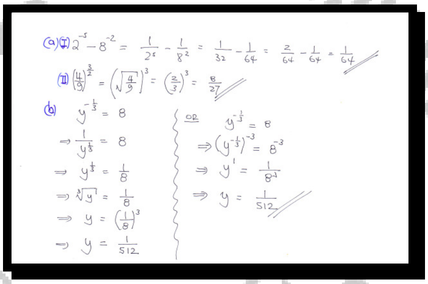
i. $2^{-5} - 8^{-2}$.

ii. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$.

b) Solve the equation

$$y^{-\frac{1}{3}} = 8.$$

$\boxed{}$, $\boxed{\frac{1}{64}}$, $\boxed{\frac{8}{27}}$, $\boxed{\frac{1}{512}}$


(a)(i) $2^{-5} - 8^{-2} = \frac{1}{2^5} - \frac{1}{8^2} = \frac{1}{32} - \frac{1}{64} = \frac{2}{64} - \frac{1}{64} = \frac{1}{64}$

(ii) $\left(\frac{4}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{4}{9}}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

(b) $y^{-\frac{1}{3}} = 8$

$\Rightarrow \frac{1}{y^{\frac{1}{3}}} = 8$

$\Rightarrow y^{\frac{1}{3}} = \frac{1}{8}$

$\Rightarrow \sqrt[3]{y} = \frac{1}{8}$

$\Rightarrow y = \left(\frac{1}{8}\right)^3$

$\Rightarrow y = \frac{1}{512}$

OR

$y^{-\frac{1}{3}} = 8$

$\Rightarrow \left(y^{-\frac{1}{3}}\right)^{-3} = 8^{-3}$

$\Rightarrow y^1 = \frac{1}{8^3}$

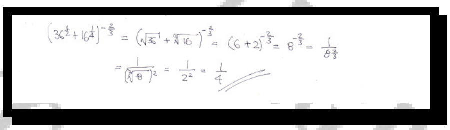
$\Rightarrow y = \frac{1}{512}$

Question 6 **(**+)**

Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$\left(36^{\frac{1}{2}} + 16^{\frac{1}{4}}\right)^{-\frac{2}{3}}.$$

$\boxed{}$, $\boxed{\frac{1}{4}}$


$\left(36^{\frac{1}{2}} + 16^{\frac{1}{4}}\right)^{-\frac{2}{3}} = \left(\sqrt{36} + \sqrt[4]{16}\right)^{-\frac{2}{3}} = (6+2)^{-\frac{2}{3}} = 8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}}$

$= \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$

Question 7 (**+)

a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

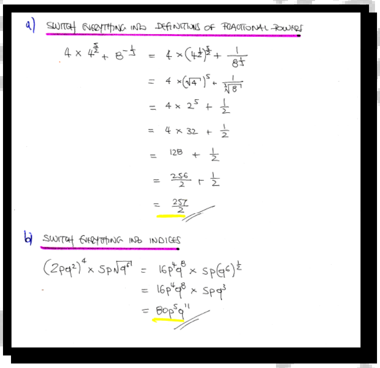
$$4\times 4^{\frac{5}{2}}+8^{-\frac{1}{3}}.$$

The answer to this part of the question must be fully supported by a detailed method, justifying each step in the evaluation.

b) Simplify fully the following expression

$$\left(2pq^2\right)^4\times 5p\sqrt{q^6}\ .$$

$\boxed{}$, $\boxed{\frac{257}{2}}$, $\boxed{80p^5q^{11}}$


a) SWITCH EVERYTHING INTO DEFINITIONS OF FRACTIONAL POWERS

$4\times 4^{\frac{5}{2}}+8^{-\frac{1}{3}} = 4\times\left(4^{\frac{1}{2}}\right)^5+\frac{1}{8^{\frac{1}{3}}}$
$= 4\times\left(\sqrt{4}\right)^5+\frac{1}{\sqrt[3]{8}}$
$= 4\times 2^5+\frac{1}{2}$
$= 4\times 32+\frac{1}{2}$
$= 128+\frac{1}{2}$
$= \frac{256}{2}+\frac{1}{2}$
$= \frac{257}{2}$

b) SWITCH EVERYTHING INTO INDICES

$\left(2pq^2\right)^4\times 5p\sqrt{q^6} = 16p^4q^8\times 5p\left(q^6\right)^{\frac{1}{2}}$
$= 16p^4q^8\times 5pq^3$
$= 80p^5q^{11}$

Question 8 (***)

a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

i. $8^{\frac{1}{3}} + 8^{-\frac{1}{3}}$.

ii. $8^{-4} \times 2^{11}$.

b) Simplify fully

$$\frac{\sqrt{9x^6y^4}}{\left(3x^2y^3\right)^2}.$$

$\boxed{}$, $\boxed{\frac{5}{2}}$, $\boxed{\frac{1}{2}}$, $\boxed{\frac{1}{3xy^4}}$

(a) (I) $8^{\frac{1}{3}} + 8^{-\frac{1}{3}} = \sqrt[3]{8} + \frac{1}{\sqrt[3]{8}} = 2 + \frac{1}{2} = \frac{5}{2}$

(II) $8^{-4} \times 2^{11} = (2^3)^{-4} \times 2^{11} = 2^{-12} \times 2^{11} = 2^{-1} = \frac{1}{2}$

(b) $\frac{\sqrt{9x^6y^4}}{(3x^2y^3)^2} = \frac{3x^3y^2}{9x^4y^6} = \frac{1}{3} x^{-1} y^{-4}$ or $\frac{1}{3xy^4}$

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Question 9 (***)

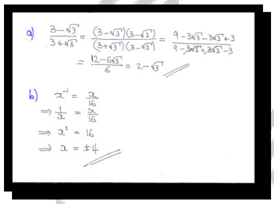
a) Simplify the following expression, writing the final answer in the form $a+b\sqrt{3}$, where a and b are integers

$$\frac{3-\sqrt{3}}{3+\sqrt{3}}.$$

b) Solve the equation

$$x^{-1}=\frac{x}{16}, \quad x \neq 0.$$

$\boxed{}$, $\boxed{2-\sqrt{3}}$, $\boxed{x=\pm 4}$



Question 10 (***)

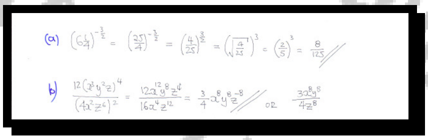
a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$\left(6\tfrac{1}{4}\right)^{-\frac{3}{2}}.$$

b) Simplify fully the following expression

$$\frac{12\left(x^3y^2z\right)^4}{\left(4x^2z^6\right)^2}.$$

$\boxed{\frac{8}{125}}$, $\boxed{\frac{3}{4}x^8y^8z^{-8} = \frac{3x^8y^8}{4z^8}}$



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Question 11 (***)

a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions, where appropriate.

i. $\dfrac{6}{2^{-2}}$.

ii. $\left(1\tfrac{7}{9}\right)^{\frac{3}{2}}$.

b) Solve the equation

$$z^{\frac{3}{2}} = 27 .$$

$\boxed{}$, $\boxed{24}$, $\boxed{\dfrac{64}{27}}$, $\boxed{z=9}$

(a) (I) $\dfrac{6}{2^{-2}} = \dfrac{6}{\frac{1}{4}} = 24$

(II) $\left(1\tfrac{7}{9}\right)^{\frac{3}{2}} = \left(\tfrac{16}{9}\right)^{\frac{3}{2}} = \left(\sqrt{\tfrac{16}{9}}\right)^{3} = \left(\tfrac{4}{3}\right)^{3} = \dfrac{64}{27}$

(b) $z^{\frac{3}{2}} = 27 \implies \left(z^{\frac{3}{2}}\right)^{\frac{2}{3}} = 27^{\frac{2}{3}}$

$\implies z^{1} = \left(\sqrt[3]{27}\right)^{2}$

$\implies z = 9$

Question 12 (***)

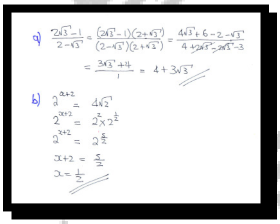
a) Simplify the following expression, writing the final answer in the form $a+b\sqrt{3}$, where a and b are integers

$$\frac{2\sqrt{3}-1}{2-\sqrt{3}}.$$

b) Solve the equation

$$2^{x+2} = 4\sqrt{2}.$$

$\boxed{4+3\sqrt{3}}$, $\boxed{x=\frac{1}{2}}$


a) $\frac{2\sqrt{3}-1}{2-\sqrt{3}} = \frac{(2\sqrt{3}-1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{4\sqrt{3}+6-2-\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}-3}$

$= \frac{3\sqrt{3}+4}{1} = 4+3\sqrt{3}$

b) $2^{x+2} = 4\sqrt{2}$

$2^{x+2} = 2^2 \times 2^{\frac{1}{2}}$

$2^{x+2} = 2^{\frac{5}{2}}$

$x+2 = \frac{5}{2}$

$x = \frac{1}{2}$

Question 13 (***)

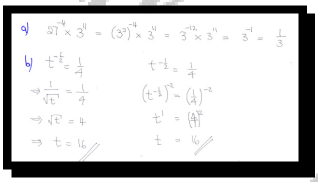
a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$27^{-4} \times 3^{11}.$$

b) Solve the equation

$$t^{-\frac{1}{2}} = \frac{1}{4}.$$

$\boxed{\frac{1}{3}}$, $\boxed{t=16}$


a) $27^{-4} \times 3^{11} = (3^3)^{-4} \times 3^{11} = 3^{-12} \times 3^{11} = 3^{-1} = \frac{1}{3}$

b) $t^{-\frac{1}{2}} = \frac{1}{4}$ | $t^{-\frac{1}{2}} = \frac{1}{4}$

$\Rightarrow \frac{1}{\sqrt{t}} = \frac{1}{4}$ | $\left(t^{-\frac{1}{2}}\right)^{-2} = \left(\frac{1}{4}\right)^{-2}$

$\Rightarrow \sqrt{t} = 4$ | $t^1 = \left(\frac{4}{1}\right)^2$

$\Rightarrow t = 16$ | $t = 16$

Question 14 (***)

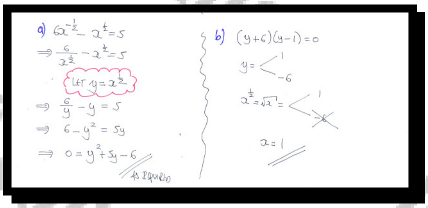
$$6x^{-\frac{1}{2}} - x^{\frac{1}{2}} = 5 .$$

a) Show that the substitution $y = x^{\frac{1}{2}}$ transforms the above indicial equation into the quadratic equation

$$y^2 + 5y - 6 = 0 .$$

b) Solve the quadratic equation and hence find the root of the **indicial** equation.

$\boxed{x = 1}$



Question 15 (***)

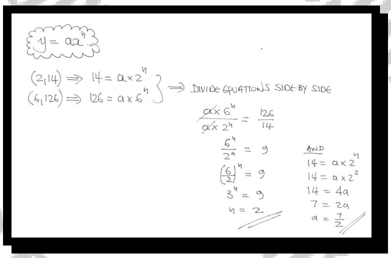
The points $(2,14)$ and $(6,126)$ lie on the curve with equation

$$y = ax^n , \ x \in \mathbb{R}$$

where a and n are non zero constants.

Find the value of a and the value of n.

$\boxed{a = \frac{7}{2}}, \ \boxed{n = 2}$



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Question 16 (***)

a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions.

i. $4^{\frac{1}{2}} + 9^{-\frac{1}{2}}$.

ii. $32^{5} \times 8^{-10}$.

b) Simplify fully the following expression

$$\sqrt{\frac{3a^{3}bc \times 6a^{2}b^{2}c^{3}}{2abc^{4}}}.$$

$\boxed{\frac{7}{3}}$, $\boxed{\frac{1}{32}}$, $\boxed{3a^{2}b}$

(a) (I) $4^{\frac{1}{2}} + 9^{-\frac{1}{2}} = \sqrt{4} + \frac{1}{\sqrt{9}} = 2 + \frac{1}{3} = \frac{7}{3}$

(II) $32^{5} \times 8^{-10} = (2^{5})^{5} \times (2^{3})^{-10} = 2^{25} \times 2^{-30} = 2^{-5} = \frac{1}{2^{5}} = \frac{1}{32}$

(b) $\sqrt{\frac{3a^{3}bc \times 6a^{2}b^{2}c^{3}}{2abc^{4}}} = \sqrt{\frac{18a^{5}b^{3}c^{4}}{2abc^{4}}} = \sqrt{9a^{4}b^{2}} = 3a^{2}b$

Question 17 (***)

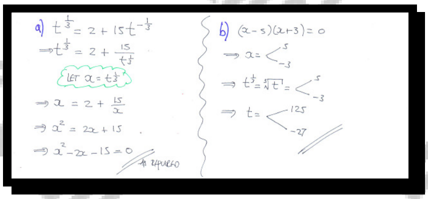
$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}.$$

a) Show that the substitution $x = t^{\frac{1}{3}}$ transforms the above indicial equation into the quadratic equation

$$x^2 - 2x - 15 = 0.$$

b) Solve the quadratic equation and hence find the two solutions of the **indicial** equation.

$\boxed{t = -27, \;\; t = 125}$



Question 18 (***)

a) Evaluate the following indicial expressions, giving the final answers as exact simplified fractions where appropriate.

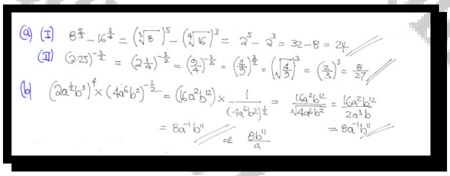
i. $8^{\frac{5}{3}} - 16^{\frac{3}{4}}$.

ii. $(2.25)^{-\frac{3}{2}}$.

b) Simplify fully the following expression

$$\left(2a^{\frac{1}{2}}b^{3}\right)^{4} \times \left(4a^{6}b^{2}\right)^{-\frac{1}{2}}.$$

$\boxed{24}$, $\boxed{\frac{8}{27}}$, $\boxed{8a^{-1}b^{11} = \frac{8b^{11}}{a}}$



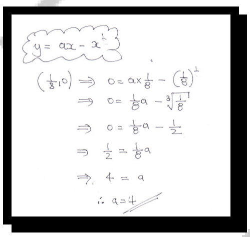
Question 19 (***)

Given that the curve with equation

$$y = ax - x^{\frac{1}{3}}, \ x \geq 0,$$

passes though the point with coordinates $\left(\frac{1}{8}, 0\right)$, find the value of the constant a.

$\boxed{a = 4}$



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Question 20 (***)

a) Evaluate the following indicial expressions, giving the answers as integers.

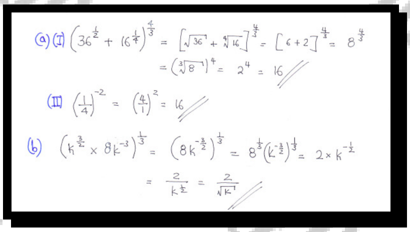
i. $\left(36^{\frac{1}{2}}+16^{\frac{1}{4}}\right)^{\frac{4}{3}}$.

ii. $\left(\frac{1}{4}\right)^{-2}$.

b) Simplify fully the following expression

$$\left(k^{\frac{3}{2}}\times 8k^{-3}\right)^{\frac{1}{3}}.$$

$\boxed{16}$, $\boxed{16}$, $\boxed{2k^{-\frac{1}{2}}=\frac{2}{\sqrt{k}}}$



(a)(i) $\left(36^{\frac{1}{2}}+16^{\frac{1}{4}}\right)^{\frac{4}{3}} = \left[\sqrt{36}+\sqrt[4]{16}\right]^{\frac{4}{3}} = \left[6+2\right]^{\frac{4}{3}} = 8^{\frac{4}{3}}$
$= \left(\sqrt[3]{8}\right)^{4} = 2^{4} = 16$

(ii) $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^{2} = 16$

(b) $\left(k^{\frac{3}{2}}\times 8k^{-3}\right)^{\frac{1}{3}} = \left(8k^{-\frac{3}{2}}\right)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(k^{-\frac{3}{2}}\right)^{\frac{1}{3}} = 2\times k^{-\frac{1}{2}}$
$= \frac{2}{k^{\frac{1}{2}}} = \frac{2}{\sqrt{k}}$

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Question 21 (***+)

a) Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{3}$.

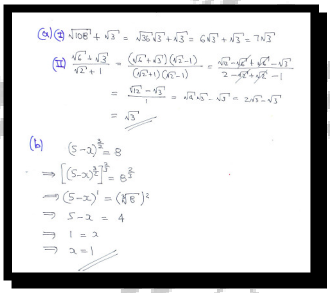
i. $\sqrt{108}+\sqrt{3}$.

ii. $\dfrac{\sqrt{6}+\sqrt{3}}{\sqrt{2}+1}$.

b) Solve the equation

$$(5-x)^{\frac{3}{2}}=8.$$

$\boxed{7\sqrt{3}}$, $\boxed{\sqrt{3}}$, $\boxed{x=1}$


(a)(i) $\sqrt{108}+\sqrt{3}=\sqrt{36}\sqrt{3}+\sqrt{3}=6\sqrt{3}+\sqrt{3}=7\sqrt{3}$

(ii) $\frac{\sqrt{6}+\sqrt{3}}{\sqrt{2}+1}=\frac{(\sqrt{6}+\sqrt{3})(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}=\frac{\sqrt{12}-\sqrt{6}+\sqrt{6}-\sqrt{3}}{2-\sqrt{2}+\sqrt{2}-1}$

$=\frac{\sqrt{12}-\sqrt{3}}{1}=\sqrt{4}\sqrt{3}-\sqrt{3}=2\sqrt{3}-\sqrt{3}$

$=\sqrt{3}$

(b) $(5-x)^{\frac{3}{2}}=8$

$\Rightarrow\left[(5-x)^{\frac{3}{2}}\right]^{\frac{2}{3}}=8^{\frac{2}{3}}$

$\Rightarrow(5-x)^{1}=(\sqrt[3]{8})^{2}$

$\Rightarrow 5-x=4$

$\Rightarrow 1=x$

$\Rightarrow x=1$

Question 22 (***+)

a) Evaluate the following indicial expressions, giving the answers as integers.

i. $\left(\frac{1}{3}\right)^{-3}$.

ii. $\frac{8^6}{32^3}$.

b) Simplify fully the following expression

$$\left(\frac{6a^7b^2 \times 9b}{2a}\right)^{\frac{1}{3}}.$$

$\boxed{108}$, $\boxed{8}$, $\boxed{3a^2b}$

(a)(I) $\left(\frac{1}{3}\right)^{-3} + 27^{\frac{4}{3}} = \left(\frac{3}{1}\right)^3 + \left(\sqrt[3]{27}\right)^4 = 27 + 3^4 = 27 + 81 = 108$

(II) $\frac{8^6}{32^3} = \frac{(2^3)^6}{(2^5)^3} = \frac{2^{18}}{2^{15}} = 2^3 = 8$

(b) $\left(\frac{6a^7b^2 \times 9b}{2a}\right)^{\frac{1}{3}} = \left(\frac{54a^7b^3}{2a}\right)^{\frac{1}{3}} = (27a^6b^3)^{\frac{1}{3}} = 3a^2b$

$(a^6)^{\frac{1}{3}} = a^2$ & $(b^3)^{\frac{1}{3}} = b$

Question 23 (***+)

a) Simplify fully each of the following expressions, writing the final answer as a single simplified surd.

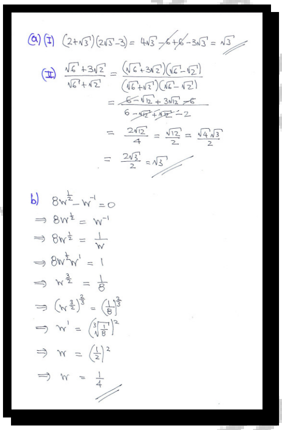
i. $\left(2+\sqrt{3}\right)\left(2\sqrt{3}-3\right)$.

ii. $\dfrac{\sqrt{6}+3\sqrt{2}}{\sqrt{6}+\sqrt{2}}$.

b) Solve the equation

$$8w^{\frac{1}{2}}-w^{-1}=0, \quad w\neq 0.$$

$\boxed{}$, $\boxed{\sqrt{3}}$, $\boxed{\sqrt{3}}$, $\boxed{w=\frac{1}{4}}$


(a) (I) $(2+\sqrt{3})(2\sqrt{3}-3)=4\sqrt{3}-6+6-3\sqrt{3}=\sqrt{3}$

(II) $\dfrac{\sqrt{6}+3\sqrt{2}}{\sqrt{6}+\sqrt{2}}=\dfrac{(\sqrt{6}+3\sqrt{2})(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$

$=\dfrac{6-\sqrt{12}+3\sqrt{12}-6}{6-\sqrt{12}+\sqrt{12}-2}$

$=\dfrac{2\sqrt{12}}{4}=\dfrac{\sqrt{12}}{2}=\dfrac{\sqrt{4}\sqrt{3}}{2}$

$=\dfrac{2\sqrt{3}}{2}=\sqrt{3}$

b) $8w^{\frac{1}{2}}-w^{-1}=0$

$\Rightarrow 8w^{\frac{1}{2}}=w^{-1}$

$\Rightarrow 8w^{\frac{1}{2}}=\dfrac{1}{w}$

$\Rightarrow 8w^{\frac{1}{2}}w^{1}=1$

$\Rightarrow w^{\frac{3}{2}}=\dfrac{1}{8}$

$\Rightarrow \left(w^{\frac{3}{2}}\right)^{\frac{2}{3}}=\left(\dfrac{1}{8}\right)^{\frac{2}{3}}$

$\Rightarrow w^{1}=\left(\sqrt[3]{\dfrac{1}{8}}\right)^{2}$

$\Rightarrow w=\left(\dfrac{1}{2}\right)^{2}$

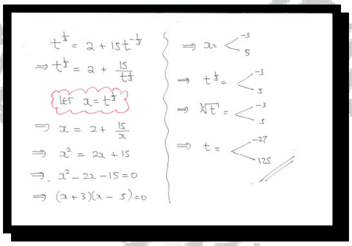
$\Rightarrow w=\dfrac{1}{4}$

Question 24 (***+)

$$t^{\frac{1}{3}} = 2 + 15t^{-\frac{1}{3}}, \quad t \neq 0.$$

Use the substitution $x = t^{\frac{1}{3}}$ to solve the above indicial equation.

$\boxed{t = -27, \; t = 125}$



Question 25 (***+)

An exponential curve has equation

$$y = ab^x, \; x \in \mathbb{R},$$

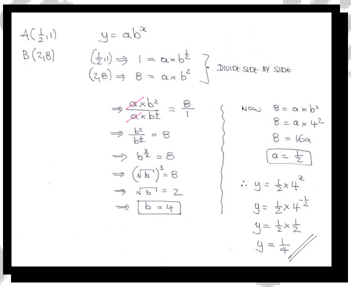
where a and b are non zero constants.

The points $A\left(\frac{1}{2}, 1\right)$, $B(2, 8)$ and $C\left(-\frac{1}{2}, k\right)$ lie on this curve.

a) Find the values of a and b.

b) Find the value of k.

$\boxed{a = \frac{1}{2}}, \; \boxed{b = 4}, \; \boxed{k = \frac{1}{4}}$



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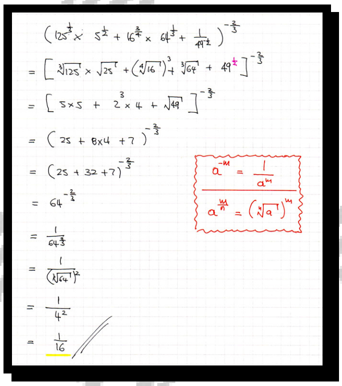
Question 26 (***+)

$$\left(125^{\frac{1}{3}} \times 25^{\frac{1}{2}} + 16^{\frac{3}{4}} \times 64^{\frac{1}{3}} + \frac{1}{49^{-\frac{1}{2}}}\right)^{-\frac{2}{3}} .$$

Evaluate the above indicial expression, giving the final answer as a simplified fraction.

You may not use any calculating aid in the above question, and detailed workings must support the answer.

$\boxed{}$, $\boxed{\frac{1}{16}}$



Question 27 (***+)

a) Simplify fully each of the following expressions, writing the final answer in terms of $\sqrt{2}$.

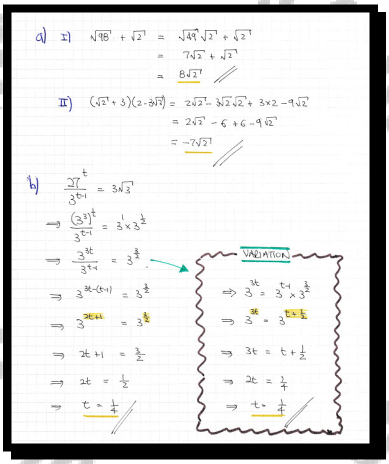
i. $\sqrt{98}+\sqrt{2}$.

ii. $\left(\sqrt{2}+3\right)\left(2-3\sqrt{2}\right)$.

b) Solve the equation

$$\frac{27^t}{3^{t-1}} = 3\sqrt{3}.$$

$\boxed{}$, $\boxed{8\sqrt{2}}$, $\boxed{-7\sqrt{2}}$, $\boxed{t=\frac{1}{4}}$



Question 28 **(***+)**

a) Evaluate the following indicial expression, giving the final answers as an exact simplified fraction.

$$16^{\frac{1}{2}} + 16^{-\frac{3}{4}}.$$

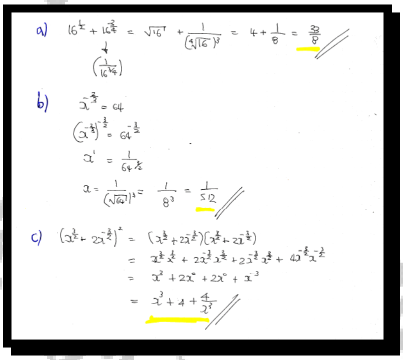
b) Solve the equation

$$x^{-\frac{2}{3}} = 64.$$

c) Simplify fully

$$\left(x^{\frac{3}{2}} + 2x^{-\frac{3}{2}}\right)^2.$$

$\boxed{}$, $\boxed{\frac{33}{8}}$, $\boxed{x = \frac{1}{512}}$, $\boxed{x^3 + 4 + \frac{4}{x^3}}$



Question 29 **(***+)**

An exponential curve has equation

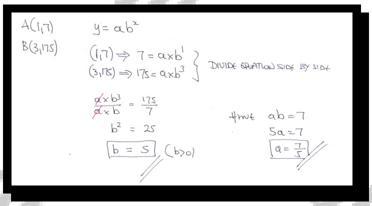
$$y = ab^x, \ x \in \mathbb{R}$$

where a and b are non zero constants.

The points $A(1,7)$ and $B(3,175)$ lie on this curve.

Given that $b > 0$, find the values of a and b.

$\boxed{a = 1.4}$, $\boxed{b = 5}$

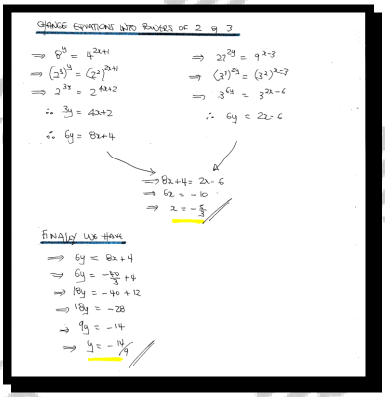


Question 30 **(***+)**

Solve the following simultaneous equations without using a calculator

$$8^y = 4^{2x+1}$$
$$27^{2y} = 9^{x-3}$$

[illegible], $\boxed{\left(-\frac{5}{3}, -\frac{14}{9}\right)}$



Question 31 (***+)

a) Solve the equation

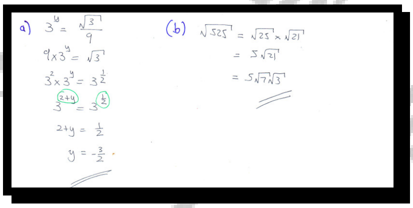
$$3^y = \frac{\sqrt{3}}{9}.$$

b) Express

$$\sqrt{525},$$

in the form $a\sqrt{b}\sqrt{c}$, where a, b and c are prime numbers.

$\boxed{}$, $\boxed{y = -\frac{3}{2}}$, $\boxed{5\sqrt{3}\sqrt{7}}$

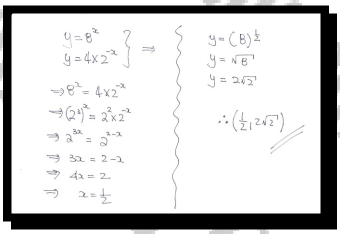


Question 32 (***+)

Find the exact coordinates of the point of intersection between the curves with equations

$$y = 8^x \quad \text{and} \quad y = 4 \times 2^{-x}.$$

$\boxed{\left(\frac{1}{2}, 2\sqrt{2}\right)}$



Question 33 (***+)

a) If x is a real number solve the following indicial equation

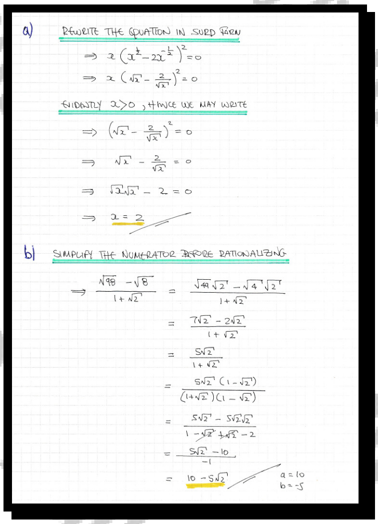
$$x\left(x^{\frac{1}{2}}-2x^{-\frac{1}{2}}\right)^{2}=0.$$

b) Express

$$\frac{\sqrt{98}-\sqrt{8}}{1+\sqrt{2}},$$

in the form $a+b\sqrt{2}$, where a and b are integers.

$\boxed{}$, $\boxed{x=2}$, $\boxed{10-5\sqrt{2}}$



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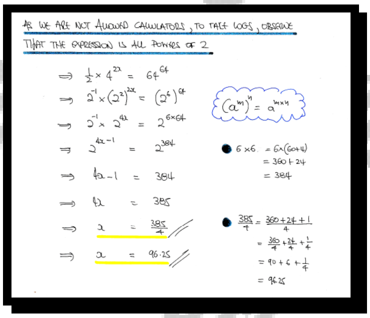
Question 34 (***+)

Find, **without** the use of any calculating aid, the solution of the equation

$$\frac{1}{2} \times 4^{2x} = 64^{64} .$$

$\boxed{x = 96.25}$



Question 35 (***+)

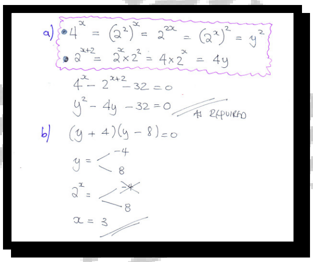
$$4^x - 2^{x+2} = 32 .$$

a) Show that the substitution $y = 2^x$ transforms the above indicial equation into the quadratic equation

$$y^2 - 4y - 32 = 0 .$$

b) Solve the quadratic equation and hence find the root of the **indicial** equation.

$\boxed{x = 3}$



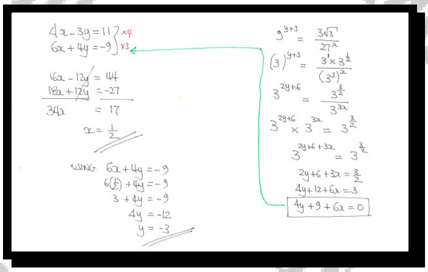
Question 36 (****)

Solve the following simultaneous equations without using a calculator.

$$4x - 3y = 11$$

$$9^{y+3} = \frac{3\sqrt{3}}{27^x}$$

$\boxed{\left(\frac{1}{2}, -3\right)}$



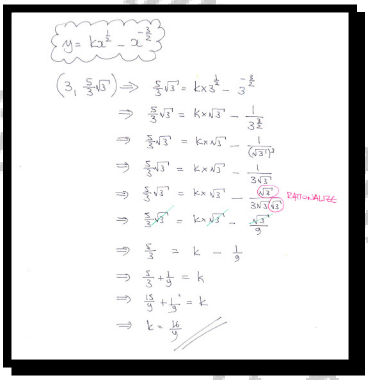
Question 37 (****)

Given that the curve with equation

$$y = kx^{\frac{1}{2}} - x^{-\frac{3}{2}}, \ x \geq 0,$$

passes though the point with coordinates $\left(3, \frac{5}{3}\sqrt{3}\right)$, show clearly that $k = \frac{16}{9}$.

$\boxed{\text{proof}}$



Question 38 (****)

The indicial equation

$$2^{x+1}+2^{3-x}=17\,,\ x\in\mathbb{R}\,,$$

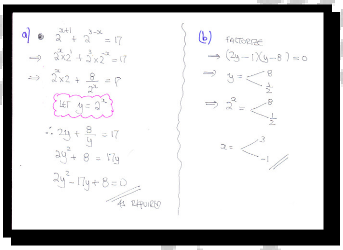
is to be solved by a suitable substitution.

a) Show clearly that the substitution $y=2^{x}$ transforms the above indicial equation into the quadratic equation

$$2y^{2}-17y+8=0\,.$$

b) Solve the quadratic equation by factorization and hence determine the two solutions of the indicial equation.

$\boxed{}$, $\boxed{x=-1,\ 3}$

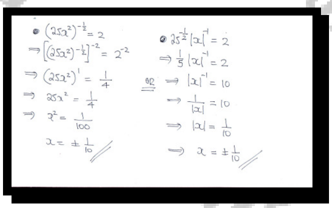


Question 39 (****)

Solve the equation

$$\left(25x^{2}\right)^{-\frac{1}{2}}=2\,,\quad x\neq 0\,.$$

$\boxed{x=\pm\frac{1}{10}}$



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Question 40 (****)

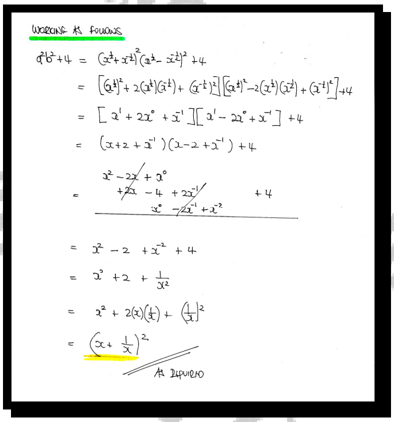
Given that

$$a = x^{\frac{1}{2}} + x^{-\frac{1}{2}} \quad \text{and} \quad b = x^{\frac{1}{2}} - x^{-\frac{1}{2}},$$

show clearly that

$$a^2 b^2 + 4 \equiv \left(x + \frac{1}{x} \right)^2.$$

[ ], [proof]


WORKING AS FOLLOWS

$a^2b^2+4 = (x^{\frac{1}{2}}+x^{-\frac{1}{2}})^2(x^{\frac{1}{2}}-x^{-\frac{1}{2}})^2+4$

$= \left[(x^{\frac{1}{2}})^2+2(x^{\frac{1}{2}})(x^{-\frac{1}{2}})+(x^{-\frac{1}{2}})^2\right]\left[(x^{\frac{1}{2}})^2-2(x^{\frac{1}{2}})(x^{-\frac{1}{2}})+(x^{-\frac{1}{2}})^2\right]+4$

$= \left[x^1+2x^0+x^{-1}\right]\left[x^1-2x^0+x^{-1}\right]+4$

$= (x+2+x^{-1})(x-2+x^{-1})+4$

$= x^2 - 2x + x^0 + 2x - 4 + 2x^{-1} + x^0 - 2x^{-1} + x^{-2} + 4$

$= x^2 - 2 + x^{-2} + 4$

$= x^2 + 2 + \frac{1}{x^2}$

$= x^2 + 2(x)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2$

$= \left(x + \frac{1}{x}\right)^2$

AS REQUIRED

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Question 41 (****)

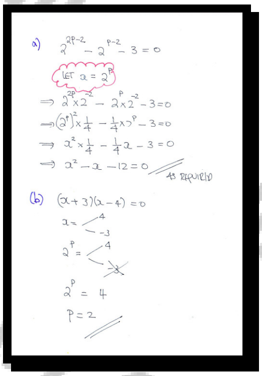
$$2^{2p-2} - 2^{p-2} - 3 = 0, \quad p \in \mathbb{R},$$

a) Show clearly that the substitution $x = 2^p$ transforms the above indicial equation into the quadratic equation

$$x^2 - x - 12 = 0.$$

b) Solve the quadratic equation and hence determine the value of p.

$\boxed{p = 2}$



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Question 42 (****)

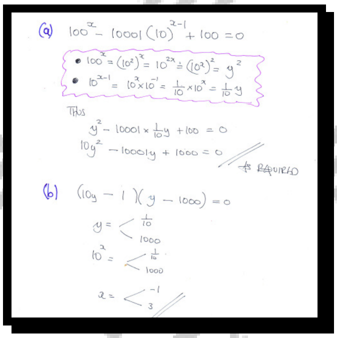
$$100^x - 10001\left(10^{x-1}\right) + 100 = 0.$$

a) Show that the substitution $y = 10^x$ transforms the above indicial equation into the quadratic equation

$$10y^2 - 10001y + 1000 = 0.$$

b) Solve the quadratic equation and hence find the two solutions of the **indicial** equation.

$x = -1, \quad x = 3$


(a) $100^x - 10001(10)^{x-1} + 100 = 0$

- $100^x = (10^2)^x = 10^{2x} = (10^x)^2 = y^2$
- $10^{x-1} = 10^x \times 10^{-1} = \frac{1}{10} \times 10^x = \frac{1}{10}y$

Thus

$y^2 - 10001 \times \frac{1}{10}y + 100 = 0$

$10y^2 - 10001y + 1000 = 0$ // AS REQUIRED

(b) $(10y - 1)(y - 1000) = 0$

$y = \frac{1}{10}, 1000$

$10^x = \frac{1}{10}, 1000$

$x = -1, 3$

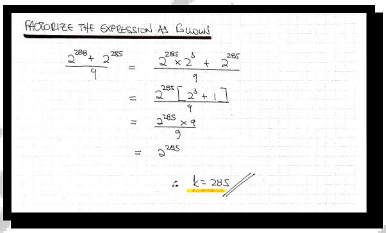
Question 43 (****)

Determine the value of k.

$$\frac{2^{288} + 2^{285}}{9} = 2^k.$$

You must show full workings.

$k = 285$


FACTORIZE THE EXPRESSION AS BELOW

$\frac{2^{288} + 2^{285}}{9} = \frac{2^{285} \times 2^3 + 2^{285}}{9}$

$= \frac{2^{285}[2^3 + 1]}{9}$

$= \frac{2^{285} \times 9}{9}$

$= 2^{285}$

$\therefore k = 285$

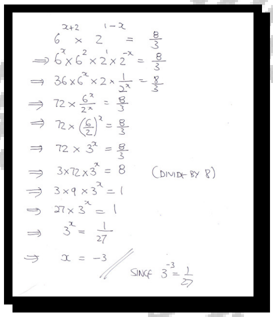
Question 44 (****+)

Solve the following indicial equation

$$6^{x+2} \times 2^{1-x} = \frac{8}{3}.$$

You must show full workings.

$\boxed{}$, $\boxed{x = -3}$

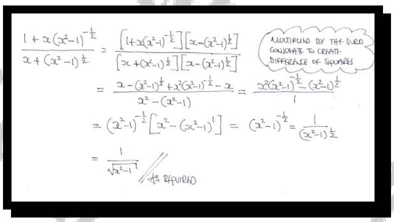

$6^{x+2} \times 2^{1-x} = \frac{8}{3}$
$\Rightarrow 6^x \times 6^2 \times 2^1 \times 2^{-x} = \frac{8}{3}$
$\Rightarrow 36 \times 6^x \times 2 \times \frac{1}{2^x} = \frac{8}{3}$
$\Rightarrow 72 \times \frac{6^x}{2^x} = \frac{8}{3}$
$\Rightarrow 72 \times \left(\frac{6}{2}\right)^x = \frac{8}{3}$
$\Rightarrow 72 \times 3^x = \frac{8}{3}$
$\Rightarrow 3 \times 72 \times 3^x = 8$ (DIVIDE BY 8)
$\Rightarrow 3 \times 9 \times 3^x = 1$
$\Rightarrow 27 \times 3^x = 1$
$\Rightarrow 3^x = \frac{1}{27}$
$\Rightarrow x = -3$ // SINCE $3^{-3} = \frac{1}{27}$

Question 45 (****+)

Show clearly that

$$\frac{1 + x\left(x^2 - 1\right)^{-\frac{1}{2}}}{x + \left(x^2 - 1\right)^{\frac{1}{2}}} \equiv \frac{1}{\sqrt{x^2 - 1}}.$$

$\boxed{}$, $\boxed{\text{proof}}$

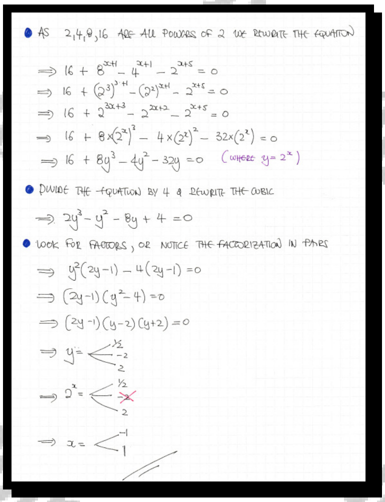

$\frac{1 + x(x^2-1)^{-\frac{1}{2}}}{x + (x^2-1)^{\frac{1}{2}}} = \frac{\left[1 + x(x^2-1)^{-\frac{1}{2}}\right]\left[x - (x^2-1)^{\frac{1}{2}}\right]}{\left[x + (x^2-1)^{\frac{1}{2}}\right]\left[x - (x^2-1)^{\frac{1}{2}}\right]}$ MULTIPLYING BY THE SURD CONJUGATE TO CREATE DIFFERENCE OF SQUARES
$= \frac{x - (x^2-1)^{\frac{1}{2}} + x^2(x^2-1)^{-\frac{1}{2}} - x}{x^2 - (x^2-1)} = \frac{x^2(x^2-1)^{-\frac{1}{2}} - (x^2-1)^{\frac{1}{2}}}{1}$
$= (x^2-1)^{-\frac{1}{2}}\left[x^2 - (x^2-1)^1\right] = (x^2-1)^{-\frac{1}{2}} = \frac{1}{(x^2-1)^{\frac{1}{2}}}$
$= \frac{1}{\sqrt{x^2-1}}$ // AS REQUIRED

Question 46 (****+)

Solve the following exponential equation

$$16+8^{x+1}-4^{x+1}-2^{x+5}=0,\ x\in\mathbb{R}.$$

, $\boxed{x=\pm 1}$



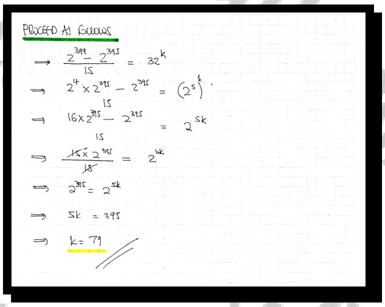
Question 47 (****+)

Determine the value of k.

$$\frac{2^{399}-2^{395}}{15}=32^{k}.$$

You must show full workings.

, $\boxed{k=79}$

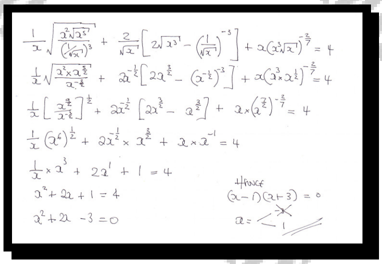


Question 48 (****+)

Solve the following equation

$$\frac{1}{x}\times\sqrt{\frac{x^2\sqrt{x^5}}{\left(\frac{1}{\sqrt{x}}\right)^3}}+\frac{2}{\sqrt{x}}\times\left[2\sqrt{x^3}-\left(\frac{1}{\sqrt{x}}\right)^{-3}\right]+x\left(x^3\sqrt{x}\right)^{-\frac{2}{7}}=4.$$

$\boxed{x=1}$



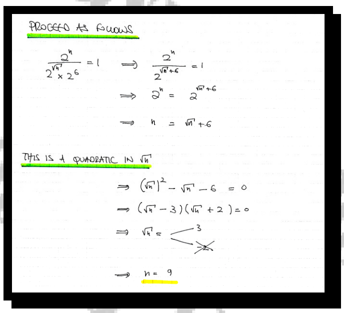
Question 49 (****+)

Solve the following indicial equation

$$\frac{2^n}{2^{\sqrt{n}}\times 2^6}=1.$$

You must show full workings.

$\boxed{}$, $\boxed{n=9}$



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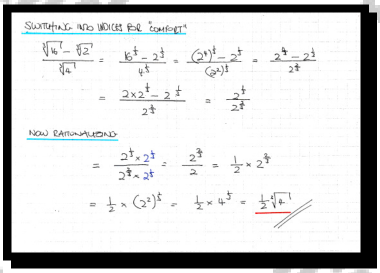
Question 50 **(****+)**

Show with a detailed method that

$$\frac{\sqrt[3]{16}-\sqrt[3]{2}}{\sqrt[3]{4}} = k\sqrt[3]{4}$$

where k is a constant to be found.

$k = \frac{1}{2}$



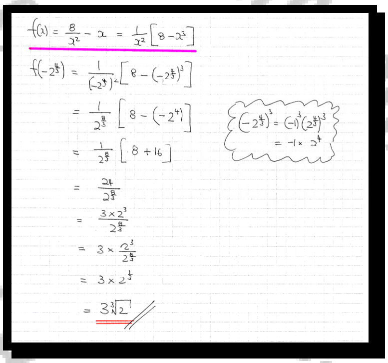
Question 51 **(*****)**

$$f(x) \equiv \frac{8}{x^2} - x, \ x \neq 0.$$

Show that $f\left(-2^{\frac{4}{3}}\right) = 3\sqrt[3]{2}$.

You must show detailed workings.

proof



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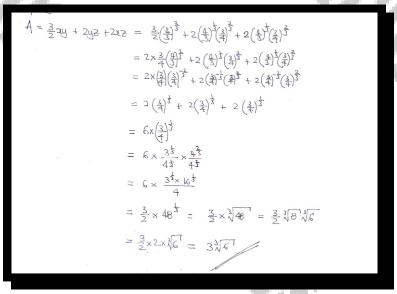
Question 52 (*****)

$$A = \frac{3}{2}xy + 2yz + 2xz\,.$$

Given that $x = \left(\frac{4}{3}\right)^{\frac{1}{3}}$, $y = \left(\frac{4}{3}\right)^{\frac{1}{3}}$ and $z = \left(\frac{3}{4}\right)^{\frac{2}{3}}$, show clearly that $A = 3\sqrt[3]{6}$

[ ], proof



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Question 53 (*****)

A curve has equation

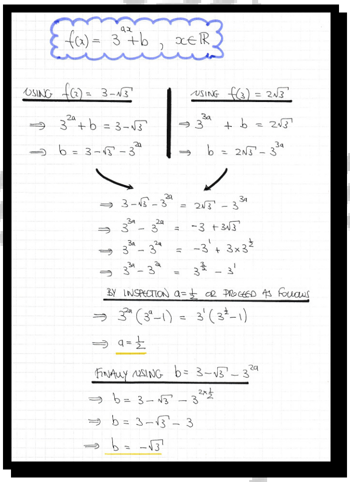
$$f(x) \equiv 3^{ax} + b, \ x \in \mathbb{R},$$

where a and b are non zero constants.

Find the value of a and the value of b, given further that

$$f(2) = 3 - \sqrt{3} \quad \text{and} \quad f(3) = 2\sqrt{3}.$$

$\boxed{a = \frac{1}{2}}$, $\boxed{b = -\sqrt{3}}$



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Question 54 (*****)

A curve has equation

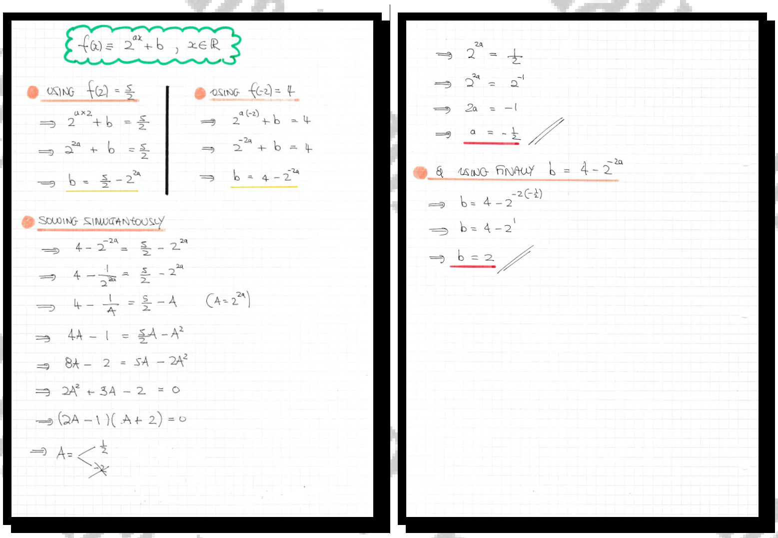
$$f(x) \equiv 2^{ax} + b, \; x \in \mathbb{R},$$

where a and b are non zero constants.

Find the value of a and the value of b, given further that

$$f(2) = \frac{5}{2} \quad \text{and} \quad f(-2) = 4.$$

$\boxed{a = -\frac{1}{2}}$, $\boxed{b = 2}$



Question 55 (*****)

A curve has equation

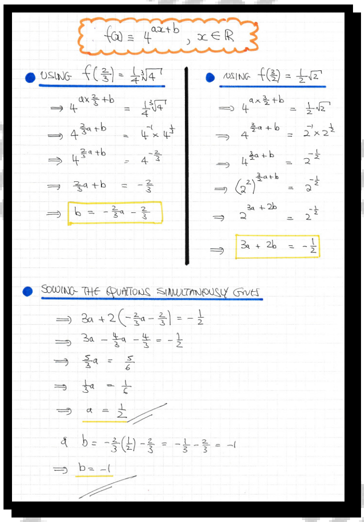
$$f(x) \equiv 4^{ax+b}, \ x \in \mathbb{R},$$

where a and b are non zero constants.

Find the value of a and the value of b, given further that

$$f\left(\frac{2}{3}\right) = \frac{1}{4}\sqrt[3]{4} \quad \text{and} \quad f\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{2}.$$

$\boxed{a = \frac{1}{2}}$, $\boxed{b = -1}$

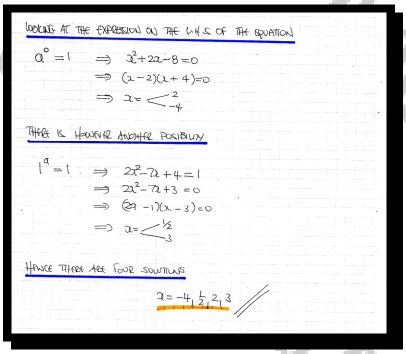


Question 56 (*****)

Find the solutions for the following equation.

$$\left(2x^2-7x+4\right)^{x^2+2x-8}=1.$$

V , [ ] , $x=-4,\ \frac{1}{2},\ 2,\ 3$



LOOKING AT THE EXPRESSION ON THE L.H.S OF THE EQUATION

$a^0=1 \Rightarrow x^2+2x-8=0$

$\Rightarrow (x-2)(x+4)=0$

$\Rightarrow x = 2,\ -4$

THERE IS HOWEVER ANOTHER POSSIBILITY

$1^a=1 \Rightarrow 2x^2-7x+4=1$

$\Rightarrow 2x^2-7x+3=0$

$\Rightarrow (2x-1)(x-3)=0$

$\Rightarrow x = \frac{1}{2},\ 3$

HENCE THERE ARE FOUR SOLUTIONS

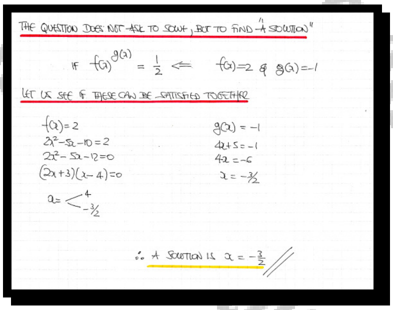
$x=-4,\ \frac{1}{2},\ 2,\ 3$

Question 57 (*****)

Find a solution for the following equation.

$$\left(2x^2 - 5x - 10\right)^{4x+5} = \frac{1}{2}.$$

V, Y.N, $x = -\frac{3}{2}$


THE QUESTION DOES NOT ASK TO SOLVE, BUT TO FIND "A SOLUTION"

IF $f(x)^{g(x)} = \frac{1}{2} \Leftarrow f(x) = 2$ & $g(x) = -1$

LET US SEE IF THESE CAN BE SATISFIED TOGETHER

$f(x) = 2$
$2x^2 - 5x - 10 = 2$
$2x^2 - 5x - 12 = 0$
$(2x+3)(x-4) = 0$
$x = 4, -\frac{3}{2}$

$g(x) = -1$
$4x + 5 = -1$
$4x = -6$
$x = -\frac{3}{2}$

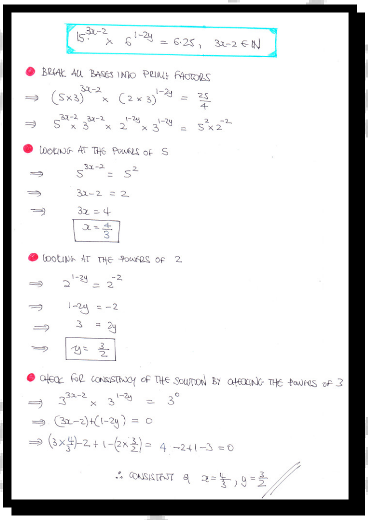
$\therefore$ A SOLUTION IS $x = -\frac{3}{2}$

Question 58 (*****)

Determine the value of x and the value of y in the following equation

$$15^{3x-2} \times 6^{1-2y} = 6.25, \quad (3x-2) \in \mathbb{N}.$$

$x = \frac{4}{3}, \ y = \frac{3}{2}$

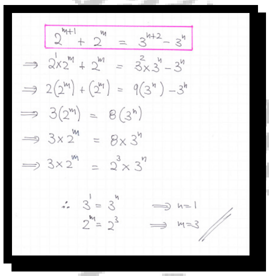


Question 59 (*****)

$$2^{m+1} + 2^m = 3^{n+2} - 3^n.$$

Given that m and n are positive integers, find the value of m and the value of n.

$m = 3$, $n = 1$



Question 60 **(*****)**

Find the term independent of x in the expansion of

$$\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}.$$

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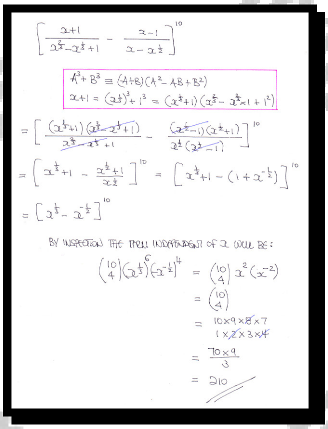

$$\left[\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right]^{10}$$

$$A^3+B^3 \equiv (A+B)(A^2-AB+B^2)$$
$$x+1=(x^{\frac{1}{3}})^3+1^3=(x^{\frac{1}{3}}+1)(x^{\frac{2}{3}}-x^{\frac{1}{3}}\times 1+1^2)$$

$$=\left[\frac{(x^{\frac{1}{3}}+1)(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1)}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}+1)}{x^{\frac{1}{2}}(x^{\frac{1}{2}}-1)}\right]^{10}$$

$$=\left[x^{\frac{1}{3}}+1-\frac{x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}}\right]^{10}=\left[x^{\frac{1}{3}}+1-(1+x^{-\frac{1}{2}})\right]^{10}$$

$$=\left[x^{\frac{1}{3}}-x^{-\frac{1}{2}}\right]^{10}$$

BY INSPECTION THE TERM INDEPENDENT OF x WILL BE:

$$\binom{10}{4}(x^{\frac{1}{3}})^6(-x^{-\frac{1}{2}})^4=\binom{10}{4}x^2(x^{-2})$$
$$=\binom{10}{4}$$
$$=\frac{10\times 9\times 8\times 7}{1\times 2\times 3\times 4}$$
$$=\frac{70\times 9}{3}$$
$$=210$$

Question 61 (*****)

Simplify the following expression.

$$\frac{x^6 - yz^4}{x^4 + xz^2\sqrt{y}}$$

Give the answer in the form $x^2 - x^r y^s z^t$, where r, s and t are constants.

$\boxed{}$, $\boxed{x^2 - x^{-1}y^{\frac{1}{2}}z^2}$