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INEQUALITIES

BASIC PRACTICE

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LINEAR INEQUALITIES

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Question 1

Solve each of the following inequalities.

a) $5(4-2x) < 30$

$x > -1$

b) $2(2x-3)+5 \geq 29$

$x \geq \frac{15}{2}$

c) $4(2x+3)+x > 47-5x$

$x > \frac{5}{2}$

d) $3(2x-3)-9 < 38-x$

$x < 8$

Handwritten solutions for the inequalities:

a) $5(4-2x) < 30$
 $\Rightarrow 20 - 10x < 30$
 $\Rightarrow -10x < 10$
 $\Rightarrow x > -1$

b) $2(2x-3)+5 \geq 29$
 $\Rightarrow 4x-6+5 \geq 29$
 $\Rightarrow 4x-1 \geq 29$
 $\Rightarrow 4x \geq 30$
 $\Rightarrow x \geq \frac{15}{2}$

c) $4(2x+3)+x > 47-5x$
 $\Rightarrow 8x+12+x > 47-5x$
 $\Rightarrow 14x+12 > 47-5x$
 $\Rightarrow 19x > 35$
 $\Rightarrow x > \frac{35}{19}$

d) $3(2x-3)-9 < 38-x$
 $\Rightarrow 6x-9-9 < 38-x$
 $\Rightarrow 6x-18 < 38-x$
 $\Rightarrow 7x < 56$
 $\Rightarrow x < 8$

Question 2

Solve each of the following inequalities.

a) $2 + 2(4x + 3) \geq 2(2x - 1) + 2x$ $x \geq -5$

b) $8 + 3(x + 3) \leq 2(2x + 1)$ $x \geq 15$

c) $6 - 4(4 - x) > 4$ $x > \frac{7}{2}$

d) $16 - 4(7 + x) \leq 8$ $x \geq -5$

Handwritten solutions for the inequalities:

a) $2 + 2(4x + 3) \geq 2(2x - 1) + 2x$
 $\Rightarrow 2 + 8x + 6 \geq 4x - 2 + 2x$
 $\Rightarrow 8x + 8 \geq 6x - 2$
 $\Rightarrow 2x \geq -10$
 $\Rightarrow x \geq -5$

b) $8 + 3(x + 3) \leq 2(2x + 1)$
 $\Rightarrow 8 + 3x + 9 \leq 4x + 2$
 $\Rightarrow 3x + 17 \leq 4x + 2$
 $\Rightarrow -x \leq -15$
 $\Rightarrow x \geq 15$

c) $6 - 4(4 - x) > 4$
 $\Rightarrow 6 - 16 + 4x > 4$
 $\Rightarrow -10 + 4x > 4$
 $\Rightarrow 4x > 14$
 $\Rightarrow x > \frac{14}{4}$
 $\Rightarrow x > \frac{7}{2}$

d) $16 - 4(7 + x) \leq 8$
 $\Rightarrow 16 - 28 - 4x \leq 8$
 $\Rightarrow -12 - 4x \leq 8$
 $\Rightarrow -4x \leq 20$
 $\Rightarrow x \geq -5$

Question 3

Solve each of the following inequalities.

a) $6 - 2(7 - 3x) \geq 8 - (3x + 7)$ $x \geq 1$

b) $13 - 2(1 - 4x) > 6 - (2 - x)$ $x > -1$

c) $21 - (3 - 2x) < 33 - 4(10 - 3x)$ $x > \frac{5}{2}$

d) $10 - (5 - 2x) \geq -11 - 2(3x - 2)$ $x \geq -\frac{3}{2}$

Handwritten solutions for the four inequalities:

a) $6 - 2(7 - 3x) \geq 8 - (3x + 7)$
 $\Rightarrow 6 - 14 + 6x \geq 8 - 3x - 7$
 $\Rightarrow -8 + 6x \geq 1 - 3x$
 $\Rightarrow 9x \geq 9$
 $\Rightarrow x \geq 1$

b) $13 - 2(1 - 4x) > 6 - (2 - x)$
 $\Rightarrow 13 - 2 + 8x > 6 - 2 + x$
 $\Rightarrow 11 + 8x > 4 + x$
 $\Rightarrow 7x > -7$
 $\Rightarrow x > -1$

c) $21 - (3 - 2x) < 33 - 4(10 - 3x)$
 $\Rightarrow 21 - 3 + 2x < 33 - 40 + 12x$
 $\Rightarrow 18 + 2x < -7 + 12x$
 $\Rightarrow 25 < 10x$
 $\Rightarrow x > \frac{5}{2}$

d) $10 - (5 - 2x) \geq -11 - 2(3x - 2)$
 $\Rightarrow 10 - 5 + 2x \geq -11 - 6x + 4$
 $\Rightarrow 5 + 2x \geq -7 - 6x$
 $\Rightarrow 8x \geq -12$
 $\Rightarrow x \geq -\frac{3}{2}$

QUADRATIC INEQUALITIES

Question 1

Solve each of the following quadratic inequalities.

a) $x^2 + 4x - 32 > 0$

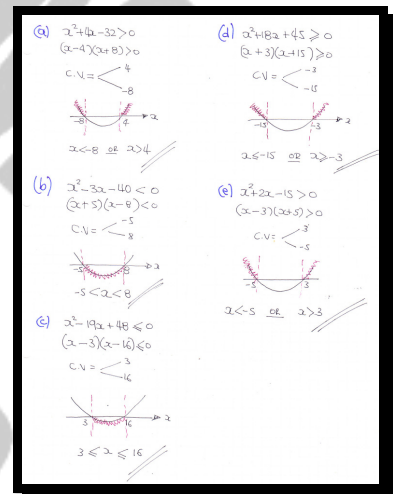
b) $x^2 - 3x - 40 < 0$

c) $x^2 - 19x + 48 \leq 0$

d) $x^2 + 18x + 45 \geq 0$

e) $x^2 + 2x - 15 > 0$

$$x < -8 \text{ or } x > 4, \quad -5 < x < 8, \quad 3 \leq x \leq 16, \quad x \leq -15 \text{ or } x \geq -3, \\ x < -5 \text{ or } x > 3$$



Question 2

Solve each of the following quadratic inequalities.

a) $x^2 + 8x + 15 \leq 0$

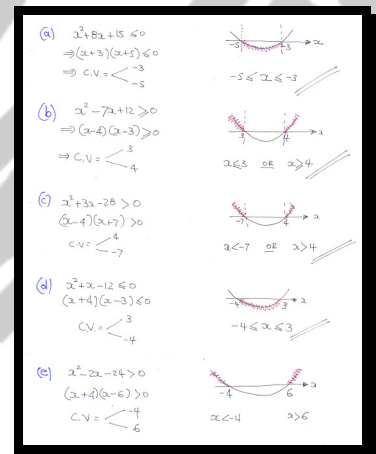
b) $x^2 - 7x + 12 \geq 0$

c) $x^2 + 3x - 28 > 0$

d) $x^2 + x - 12 \leq 0$

e) $x^2 - 2x - 24 > 0$

$-5 \leq x \leq -3$, $x \leq 3$ or $x \geq 4$, $x < -7$ or $x > 4$, $-4 \leq x \leq 3$, $x < -4$ or $x > 6$



Question 3

Solve each of the following quadratic inequalities.

a) $x^2 - 14x + 24 < 0$

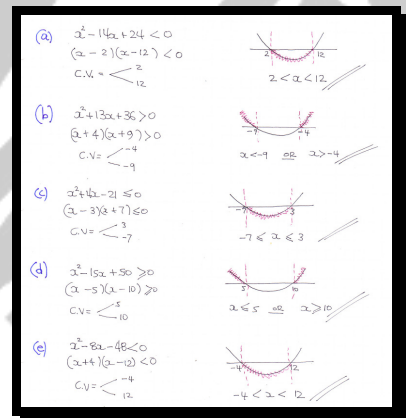
b) $x^2 + 13x + 36 > 0$

c) $x^2 + 4x - 21 \leq 0$

d) $x^2 - 15x + 50 \geq 0$

e) $x^2 - 8x - 48 < 0$

$2 < x < 12$, $x < -9$ or $x > -4$, $-7 \leq x \leq 3$, $x \leq 5$ or $x \geq 10$, $-4 < x < 12$



Question 4

Solve each of the following quadratic inequalities.

a) $x^2 + 13x - 30 < 0$

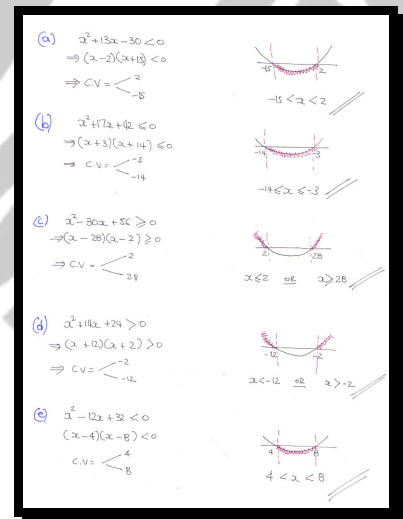
b) $x^2 + 17x + 42 \leq 0$

c) $x^2 - 30x + 56 \geq 0$

d) $x^2 + 14x + 24 > 0$

e) $x^2 - 12x + 32 < 0$

$$\boxed{-15 < x < 2}, \quad \boxed{-14 \leq x \leq -3}, \quad \boxed{x \leq 2 \text{ or } x \geq 28}, \quad \boxed{x < -12 \text{ or } x > -2}, \quad \boxed{4 < x < 8}$$



Question 5

Solve each of the following quadratic inequalities.

a) $2x^2 - 5x + 3 > 0$

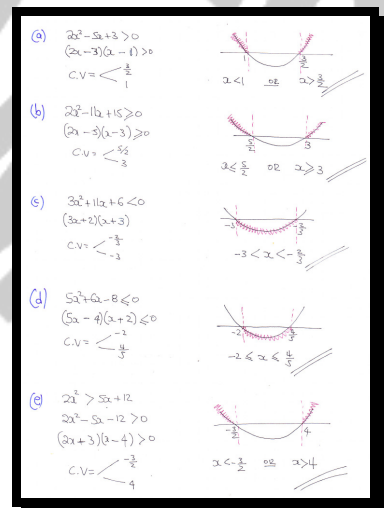
b) $2x^2 - 11x + 15 \geq 0$

c) $3x^2 + 11x + 6 < 0$

d) $5x^2 + 6x - 8 \leq 0$

e) $2x^2 > 5x + 12$

$$x < 1 \text{ or } x > \frac{3}{2}, \quad x \leq \frac{5}{2} \text{ or } x \geq 3, \quad -3 < x < -\frac{2}{3}, \quad -2 \leq x \leq \frac{4}{3}, \quad x < -\frac{3}{2} \text{ or } x > 4$$



Question 6

Solve each of the following quadratic inequalities.

a) $3x^2 + 8 \leq 10x$

b) $5x^2 < 6 - 7x$

c) $5x^2 \leq 4 - 19x$

d) $4(x^2 + 2x) + 3 > 0$

e) $6(x^2 + 1) > 13x$

$-\frac{2}{3} \leq x \leq 4$, $-2 < x < \frac{3}{5}$, $-4 \leq x \leq \frac{1}{5}$, $x < -\frac{3}{2}$ or $x > -\frac{1}{2}$, $x < \frac{2}{3}$ or $x > \frac{3}{2}$

Handwritten solutions for the quadratic inequalities:

- a) $3x^2 + 8 \leq 10x$
 $3x^2 - 10x + 8 \leq 0$
 $(3x + 2)(x - 4) \leq 0$
 $C.V = -\frac{2}{3}, 4$
 $-\frac{2}{3} \leq x \leq 4$
- b) $5x^2 < 6 - 7x$
 $5x^2 + 7x - 6 < 0$
 $(5x - 3)(x + 2) < 0$
 $C.V = \frac{3}{5}, -2$
 $-2 < x < \frac{3}{5}$
- c) $5x^2 \leq 4 - 19x$
 $5x^2 + 19x - 4 \leq 0$
 $(5x - 1)(x + 4) \leq 0$
 $C.V = \frac{1}{5}, -4$
 $-4 \leq x \leq \frac{1}{5}$
- d) $4(x^2 + 2x) + 3 > 0$
 $4x^2 + 8x + 3 > 0$
 $(2x + 1)(2x + 3) > 0$
 $C.V = -\frac{1}{2}, -\frac{3}{2}$
 $x < -\frac{3}{2}$ or $x > -\frac{1}{2}$
- e) $6(x^2 + 1) > 13x$
 $6x^2 + 6 > 13x$
 $6x^2 - 13x + 6 > 0$
 $(2x - 3)(3x - 2) > 0$
 $C.V = \frac{2}{3}, \frac{3}{2}$
 $x < \frac{2}{3}$ or $x > \frac{3}{2}$

Question 7

Solve each of the following quadratic inequalities.

a) $2x^2 > x + 10$

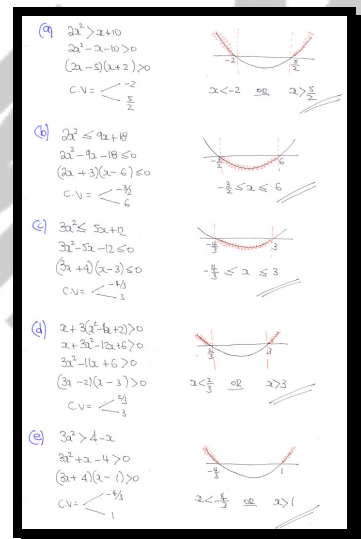
b) $2x^2 \leq 9x + 18$

c) $3x^2 \leq 5x + 12$

d) $x + 3(x^2 - 4x + 2) > 0$

e) $3x^2 > 4 - x$

$$x < -2 \text{ or } x > \frac{5}{2}, \quad -\frac{3}{2} \leq x \leq 6, \quad -\frac{4}{3} \leq x \leq 3, \quad x < \frac{2}{3} \text{ or } x > 3, \quad x < -\frac{4}{3} \text{ or } x > 1$$



Question 8

Solve each of the following quadratic inequalities.

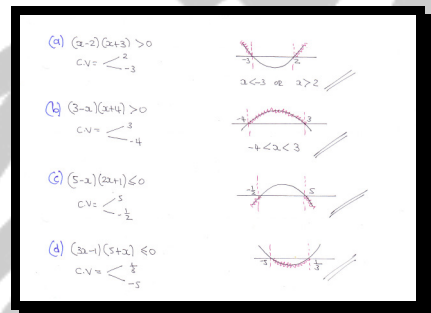
a) $(x-2)(x+3) > 0$

b) $(3-x)(x+4) > 0$

c) $(5-x)(2x+1) \leq 0$

d) $(3x-1)(5+x) \leq 0$

$x < -3$ or $x > 2$, $-4 < x < 3$, $x \leq -\frac{1}{2}$ or $x \geq 5$, $-5 \leq x \leq \frac{1}{3}$



Question 9

Solve each of the following quadratic inequalities.

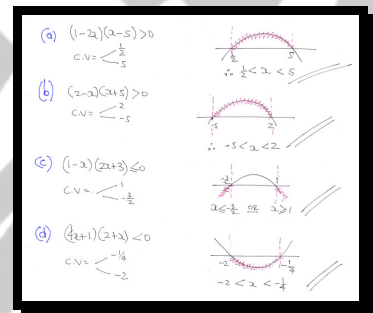
a) $(1-2x)(x-5) > 0$

b) $(2-x)(x+5) > 0$

c) $(1-x)(2x+3) \leq 0$

d) $(4x-1)(2+x) < 0$

$$\boxed{\frac{1}{2} < x < 5}, \quad \boxed{-5 < x < 2}, \quad \boxed{x < -\frac{3}{2} \text{ or } x > 1}, \quad \boxed{-2 < x < -\frac{1}{4}}$$



Question 10

Solve each of the following quadratic inequalities.

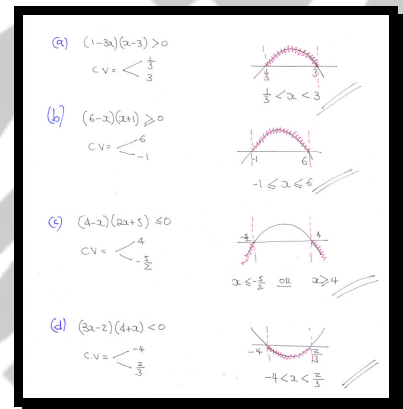
a) $(1-3x)(x-3) > 0$

b) $(6-x)(x+1) \geq 0$

c) $(4-x)(2x+5) \leq 0$

d) $(3x-2)(4+x) < 0$

$\boxed{\frac{1}{3} < x < 3}$, $\boxed{-1 \leq x \leq 6}$, $\boxed{x \leq -\frac{5}{2} \text{ or } x \geq 4}$, $\boxed{-4 < x < \frac{2}{3}}$



Question 11

Solve each of the following quadratic inequalities.

a) $(2x-3)(x+4) < x(x+6)$

b) $(x+2)(2x-7) \geq (x-2)(x+4)$

c) $(x-6)(2x-7) > 2(x+1)$

d) $(x+1)^2 \leq 4x+9$

$$\boxed{-3 < x < 4}, \quad \boxed{x \leq -1 \text{ or } x \geq 6}, \quad \boxed{x < \frac{5}{2} \text{ or } x > 8}, \quad \boxed{-2 < x < 4}$$

(a) $(2x-3)(x+4) < x(x+6)$
 $\Rightarrow 2x^2 + 5x - 12 < x^2 + 6x$
 $\Rightarrow x^2 - x - 12 < 0$
 $\Rightarrow (x-4)(x+3) < 0$
 $\therefore \text{C.V.} = \begin{matrix} 4 \\ -3 \end{matrix}$
 $-3 < x < 4$

(b) $(x+2)(2x-7) \geq (x-2)(x+4)$
 $\Rightarrow 2x^2 - 3x - 14 \geq x^2 + 2x - 8$
 $\Rightarrow x^2 - 5x - 6 \geq 0$
 $\Rightarrow (x+1)(x-6) \geq 0$
 $\therefore \text{C.V.} = \begin{matrix} -1 \\ 6 \end{matrix}$
 $x \leq -1 \text{ or } x \geq 6$

(c) $(x-6)(2x-7) > 2(x+1)$
 $\Rightarrow 2x^2 - 7x - 12x + 42 > 2x + 2$
 $\Rightarrow 2x^2 - 21x + 40 > 0$
 $\Rightarrow (2x-5)(x-8) > 0$
 $\therefore \text{C.V.} = \begin{matrix} \frac{5}{2} \\ 8 \end{matrix}$
 $x < \frac{5}{2} \text{ or } x > 8$

(d) $(x+1)^2 \leq 4x+9$
 $\Rightarrow x^2 + 2x + 1 \leq 4x + 9$
 $\Rightarrow x^2 - 2x - 8 \leq 0$
 $\Rightarrow (x+2)(x-4) \leq 0$
 $\therefore \text{C.V.} = \begin{matrix} -2 \\ 4 \end{matrix}$
 $-2 \leq x \leq 4$

Question 12

Solve each of the following quadratic inequalities.

a) $(2x+1)(x-2) > 2(x+5)$

b) $(x-1)(2x+3) \leq 2(x+6)$

c) $(x+5)(2x-1) \geq (x+1)(x+7)$

d) $x(2x-11) < (x-4)(x+2)$

$$x < -\frac{3}{2} \text{ or } x > 4, \quad -\frac{5}{2} \leq x \leq 3, \quad x \leq -4 \text{ or } x \geq 3, \quad 1 < x < 8$$

Handwritten solutions for the four quadratic inequalities:

(a) $(2x+1)(x-2) > 2(x+5)$
 $\Rightarrow 2x^2 + x - 4x - 2 > 2x + 10$
 $\Rightarrow 2x^2 - 3x - 12 > 0$
 $\Rightarrow (2x+3)(x-4) > 0$
 $\Rightarrow C.V = -\frac{3}{2}$
 Diagram: Number line with roots at $-\frac{3}{2}$ and 4 . The region between the roots is shaded, indicating $-\frac{3}{2} < x < 4$.

(b) $(x-1)(2x+3) \leq 2(x+6)$
 $\Rightarrow 2x^2 + 3x - 2x - 3 \leq 2x + 12$
 $\Rightarrow 2x^2 - 2 - 15 \leq 0$
 $\Rightarrow (2x+5)(x-3) \leq 0$
 $\Rightarrow C.V = -\frac{5}{2}$
 Diagram: Number line with roots at $-\frac{5}{2}$ and 3 . The region between the roots is shaded, indicating $-\frac{5}{2} \leq x \leq 3$.

(c) $(x+5)(2x-1) \geq (x+1)(x+7)$
 $\Rightarrow 2x^2 + 10x - x - 5 \geq x^2 + 8x + 7$
 $\Rightarrow x^2 + 2x - 12 \geq 0$
 $\Rightarrow (x-3)(x+4) \geq 0$
 $\Rightarrow C.V = -4$
 Diagram: Number line with roots at -4 and 3 . The regions outside the roots are shaded, indicating $x \leq -4$ or $x \geq 3$.

(d) $x(2x-11) < (x-4)(x+2)$
 $\Rightarrow 2x^2 - 11x < x^2 - 4x + 2x - 8$
 $\Rightarrow x^2 - 9x + 8 < 0$
 $\Rightarrow (x-8)(x-1) < 0$
 $\Rightarrow C.V = 1$
 Diagram: Number line with roots at 1 and 8 . The region between the roots is shaded, indicating $1 < x < 8$.

Question 13

Solve each of the following quadratic inequalities.

a) $(2x+3)(4x-5) > (2x+5)(2x-1)$

b) $(3x+2)(6x-7) > (3x-2)(3x+4)$

c) $(3x+8)(6x+5) \leq (3x+4)(3x+10)$

d) $(2x+3)(x+7) \leq (x+3)(x+9)$

$$x < -1 \text{ or } x > \frac{5}{2}, \quad x < -\frac{1}{3} \text{ or } x > 2, \quad -\frac{7}{3} \leq x \leq 0, \quad -6 \leq x \leq 1$$

(a) $(2x+3)(4x-5) > (2x+5)(2x-1)$
 $8x^2 - 2x - 15 > 4x^2 + 8x - 5$
 $4x^2 - 6x - 10 > 0$
 $2x^2 - 3x - 5 > 0$
 $(2x-5)(x+1) > 0$
C.V. = $-\frac{1}{2}$
 $x < -1$ or $x > \frac{5}{2}$

(b) $(3x+2)(6x-7) > (3x-2)(3x+4)$
 $18x^2 - 14x - 14 > 9x^2 + 6x - 8$
 $9x^2 - 15x - 6 > 0$
 $3x^2 - 5x - 2 > 0$
 $(3x+1)(x-2) > 0$
C.V. = $-\frac{1}{3}$
 $x < -\frac{1}{3}$ or $x > 2$

(c) $(3x+8)(6x+5) \leq (3x+4)(3x+10)$
 $18x^2 + 63x + 40 \leq 9x^2 + 42x + 40$
 $9x^2 + 21x \leq 0$
 $3x^2 + 7x \leq 0$
 $x(3x+7) \leq 0$
C.V. = $-\frac{7}{3}$
 $-\frac{7}{3} \leq x \leq 0$

(d) $(2x+3)(x+7) \leq (x+3)(x+9)$
 $2x^2 + 17x + 21 \leq x^2 + 12x + 27$
 $x^2 + 5x - 6 \leq 0$
 $(x+6)(x-1) \leq 0$
C.V. = -6
 $-6 \leq x \leq 1$