STRAIGHT LINE COORDINATE GEOMETRY

(118 EXAM QUESTIONS)

41 BASIC QUESTIONS

Question 1 (**)

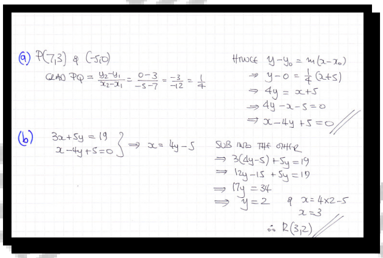
The points P and Q have coordinates $(7,3)$ and $(-5,0)$, respectively.

a) Determine an equation for the straight line PQ, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

The straight line RT with equation $3x+5y=19$ intersects the straight line PQ at the point R.

b) Find the coordinates of R.

$\boxed{x-4y+5=0}$, $\boxed{R(3,2)}$



Question 2 **(**)**

The straight line l_1 passes through the points $A(-1,3)$ and $B(2,-3)$.

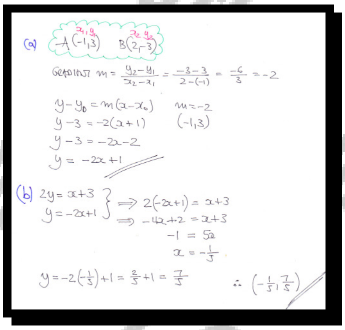
a) Find an equation for l_1.

The straight line l_2 has equation

$$2y = x + 3.$$

b) Find the exact coordinates of the point of intersection between l_1 and l_2.

$\boxed{y = 1 - 2x}$, $\boxed{\left(-\frac{1}{5}, \frac{7}{5}\right)}$



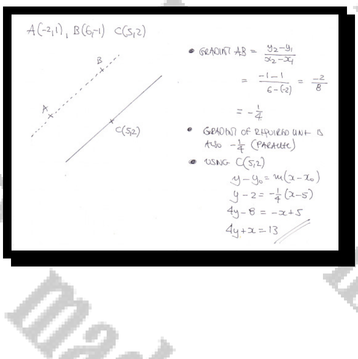
Question 3 **(**)**

The coordinates of three points are given below

$$A(-2,1), \quad B(6,-1) \quad \text{and} \quad C(5,2).$$

Determine the equation of the straight line which passes through C and is parallel to AB, giving the answer in the form $ax + by = c$, where a, b and c are integers.

$\boxed{x + 4y = 13}$



Question 4 (**)

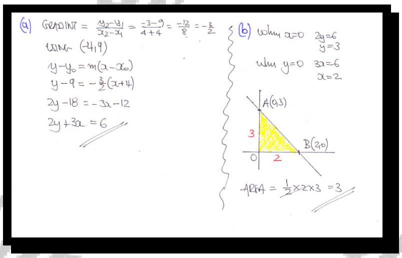
The straight line l passes through the points with coordinates $(-4,9)$ and $(4,-3)$.

a) Find an equation for l, giving the answer in the form $ax+by=c$ where a, b and c are integers.

The straight line l meets the coordinate axes at the points A and B.

b) Determine the area of the triangle OAB, where O is the origin.

$\boxed{3x+2y=6}$, $\boxed{\text{area}=3}$



Question 5 (**)

The straight line l_1 passes through the points $A(5,4)$ and $B(13,0)$.

a) Find an equation of l_1, in the form $ax+by=c$, where a, b and c are integers.

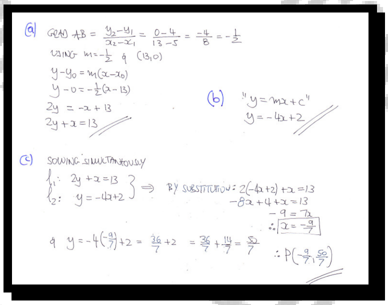
The straight line l_2 passes through the point $C(0,2)$ and has gradient -4.

b) Write down an equation of l_2.

The point P is the intersection of l_1 and l_2.

c) Determine the exact coordinates of P.

$\boxed{x+2y=13}$, $\boxed{y=2-4x}$, $\boxed{P\left(-\frac{9}{7},\frac{50}{7}\right)}$



Question 6 (**)

The straight line l_1 passes through the points $A(3,17)$ and $B(13,-3)$.

a) Find an equation of l_1.

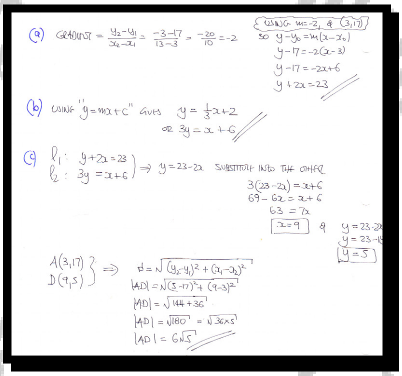
The straight line l_2 has gradient $\frac{1}{3}$ and passes through the point $C(0,2)$.

b) Find an equation of l_2.

The two lines, l_1 and l_2, intersect at the point D.

c) Show that the length of AD is $k\sqrt{5}$, where k is an integer.

$\boxed{y+2x=23}$, $\boxed{3y=x+6}$, $\boxed{k=6}$


(a) GRADIENT $= \frac{y_2-y_1}{x_2-x_1} = \frac{-3-17}{13-3} = \frac{-20}{10} = -2$

USING $m=-2$, & $(3,17)$

SO $y-y_0 = m(x-x_0)$

$y-17 = -2(x-3)$

$y-17 = -2x+6$

$y+2x = 23$

(b) USING "$y=mx+c$" GIVES $y = \frac{1}{3}x+2$

OR $3y = x+6$

(c) $l_1: y+2x=23$, $l_2: 3y = x+6$ $\Rightarrow$ $y = 23-2x$ SUBSTITUTE INTO THE OTHER

$3(23-2x) = x+6$

$69-6x = x+6$

$63 = 7x$

$\boxed{x=9}$ & $y = 23-2$ … $y = 23-1$ … $\boxed{y=5}$

$A(3,17)$, $D(9,5)$ $\Rightarrow$ $d = \sqrt{(y_2-y_1)^2+(x_1-x_2)^2}$

$|AD| = \sqrt{(5-17)^2+(9-3)^2}$

$|AD| = \sqrt{144+36}$

$|AD| = \sqrt{180} = \sqrt{36\times5}$

$|AD| = 6\sqrt{5}$

Question 7 (**)

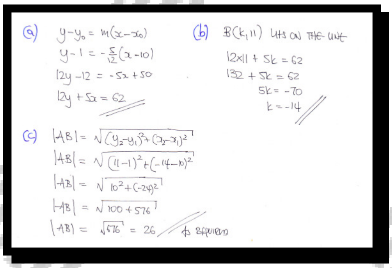
The straight line l has a gradient of $-\frac{5}{12}$, and passes through the points $A(10,1)$ and $B(k,11)$, where k is a constant.

a) Find an equation of l, in the form $ax+by=c$, where a, b and c are integers.

b) Determine the value of k.

c) Hence show that the distance AB is 26 units.

$\boxed{5x+12y=62}$, $\boxed{k=-14}$


(a) $y-y_0=m(x-x_0)$
$y-1=-\frac{5}{12}(x-10)$
$12y-12=-5x+50$
$12y+5x=62$

(b) $B(k,11)$ lies on the line
$12\times11+5k=62$
$132+5k=62$
$5k=-70$
$k=-14$

(c) $|AB|=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$
$|AB|=\sqrt{(11-1)^2+(-14-10)^2}$
$|AB|=\sqrt{10^2+(-24)^2}$
$|AB|=\sqrt{100+576}$
$|AB|=\sqrt{676}=26$ as required

Question 8 (**)

The points A and B have coordinates $(1,1)$ and $(5,7)$, respectively.

a) Find an equation for the straight line l_1 which passes through A and B.

The straight line l_2 with equation

$$2x+3y=18$$

meets l_1 at the point C.

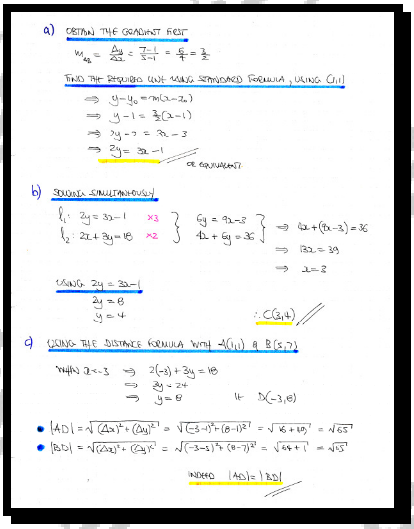
b) Determine the coordinates of C.

The point D, where $x=-3$, lies on l_2.

c) Show clearly that

$$|AD|=|BD|.$$

$\boxed{}$, $\boxed{2y=3x-1}$, $\boxed{C(3,4)}$

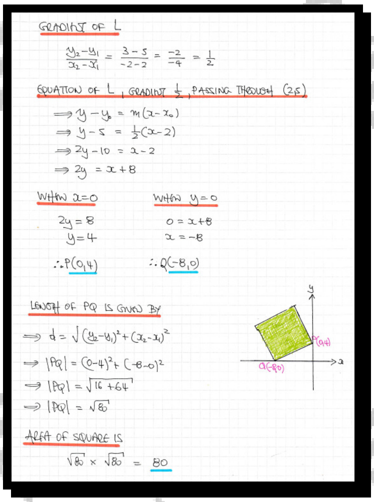


Question 9 (**+)

The straight line L passes through the points $(2,5)$ and $(-2,3)$, and meets the coordinate axes at the points P and Q.

Find the area of a square whose side is PQ.

$\boxed{\text{area} = 80}$


GRADIENT OF L

$$\frac{y_2-y_1}{x_2-x_1} = \frac{3-5}{-2-2} = \frac{-2}{-4} = \frac{1}{2}$$

EQUATION OF L, GRADIENT $\frac{1}{2}$, PASSING THROUGH (2,5)

$$\Rightarrow y - y_0 = m(x - x_0)$$
$$\Rightarrow y - 5 = \tfrac{1}{2}(x-2)$$
$$\Rightarrow 2y - 10 = x - 2$$
$$\Rightarrow 2y = x + 8$$

WHEN $x=0$: $2y = 8$, $y = 4$, $\therefore P(0,4)$

WHEN $y=0$: $0 = x+8$, $x = -8$, $\therefore Q(-8,0)$

LENGTH OF PQ IS GIVEN BY

$$\Rightarrow d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$$
$$\Rightarrow |PQ| = (0-4)^2 + (-8-0)^2$$
$$\Rightarrow |PQ| = \sqrt{16+64}$$
$$\Rightarrow |PQ| = \sqrt{80}$$

AREA OF SQUARE IS

$$\sqrt{80} \times \sqrt{80} = 80$$

Question 10 **(**+)**

The straight line l_1 passes through the points $A(-1,-1)$ and $B(k,5)$, where k is a constant.

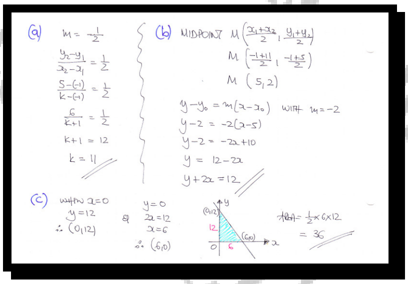
a) Given that the gradient of l_1 is $\frac{1}{2}$ show that $k = 11$.

The straight line l_2 passes through the midpoint of AB and is perpendicular to l_1.

b) Determine an equation of l_2, giving the answer in the form $ax + by = c$, where a, b and c are integers.

c) Calculate the area of the triangle enclosed by l_2 and the coordinate axes.

$\boxed{2x + y = 12}$, $\boxed{\text{area} = 36}$



Question 11 (**+)

The straight lines l_1 and l_2 have equations

$$l_1 : \ 2x + y = 10,$$

$$l_2 : \ 3x - 4y = 10.$$

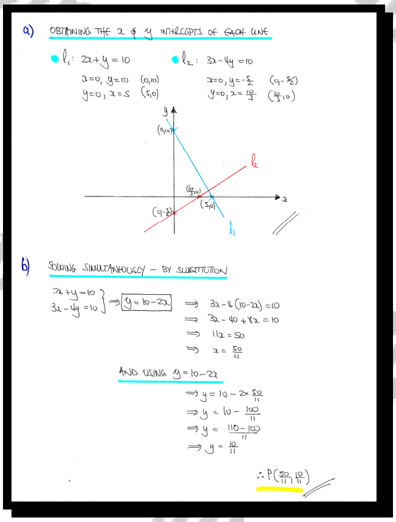
a) Sketch l_1 and l_2 in a single set of axes.

The sketch must include the coordinates of all the points where each of these straight lines meet the coordinate axes.

The two lines intersect at the point P.

b) Use algebra to determine the exact coordinates of P.

$\boxed{}$, $\boxed{P\left(\frac{50}{11}, \frac{10}{11}\right)}$



Question 12 **(**+)**

The points A and B have coordinates $(-1,4)$ and $B(3,-2)$, respectively.

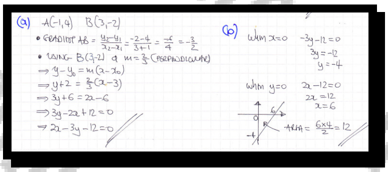
a) Find an equation of the straight line L which is perpendicular to the straight line AB and passes through the point B, giving the answer in the form $ax+by+c=0$ where a, b and c are integers.

L meets the coordinate axes at the points C and D.

The point O represents the origin.

b) Find the area of the triangle OCD.

$\boxed{2x-3y-12=0}$, $\boxed{\text{area}=12}$



Question 13 (+)**

The straight line l_1 has equation

$$3x - 2y = 1.$$

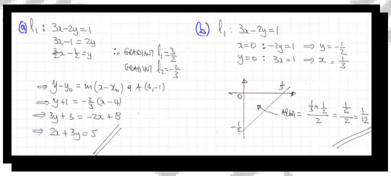
a) Find an equation of the straight line l_2 which is perpendicular to l_1 and passes through the point $A(4,-1)$, giving the answer in the form $ax + by = c$ where a, b and c are integers.

The straight line l_1 meets the coordinate axes at the points P and Q.

The point O represents the origin.

b) Show that the area of the triangle OPQ is $\frac{1}{12}$ of a square unit.

$\boxed{}$, $\boxed{2x + 3y = 5}$



Question 14 (**+)

The straight line L_1 passes through the points $A(-6,4)$ and $B(3,16)$.

a) Find an equation for L_1.

The straight line L_2 passes through the points $C(9,-1)$ and $D(-7,11)$.

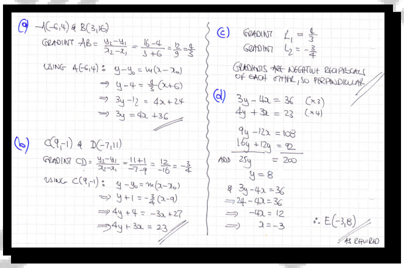
b) Find an equation for L_2.

c) Show that L_1 is perpendicular to L_2

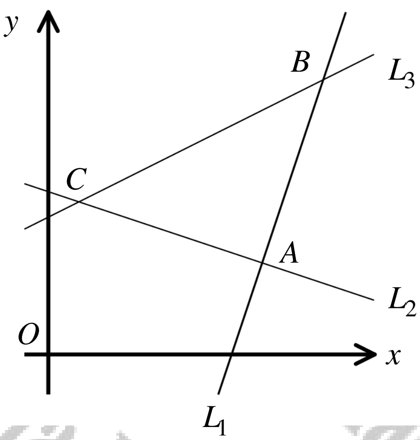
The point E is the of intersection of L_1 and L_2.

d) Show that the coordinates of E are $(-3,8)$.

$\boxed{3y-4x=36}$, $\boxed{4y+3x=23}$



Question 15 (**+)



The figure above shows three straight lines L_1, L_2 and L_3.

a) Find an equation of the straight line L_1, given that it passes through the points $A(7,3)$ and $B(9,9)$.

L_2 is perpendicular to L_1 and passes through A.

b) Find an equation of L_2.

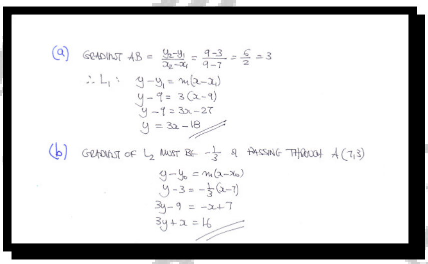
L_3 meets L_1 at the point B and L_2 at the point C.

The equation of L_3 is $y = \dfrac{x+9}{2}$.

c) Determine the coordinates of C.

d) Show that the triangle ABC is isosceles.

$\boxed{}$, $\boxed{y = 3x - 18}$, $\boxed{3y + x = 16}$, $\boxed{C(1,5)}$



(a) GRADIENT AB = $\frac{y_2-y_1}{x_2-x_1} = \frac{9-3}{9-7} = \frac{6}{2} = 3$

$\therefore L_1$: $y - y_1 = m(x - x_1)$

$y - 9 = 3(x-9)$

$y - 9 = 3x - 27$

$y = 3x - 18$

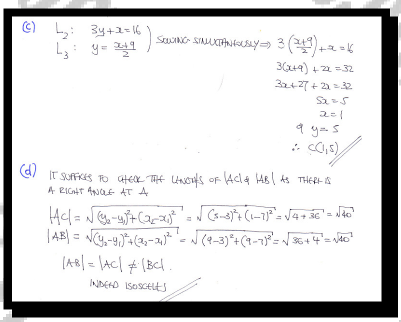
(b) GRADIENT OF L_2 MUST BE $-\frac{1}{3}$ & PASSING THROUGH $A(7,3)$

$y - y_0 = m(x - x_0)$

$y - 3 = -\frac{1}{3}(x-7)$

$3y - 9 = -x + 7$

$3y + x = 16$



(c) L_2: $3y + x = 16$, L_3: $y = \frac{x+9}{2}$ SOLVING SIMULTANEOUSLY $\Rightarrow 3\left(\frac{x+9}{2}\right) + x = 16$

$3(x+9) + 2x = 32$

$3x + 27 + 2x = 32$

$5x = 5$

$x = 1$

& $y = 5$

$\therefore C(1,5)$

(d) IT SUFFICES TO CHECK THE LENGTHS OF $|AC|$ & $|AB|$ AS THERE IS A RIGHT ANGLE AT A

$|AC| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(5-3)^2 + (1-7)^2} = \sqrt{4+36} = \sqrt{40}$

$|AB| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(9-3)^2 + (9-7)^2} = \sqrt{36+4} = \sqrt{40}$

$|AB| = |AC| \neq |BC|$.

INDEED ISOSCELES

Question 16 **(**+)**

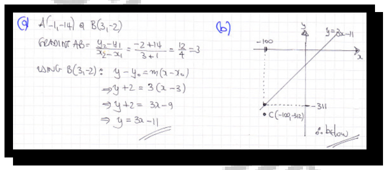
The straight line L passes through the points $A(-1,-14)$ and $B(3,-2)$.

a) Find an equation for L, giving the answer in the form $y = mx + c$.

The point C has coordinates $(-100,-312)$.

b) Determine, by calculation, whether C lies above L or below L.

$\boxed{}$, $\boxed{y = 3x - 11}$, $\boxed{\text{below}}$



Question 17 (**+)

The straight line l_1 passes through the points $A(1,-1)$ and $B(7,8)$.

a) Find an equation of l_1.

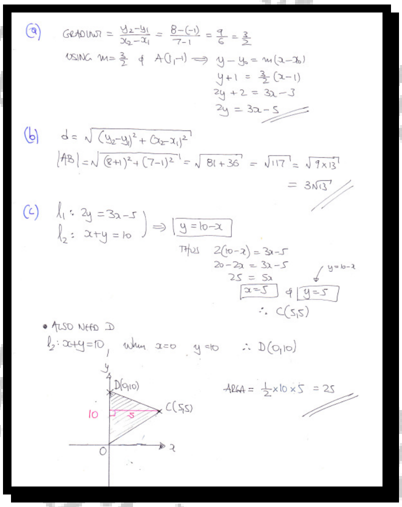
b) Show that the length of AB is $k\sqrt{13}$, where k is an integer.

The straight line l_2, whose equation is $x+y=10$, meets l_1 at the point C.

The point D is the y intercept of l_2.

c) Determine the area of the triangle OCD, where O is the origin.

$\boxed{2y=3x-5}$, $\boxed{k=3}$, $\boxed{\text{area}=25}$


(a) GRADIENT $= \frac{y_2-y_1}{x_2-x_1} = \frac{8-(-1)}{7-1} = \frac{9}{6} = \frac{3}{2}$

USING $m=\frac{3}{2}$ & $A(1,-1) \Rightarrow y-y_0 = m(x-x_0)$

$y+1 = \frac{3}{2}(x-1)$

$2y+2 = 3x-3$

$2y = 3x-5$

(b) $d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$|AB| = \sqrt{(8+1)^2+(7-1)^2} = \sqrt{81+36} = \sqrt{117} = \sqrt{9\times13}$

$= 3\sqrt{13}$

(c) $l_1: 2y=3x-5$, $l_2: x+y=10$ $\Rightarrow$ $\boxed{y=10-x}$

THUS $2(10-x) = 3x-5$

$20-2x = 3x-5$

$25 = 5x$

$\boxed{x=5}$ & $\boxed{y=5}$ ($y=10-x$)

$\therefore C(5,5)$

• ALSO NEED D

$l_2: x+y=10$, when $x=0$ $y=10$ $\therefore D(0,10)$

AREA $= \frac{1}{2}\times10\times5 = 25$

Question 18 (**+)

The points A and B have coordinates $(2,3)$ and $(6,1)$, respectively.

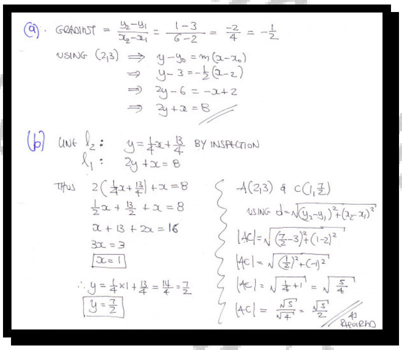
a) Find an equation of the straight line l_1 which passes through A and B.

The line l_2 has gradient $\frac{1}{4}$ and meets the y axis at the point $\left(0,\frac{13}{4}\right)$.

The two lines, l_1 and l_2, intersect at the point C.

b) Show clearly that the length of AC is exactly $\frac{1}{2}\sqrt{5}$.

$\boxed{2y + x = 8}$



Question 19 (***)

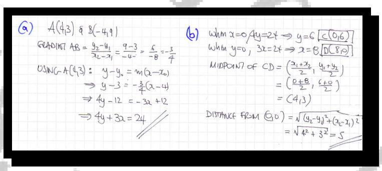
The straight line l passes through the points $A(4,3)$ and $B(-4,9)$.

a) Find an equation for l.

l meets the coordinate axes at the points C and D.

b) Show that the midpoint of CD is at a distance of 5 units from the origin.

$\boxed{4y + 3x = 24}$



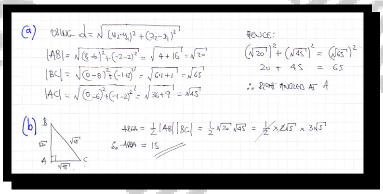
Question 20 (***)

A triangle has vertices $A(2,6)$, $B(-2,8)$ and $C(-1,0)$.

a) Show that the triangle is right angled.

b) Calculate the area of the triangle.

$\boxed{\text{area} = 15}$



Question 21 (***)

The straight line l_1 has gradient 2 and passes through the point $(8,3)$.

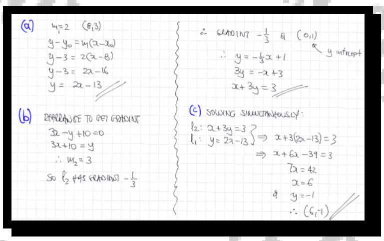
a) Find an equation for l_1.

The straight line l_2 is perpendicular to the line with equation $3x - y + 10 = 0$ and crosses the y axis at $(0,1)$.

b) Determine an equation for l_2, giving the answer in the form $ax + by = c$, where a, b and c are integers.

c) Determine the coordinates of the point of intersection between l_1 and l_2.

$\boxed{y = 2x - 13}$, $\boxed{x + 3y = 3}$, $\boxed{(6,-1)}$



Question 22 (***)

The points A and B have coordinates $(3,-5)$ and $B(1,1)$, respectively.

a) Find an equation of the straight line through A and B, in the form $ax+by=c$ where a, b and c are integers.

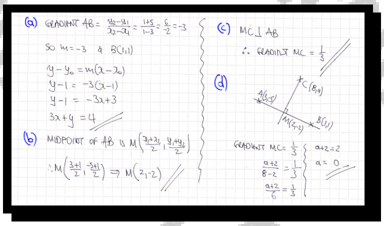
The midpoint of AB is M, and the line segment MC is perpendicular to AB.

b) Find the coordinates of M.

c) State the gradient of MC.

d) Given that C has coordinates $(8,a)$, find the value of a.

$\boxed{3x+y=4}$, $\boxed{M(2,-2)}$, $\boxed{m=\frac{1}{3}}$, $\boxed{a=0}$



Question 23 **(***)**

The straight line L_1 passes through the point $A(-1,2)$ and is parallel to the line that joins the points $P(7,4)$ and $Q(3,8)$.

a) Show that an equation for L_1 is

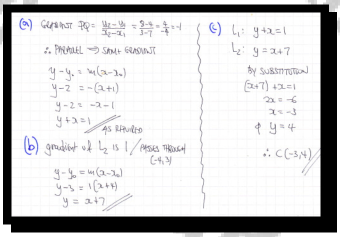
$$x + y = 1.$$

The straight line L_2 is perpendicular to L_1 and passes through the point $B(-4,3)$, intersecting L_1 at the point C.

b) Find an equation for L_2.

c) Determine the coordinates of C.

$\boxed{y = x + 7}$, $\boxed{C(-3,4)}$



Question 24 **(***)**

The straight line L_1 passes through the points $A(13,5)$ and $B(9,2)$.

a) Find an equation for L_1.

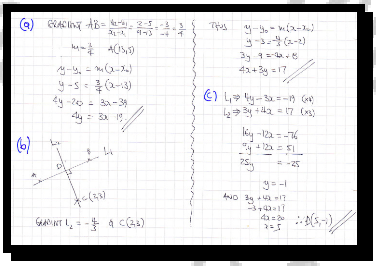
The point D lies on L_1 and the point C has coordinates $(2,3)$.

The straight line L_2 passes through C and D, and is perpendicular to L_1.

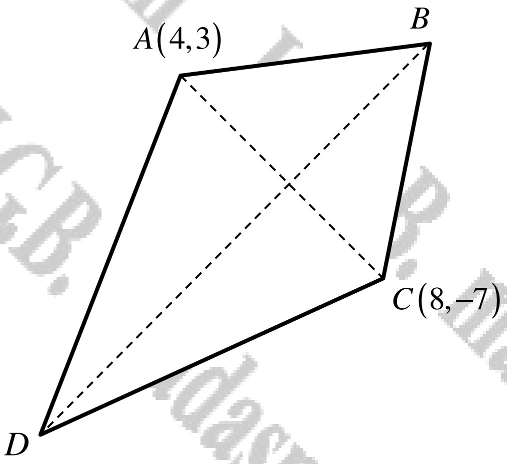
b) Determine an equation for L_2, giving the answer in the form $ax+by=c$, where a, b and c are integers.

c) Find the coordinates of D.

$\boxed{4y=3x-19}$, $\boxed{3y+4x=17}$, $\boxed{D(5,-1)}$



Question 25 (***)

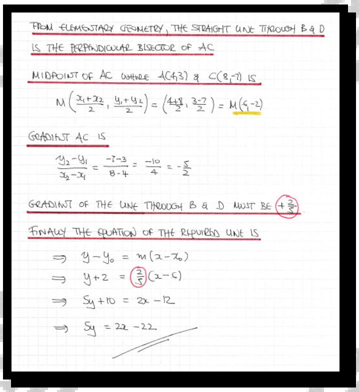


The figure above shows a kite $ABCD$, where the vertices A and C have coordinates $(4,3)$ and $(8,-7)$, respectively.

The diagonal BD is a line of symmetry of the kite.

Find an equation for the diagonal BD.

$\boxed{}$, $\boxed{5y = 2x - 22}$



FROM ELEMENTARY GEOMETRY, THE STRAIGHT LINE THROUGH B & D IS THE PERPENDICULAR BISECTOR OF AC

MIDPOINT OF AC WHERE A(4,3) & C(8,−7) IS

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{4+8}{2}, \frac{3-7}{2}\right) = M(6,-2)$$

GRADIENT AC IS

$$\frac{y_2-y_1}{x_2-x_1} = \frac{-7-3}{8-4} = \frac{-10}{4} = -\frac{5}{2}$$

GRADIENT OF THE LINE THROUGH B & D MUST BE $+\frac{2}{5}$

FINALLY THE EQUATION OF THE REQUIRED LINE IS

$$\Rightarrow y - y_0 = m(x - x_0)$$
$$\Rightarrow y + 2 = \frac{2}{5}(x - 6)$$
$$\Rightarrow 5y + 10 = 2x - 12$$
$$\Rightarrow 5y = 2x - 22$$

Question 26 **(***)**

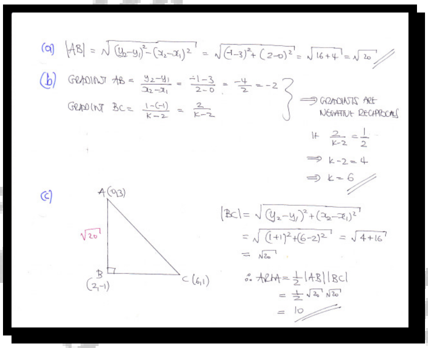
The points $A(0,3)$, $B(2,-1)$ and $C(k,1)$ are given, where k is a constant.

a) Find the exact length of AB.

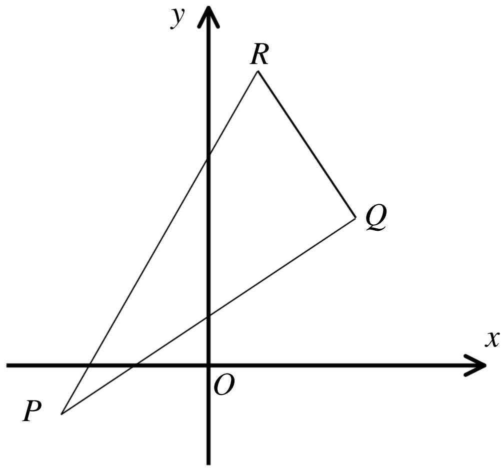
b) Given that AB is perpendicular to BC, find the value of k.

c) Determine the area of the triangle ABC.

$\boxed{|AB| = \sqrt{20} = 2\sqrt{5}}$, $\boxed{k=6}$, $\boxed{\text{area} = 10}$



Question 27 (***)



The figure above shows the right angled triangle PQR, whose vertices are located at $P(-6,-2)$ and $Q(6,6)$.

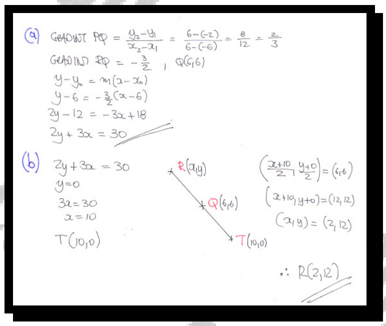
It is further given that $\measuredangle PQR = 90°$.

a) Find an equation for the straight line l, which passes through Q and R.

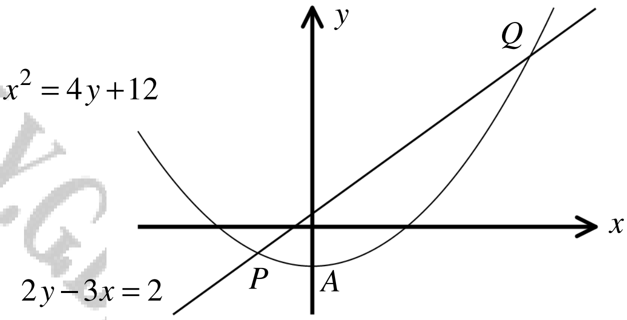
The straight line l meets the x axis at the point T.

b) Given that Q is the midpoint of RT, determine the coordinates of R.

$\boxed{}$, $\boxed{2y+3x=30}$, $\boxed{R(2,12)}$



Question 28 (***)



The figure above shows the curve C and the straight line L, with respective equations

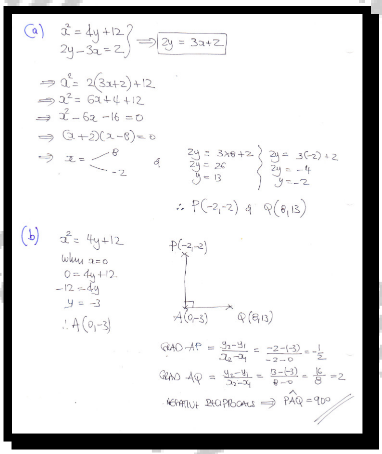
$$x^2 = 4y + 12 \quad \text{and} \quad 2y - 3x = 2.$$

C meets the y axis at the point A, while C and L intersect each other at the points P and Q.

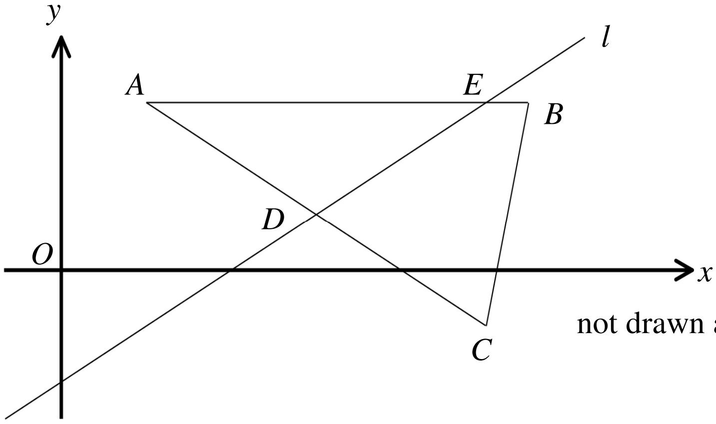
a) Find the coordinates of P and the coordinates of Q.

b) Show clearly that $\measuredangle PAQ = 90°$.

$\boxed{}$, $\boxed{P(-2,-2)}$, $\boxed{Q(8,13)}$



Question 29 (***)



not drawn accurately

The figure above shows a triangle with vertices at $A(2,6)$, $B(11,6)$ and $C(p,q)$.

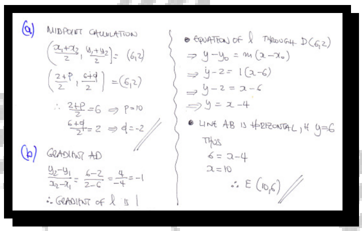
a) Given that the point $D(6,2)$ is the midpoint of AC, determine the value of p and the value of q.

The straight line l, passes through D and is perpendicular to AC.

The point E is the intersection of l and AB.

b) Find the coordinates of E.

$\boxed{}$, $\boxed{p=10}$, $\boxed{q=-2}$, $\boxed{E(10,6)}$



Question 30 **(***)**

The straight line PQ has equation

$$5x - 2y + 7 = 0.$$

The points P and Q have coordinates $(-1,1)$ and $(1,k)$, respectively.

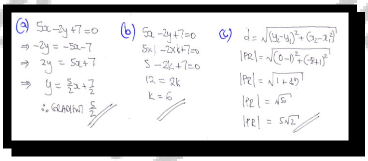
Calculate, showing a clear method …

a) … the gradient of PQ.

b) … the value of k.

c) … the distance PR, where R is the point $(-8,0)$.

$\boxed{\text{gradient} = \frac{5}{2}}$, $\boxed{k = 6}$, $\boxed{|PR| = 5\sqrt{2}}$


(a) $5x-2y+7=0$
$\Rightarrow -2y = -5x-7$
$\Rightarrow 2y = 5x+7$
$\Rightarrow y = \frac{5}{2}x + \frac{7}{2}$
$\therefore$ GRADIENT $\frac{5}{2}$

(b) $5x-2y+7=0$
$5\times 1 - 2\times k + 7 = 0$
$5 - 2k + 7 = 0$
$12 = 2k$
$k = 6$

(c) $d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$
$|PR| = \sqrt{(0-1)^2 + (-8+1)^2}$
$|PR| = \sqrt{1+49}$
$|PR| = \sqrt{50}$
$|PR| = 5\sqrt{2}$

Question 31 (***)

The points A and B have coordinates $(-1,5)$ and $(7,11)$, respectively.

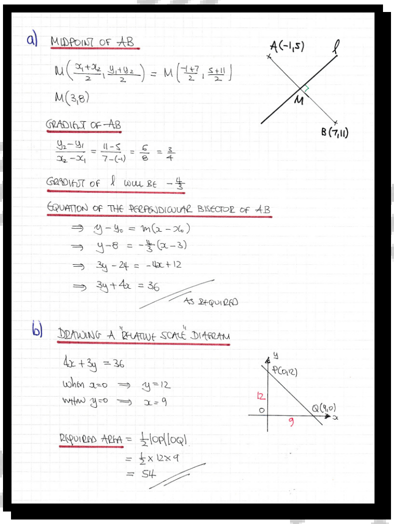
a) Show that the equation of the perpendicular bisector of AB is

$$4x+3y=36.$$

The perpendicular bisector of AB crosses the coordinate axes at the points P and Q.

b) Find the area of the triangle OPQ, where O is the origin.

$\boxed{}$, $\boxed{\text{area}=54}$


a) MIDPOINT OF AB

$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=M\left(\frac{-1+7}{2},\frac{5+11}{2}\right)$

$M(3,8)$

GRADIENT OF AB

$\frac{y_2-y_1}{x_2-x_1}=\frac{11-5}{7-(-1)}=\frac{6}{8}=\frac{3}{4}$

GRADIENT OF l WILL BE $-\frac{4}{3}$

EQUATION OF THE PERPENDICULAR BISECTOR OF AB

$\Rightarrow y-y_0=m(x-x_0)$

$\Rightarrow y-8=-\frac{4}{3}(x-3)$

$\Rightarrow 3y-24=-4x+12$

$\Rightarrow 3y+4x=36$

AS REQUIRED

b) DRAWING A "RELATIVE SCALE" DIAGRAM

$4x+3y=36$

when $x=0 \Rightarrow y=12$

when $y=0 \Rightarrow x=9$

REQUIRED AREA $=\frac{1}{2}|OP||OQ|$

$=\frac{1}{2}\times 12\times 9$

$=54$

Question 32 (***)

The points A, B and C have coordinates $(-1,5)$, $(7,-1)$ and $(6,6)$, respectively.

a) Find an equation of the straight line through A and B, giving the answer in the form $ax+by=c$, where a, b and c are integers.

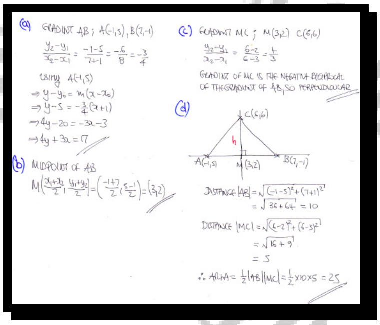
The midpoint of AB is the point M.

b) Find the coordinates of M.

c) Show that MC is perpendicular to AB.

d) Calculate the area of the triangle ABC.

$\boxed{3x+4y=17}$, $\boxed{M(3,2)}$, $\boxed{\text{area}=25}$



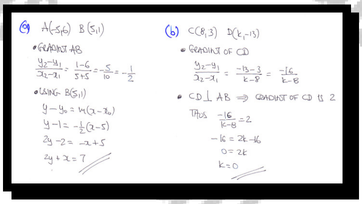
Question 33 **(***)**

The points A, B, C and D have coordinates $(-5,6)$, $(5,1)$, $(8,3)$ and $(k,-13)$, respectively, where k is a constant.

a) Find an equation of the straight line through A and B.

b) Given that CD is perpendicular to AB, find the value of k.

$\boxed{x+2y=7}$, $\boxed{k=0}$



Question 34 (***)

Relative to a fixed origin O the points A, B and C have respective coordinates $(-1,3)$, $(1,11)$ and $(13,k)$, where k is a constant.

a) Find the length of AB, in the form $a\sqrt{17}$, where a is an integer.

b) Given the length of BC is $3\sqrt{17}$, determine the possible values of k.

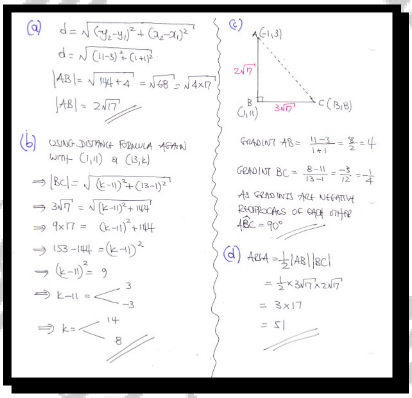
The actual value of k is in fact the smaller of the two values found in part **(b)**.

c) Show clearly that

$$\measuredangle ABC = 90° .$$

d) Calculate the area of the triangle ABC.

$\boxed{|AB| = 2\sqrt{17}}$, $\boxed{k = 8 \text{ or } 14}$, $\boxed{\text{area} = 51}$



(a) $d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$d = \sqrt{(11-3)^2+(1+1)^2}$

$|AB| = \sqrt{144+4} = \sqrt{68} = \sqrt{4\times17}$

$|AB| = 2\sqrt{17}$

(b) USING DISTANCE FORMULA AGAIN WITH $(1,11)$ & $(13,k)$

$\Rightarrow |BC| = \sqrt{(k-11)^2+(13-1)^2}$

$\Rightarrow 3\sqrt{17} = \sqrt{(k-11)^2+144}$

$\Rightarrow 9\times17 = (k-11)^2+144$

$\Rightarrow 153-144 = (k-11)^2$

$\Rightarrow (k-11)^2 = 9$

$\Rightarrow k-11 = 3$ or -3

$\Rightarrow k = 14$ or 8

(c)

GRADIENT $AB = \frac{11-3}{1+1} = \frac{8}{2} = 4$

GRADIENT $BC = \frac{8-11}{13-1} = \frac{-3}{12} = -\frac{1}{4}$

AS GRADIENTS ARE NEGATIVE RECIPROCALS OF EACH OTHER $\hat{ABC} = 90°$

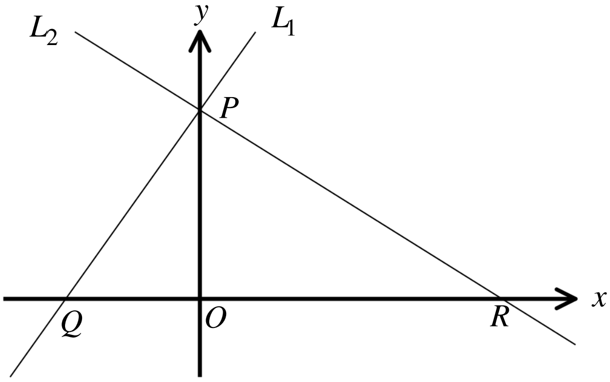
(d) AREA $= \frac{1}{2}|AB||BC|$

$= \frac{1}{2}\times3\sqrt{17}\times2\sqrt{17}$

$= 3\times17$

$= 51$

Question 35 (***)



The figure above shows the straight line L_1 with equation

$$3x - 2y + 18 = 0.$$

The straight line L_2 is perpendicular to L_1 and the two lines meet each other at the point $P(0, p)$.

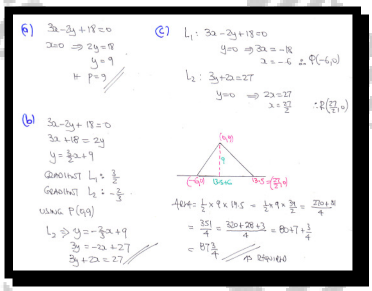
The straight lines L_1 and L_2 cross the x axis at the points Q and R, respectively.

a) Find the value of p.

b) Determine an equation for L_2.

c) Show that the area of the triangle PQR is 87.75 square units.

$\boxed{}$, $\boxed{p = 9}$, $\boxed{2x + 3y = 27}$



Question 36 (***)

The straight line l_1 passes through the points $A(1,2)$ and $B(7,-2)$.

a) Determine an equation for l_1, giving the answer in the form $ax+by=c$, where a, b and c are integers.

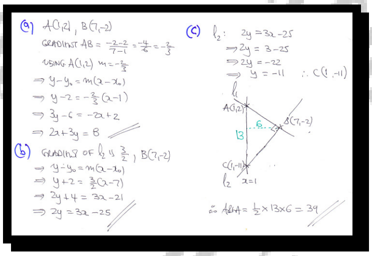
The straight line l_2 passes through B and is perpendicular to l_1.

b) Find an equation for l_2.

l_2 meets the straight line with equation $x=1$ at the point C.

c) Calculate the area of the triangle ABC.

$\boxed{}$, $\boxed{2x+3y=8}$, $\boxed{3x-2y-25=0}$, $\boxed{\text{area}=39}$



Question 37 **(***)**

The straight line l_1 passes through the points $A(8,-2)$ and $B(10,1)$.

a) Determine an equation of l_1 giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

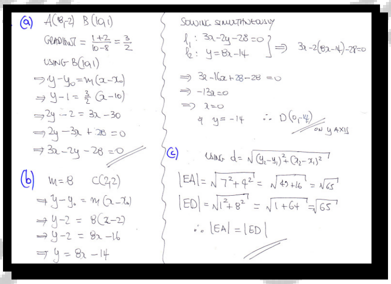
The straight line l_2 has gradient 8 and passes through the point $C(2,2)$. The two lines meet at the point D.

b) Show that D lies on the y axis.

The point E has coordinates $(1,-6)$.

c) Show clearly that $|EA|=|ED|$.

$$\boxed{3x-2y-28=0}$$



Question 38 (***)

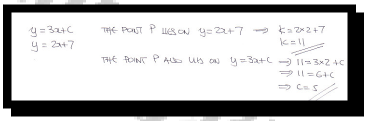
The straight lines with equations

$$y = 3x + c \quad \text{and} \quad y = 2x + 7$$

intersect at the point $P(2,k)$, where c and k are constants.

Find the value of c and the value of k.

$\boxed{}$, $\boxed{c = 5, \ \ k = 11}$



Question 39 (***)

The straight line AB has equation $3x + 4y = 9$ and C is the point $(6,4)$.

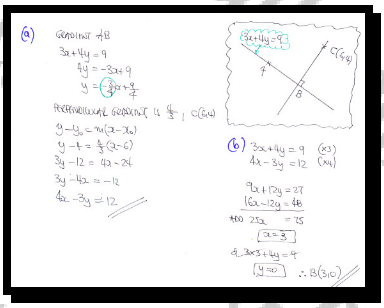
The straight line BC is perpendicular to AB.

Find, showing a clear method, …

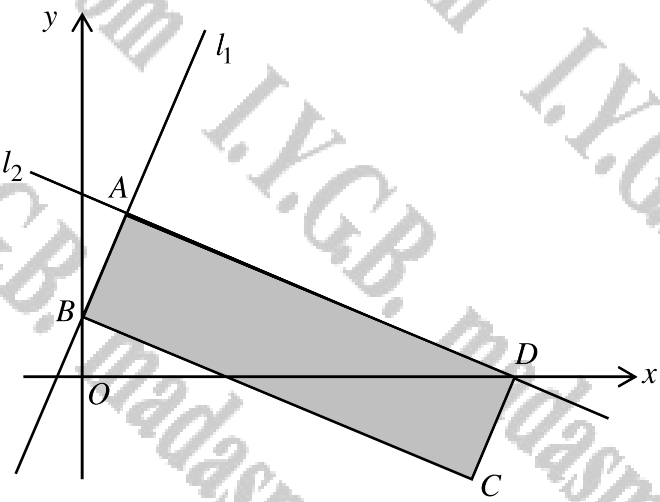
a) … an equation for the straight line BC, giving the answer in the form $ax + by = c$, where a, b and c are integers.

b) … the coordinates of B.

$\boxed{4x - 3y = 12}$, $\boxed{B(3,0)}$



Question 40 (***)



The straight line l_1 has equation

$$3x - 2y + 6 = 0,$$

and crosses the y axis at the point B.

a) Find the gradient of l_1.

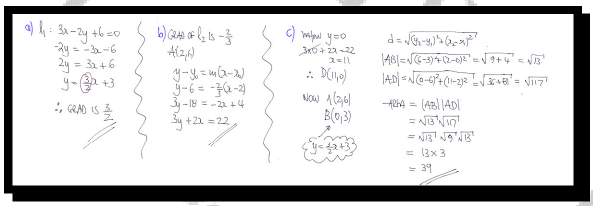
The straight line l_2 intersects l_1 at the point $A(2,6)$ and crosses the x axis at the point D.

b) Given that $\measuredangle BAD = 90°$, find an equation of l_2.

The point C is such so that $ABCD$ is a rectangle, as shown in the figure above.

c) Calculate the area of the rectangle $ABCD$.

$\boxed{}$, $\boxed{\text{gradient} = \frac{3}{2}}$, $\boxed{3y + 2x = 22}$, $\boxed{\text{area} = 39}$



Question 41 (***)

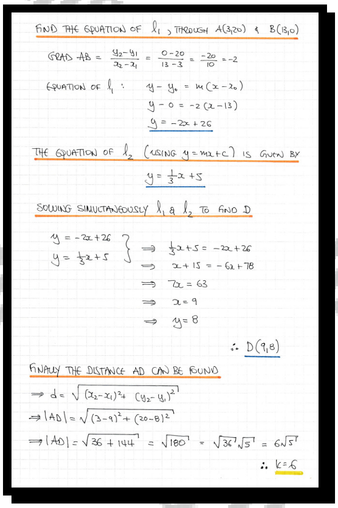
The straight line l_1 passes through the points $A(3,20)$ and $B(13,0)$.

The straight line l_2 has gradient $\frac{1}{3}$ and passes through the point $C(0,5)$.

The point D is the intersection of l_1 and l_2.

Show that the length of AD is $k\sqrt{5}$, where k is an integer.

$\boxed{}$, $\boxed{k=6}$


FIND THE EQUATION OF l_1, THROUGH $A(3,20)$ & $B(13,0)$

$$\text{GRAD } AB = \frac{y_2-y_1}{x_2-x_1} = \frac{0-20}{13-3} = \frac{-20}{10} = -2$$

EQUATION OF l_1:

$$y - y_0 = m(x - x_0)$$
$$y - 0 = -2(x-13)$$
$$y = -2x+26$$

THE EQUATION OF l_2 (USING $y = mx+c$) IS GIVEN BY

$$y = \frac{1}{3}x + 5$$

SOLVING SIMULTANEOUSLY l_1 & l_2 TO FIND D

$$\left.\begin{array}{l} y = -2x+26 \\ y = \frac{1}{3}x+5 \end{array}\right\} \Rightarrow \frac{1}{3}x + 5 = -2x+26$$
$$\Rightarrow x + 15 = -6x + 78$$
$$\Rightarrow 7x = 63$$
$$\Rightarrow x = 9$$
$$\Rightarrow y = 8$$

$\therefore D(9,8)$

FINALLY THE DISTANCE AD CAN BE FOUND

$$\Rightarrow d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$\Rightarrow |AD| = \sqrt{(3-9)^2 + (20-8)^2}$$
$$\Rightarrow |AD| = \sqrt{36+144} = \sqrt{180} = \sqrt{36}\sqrt{5} = 6\sqrt{5}$$

$\therefore k = 6$

44 STANDARD QUESTIONS

Question 1 (***+)

The points A and B have coordinates $(-4,4)$ and $(2,6)$, respectively.

The straight line L_1 passes through the point B and is perpendicular to the straight line which passes through A and B.

a) Find an equation of L_1.

L_1 meets the y axis at the point C.

b) Show by calculation that

$$|AB| = |BC|.$$

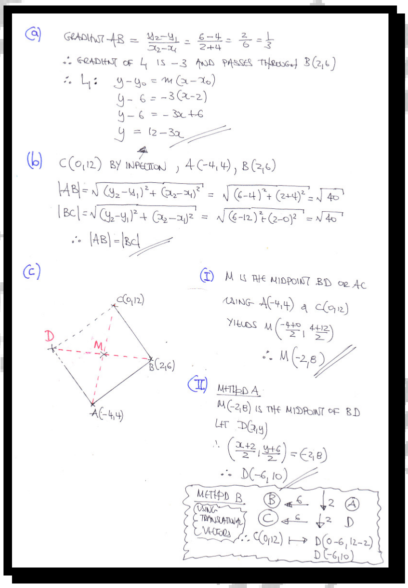
The quadrilateral $ABCD$ is a square whose diagonals intersect at the point M.

c) Determine …

i. … the coordinates of M.

ii. … the coordinates of D.

$\boxed{}$, $\boxed{y = 12 - 3x}$, $\boxed{M(-2,8)}$, $\boxed{D(-6,10)}$



Question 2 (***+)

The straight line l_1 crosses the coordinate axes at the points $A(-6,0)$ and $B(0,18)$.

a) Determine an equation for l_1, giving the answer in the form $y = mx + c$, where m and c are constants.

The straight line l_2 has equation

$$x - 7y = 14,$$

and meets l_1 at the point D.

b) Find the coordinates of D.

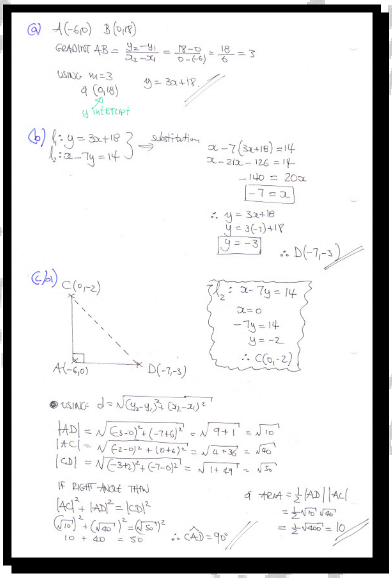
l_2 meets the y axis at the point C.

c) Show clearly that

$$\measuredangle CAD = 90°.$$

d) Calculate the area of the triangle CAD.

$\boxed{y = 3x + 18}$, $\boxed{D(-7,-3)}$, $\boxed{\text{area} = 10}$



Question 3 (***+)

The straight line L passes through the points $A(-2,2)$ and $B(1,3)$.

a) Find an equation for L.

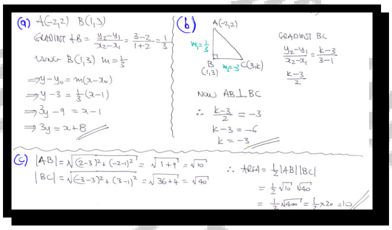
The point C has coordinates $(3,k)$.

Given that the angle ABC is $90°$, find …

b) … the value of k.

c) … the area of the triangle ABC.

$\boxed{x-3y+8=0}$, $\boxed{k=-3}$, $\boxed{\text{area}=10}$



Question 4 (***+)

The straight line segment joining the points with coordinates $(-7,k)$ and $(-2,-11)$, where k is a constant, has gradient -2.

a) Determine the value of k.

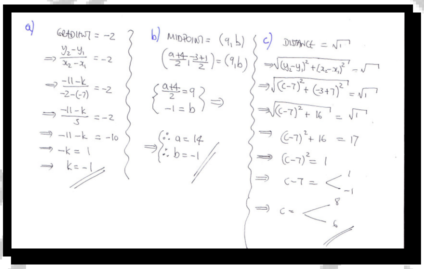
The midpoint of the straight line segment joining the points with coordinates $(a,-3)$ and $(4,1)$ has coordinates $(9,b)$, where a and b are constants.

b) Find the value of a and the value of b.

The straight line segment joining the points with coordinates $(-7,7)$ and $(-3,c)$, where c is a constant, has length $\sqrt{17}$.

c) Determine the possible values of c.

$\boxed{}$, $\boxed{k=-1}$, $\boxed{a=14}$, $\boxed{b=-1}$, $\boxed{c=6,\ 8}$



Question 5 (***+)

The points $A(-1,3)$, $B(3,1)$ and $C(5,5)$ are given.

a) Show that AB is perpendicular to BC.

b) Find an equation for the straight line which passes through B and C.

The straight line through A and C has equation

$$x-3y+10=0.$$

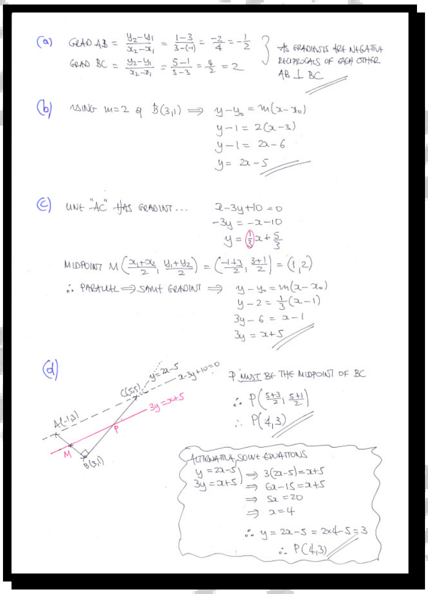
The midpoint of AB is the point M.

c) Determine an equation of the straight line which passes through M and is parallel to the straight line through A and C.

The straight line which passes through M and is parallel to the straight line through A and C, meets BC at the point P.

d) **State** the coordinates of P.

$\boxed{}$, $\boxed{y=2x-5}$, $\boxed{3y=x+5}$, $\boxed{P(4,3)}$



Question 6 (***+)

The points A, B, C and D have coordinates $(0,4)$, $(2,8)$, $(3,0)$ and $(6,k)$, respectively, where k is a constant.

The straight line L_1 passes through the points A and B, and the straight line L_2 passes through the points C and D.

a) Given that L_1 is parallel to L_2, find an equation for L_2.

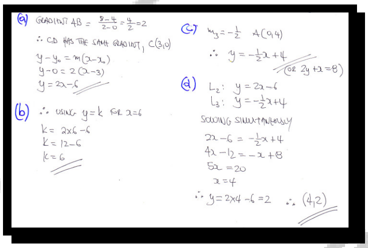
b) Find the value of k.

The straight line L_3 passes through the point A is perpendicular to L_2.

c) Determine an equation for L_3.

d) Show that L_2 and L_3 meet at the point with coordinates $(4,2)$.

$\boxed{y = 2x - 6}$, $\boxed{k = 6}$, $\boxed{x + 2y = 8}$


(a) GRADIENT AB $= \frac{8-4}{2-0} = \frac{4}{2} = 2$

∴ CD HAS THE SAME GRADIENT, C(3,0)

$y - y_0 = m(x - x_0)$

$y - 0 = 2(x-3)$

$y = 2x - 6$

(b) ∴ USING $y = k$ FOR $x = 6$

$k = 2\times6 - 6$

$k = 12 - 6$

$k = 6$

(c) $m_3 = -\frac{1}{2}$ A(0,4)

∴ $y = -\frac{1}{2}x + 4$ (OR $2y + x = 8$)

(d) L_2: $y = 2x - 6$

L_3: $y = -\frac{1}{2}x + 4$

SOLVING SIMULTANEOUSLY

$2x - 6 = -\frac{1}{2}x + 4$

$4x - 12 = -x + 8$

$5x = 20$

$x = 4$

∴ $y = 2\times4 - 6 = 2$ ∴ (4,2)

Question 7 **(***+)**

The straight line l_1 passes through the points $A(0,3)$ and $B(12,9)$.

a) Find an equation for l_1.

The straight line l_2 passes through the point $C(11,1)$ and is perpendicular to l_1.

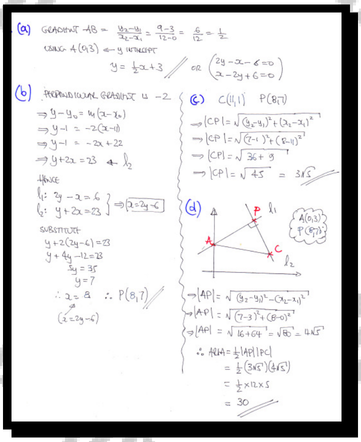
The two lines intersect at the point P.

b) Calculate the coordinates of P.

c) Determine the length of CP.

d) Hence, or otherwise, show that the area of the triangle APC is 30 square units.

$\boxed{}$, $\boxed{x-2y+6=0}$, $\boxed{P(8,7)}$, $\boxed{|CP|=3\sqrt{5}}$



Question 8 (***+)

The straight lines l_1 and l_2 have respective equations

$$x + 4y + 5 = 0 \text{ and } y = 2x - 2 .$$

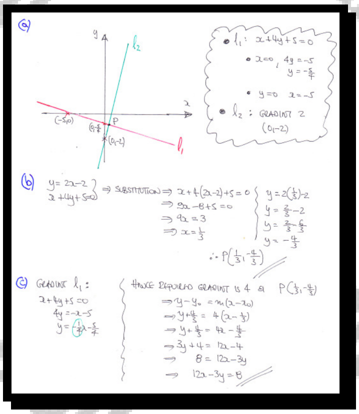
These two lines intersect at the point P.

a) Sketch l_1 and l_2 on the same diagram, showing clearly all the points where each of these lines meet the coordinate axes.

b) Calculate the exact coordinates of P.

c) Determine an equation of the straight line which passes through P, and is perpendicular to l_1.
Give the answer in the form $ax + by = c$, where a, b and c are integers.

$\boxed{}$, $\boxed{P\left(\frac{1}{3}, -\frac{4}{3}\right)}$, $\boxed{12x - 3y = 8}$



Question 9 **(***+)**

The points A, B and C have coordinates $(-6,5)$, $(0,7)$ and $(8,3)$, respectively.

The straight line L_1 is parallel to BC and passes through the point A.

a) Show that an equation for L_1 is

$$x+2y=4.$$

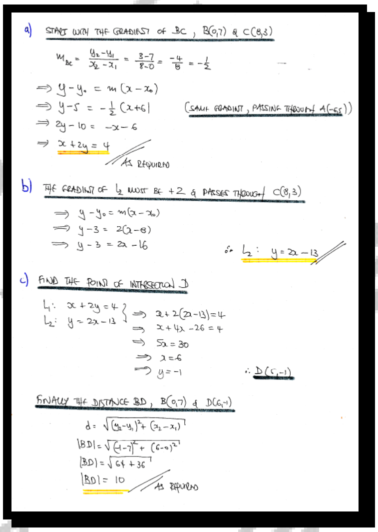
The straight line L_2 passes through the point C is perpendicular to BC.

b) Find an equation for L_2.

The lines L_1 and L_2 meet at the point D.

c) Show that the distance BD is 10 units.

$\boxed{}$, $\boxed{y=2x-13}$



a) START WITH THE GRADIENT OF BC, B(0,7) & C(8,3)

$m_{BC} = \frac{y_2-y_1}{x_2-x_1} = \frac{3-7}{8-0} = \frac{-4}{8} = -\frac{1}{2}$

$\Rightarrow y-y_0 = m(x-x_0)$

$\Rightarrow y-5 = -\frac{1}{2}(x+6)$ (SAME GRADIENT, PASSING THROUGH A(-6,5))

$\Rightarrow 2y-10 = -x-6$

$\Rightarrow x+2y=4$ AS REQUIRED

b) THE GRADIENT OF L_2 MUST BE +2 & PASSES THROUGH C(8,3)

$\Rightarrow y-y_0 = m(x-x_0)$

$\Rightarrow y-3 = 2(x-8)$

$\Rightarrow y-3 = 2x-16$

$\therefore L_2: y=2x-13$

c) FIND THE POINT OF INTERSECTION D

$L_1: x+2y=4$, $L_2: y=2x-13$

$\Rightarrow x+2(2x-13)=4$

$\Rightarrow x+4x-26=4$

$\Rightarrow 5x=30$

$\Rightarrow x=6$

$\Rightarrow y=-1$ $\therefore D(6,-1)$

FINALLY THE DISTANCE BD, B(0,7) & D(6,-1)

$d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$|BD| = \sqrt{(-1-7)^2+(6-0)^2}$

$|BD| = \sqrt{64+36}$

$|BD| = 10$ AS REQUIRED

Question 10 (***+)

The points $A(-1,-1)$, $B(8,2)$ and $C(0,1)$ are given.

a) Find an equation for the straight line L_1, which passes through A and B.

The straight line L_2 has gradient -3 and passes through C.

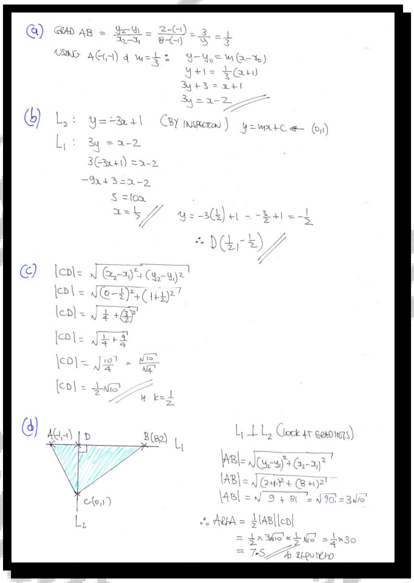
L_1 and L_2 intersect at the point D.

b) Find the exact coordinates of D.

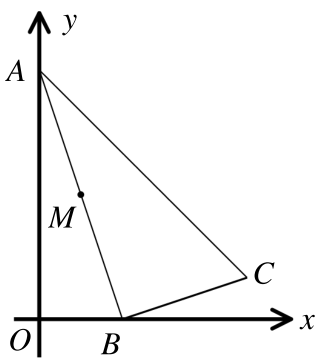
c) Find the exact length of CD, giving the answer in the form of $k\sqrt{10}$, where k is a constant.

d) Hence, or otherwise, show clearly that the area of the triangle ABC is 7.5 square units.

$\boxed{3y = x - 2}$, $\boxed{D\left(\frac{1}{2}, -\frac{1}{2}\right)}$, $\boxed{|CD| = \frac{1}{2}\sqrt{10}}$



Question 11 (***+)



The figure above shows the points $A(0,12)$, B, $C(10,2)$ and $M(2,6)$.

a) Given that M is the midpoint of AB, state the coordinates of B.

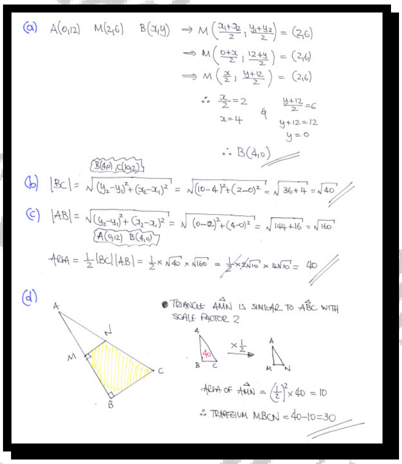
b) Find the exact length of BC.

c) Given that $\measuredangle ABC = 90°$, find the area of the triangle ABC, giving the final answer as an integer.

The straight line through M and perpendicular to AB meets AC at the point N.

d) Determine the area of the trapezium $MBCN$.

$\boxed{}$, $\boxed{B(4,0)}$, $\boxed{|BC| = \sqrt{40}}$, $\boxed{\text{area of } \triangle ABC = 40}$, $\boxed{\text{area of } \triangle MBCN = 30}$



Question 12 (***+)

The straight line L_1 has gradient 3 and y intercept of -8.

The straight line L_2 is perpendicular to L_1 and passes through the point $(7,3)$.

The point P is the intersection of L_1 and L_2.

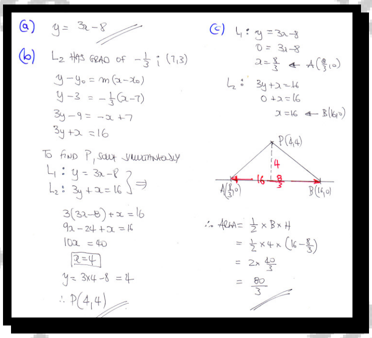
a) Write down an equation for L_1.

b) Find the coordinates of P.

L_1 and L_2 meet the x axis at the points A and B, respectively.

c) Determine the exact area of the triangle APB.

$\boxed{y = 3x - 8}$, $\boxed{P(4,4)}$, $\boxed{\text{area} = \frac{80}{3}}$



Question 13 (***+)

The straight line L_1 passes through the points $A(1,4)$ and $B(3,9)$.

a) Find an equation for L_1, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

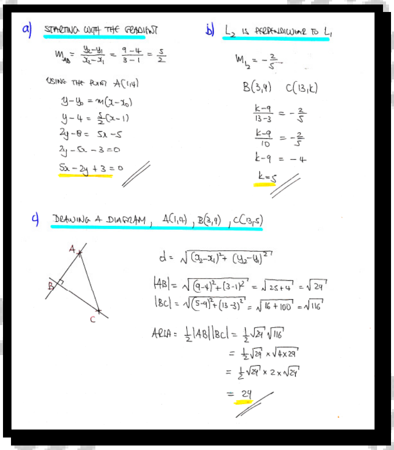
The straight line L_2 is perpendicular to L_1 and passes through the points B and C.

Given the point C has coordinates $(13,k)$, find …

b) … the value of k.

c) … the area of the triangle ABC.

$\boxed{5x-2y+3=0}$, $\boxed{k=5}$, $\boxed{\text{area}=29}$



Question 14 (***+)

The straight line PQ has equation

$$5x + 3y = 18$$

and P is the point with coordinates $(9, -9)$.

a) Find an equation for the straight line that is perpendicular to the line PQ and passing through the point P, giving the answer in the form $ax + by = c$, where a, b and c are integers.

The straight line QR has equation

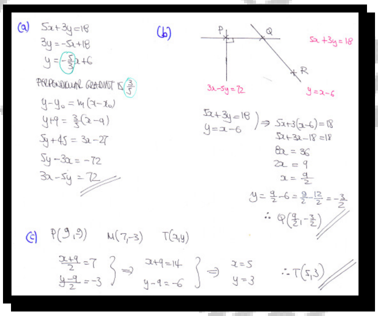
$$y = x - 6.$$

b) Find the exact coordinates of Q.

The point $M(7, -3)$ is the midpoint of PT.

c) Find the coordinates of T.

$\boxed{3x - 5y = 72}$, $\boxed{Q\left(\frac{9}{2}, -\frac{3}{2}\right)}$, $\boxed{T(5,3)}$



Question 15 (***+)

The straight line l_1 passes through the points $A(-2,-3)$ and $B(1,-12)$.

a) Find an equation for l_1.

The point M is the midpoint of AB.

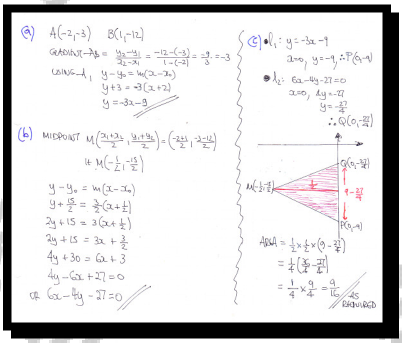
The straight line l_2 passes through M and has gradient $\frac{3}{2}$.

b) Find an equation for l_2.

l_1 and l_2 cross the y axis at the points P and Q, respectively.

c) Show that the area of the triangle PMQ is $\frac{9}{16}$.

$\boxed{y = -3x - 9}$, $\boxed{6x - 4y - 27 = 0}$



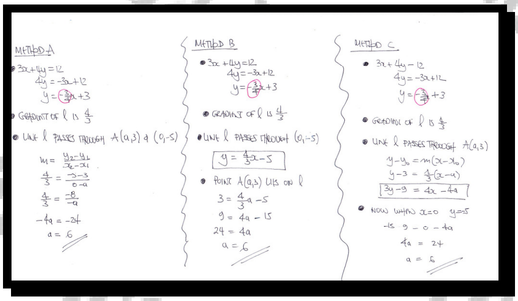
Question 16 (***+)

The straight line l passes through the point $A(a,3)$, where a is a constant, and is perpendicular to the line with equation

$$3x+4y=12.$$

Given that l crosses the y axis at $(0,-5)$, find the value of a.

$\boxed{a=6}$

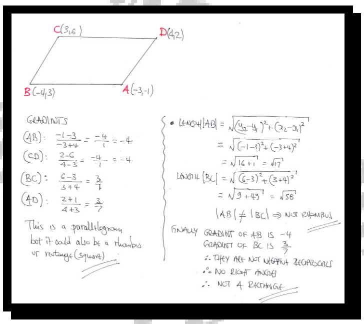


Question 17 (***+)

The points $A(-3,-1)$, $B(-4,3)$, $C(3,6)$ and $D(4,2)$ form the vertices of the quadrilateral $ABCD$.

By calculating relevant gradients and lengths show that $ABCD$ is a parallelogram but not a rhombus or a rectangle.

proof



Question 18 (***+)

A triangle has vertices at $A(1,0)$, $B(9,4)$ and $C(k,6)$.

a) Given that $\measuredangle ABC = 90°$, show that $k = 8$.

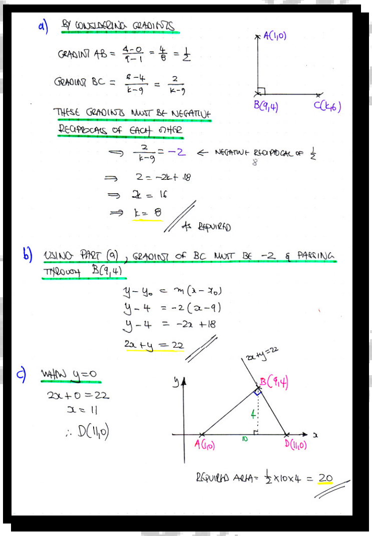
b) Find an equation of the straight line BC, giving the answer in the form $ax + by = c$, where a, b and c are integers.

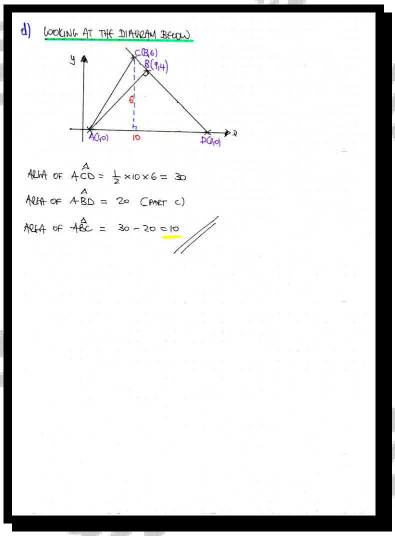
The line BC meets the x axis at the point D.

c) Find the area of the triangle ABD.

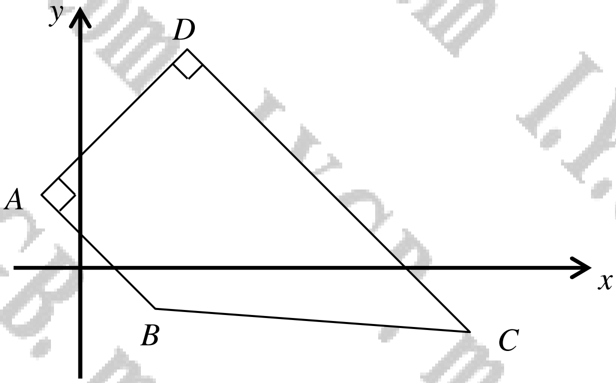
d) Hence, or otherwise, determine the area of the triangle ABC.

$\boxed{}$, $\boxed{2x + y = 22}$, $\boxed{\text{area } ABD = 20}$, $\boxed{\text{area } ABC = 10}$





Question 19 (***+)



The figure above shows a trapezium $ABCD$.

The side AB is parallel to CD and the angles BAD and ADC are both right angles.

The coordinates of D are $(4,7)$, and the straight line through A and B has equation

$$5x + 4y = 7.$$

a) Show that an equation for the straight line through C and D is

$$5x + 4y = 48.$$

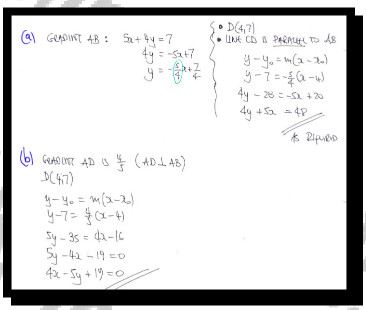
b) Find an equation for the straight line through A and D.

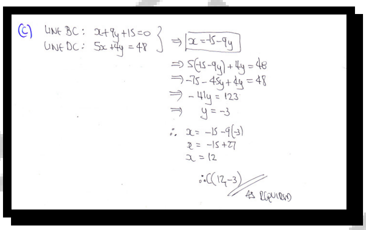
The straight line through B and C has equation

$$x + 9y + 15 = 0.$$

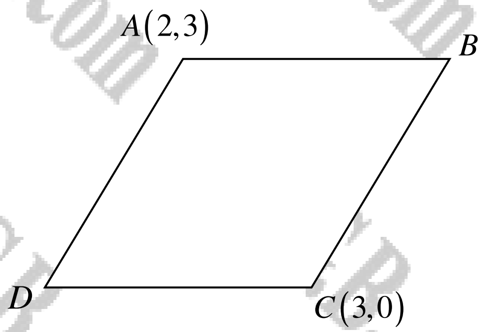
c) Show that the coordinates of C are $(12,-3)$.

$\boxed{}$, $\boxed{4x - 5y + 19 = 0}$





Question 20 (***+)



The figure above shows a rhombus $ABCD$, where the vertices A and C have coordinates $(2,3)$ and $(3,0)$, respectively.

a) Show that an equation of the diagonal BD is

$$x - 3y + 2 = 0.$$

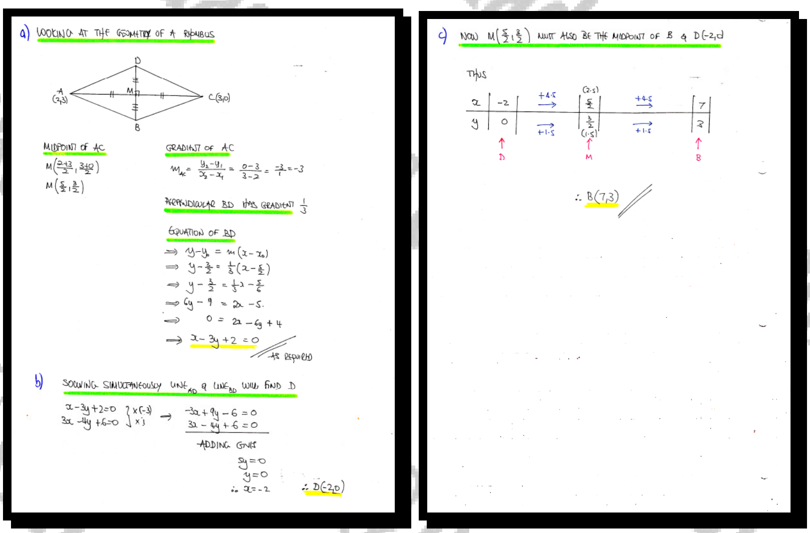
b) Given that an equation of the line through A and D is

$$3x - 4y + 6 = 0,$$

find the coordinates of D.

c) State the coordinates of B.

[ ], $D(-2,0)$, $B(7,3)$



Question 21 (***+)

The straight line l_1 passes through the point $(10,-3)$ and has gradient $\frac{1}{3}$.

a) Find an equation for l_1, in the form $ax+by+c=0$, where a, b and c are integers.

The straight line l_2 has gradient of -2 and y intercept of 3.

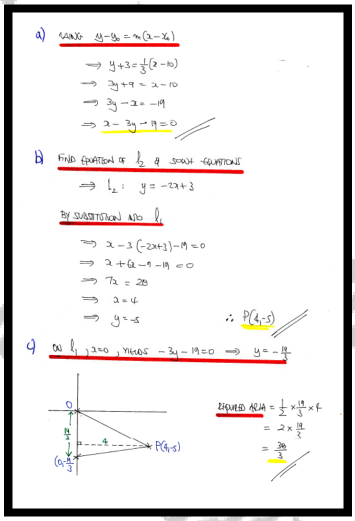
l_1 and l_2 intersect at the point P.

b) Determine the coordinates of P.

l_1 meets the y axis at the point Q.

c) Calculate the exact area of the triangle OPQ, where O is the origin.

[ ], $x-3y-19=0$, $P(4,-5)$, $\text{area} = \frac{38}{3}$



Question 22 (***+)

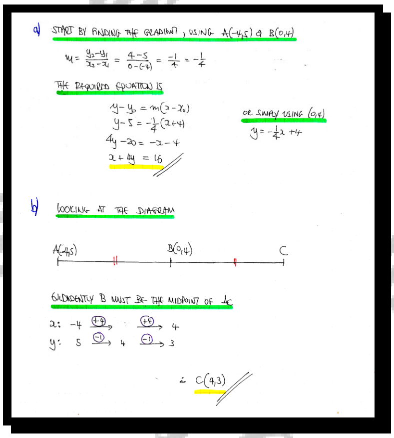
The points A and B have coordinates $(-4,5)$ and $(0,4)$, respectively.

a) Find an equation of the straight line which passes through A and B.

The point C lies on the straight line through A and B, so that the distance of AB is the same as the distance of BC.

b) Find the coordinates of C.

MP1-I, $\boxed{x+4y=16}$, $\boxed{C(4,3)}$



Question 23 (***+)

The distance of the point A from the origin O is exactly $10\sqrt{2}$.

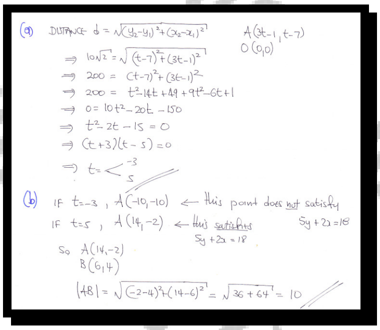
a) Given the coordinates of A are $(3t-1, t-7)$, where t is a constant, determine the possible values of t.

It is further given that A lies on the straight line with equation

$$5y + 2x = 18.$$

b) Calculate the distance AB if B has coordinates $(6,4)$.

$\boxed{}$, $\boxed{k = -3 \text{ or } 5}$, $\boxed{|AB| = 10}$


(a) DISTANCE $d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$ $A(3t-1, t-7)$ $O(0,0)$

$\Rightarrow 10\sqrt{2} = \sqrt{(t-7)^2+(3t-1)^2}$

$\Rightarrow 200 = (t-7)^2+(3t-1)^2$

$\Rightarrow 200 = t^2-14t+49+9t^2-6t+1$

$\Rightarrow 0 = 10t^2-20t-150$

$\Rightarrow t^2-2t-15=0$

$\Rightarrow (t+3)(t-5)=0$

$\Rightarrow t = -3, 5$

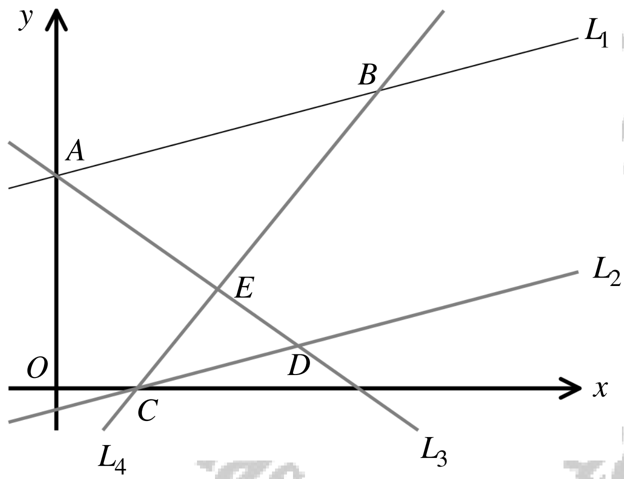
(b) IF $t=-3$, $A(-10,-10)$ ← this point does not satisfy $5y+2x=18$

IF $t=5$, $A(14,-2)$ ← this satisfies $5y+2x=18$

So $A(14,-2)$, $B(6,4)$

$|AB| = \sqrt{(-2-4)^2+(14-6)^2} = \sqrt{36+64} = 10$

Question 24 (***+)



The figure above shows the points $A(0,5)$, $B(4,7)$, $C(1,0)$ and $D(3,1)$.

The straight line L_1 passes through A and B, and the straight line L_2 passes through C and D.

a) Find an equation for L_1.

b) Show that L_2 is parallel to L_1.

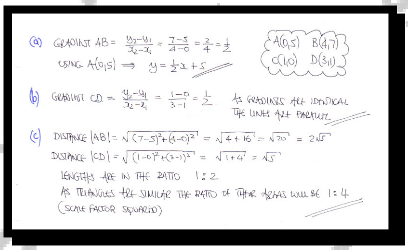
The straight line L_3 passes through A and D, and the straight line L_4 passes through B and C.

The lines L_3 and L_4 intersect at the point E.

c) Determine the ratio of the area of the triangle ABE to that of ECD.

The individual calculations of these areas are not needed for this part.

$\boxed{}$, $\boxed{y = \frac{1}{2}x + 5}$, $\boxed{4:1}$



Question 25 (***+)

The straight line l_1 with equation $3x-2y=5$ crosses the y axis at the point P, and the point Q has coordinates $(6,-2)$.

a) Find the exact coordinates of the midpoint of PQ.

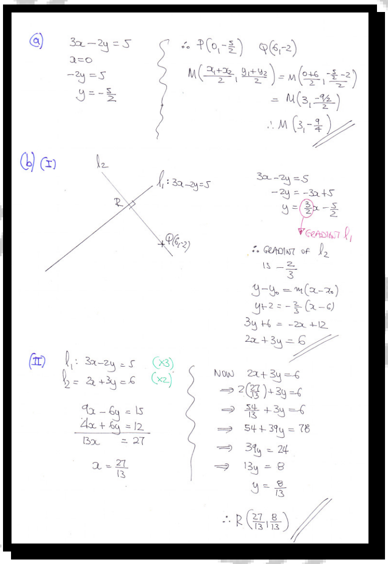
The straight line l_2 passes through the point Q and is perpendicular to l_1. The two lines intersect at the point R.

b) Find, showing a clear method, …

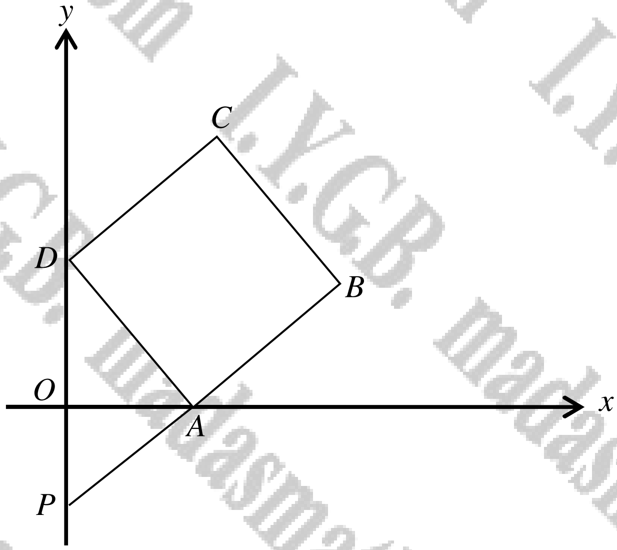
i. … an equation for l_2, giving the answer in the form $ax+by=c$, where a, b and c are integers.

ii. ... the exact coordinates of R.

$\boxed{M\left(3,-\frac{9}{4}\right)}$, $\boxed{2x+3y=6}$, $\boxed{R\left(\frac{27}{13},\frac{8}{13}\right)}$



Question 26 (***+)

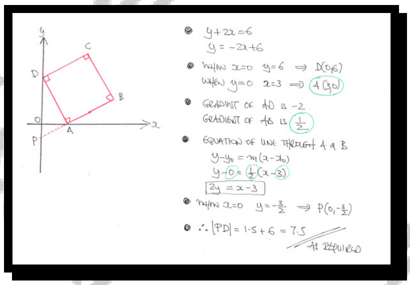


The figure above shows the square $ABCD$, where the vertices of A and D lie on the x axis and the y axis, respectively.

The point P lies on the y axis so that PAB is a straight line.

Given that the equation of the straight line through A and D is $y + 2x = 6$, show clearly that the distance PD is 7.5 units.

[ ] , [proof]



Question 27 **(***+)**

The straight line l_1 passes through the points $A(-4,-7)$ and $B(4,9)$.

The straight line l_2 has equation

$$y = \frac{1}{2}x + 4,$$

and meets the l_1 at the point C.

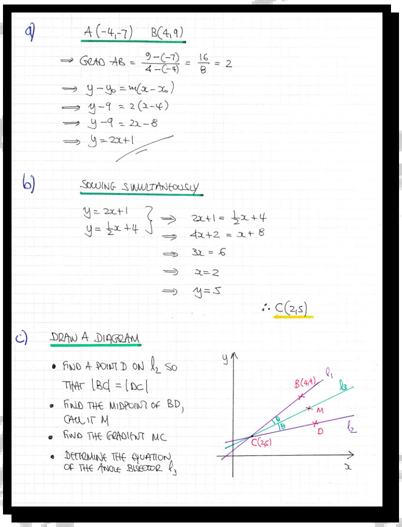
a) Determine an equation for l_1.

b) Find the coordinates of C.

The straight line l_3 is the bisector of the angle formed by l_1 and l_2.

c) Given that l_3 has positive gradient, determine an equation for l_3.

$\boxed{}$, $\boxed{y = 2x + 1}$, $\boxed{C(2,5)}$, $\boxed{y = x + 3}$


a) $A(-4,-7)$ $B(4,9)$

$\Rightarrow$ GRAD AB $= \frac{9-(-7)}{4-(-4)} = \frac{16}{8} = 2$

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 9 = 2(x-4)$

$\Rightarrow y - 9 = 2x - 8$

$\Rightarrow y = 2x + 1$

b) SOLVING SIMULTANEOUSLY

$y = 2x+1$, $y = \frac{1}{2}x + 4$ $\Rightarrow 2x + 1 = \frac{1}{2}x + 4$

$\Rightarrow 4x + 2 = x + 8$

$\Rightarrow 3x = 6$

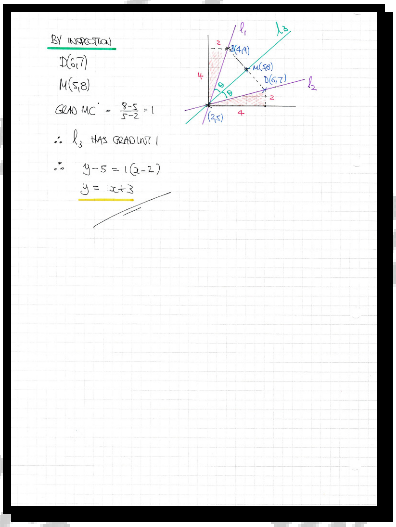
$\Rightarrow x = 2$

$\Rightarrow y = 5$

$\therefore C(2,5)$

c) DRAW A DIAGRAM

- FIND A POINT D ON l_2 SO THAT $|BC| = |DC|$
- FIND THE MIDPOINT OF BD, CALL IT M
- FIND THE GRADIENT MC
- DETERMINE THE EQUATION OF THE ANGLE BISECTOR l_3


BY INSPECTION

D(6,7)

M(5,8)

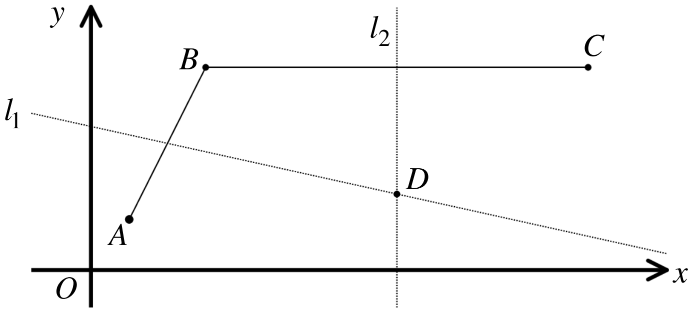
GRAD MC $= \frac{8-5}{5-2} = 1$

$\therefore l_3$ HAS GRADIENT 1

$\therefore y - 5 = 1(x-2)$

$y = x + 3$

Question 28 (***+)



The points $A(1,2)$, $B(3,8)$ and $C(13,8)$ are shown in the figure above.

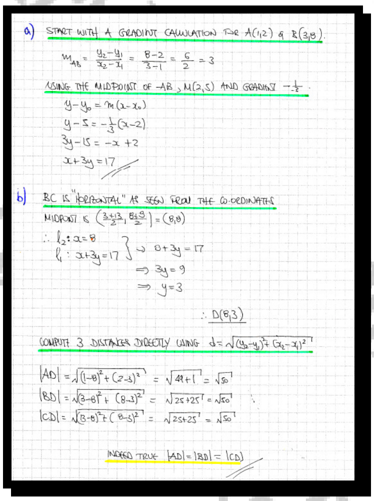
The straight lines l_1 and l_2 are the perpendicular bisectors of the straight line segments AB and BC, respectively.

a) Find an equation for l_1.

The point D is the intersection of l_1 and l_2.

b) Show by a direct algebraic method that D is equidistant from A, B and C.
You may not use any circle theorems in this part of the question.

[ ], $\boxed{x+3y=17}$



a) START WITH A GRADIENT CALCULATION FOR $A(1,2)$ & $B(3,8)$.

$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{8-2}{3-1} = \frac{6}{2} = 3$

USING THE MIDPOINT OF AB, $M(2,5)$ AND GRADIENT $-\frac{1}{3}$

$y - y_0 = m(x - x_0)$
$y - 5 = -\frac{1}{3}(x-2)$
$3y - 15 = -x + 2$
$x + 3y = 17$

b) BC IS "HORIZONTAL" AS SEEN FROM THE CO-ORDINATES
MIDPOINT IS $\left(\frac{3+13}{2}, \frac{8+8}{2}\right) = (8,8)$

$\therefore l_2 : x = 8$
$l_1 : x + 3y = 17$ $\Rightarrow 8 + 3y = 17 \Rightarrow 3y = 9 \Rightarrow y = 3$

$\therefore D(8,3)$

COMPUTE 3 DISTANCES DIRECTLY USING $d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$|AD| = \sqrt{(1-8)^2 + (2-3)^2} = \sqrt{49+1} = \sqrt{50}$
$|BD| = \sqrt{(3-8)^2 + (8-3)^2} = \sqrt{25+25} = \sqrt{50}$
$|CD| = \sqrt{(13-8)^2 + (8-3)^2} = \sqrt{25+25} = \sqrt{50}$

INDEED TRUE $|AD| = |BD| = |CD|$

Question 29 (****)

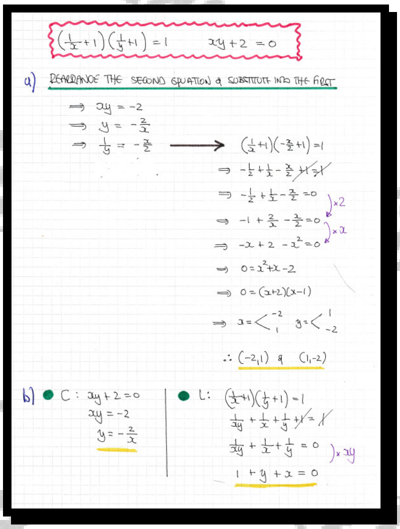
A straight line L and a curve C have the following equations.

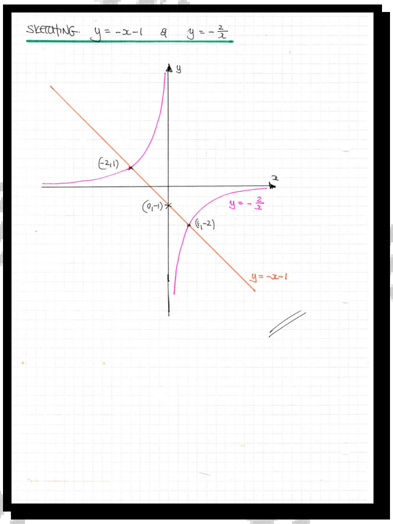
$$L: \left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)=1 \quad \text{and} \quad C: xy+2=0.$$

a) Find the coordinates of the points of intersection between L and C.

b) Sketch the graphs of L and C in the same set of axes.

[illegible], $(-2,1)$, $(1,-2)$





Question 30 (****)

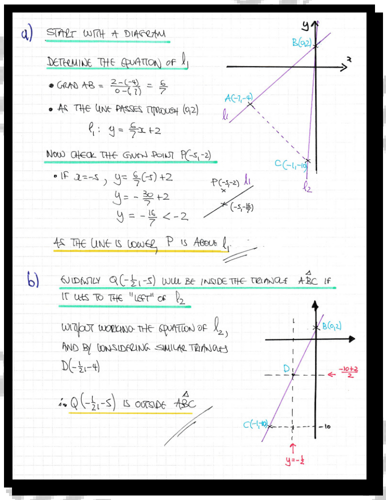
The straight line l_1 passes through the point $A(-7,-4)$ and meets the y axis at the point $B(0,2)$.

a) Determine, with full justification, whether the point $P(-5,-2)$ lies above l_1 or below l_1.

The straight line l_2 passes through the point $C(-1,-10)$ and meets the l_1 at B.

b) Determine, with full justification, whether the point $Q\left(-\frac{1}{2},-5\right)$ lies inside or outside the triangle ABC.

MP1-, P is above l_1, Q is outside ABC



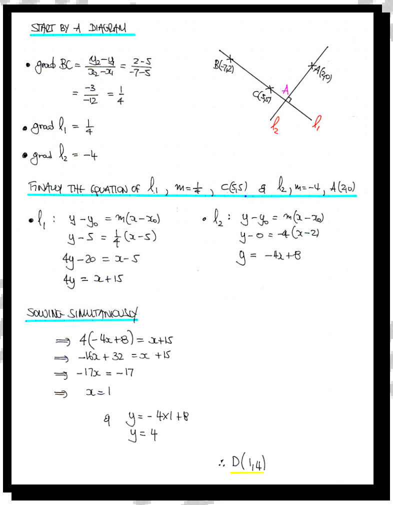
Question 31 (****)

The points A, B and C have coordinates $(2,0)$, $(-7,2)$ and $(5,5)$, respectively.

The straight line through A, which is perpendicular to the straight line BC, intersects BC at the point D.

Find the coordinates of D.

$\boxed{}$, $\boxed{D(1,4)}$

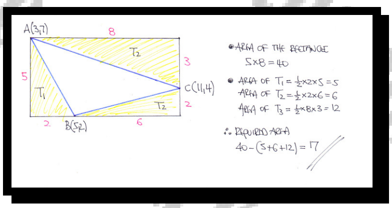


Question 32 (****)

The points $A(3,7)$, $B(5,2)$ and $C(11,4)$ are given.

Calculate the area of the triangle ABC.

$\boxed{}$, $\boxed{\text{area} = 17}$



Question 33 (****)

The points A and B have coordinates $\left(1,4\sqrt{3}\right)$ and $\left(-3+\sqrt{3},3\right)$, respectively.

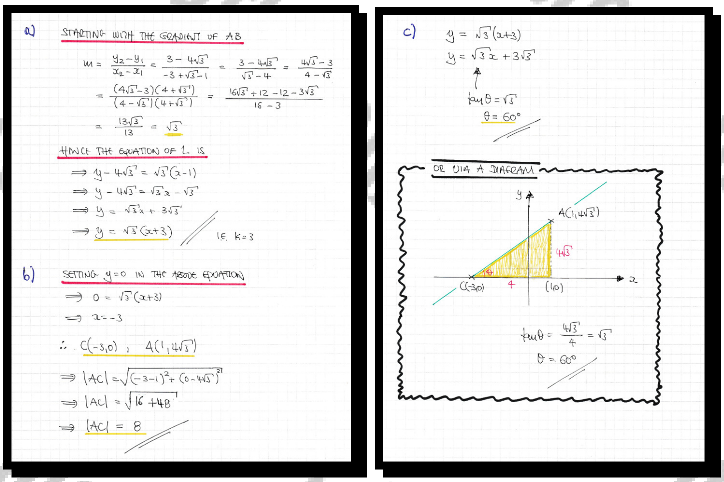
a) Find an equation for the straight line L which passes through A and B, giving the answer in the form $y=\sqrt{3}\left(x+k\right)$, where k is an integer.

L meets the x axis at the point C.

b) Determine the length of AC.

c) Calculate the acute angle between L and the x axis.

[ ], $y=\sqrt{3}x+3\sqrt{3}$, $\left|AC\right|=8$, $60°$



Question 34 (****)

The points A and C are the diagonally opposite vertices of a square $ABCD$.

The straight line l_1 with equation

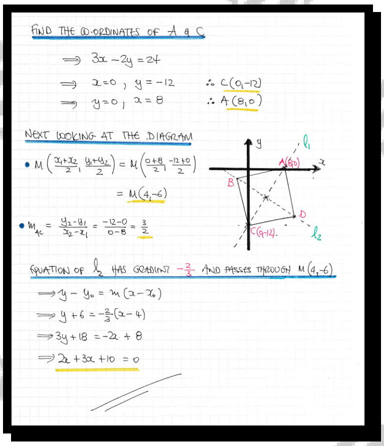
$$3x - 2y = 24,$$

meets the x and y axes at A and C, respectively.

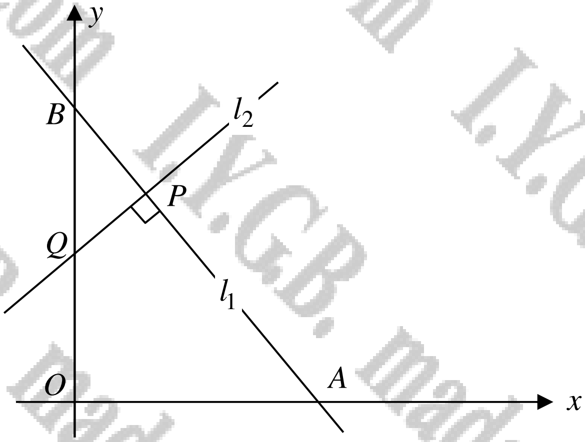
The straight line l_2 passes through B and D.

Determine an equation of l_2.

$\boxed{}$, $\boxed{2x + 3y + 10 = 0}$



Question 35 (****)



The straight line l_1 passes though the points $A(15,0)$ and $B(0,30)$.

a) Determine an equation for l_1.

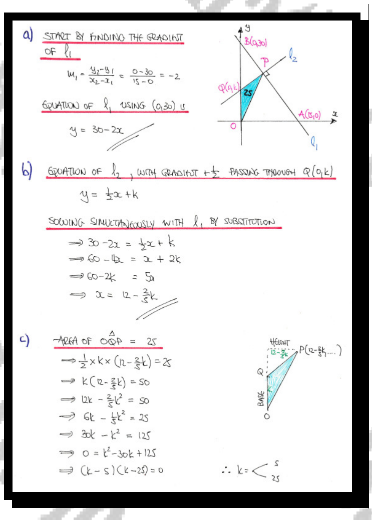
The straight line l_2 is perpendicular to l_1 and passes though the point $Q(0,k)$, where k is a positive constant.

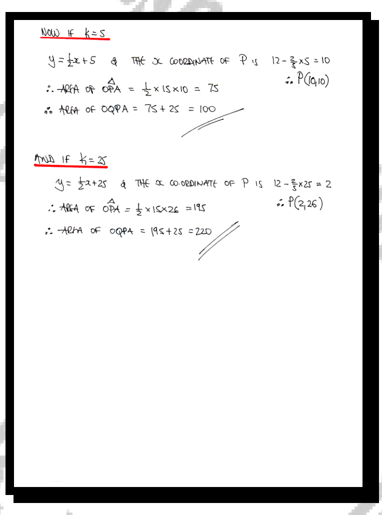
The point P is the intersection between l_1 and l_2.

b) Find, in terms of k, the x coordinate of P.

c) Given further that the area of the triangle OQP is 25, where O is the origin, determine the possible area of the quadrilateral $OQPA$.

[ ] , $\boxed{y = 30 - 2x}$, $\boxed{x = 12 - \frac{2}{5}k}$, $\boxed{\text{area} = 220 \text{ or } 100}$





Question 36 (****)

The straight line l passes through the point $A(3,4)$ and has gradient $\frac{1}{2}$.

a) Find an equation of l, giving the answer in the form $y = mx + c$, where m and c are constants.

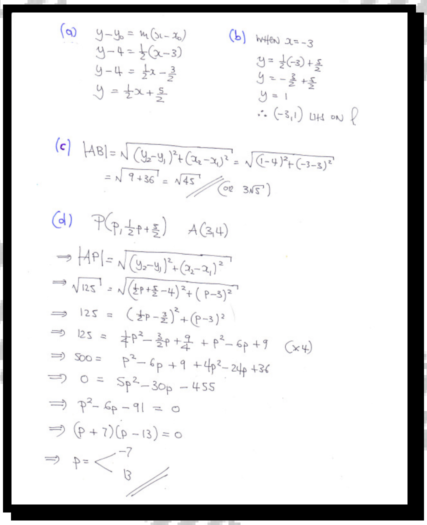
b) Show that $B(-3,1)$ also lies on l.

c) Calculate, in exact surd form, the distance of AB.

The point P lies on l and has x coordinate p, where p is a constant.

d) Given that the distance AP is $\sqrt{125}$, determine the possible values of p.

$\boxed{}$, $\boxed{y = \frac{1}{2}x + \frac{5}{2}}$, $\boxed{|AB| = 3\sqrt{5} = \sqrt{45}}$, $\boxed{k = -7, 13}$


(a) $y - y_0 = m(x - x_0)$
$y - 4 = \frac{1}{2}(x-3)$
$y - 4 = \frac{1}{2}x - \frac{3}{2}$
$y = \frac{1}{2}x + \frac{5}{2}$

(b) WHEN $x = -3$
$y = \frac{1}{2}(-3) + \frac{5}{2}$
$y = -\frac{3}{2} + \frac{5}{2}$
$y = 1$
$\therefore (-3,1)$ LIES ON l

(c) $|AB| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(1-4)^2 + (-3-3)^2}$
$= \sqrt{9+36} = \sqrt{45}$ (OR $3\sqrt{5}$)

(d) $P(p, \frac{1}{2}p + \frac{5}{2})$ $A(3,4)$
$\Rightarrow |AP| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$
$\Rightarrow \sqrt{125} = \sqrt{(\frac{1}{2}p + \frac{5}{2} - 4)^2 + (p-3)^2}$
$\Rightarrow 125 = (\frac{1}{2}p - \frac{3}{2})^2 + (p-3)^2$
$\Rightarrow 125 = \frac{1}{4}p^2 - \frac{3}{2}p + \frac{9}{4} + p^2 - 6p + 9$ $(\times 4)$
$\Rightarrow 500 = p^2 - 6p + 9 + 4p^2 - 24p + 36$
$\Rightarrow 0 = 5p^2 - 30p - 455$
$\Rightarrow p^2 - 6p - 91 = 0$
$\Rightarrow (p+7)(p-13) = 0$
$\Rightarrow p = -7, 13$

Question 37 (****)

The straight line l passes through the points $A(6,0)$ and $B(10,8)$.

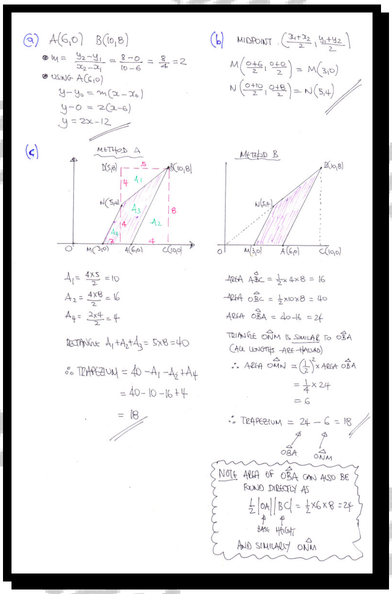
a) Find an equation for l, giving the answer in the form $y = mx + c$, where m and c are constants.

The midpoint of OA is M and the midpoint of OB is N, where O is the origin.

b) State the coordinates of M and N.

c) Determine the area of the trapezium $ABNM$.

[ ], [$y = 2x - 12$], [$M(3,0)$], [$N(5,4)$], [area $= 18$]



Question 38 (****)

The straight line l_1 has equation $2x + y - 18 = 0$ and crosses the x axis at the point P.

The straight line l_2 is parallel to l_1 and passes through the point $Q(-4,6)$.

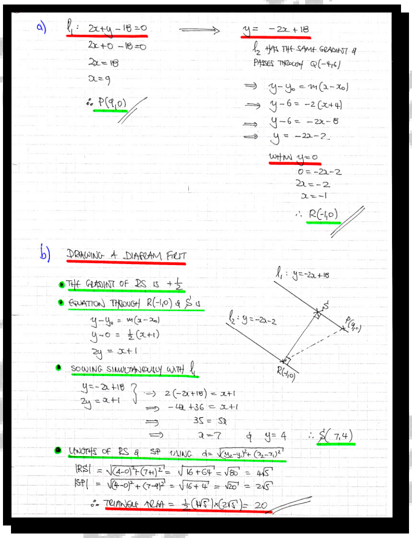
The point R is the x intercept of l_2.

a) Determine the coordinates of P and R.

The point S lies on l_1 so that RS is perpendicular to l_1.

b) Calculate the area of the triangle PRS.

$\boxed{}$, $\boxed{P(9,0)}$, $\boxed{R(-1,0)}$, $\boxed{\text{area} = 20}$


a) $l_1: 2x+y-18=0 \implies y = -2x+18$

$2x+0-18=0$

$2x=18$

$x=9$

$\therefore P(9,0)$

l_2 HAS THE SAME GRADIENT & PASSES THROUGH $Q(-4,6)$

$\implies y-y_0 = m(x-x_0)$

$\implies y-6 = -2(x+4)$

$\implies y-6 = -2x-8$

$\implies y = -2x-2$

WHEN $y=0$

$0=-2x-2$

$2x=-2$

$x=-1$

$\therefore R(-1,0)$

b) DRAWING A DIAGRAM FIRST

- THE GRADIENT OF RS IS $+\frac{1}{2}$
- EQUATION THROUGH $R(-1,0)$ & S IS

$y-y_0 = m(x-x_0)$

$y-0 = \frac{1}{2}(x+1)$

$2y = x+1$

- SOLVING SIMULTANEOUSLY WITH l_1

$\left.\begin{array}{l} y=-2x+18 \\ 2y=x+1 \end{array}\right\} \implies 2(-2x+18) = x+1$

$\implies -4x+36 = x+1$

$\implies 35 = 5x$

$\implies x=7$ & $y=4$ $\therefore S(7,4)$

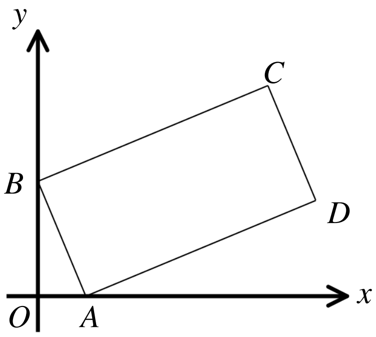
- LENGTHS OF RS & SP USING $d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$|RS| = \sqrt{(4-0)^2+(7+1)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$

$|SP| = \sqrt{(4-0)^2+(7-9)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

$\therefore$ TRIANGLE AREA $= \frac{1}{2}(4\sqrt{5})\times(2\sqrt{5}) = 20$

Question 39 (****)

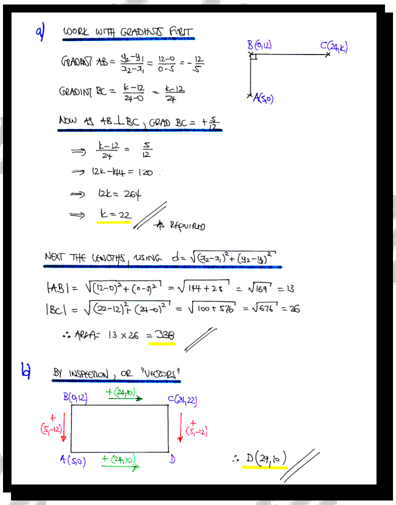


The figure above shows the rectangle $ABCD$, with $A(5,0)$, $B(0,12)$ and $C(24,k)$.

a) Show that $k = 22$ and hence calculate the area of the rectangle $ABCD$.

b) Determine the coordinates of D.

$\boxed{}$, $\boxed{\text{area} = 338}$, $\boxed{D(29,10)}$



a) WORK WITH GRADIENTS FIRST

GRADIENT $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{12-0}{0-5} = -\frac{12}{5}$

GRADIENT $BC = \frac{k-12}{24-0} = \frac{k-12}{24}$

NOW AS $AB \perp BC$, GRAD $BC = +\frac{5}{12}$

$\Rightarrow \frac{k-12}{24} = \frac{5}{12}$

$\Rightarrow 12k - 144 = 120$

$\Rightarrow 12k = 264$

$\Rightarrow k = 22$ // AS REQUIRED

NEXT THE LENGTHS, USING $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|AB| = \sqrt{(12-0)^2+(0-5)^2} = \sqrt{144+25} = \sqrt{169} = 13$

$|BC| = \sqrt{(22-12)^2+(24-0)^2} = \sqrt{100+576} = \sqrt{676} = 26$

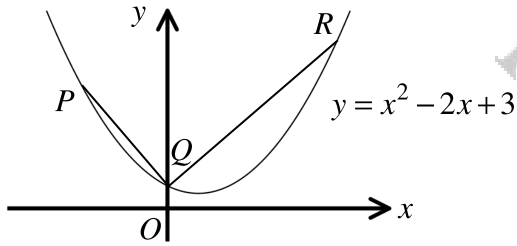
$\therefore$ AREA $= 13 \times 26 = 338$ //

b) BY INSPECTION, OR "VECTORS"

B(0,12), C(24,22), A(5,0), D; $+(24,10)$, $+(5,-12)$

$\therefore D(29,10)$ //

Question 40 (****)



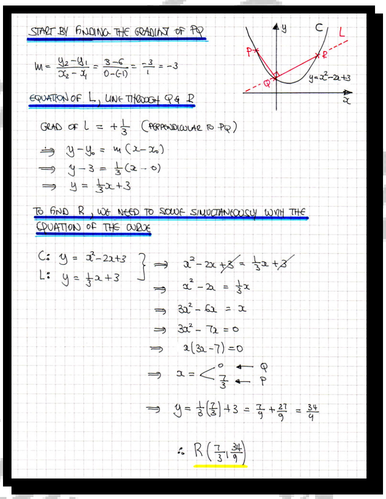
The figure above shows the curve C with equation

$$y = x^2 - 2x + 3.$$

The points $P(-1,6)$, $Q(0,3)$ and R all lie on C.

Given that $\measuredangle PQR = 90°$, determine the exact coordinates of R.

$R\left(\frac{7}{3}, \frac{34}{9}\right)$


START BY FINDING THE GRADIENT OF PQ

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-6}{0-(-1)} = \frac{-3}{1} = -3$

EQUATION OF L, LINE THROUGH Q & R

GRAD OF L $= +\frac{1}{3}$ (PERPENDICULAR TO PQ)

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 3 = \frac{1}{3}(x - 0)$

$\Rightarrow y = \frac{1}{3}x + 3$

TO FIND R, WE NEED TO SOLVE SIMULTANEOUSLY WITH THE EQUATION OF THE CURVE

C: $y = x^2 - 2x + 3$
L: $y = \frac{1}{3}x + 3$

$\Rightarrow x^2 - 2x + 3 = \frac{1}{3}x + 3$

$\Rightarrow x^2 - 2x = \frac{1}{3}x$

$\Rightarrow 3x^2 - 6x = x$

$\Rightarrow 3x^2 - 7x = 0$

$\Rightarrow x(3x - 7) = 0$

$\Rightarrow x = 0 \leftarrow Q, \quad \frac{7}{3} \leftarrow P$

$\Rightarrow y = \frac{1}{3}\left(\frac{7}{3}\right) + 3 = \frac{7}{9} + \frac{27}{9} = \frac{34}{9}$

$\therefore R\left(\frac{7}{3}, \frac{34}{9}\right)$

Question 41 **(****)**

The straight lines l_1 and l_2, with respective equations

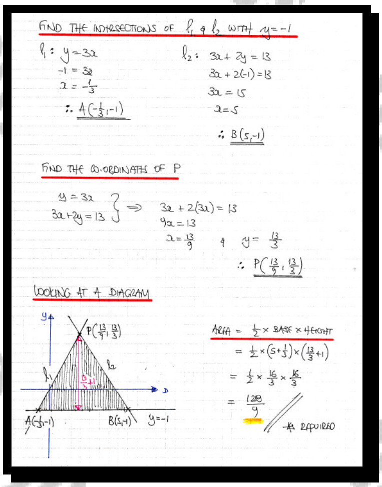
$$y = 3x \qquad \text{and} \qquad 3x + 2y = 13,$$

intersect at the point P.

The points A and B, are the points of intersection of the straight line with equation $y = -1$ with l_1 and l_2, respectively.

Show that the area of the triangle ABP is $\frac{128}{9}$ square units.

$\boxed{}$, $\boxed{m = -\frac{3}{2}}$, $\boxed{P\left(\frac{13}{9}, \frac{13}{3}\right)}$



Question 42 (****)

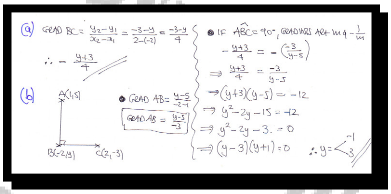
The points A, B and C have coordinates $(1,5)$, $(-2, y)$ and $(2,-3)$, respectively.

a) Find, in terms of y, the gradient of BC.

The angle ABC is $90°$.

b) Determine the possible values of y.

$$-\frac{y+3}{4}, \quad y=-1, \quad y=3$$



Question 43 (****)

The rectangle $ABCD$ has three of its vertices located at $A(5,10)$, $B(3,k)$ and $C(9,2)$, where k is a constant.

a) Show that

$$(10-k)(2-k)+12=0.$$

and hence determine the two possible values of k.

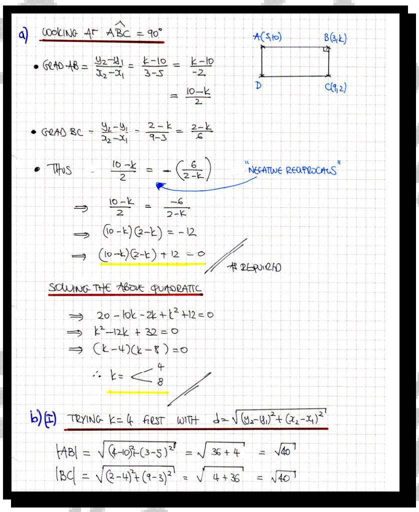
It is further given that the rectangle $ABCD$ reduces to a square, for one of the two values of k found in part **(a)**.

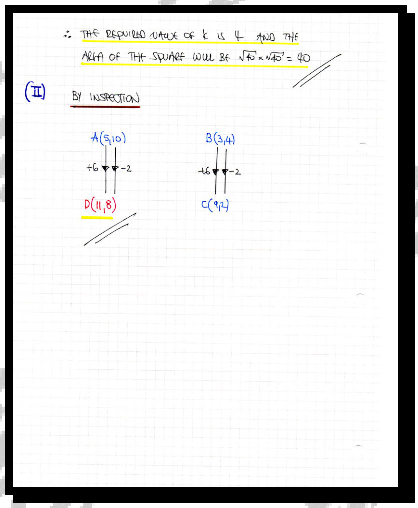
For the square $ABCD$ ...

b) ... determine its area.

c) ... state the coordinates of D.

$\boxed{}$, $\boxed{k=8}$, $\boxed{\text{area}=40}$, $\boxed{D(11,8)}$





Question 44 (****)

The straight line l_1 has gradient m and has x intercept 6.

a) Show clearly that the equation of l_1 can be written as $y = m(x-6)$.

The straight line l_2, with equation $y = 2x+9$, meets l_1 at the point A.

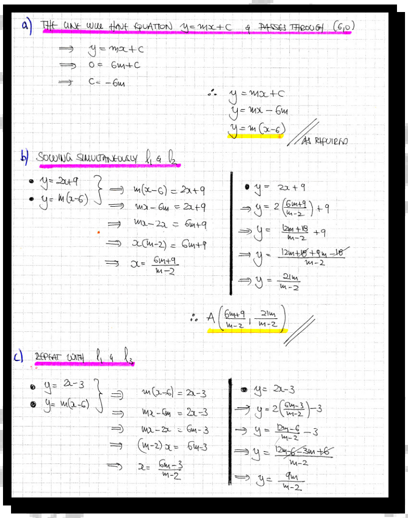
b) Show that the coordinates of A are $\left(\dfrac{6m+9}{m-2}, \dfrac{21m}{m-2}\right)$

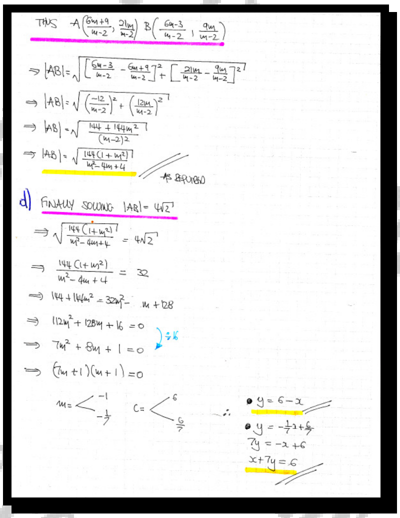
The straight line l_3, with equation $y = 2x-3$, meets l_1 at the point B.

c) Show that the distance AB is $\sqrt{\dfrac{144(1+m^2)}{m^2+4m+4}}$

d) Given further that distance AB is $4\sqrt{2}$ find the two possible equations of l_1.

$\boxed{}$, $\boxed{y = 6-x}$, $\boxed{x+7y=6}$





16 HARD QUESTIONS

Question 1 **(****+)**

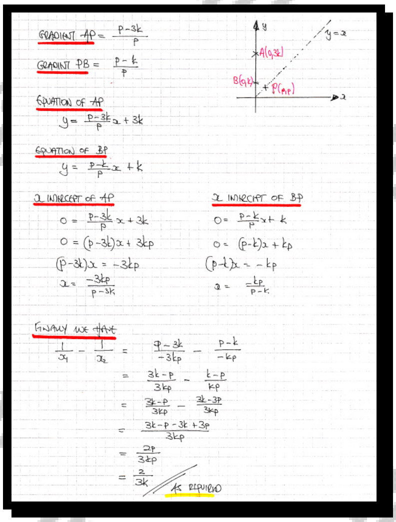
The points $A(0,3k)$ and $B(0,k)$ are given, where k is a non zero constant.

The point P lies on the straight line with equation $y=x$, so that both straight lines, AP and BP, have negative gradient.

The straight line through A and P meets the x axis at x_1 and the straight line through B and P meets the x axis at x_2.

Show that $\dfrac{1}{x_1}-\dfrac{1}{x_2}=\dfrac{2}{3k}$.

proof



GRADIENT AP $= \dfrac{p-3k}{p}$

GRADIENT PB $= \dfrac{p-k}{p}$

EQUATION OF AP

$y=\dfrac{p-3k}{p}x+3k$

EQUATION OF BP

$y=\dfrac{p-k}{p}x+k$

x INTERCEPT OF AP

$0=\dfrac{p-3k}{p}x+3k$

$0=(p-3k)x+3kp$

$(p-3k)x=-3kp$

$x=\dfrac{-3kp}{p-3k}$

x INTERCEPT OF BP

$0=\dfrac{p-k}{p}x+k$

$0=(p-k)x+kp$

$(p-k)x=-kp$

$x=\dfrac{-kp}{p-k}$

FINALLY WE HAVE

$\dfrac{1}{x_1}-\dfrac{1}{x_2}=\dfrac{p-3k}{-3kp}-\dfrac{p-k}{-kp}$

$=\dfrac{3k-p}{3kp}-\dfrac{k-p}{kp}$

$=\dfrac{3k-p}{3kp}-\dfrac{3k-3p}{3kp}$

$=\dfrac{3k-p-3k+3p}{3kp}$

$=\dfrac{2p}{3kp}$

$=\dfrac{2}{3k}$ AS REQUIRED

Question 2 (****+)

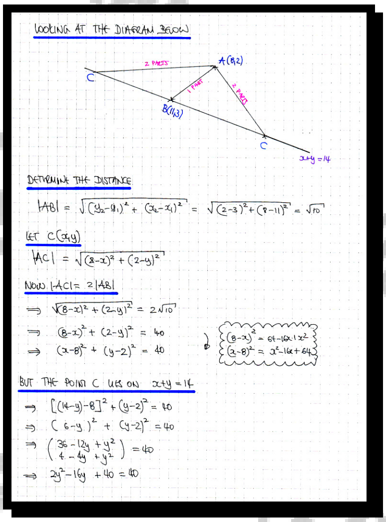
The points A and B have coordinates $(8,2)$ and $(11,3)$, respectively.

The point C lies on the straight line with equation

$$y + x = 14.$$

Given further that the distance AC is twice as large as the distance AB, determine the two possible sets of coordinates of C.

Y N F, $\boxed{C(6,8) \ \cup \ C(14,0)}$



LOOKING AT THE DIAGRAM BELOW

DETERMINE THE DISTANCE

$|AB| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(2-3)^2+(8-11)^2} = \sqrt{10}$

LET $C(x,y)$

$|AC| = \sqrt{(8-x)^2 + (2-y)^2}$

NOW $|AC| = 2|AB|$

$\Rightarrow \sqrt{(8-x)^2 + (2-y)^2} = 2\sqrt{10}$

$\Rightarrow (8-x)^2 + (2-y)^2 = 40$

$\Rightarrow (x-8)^2 + (y-2)^2 = 40$

$\left\{ \begin{array}{l} (8-x)^2 = 64-16x+x^2 \\ (x-8)^2 = x^2-16x+64 \end{array} \right\}$

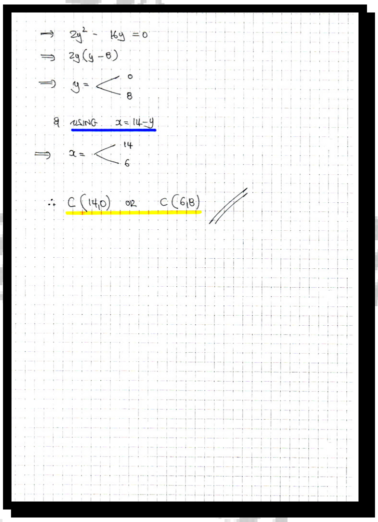
BUT THE POINT C LIES ON $x+y=14$

$\Rightarrow [(14-y)-8]^2 + (y-2)^2 = 40$

$\Rightarrow (6-y)^2 + (y-2)^2 = 40$

$\Rightarrow \begin{pmatrix} 36-12y+y^2 \\ 4-4y+y^2 \end{pmatrix} = 40$

$\Rightarrow 2y^2-16y+40 = 40$



$\Rightarrow 2y^2 - 16y = 0$

$\Rightarrow 2y(y-8)$

$\Rightarrow y = \left< \begin{array}{l} 0 \\ 8 \end{array} \right.$

& USING $x = 14-y$

$\Rightarrow x = \left< \begin{array}{l} 14 \\ 6 \end{array} \right.$

$\therefore C(14,0)$ OR $C(6,8)$

Question 3 (****+)

The points A and B have coordinates $(-3,10)$ and $(9,6)$, respectively.

a) Find an equation for the straight line L_1 which passes through the point B and is perpendicular to AB, in the form $y = mx + c$, where m and c are constants.

L_1 crosses the x axis at the point C.

b) Determine the value of $\tan\theta$, where θ is the angle that BC makes with the positive x axis.

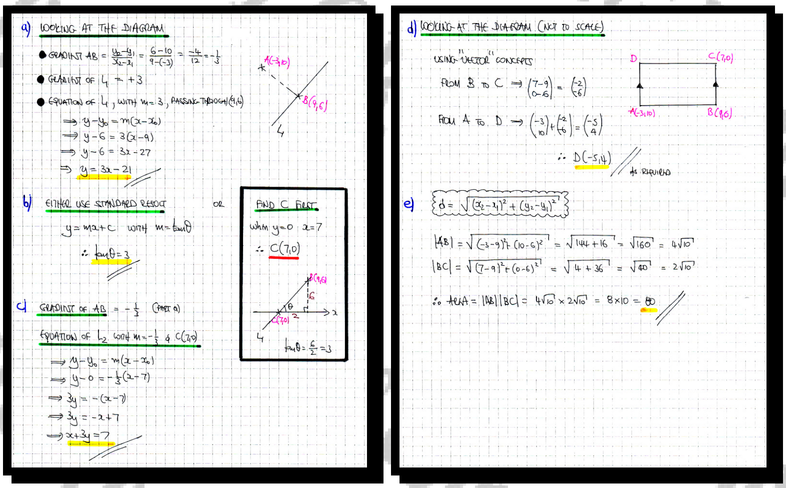
c) Find an equation for the straight line L_2 which passes through the point C and is parallel to AB, in the form $ax + by = c$, where a, b and c are integers.

The point D is such so that $ABCD$ is a rectangle.

d) Show that the coordinates of D are $(-5,4)$.

e) Find the area of the rectangle $ABCD$.

[ ], $\boxed{y = 3x - 21}$, $\boxed{\tan\theta = 3}$, $\boxed{x + 3y = 7}$, $\boxed{\text{area} = 80}$



Question 4 (****+)

The points A and C have coordinates $(3,2)$ and $(5,6)$, respectively.

a) Find an equation for the perpendicular bisector of AC, giving the answer in the form $ax+by=c$, where a, b and c are integers.

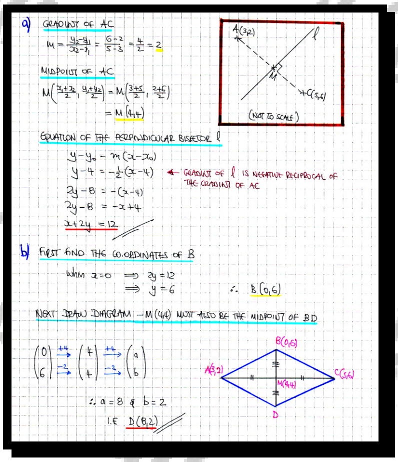
The perpendicular bisector of AC crosses the y axis at the point B.

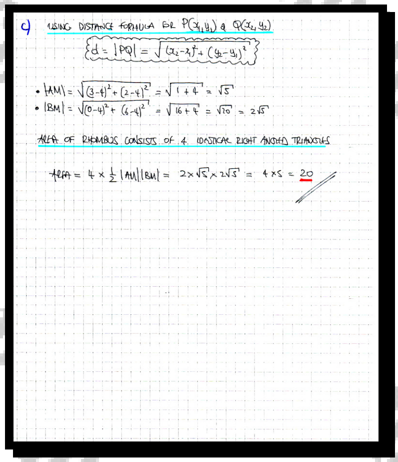
The point D is such so that $ABCD$ is a rhombus.

b) Show that the coordinates of D are $(8,2)$.

c) Calculate the area of the rhombus $ABCD$.

$\boxed{}$, $\boxed{x+2y=12}$, $\boxed{\text{area}=20}$





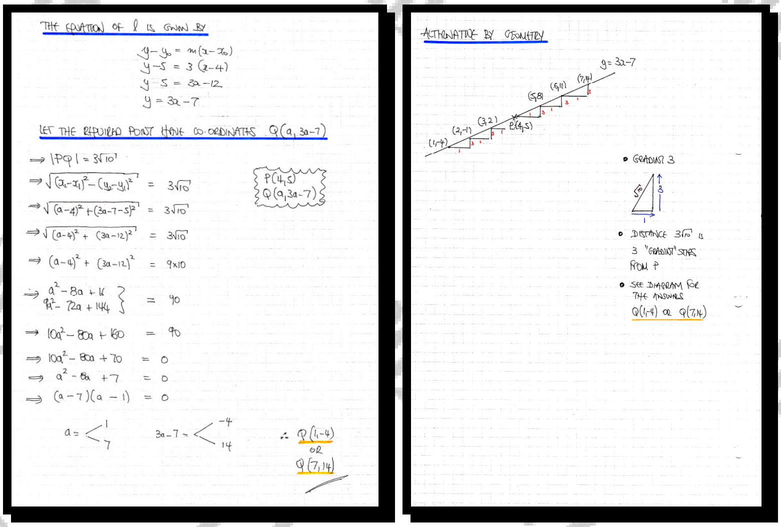
Question 5 (****+)

The straight line l passes through the point $P(4,5)$ and has gradient 3.

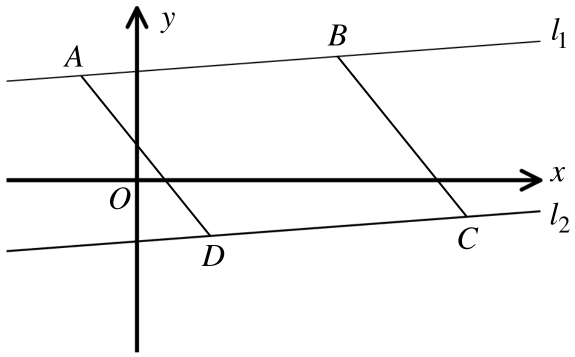
The point Q also lies on l so that the distance PQ is $3\sqrt{10}$.

Determine the coordinates of the **two** possible positions of Q.

$\boxed{}$, $\boxed{Q(7,14) \text{ or } Q(1,-4)}$



Question 6 (****+)



The figure above shows a parallelogram $ABCD$.

The straight line l_1 passes through $A(-1,3)$ and $B(4,4)$.

a) Find an equation for l_1.

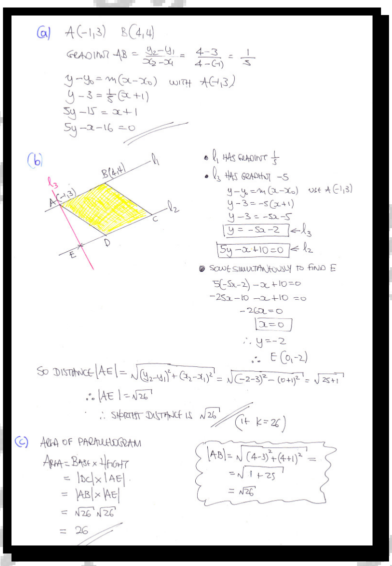
Give the answer in the form $ax+by+c=0$, where a, b and c are integers.

The points C and D lie on the straight line l_2, which has equation $5y-x+10=0$.

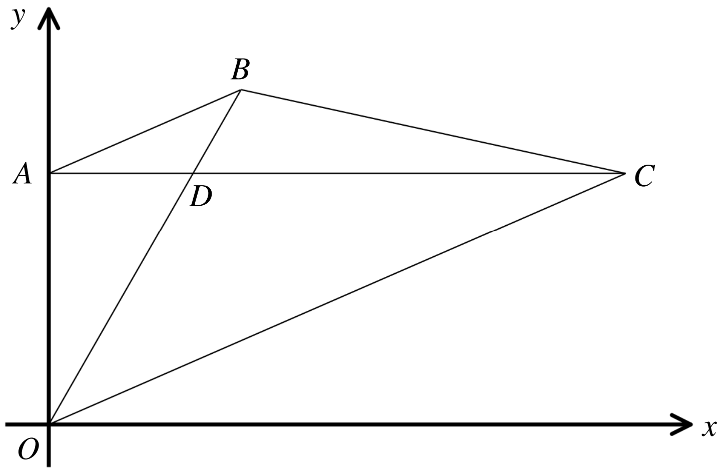
b) Show that the distance between l_1 and l_2 is $\sqrt{k}$, where k is an integer.

c) Hence, find the area of the parallelogram $ABCD$.

$\boxed{}$, $\boxed{5y-x-16=0}$, $\boxed{k=26}$, $\boxed{\text{area}=26}$



Question 7 (****+)



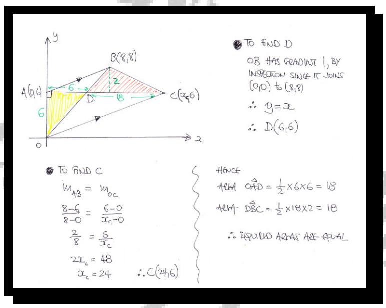
The figure above shows a trapezium $OABC$, where O is the origin, whose side AB is parallel to the side OC.

The diagonal AC is horizontal and the points A and B have coordinates $(0,6)$ and $(8,8)$, respectively.

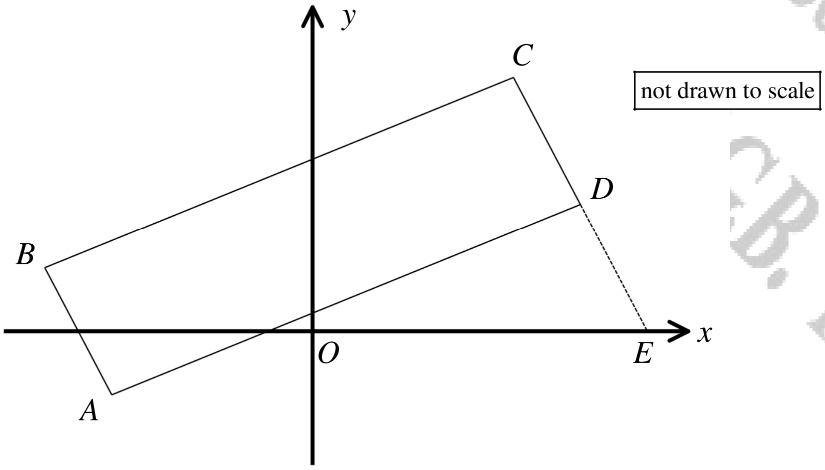
The diagonals of the trapezium meet at the point D.

Show by direct area calculations that the area of the triangle BCD is equal to the area of the triangle OAD.

[ ], [proof]



Question 8 (****+)



A parallelogram has vertices at $A(-3,-2)$, $B(-4,2)$, $C(3,8)$ and $D(4,4)$.

a) Show that $ABD = 90°$ and hence find the area of the parallelogram $ABCD$.

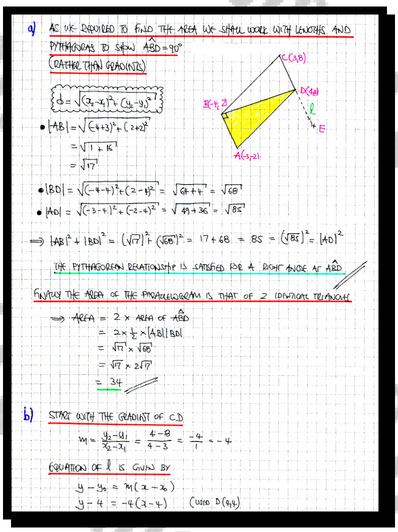
The side CD is extended so that it meets the x axis at the point E.

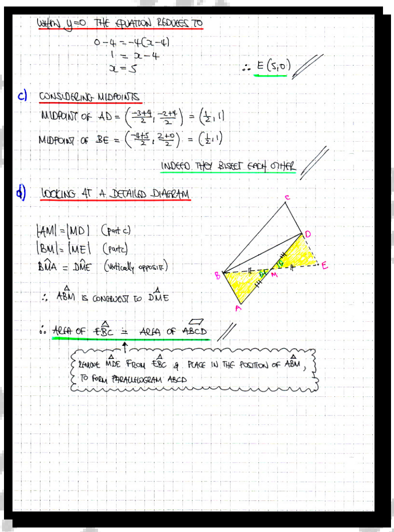
b) Find the coordinates of E.

c) Show that EB and AD bisect each other.

d) By considering two suitable congruent triangles and without any direct area calculations, show that the area of the triangle EBC is equal to the area of the parallelogram $ABCD$.

[ ], [area $= 34$], [$E(5,0)$]





Question 9 (****+)

The straight line l_1 passes through the points $A(2,1)$ and $B(k,8)$, where k is a constant.

a) Given the gradient of l_1 is $\frac{7}{2}$ show that $k=4$.

b) Find an equation for l_1.

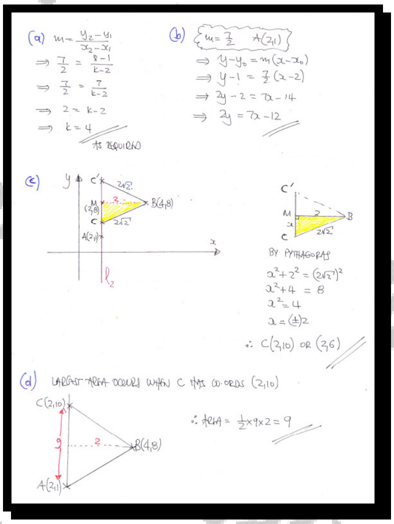
The straight line l_2 is parallel to the y axis and passing through A.

The point C lies on l_2 so that the length of BC is exactly $2\sqrt{2}$.

c) Find the possible coordinates of C.

d) Determine the largest possible area of the triangle ABC.

$\boxed{}$, $\boxed{2y=7x-12}$, $\boxed{C(2,6) \text{ or } C(2,10)}$, $\boxed{\text{area}=9}$



Question 10 (****+)

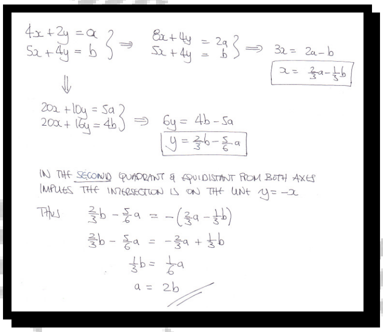
The straight lines L_1 and L_2 have respective equations

$$4x + 2y = a \quad \text{and} \quad 5x + 4y = b .$$

It is given that L_1 and L_2 meet at the point P.

Express a in terms of b, given further that P lies in the second quadrant and is equidistant from the coordinate axes.

$\boxed{}$, $\boxed{a = 2b}$



Question 11 (****+)

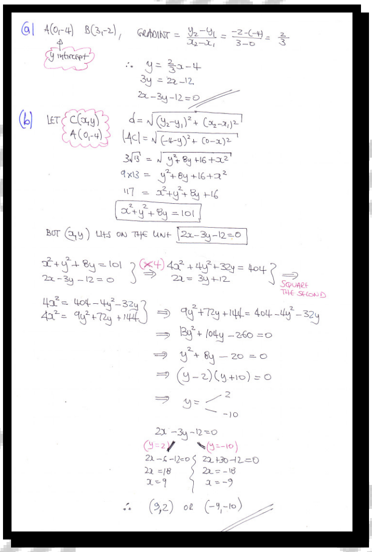
The points A and B have coordinates $(0,-4)$ and $(3,-2)$, respectively.

a) Determine an equation for the straight line l which passes through the points A and B, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

The point C lies on l, so that the distance AC is $3\sqrt{13}$ units.

b) Show, by a complete algebraic solution, that one possible set of coordinates for C are $(9,2)$ and find the other set.

$\boxed{}$, $\boxed{2x-3y-12=0}$, $\boxed{C(-9,-10)}$


(a) $A(0,-4)$ $B(3,-2)$, GRADIENT $= \frac{y_2-y_1}{x_2-x_1} = \frac{-2-(-4)}{3-0} = \frac{2}{3}$

y intercept

$\therefore\ y = \frac{2}{3}x - 4$

$3y = 2x - 12$

$2x - 3y - 12 = 0$

(b) LET $C(x,y)$, $A(0,-4)$

$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$|AC| = \sqrt{(-4-y)^2 + (0-x)^2}$

$3\sqrt{13} = \sqrt{y^2+8y+16+x^2}$

$9 \times 13 = y^2+8y+16+x^2$

$117 = x^2+y^2+8y+16$

$\boxed{x^2+y^2+8y=101}$

BUT (x,y) LIES ON THE LINE $\boxed{2x-3y-12=0}$

$\left.\begin{array}{l} x^2+y^2+8y=101 \\ 2x-3y-12=0 \end{array}\right\} \overset{(\times 4)}{\Rightarrow} \left.\begin{array}{l} 4x^2+4y^2+32y=404 \\ 2x=3y+12 \end{array}\right\} \Rightarrow$ SQUARE THE SECOND

$\left.\begin{array}{l} 4x^2 = 404-4y^2-32y \\ 4x^2 = 9y^2+72y+144 \end{array}\right\} \Rightarrow 9y^2+72y+144 = 404-4y^2-32y$

$\Rightarrow 13y^2+104y-260=0$

$\Rightarrow y^2+8y-20=0$

$\Rightarrow (y-2)(y+10)=0$

$\Rightarrow y = 2$ or -10

$2x-3y-12=0$

$(y=2)$: $2x-6-12=0$, $2x=18$, $x=9$

$(y=-10)$: $2x+30-12=0$, $2x=-18$, $x=-9$

$\therefore\ (9,2)$ OR $(-9,-10)$

Question 12 (****+)

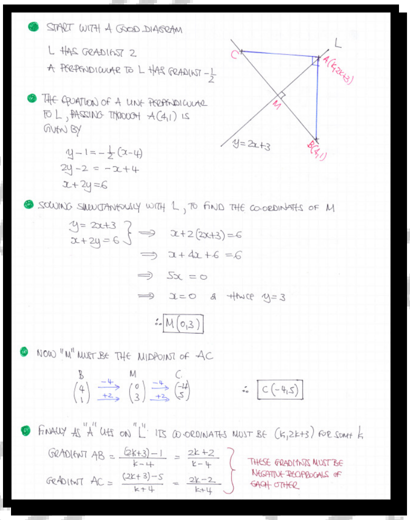
The point A lies on the straight line L with equation

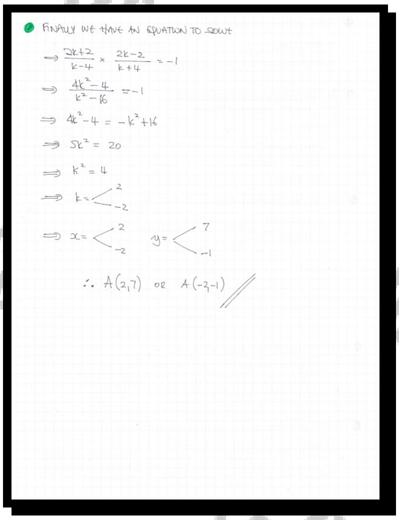
$$y = 2x + 3.$$

The point B has coordinates $(4,1)$ and the point C is the reflection of A about L.

Determine the possible coordinates of A, given that AB is perpendicular to AC.

$A(2,7)$ or $A(-2,-1)$



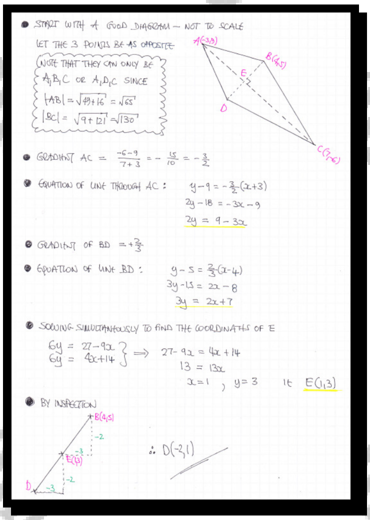


Question 13 **(****+)**

The points $A(-3,9)$, $B(4,5)$ and $C(7,-6)$ are three vertices of the kite $ABCD$.

Determine the coordinates of D.

SP, $D(-2,1)$



- START WITH A GOOD DIAGRAM – NOT TO SCALE

LET THE 3 POINTS BE AS OPPOSITE

(NOTE THAT THEY CAN ONLY BE A, B, C OR A, D, C SINCE $|AB| = \sqrt{49+16} = \sqrt{65}$, $|BC| = \sqrt{9+121} = \sqrt{130}$)

- GRADIENT AC $= \frac{-6-9}{7+3} = -\frac{15}{10} = -\frac{3}{2}$
- EQUATION OF LINE THROUGH AC: $y-9 = -\frac{3}{2}(x+3)$, $2y-18 = -3x-9$, $2y = 9-3x$
- GRADIENT OF BD $= +\frac{2}{3}$
- EQUATION OF LINE BD: $y-5 = \frac{2}{3}(x-4)$, $3y-15 = 2x-8$, $3y = 2x+7$
- SOLVING SIMULTANEOUSLY TO FIND THE COORDINATES OF E

$\left.\begin{array}{l} 6y = 27-9x \\ 6y = 4x+14 \end{array}\right\} \Rightarrow 27-9x = 4x+14$, $13 = 13x$, $x=1$, $y=3$ ie $E(1,3)$

- BY INSPECTION

$\therefore D(-2,1)$

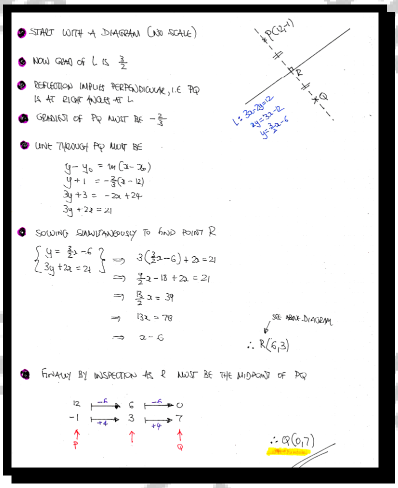
Question 14 (****+)

The straight line L has equation

$$3x - 2y = 12.$$

Find the coordinates of the point Q, where Q is the reflection of the point $P(12,-1)$.

V, , $Q(0,7)$

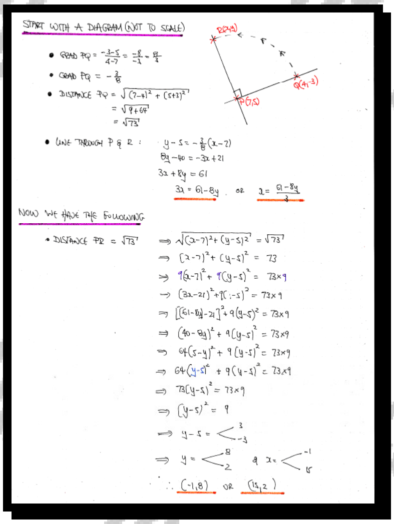


Question 15 (****+)

The points $P(7,5)$ and $Q(4,-3)$ are given.

The point Q is rotated by $90°$ anticlockwise about the point P.

V, [ ], $R(15,2)$

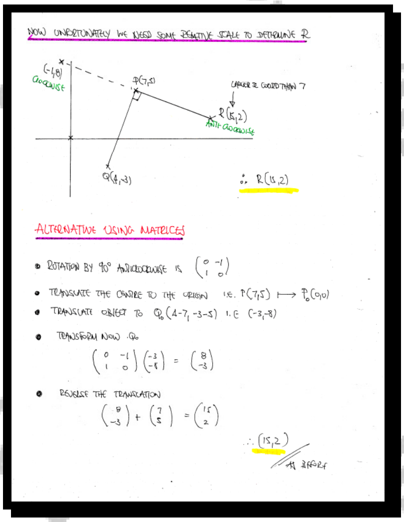

START WITH A DIAGRAM (NOT TO SCALE)

- GRAD $PQ = \frac{-3-5}{4-7} = \frac{-8}{-3} = \frac{8}{3}$
- GRAD $PR = -\frac{3}{8}$
- DISTANCE $PQ = \sqrt{(7-4)^2+(5+3)^2} = \sqrt{9+64} = \sqrt{73}$
- LINE THROUGH P & R: $y-5 = -\frac{3}{8}(x-7)$, $8y-40 = -3x+21$, $3x+8y=61$, $3x = 61-8y$ OR $x = \frac{61-8y}{3}$

NOW WE HAVE THE FOLLOWING

- DISTANCE $PR = \sqrt{73}$ $\Rightarrow \sqrt{(x-7)^2+(y-5)^2} = \sqrt{73}$
$\Rightarrow (x-7)^2+(y-5)^2 = 73$
$\Rightarrow 9(x-7)^2+9(y-5)^2 = 73\times9$
$\Rightarrow (3x-21)^2+9(y-5)^2 = 73\times9$
$\Rightarrow [(61-8y)-21]^2+9(y-5)^2 = 73\times9$
$\Rightarrow (40-8y)^2+9(y-5)^2 = 73\times9$
$\Rightarrow 64(5-y)^2+9(y-5)^2 = 73\times9$
$\Rightarrow 64(y-5)^2+9(y-5)^2 = 73\times9$
$\Rightarrow 73(y-5)^2 = 73\times9$
$\Rightarrow (y-5)^2 = 9$
$\Rightarrow y-5 = 3$ or -3
$\Rightarrow y = 8$ or 2 & $x = -1$ or 15

$\therefore (-1,8)$ OR $(15,2)$


NOW UNFORTUNATELY WE NEED SOME RELATIVE SCALE TO DETERMINE R

$\therefore R(15,2)$

ALTERNATIVE USING MATRICES

- ROTATION BY $90°$ ANTICLOCKWISE IS $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$
- TRANSLATE THE CENTRE TO THE ORIGIN I.E. $P(7,5) \mapsto P_0(0,0)$
- TRANSLATE OBJECT TO $Q_0(4-7,-3-5)$ I.E. $(-3,-8)$
- TRANSFORM NOW Q_0
$\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}-3\\-8\end{pmatrix} = \begin{pmatrix}8\\-3\end{pmatrix}$
- REVERSE THE TRANSLATION
$\begin{pmatrix}8\\-3\end{pmatrix}+\begin{pmatrix}7\\5\end{pmatrix} = \begin{pmatrix}15\\2\end{pmatrix}$

$\therefore (15,2)$

Question 16 (****+)

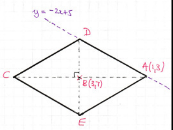
The point $A(1,3)$ is one of the vertices of a rhombus whose centre is $B(3,7)$.

One of the sides of the rhombus lies on the straight line with equation

$$y = 5 - 2x.$$

Determine the coordinates of the other three vertices of this rhombus.

$\boxed{}$, $\boxed{(5,11),\ \left(-\frac{7}{3},\frac{29}{3}\right),\ \left(\frac{25}{3},\frac{13}{3}\right)}$


START BY DRAWING A DIAGRAM, AND NOTE THAT $A(1,3)$ LIES ON THE LINE WITH EQUATION $y=-2x+5$

BY THE GEOMETRICAL PROPERTIES OF RHOMBUSES B, MUST BE THE MIDPOINT OF CA

A B C

$\begin{pmatrix}1\\3\end{pmatrix} \begin{matrix}+2\\+4\end{matrix} \begin{pmatrix}3\\7\end{pmatrix} \begin{matrix}+2\\+4\end{matrix} \begin{pmatrix}5\\11\end{pmatrix}$

$\therefore C(5,11)$

NEXT FIND THE GRADIENT OF BA

GRAD $AB = \frac{7-3}{3-1} = \frac{4}{2} = 2$

$\therefore$ GRAD $BD = -\frac{1}{2}$

$\therefore$ EQUATION OF BD $\quad y-7 = -\frac{1}{2}(x-3)$

$2y-14 = -x+3$

$x+2y = 17$

SOLVING SIMULTANEOUSLY WITH $y=-2x+5$, BY SUBSTITUTION

$x+2y = 17$

$x+2(-2x+5) = 17$

$x-4x+10 = 17$

$-7 = 3x$

$x = -\frac{7}{3}$ & $y = -2\left(-\frac{7}{3}\right)+5 = \frac{14}{3}+5 = \frac{29}{3}$

$\therefore D\left(-\frac{7}{3},\frac{29}{3}\right)$

FINALLY TO FIND THE CO-ORDINATES OF E, WE USE MIDPOINT CONSIDERATIONS AS BEFORE

D B E

$\begin{pmatrix}-\frac{7}{3}\\ \frac{29}{3}\end{pmatrix} \quad \begin{pmatrix}3\\7\end{pmatrix} \quad \begin{pmatrix}x\\y\end{pmatrix}$

$\frac{x-\frac{7}{3}}{2} = 3$ & $\frac{y+\frac{29}{3}}{2} = 7$

$x-\frac{7}{3} = 6$ $\quad y+\frac{29}{3} = 14$

$3x-7 = 18$ $\quad 3y+29 = 42$

$3x = 25$ $\quad 3y = 13$

$x = \frac{25}{3}$ $\quad y = \frac{13}{3}$

$\therefore E\left(\frac{25}{3},\frac{13}{3}\right)$

Question 17 (****+)

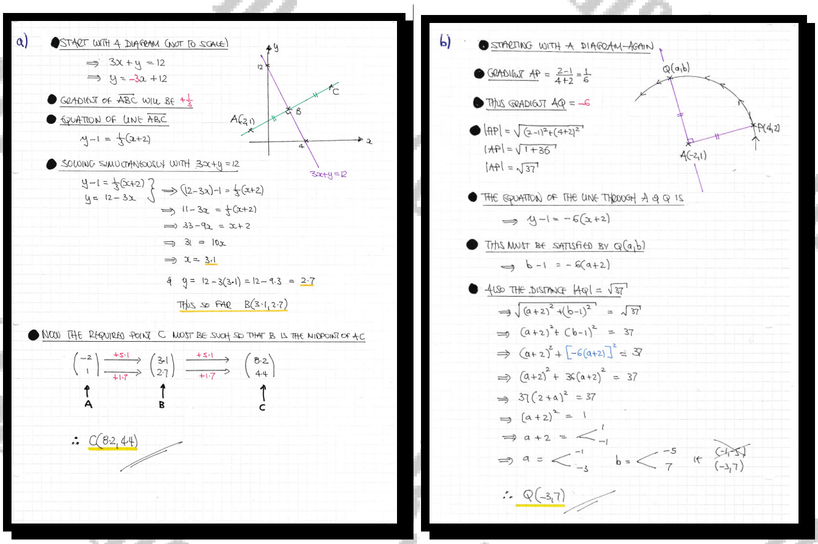
The point A has coordinates $(-2,1)$.

a) Find the coordinates of the point of reflection of A about the straight line with equation $3x + y = 12$.

The point P, whose coordinates are $(4,2)$, is rotated about A by $90°$ anticlockwise, onto the point Q.

b) Determine the coordinates of Q.

$\boxed{}$, $\boxed{(8.2,4.4)}$, $\boxed{Q(-3,7)}$



Question 18 (****+)

The point $A(a,b)$, where a and b are constants, lie on the straight line with equation

$$y = 2x + 1.$$

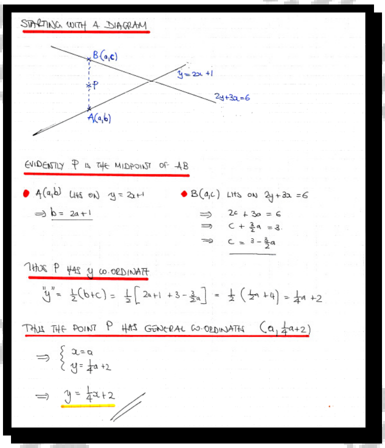
The point $B(a,c)$, where c is a constant, lie on the straight line with equation

$$2y + 3x = 6.$$

The point $P\left[a, \frac{1}{2}(b+c)\right]$ lies on the straight line L.

Determine an equation of L.

$\boxed{}$, $\boxed{y = \frac{1}{4}x + 2}$



EVIDENTLY P IS THE MIDPOINT OF AB

- A(a,b) LIES ON $y = 2x+1$ $\Rightarrow b = 2a+1$
- B(a,c) LIES ON $2y+3x=6$ $\Rightarrow 2c + 3a = 6$ $\Rightarrow c + \frac{3}{2}a = 3$ $\Rightarrow c = 3 - \frac{3}{2}a$

THUS P HAS y CO-ORDINATE

$$\text{"}y\text{"} = \frac{1}{2}(b+c) = \frac{1}{2}\left[2a+1+3-\frac{3}{2}a\right] = \frac{1}{2}\left(\frac{1}{2}a+4\right) = \frac{1}{4}a+2$$

THUS THE POINT P HAS GENERAL CO-ORDINATES $\left(a, \frac{1}{4}a+2\right)$

$$\Rightarrow \begin{cases} x = a \\ y = \frac{1}{4}a+2 \end{cases}$$

$$\Rightarrow y = \frac{1}{4}x+2$$

17 ENRICHMENT QUESTIONS

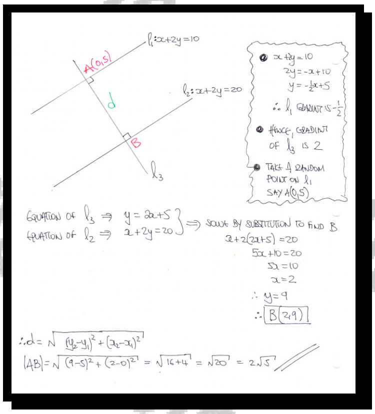
Question 1 (*****)

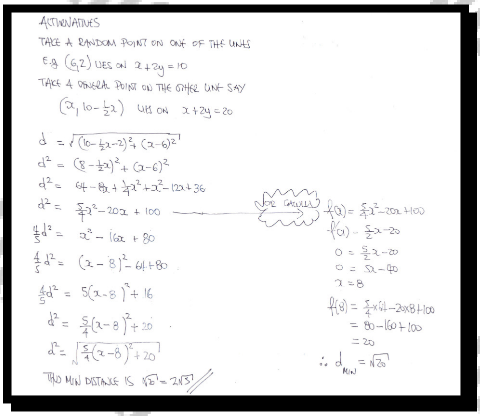
Find the shortest distance between the parallel lines with equations

$$x + 2y = 10 \quad \text{and} \quad x + 2y = 20.$$

[*You may not use a standard formula which determines the shortest distance of a point from a straight line in this question*]

$\boxed{}$, $\boxed{2\sqrt{5}}$





Question 2 (*****)

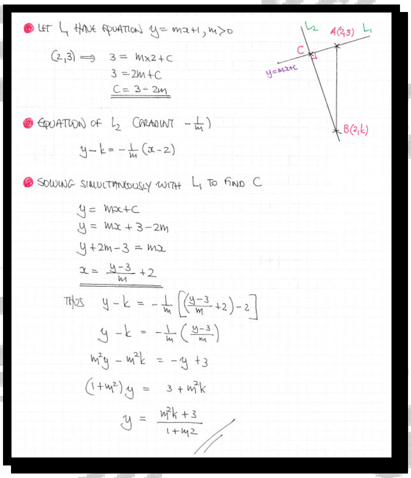
The straight line L_1, has gradient m, $m>0$ and passes through the point $A(2,3)$.

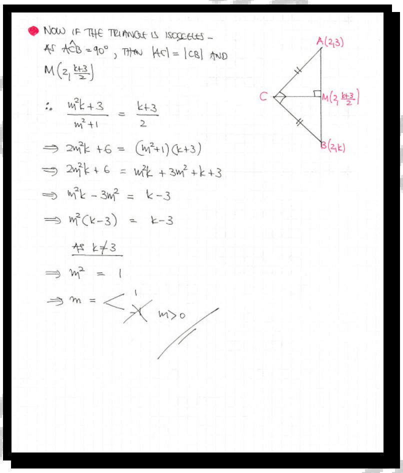
Another straight line L_2 is perpendicular to L_1 and passes through the point $B(2,k)$, $k \neq 3$.

The point C is the intersection of L_1 and L_2.

Determine the y coordinate of C, in terms of k and m, and given further that the triangle ABC is isosceles, prove that $m=1$.

SPM, $\boxed{y=\dfrac{km^2+3}{m^2+1}}$





Question 3 (*****)

The variables x and y are such so that

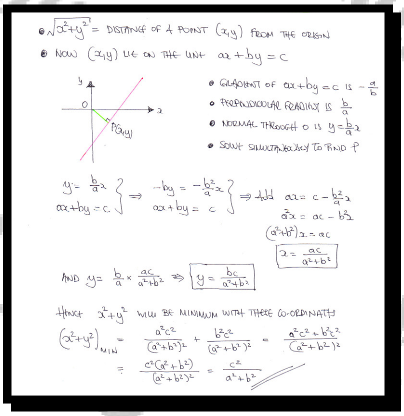
$$ax + by = c\,,$$

where a, b and c are non zero constants.

Show that the minimum value of $x^2 + y^2$ is

$$\frac{c^2}{a^2 + b^2}.$$

[ ], [proof]



- $\sqrt{x^2+y^2}$ = DISTANCE OF A POINT (x,y) FROM THE ORIGIN
- NOW (x,y) LIE ON THE LINE $ax+by=c$

- GRADIENT OF $ax+by=c$ IS $-\frac{a}{b}$
- PERPENDICULAR GRADIENT IS $\frac{b}{a}$
- NORMAL THROUGH O IS $y=\frac{b}{a}x$
- SOLVE SIMULTANEOUSLY TO FIND P

$$\left.\begin{array}{l} y=\frac{b}{a}x \\ ax+by=c \end{array}\right\} \Rightarrow \left.\begin{array}{l} -by=-\frac{b^2}{a}x \\ ax+by=c \end{array}\right\} \Rightarrow \text{Add } ax=c-\frac{b^2}{a}x$$

$$a^2x = ac - b^2x$$

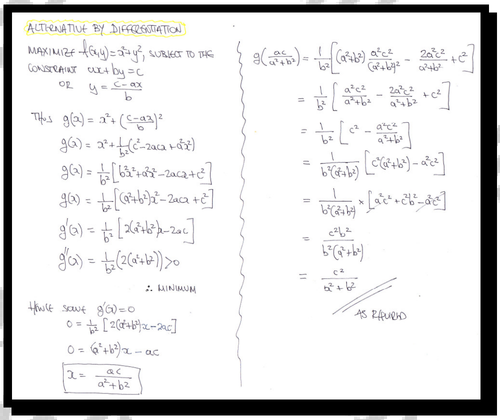
$$(a^2+b^2)x = ac$$

$$\boxed{x=\frac{ac}{a^2+b^2}}$$

AND $y=\frac{b}{a}\times\frac{ac}{a^2+b^2} \Rightarrow \boxed{y=\frac{bc}{a^2+b^2}}$

HENCE x^2+y^2 WILL BE MINIMUM WITH THESE CO-ORDINATES

$$(x^2+y^2)_{MIN} = \frac{a^2c^2}{(a^2+b^2)^2}+\frac{b^2c^2}{(a^2+b^2)^2} = \frac{a^2c^2+b^2c^2}{(a^2+b^2)^2} = \frac{c^2(a^2+b^2)}{(a^2+b^2)^2} = \frac{c^2}{a^2+b^2}$$



ALTERNATIVE BY DIFFERENTIATION

MAXIMIZE $f(x,y)=x^2+y^2$, SUBJECT TO THE CONSTRAINT $ax+by=c$

OR $y=\frac{c-ax}{b}$

THUS $g(x)=x^2+\left(\frac{c-ax}{b}\right)^2$

$$g(x)=x^2+\frac{1}{b^2}(c^2-2acx+a^2x^2)$$

$$g(x)=\frac{1}{b^2}\left[b^2x^2+a^2x^2-2acx+c^2\right]$$

$$g(x)=\frac{1}{b^2}\left[(a^2+b^2)x^2-2acx+c^2\right]$$

$$g'(x)=\frac{1}{b^2}\left[2(a^2+b^2)x-2ac\right]$$

$$g''(x)=\frac{1}{b^2}\left(2(a^2+b^2)\right)>0$$

$\therefore$ MINIMUM

HENCE SOLVE $g'(x)=0$

$$0=\frac{1}{b^2}\left[2(a^2+b^2)x-2ac\right]$$

$$0=(a^2+b^2)x-ac$$

$$\boxed{x=\frac{ac}{a^2+b^2}}$$

$$g\left(\frac{ac}{a^2+b^2}\right)=\frac{1}{b^2}\left[(a^2+b^2)\frac{a^2c^2}{(a^2+b^2)^2}-\frac{2a^2c^2}{a^2+b^2}+c^2\right]$$

$$=\frac{1}{b^2}\left[\frac{a^2c^2}{a^2+b^2}-\frac{2a^2c^2}{a^2+b^2}+c^2\right]$$

$$=\frac{1}{b^2}\left[c^2-\frac{a^2c^2}{a^2+b^2}\right]$$

$$=\frac{1}{b^2(a^2+b^2)}\left[c^2(a^2+b^2)-a^2c^2\right]$$

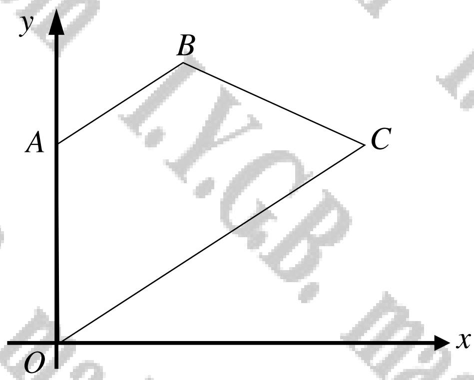
$$=\frac{1}{b^2(a^2+b^2)}\times\left[a^2c^2+c^2b^2-a^2c^2\right]$$

$$=\frac{c^2b^2}{b^2(a^2+b^2)}$$

$$=\frac{c^2}{a^2+b^2}$$

AS REQUIRED

Question 4 (*****)



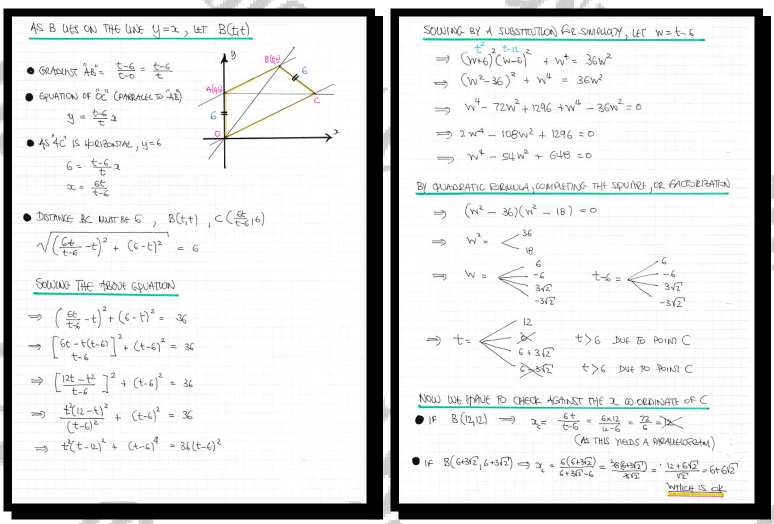
The figure above shows an isosceles trapezium $OABC$, where O is the origin.

It is further given that

- the coordinates of A are $(0,6)$,
- the sides AB and OC are parallel,
- $|OA| = |BC|$,
- the diagonal AC is parallel to the x axis.

If B lies on the line with equation $y = x$, determine, in exact simplified surd form, the coordinates of B.

$\boxed{B\left(6+3\sqrt{2},6+3\sqrt{2}\right)}$



Question 5 (*****)

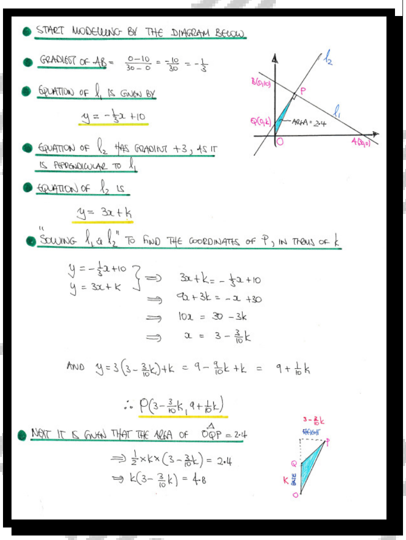
The straight line l_1 passes though the points $A(30,0)$ and $B(0,10)$.

The straight line l_2 is perpendicular to l_1.

The point P is the intersection between l_1 and l_2, and the point Q is the point where l_2 meets the y axis.

Given further that the area of the triangle OQP is 2.4, where O is the origin, find the possible area of the quadrilateral $AQPA$.

| | , | area $= 140.4$ or 149.4 |



- START MODELLING BY THE DIAGRAM BELOW
- GRADIENT OF AB $= \frac{0-10}{30-0} = \frac{-10}{30} = -\frac{1}{3}$
- EQUATION OF l_1 IS GIVEN BY

$y = -\frac{1}{3}x + 10$

- EQUATION OF l_2 HAS GRADIENT $+3$, AS IT IS PERPENDICULAR TO l_1
- EQUATION OF l_2 IS

$y = 3x + k$

- "SOLVING l_1 & l_2" TO FIND THE COORDINATES OF P, IN TERMS OF k

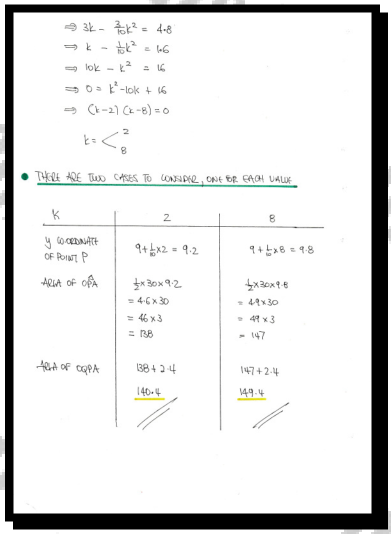
$y = -\frac{1}{3}x + 10$, $y = 3x + k$ $\Rightarrow 3x + k = -\frac{1}{3}x + 10$
$\Rightarrow 9x + 3k = -x + 30$
$\Rightarrow 10x = 30 - 3k$
$\Rightarrow x = 3 - \frac{3}{10}k$

AND $y = 3\left(3 - \frac{3}{10}k\right) + k = 9 - \frac{9}{10}k + k = 9 + \frac{1}{10}k$

$\therefore P\left(3 - \frac{3}{10}k, 9 + \frac{1}{10}k\right)$

- NEXT IT IS GIVEN THAT THE AREA OF $\overset{\triangle}{OQP} = 2.4$

$\Rightarrow \frac{1}{2} \times k \times \left(3 - \frac{3}{10}k\right) = 2.4$
$\Rightarrow k\left(3 - \frac{3}{10}k\right) = 4.8$



$\Rightarrow 3k - \frac{3}{10}k^2 = 4.8$
$\Rightarrow k - \frac{1}{10}k^2 = 1.6$
$\Rightarrow 10k - k^2 = 16$
$\Rightarrow 0 = k^2 - 10k + 16$
$\Rightarrow (k-2)(k-8) = 0$

$k = 2$ or 8

- THERE ARE TWO CASES TO CONSIDER, ONE FOR EACH VALUE

K	2	8
y CO-ORDINATE OF POINT P	$9 + \frac{1}{10} \times 2 = 9.2$	$9 + \frac{1}{10} \times 8 = 9.8$
AREA OF $\overset{\triangle}{OPA}$	$\frac{1}{2} \times 30 \times 9.2 = 4.6 \times 30 = 46 \times 3 = 138$	$\frac{1}{2} \times 30 \times 9.8 = 4.9 \times 30 = 49 \times 3 = 147$
AREA OF OQPA	$138 + 2.4$ 140.4	$147 + 2.4$ 149.4

Question 6 (*****)

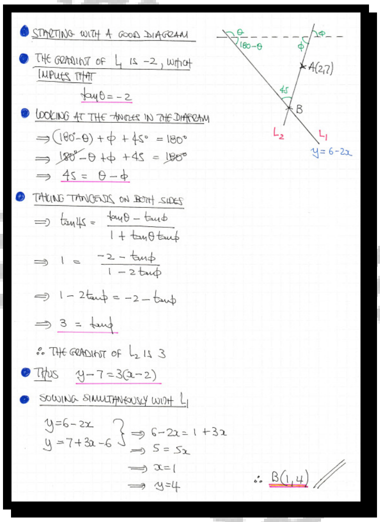
The straight line L_1 has equation

$$y = 6 - 2x,$$

The straight line L_2 passes through the point $A(2,7)$ and meets L_1 at the point B.

Given that L_1 and L_2 intersect each other at $45°$, determine the coordinates of B.

$\boxed{}$, $\boxed{B(1,4)}$


• STARTING WITH A GOOD DIAGRAM

• THE GRADIENT OF L_1 IS -2, WHICH IMPLIES THAT

$\tan\theta = -2$

• LOOKING AT THE ANGLES IN THE DIAGRAM

$\Rightarrow (180° - \theta) + \phi + 45° = 180°$

$\Rightarrow 180° - \theta + \phi + 45 = 180°$

$\Rightarrow 45 = \theta - \phi$

• TAKING TANGENTS ON BOTH SIDES

$\Rightarrow \tan 45 = \dfrac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$

$\Rightarrow 1 = \dfrac{-2 - \tan\phi}{1 - 2\tan\phi}$

$\Rightarrow 1 - 2\tan\phi = -2 - \tan\phi$

$\Rightarrow 3 = \tan\phi$

$\therefore$ THE GRADIENT OF L_2 IS 3

• THUS $y - 7 = 3(x - 2)$

• SOLVING SIMULTANEOUSLY WITH L_1

$\left.\begin{array}{l} y = 6 - 2x \\ y = 7 + 3x - 6 \end{array}\right\} \Rightarrow 6 - 2x = 1 + 3x$

$\Rightarrow 5 = 5x$

$\Rightarrow x = 1$

$\Rightarrow y = 4$

$\therefore B(1,4)$

Question 7 (*****)

The straight lines L_1 and L_2 have respective equations

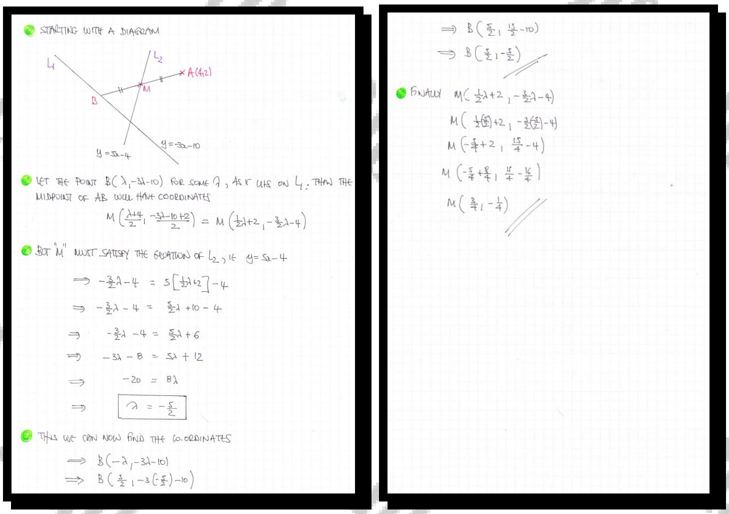
$$y = -3x - 10 \qquad \text{and} \qquad y = 5x - 4 .$$

The point A has coordinates $(4,2)$.

The point B lies on L_1, so that the midpoint M of the straight line segment AB, lies on L_2.

Determine the coordinates of B and the coordinates of M.

$$\boxed{}, \quad \boxed{B\left(\frac{5}{2},\frac{5}{2}\right), \quad M\left(\frac{3}{4},-\frac{1}{4}\right)}$$



STARTING WITH A DIAGRAM

LET THE POINT $B(\lambda, -3\lambda-10)$ FOR SOME λ, AS IT LIES ON L_1. THEN THE MIDPOINT OF AB WILL HAVE COORDINATES

$$M\left(\frac{\lambda+4}{2}, \frac{-3\lambda-10+2}{2}\right) = M\left(\tfrac{1}{2}\lambda+2, -\tfrac{3}{2}\lambda-4\right)$$

BUT "M" MUST SATISFY THE EQUATION OF L_2, IE $y=5x-4$

$$\Rightarrow -\tfrac{3}{2}\lambda - 4 = 5\left[\tfrac{1}{2}\lambda+2\right]-4$$
$$\Rightarrow -\tfrac{3}{2}\lambda - 4 = \tfrac{5}{2}\lambda + 10 - 4$$
$$\Rightarrow -\tfrac{3}{2}\lambda - 4 = \tfrac{5}{2}\lambda + 6$$
$$\Rightarrow -3\lambda - 8 = 5\lambda + 12$$
$$\Rightarrow -20 = 8\lambda$$
$$\Rightarrow \boxed{\lambda = -\tfrac{5}{2}}$$

THUS WE CAN NOW FIND THE CO-ORDINATES

$$\Rightarrow B(-\lambda, -3\lambda-10)$$
$$\Rightarrow B\left(\tfrac{5}{2}, -3\left(-\tfrac{5}{2}\right)-10\right)$$
$$\Rightarrow B\left(\tfrac{5}{2}, \tfrac{15}{2}-10\right)$$
$$\Rightarrow B\left(\tfrac{5}{2}, -\tfrac{5}{2}\right)$$

FINALLY $M\left(\tfrac{1}{2}\lambda+2, -\tfrac{3}{2}\lambda-4\right)$

$$M\left(\tfrac{1}{2}\left(\tfrac{5}{2}\right)+2, -\tfrac{3}{2}\left(\tfrac{5}{2}\right)-4\right)$$
$$M\left(-\tfrac{5}{4}+2, \tfrac{15}{4}-4\right)$$
$$M\left(-\tfrac{5}{4}+\tfrac{8}{4}, \tfrac{15}{4}-\tfrac{16}{4}\right)$$
$$M\left(\tfrac{3}{4}, -\tfrac{1}{4}\right)$$

Question 8 (*****)

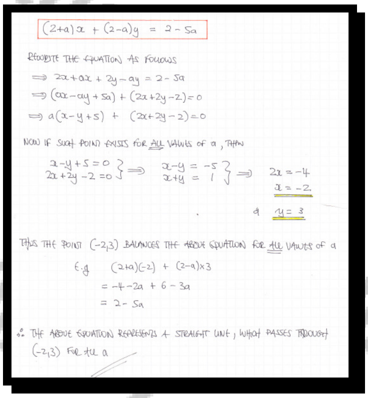
The straight line l has equation

$$(2+a)x+(2-a)y=2-5a,$$

where a is a constant.

Show that l passes through a fixed point P for all values of a.

[ ] , [proof]



$(2+a)x + (2-a)y = 2 - 5a$

REWRITE THE EQUATION AS FOLLOWS

$\Rightarrow 2x + ax + 2y - ay = 2 - 5a$

$\Rightarrow (ax - ay + 5a) + (2x + 2y - 2) = 0$

$\Rightarrow a(x - y + 5) + (2x + 2y - 2) = 0$

NOW IF SUCH POINT EXISTS FOR ALL VALUES OF a, THEN

$\left.\begin{array}{l} x - y + 5 = 0 \\ 2x + 2y - 2 = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x - y = -5 \\ x + y = 1 \end{array}\right\} \Rightarrow 2x = -4$

$x = -2$

& $y = 3$

THUS THE POINT $(-2,3)$ BALANCES THE ABOVE EQUATION FOR ALL VALUES OF a

E.g. $(2+a)(-2) + (2-a)\times 3$

$= -4 - 2a + 6 - 3a$

$= 2 - 5a$

$\therefore$ THE ABOVE EQUATION REPRESENTS A STRAIGHT LINE, WHICH PASSES THROUGH $(-2,3)$ FOR ALL a

Question 9 (*****)

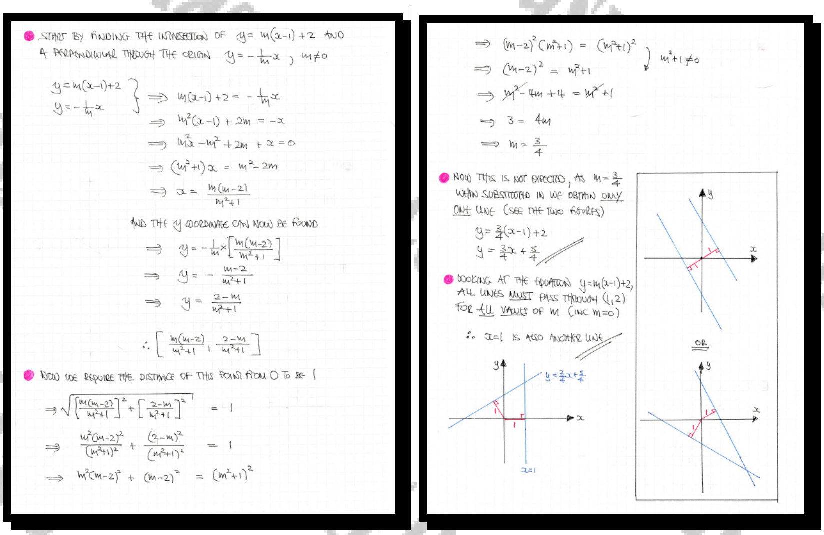
A family of straight lines, has equation

$$y = m(x-1)+2, \quad x \in \mathbb{R},$$

where m is a parameter.

From the above family of straight lines, determine the equations of any straight lines whose distance from the origin O is 1 unit.

SP , $\boxed{y = \frac{1}{4}(3x+5)}$, $\boxed{x=1}$



Question 10 (*****)

The straight lines l_1 and l_2 have respective equations

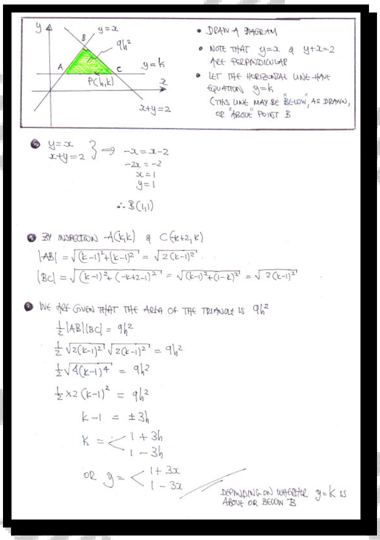
$$y = x \quad \text{and} \quad x + y = 2 .$$

The straight line l_3 is parallel to the x axis and passes through the point $P(h,k)$.

It is further given that the point B is the intersection of l_1 and l_2, the point A is the intersection of l_1 and l_3 and the point C is the intersection of l_2 and l_3,

Find the locus of P if the area of the triangle ABC is $9h^2$.

SP-U, $y = 1 \pm 3x$



- DRAW A DIAGRAM
- NOTE THAT $y=x$ & $y+x=2$ ARE PERPENDICULAR
- LET THE HORIZONTAL LINE HAVE EQUATION $y=k$ (THIS LINE MAY BE "BELOW", AS DRAWN, OR "ABOVE" POINT B

- $y=x$, $x+y=2$ $\Rightarrow$ $-x = x-2$, $-2x=-2$, $x=1$, $y=1$ $\therefore B(1,1)$

- BY INSPECTION $A(k,k)$ & $C(-k+2,k)$

$|AB| = \sqrt{(k-1)^2+(k-1)^2} = \sqrt{2(k-1)^2}$

$|BC| = \sqrt{(k-1)^2+(-k+2-1)^2} = \sqrt{(k-1)^2+(1-k)^2} = \sqrt{2(k-1)^2}$

- WE ARE GIVEN THAT THE AREA OF THE TRIANGLE IS $9h^2$

$\frac{1}{2}|AB||BC| = 9h^2$

$\frac{1}{2}\sqrt{2(k-1)^2}\sqrt{2(k-1)^2} = 9h^2$

$\frac{1}{2}\sqrt{4(k-1)^4} = 9h^2$

$\frac{1}{2}\times 2(k-1)^2 = 9h^2$

$k-1 = \pm 3h$

$k = 1+3h$ or $1-3h$

OR $y = 1+3x$ or $1-3x$ — DEPENDING ON WHETHER $y=k$ IS ABOVE OR BELOW B

Question 11 (*****)

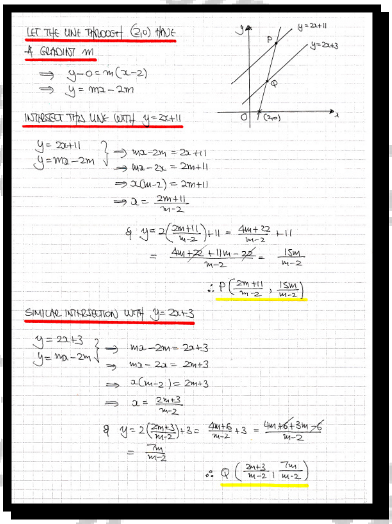
The straight parallel lines l_1 and l_2 have respective equations

$$y = 2x + 11 \qquad \text{and} \qquad y = 3x + 3 .$$

The straight line l_3, passing through the point $P(2,0)$, intersects l_1 and l_2 at the points P and Q respectively.

Given that $|PQ| = 8$ determine the possible equation of l_3.

$\boxed{}$, $\boxed{x = 2}$, $\boxed{y = \frac{3}{4}x - \frac{3}{2}}$



LET THE LINE THROUGH $(2,0)$ HAVE A GRADIENT m

$\Rightarrow y - 0 = m(x-2)$
$\Rightarrow y = mx - 2m$

INTERSECT THIS LINE WITH $y = 2x+11$

$\left.\begin{array}{l} y = 2x+11 \\ y = mx-2m \end{array}\right\} \Rightarrow mx - 2m = 2x + 11$
$\Rightarrow mx - 2x = 2m + 11$
$\Rightarrow x(m-2) = 2m+11$
$\Rightarrow x = \frac{2m+11}{m-2}$

& $y = 2\left(\frac{2m+11}{m-2}\right) + 11 = \frac{4m+22}{m-2} + 11 = \frac{4m+22+11m-22}{m-2} = \frac{15m}{m-2}$

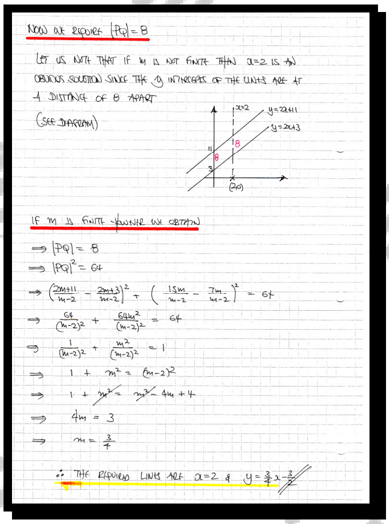
$\therefore P\left(\frac{2m+11}{m-2}, \frac{15m}{m-2}\right)$

SIMILAR INTERSECTION WITH $y = 2x+3$

$\left.\begin{array}{l} y = 2x+3 \\ y = mx-2m \end{array}\right\} \Rightarrow mx - 2m = 2x + 3$
$\Rightarrow mx - 2x = 2m + 3$
$\Rightarrow x(m-2) = 2m+3$
$\Rightarrow x = \frac{2m+3}{m-2}$

& $y = 2\left(\frac{2m+3}{m-2}\right) + 3 = \frac{4m+6}{m-2} + 3 = \frac{4m+6+3m-6}{m-2} = \frac{7m}{m-2}$

$\therefore Q\left(\frac{2m+3}{m-2}, \frac{7m}{m-2}\right)$



NOW WE REQUIRE $|PQ| = 8$

LET US NOTE THAT IF m IS NOT FINITE THEN $x=2$ IS AN OBVIOUS SOLUTION SINCE THE y INTERCEPTS OF THE LINES ARE AT A DISTANCE OF 8 APART

(SEE DIAGRAM)

IF m IS FINITE HOWEVER WE OBTAIN

$\Rightarrow |PQ| = 8$
$\Rightarrow |PQ|^2 = 64$
$\Rightarrow \left(\frac{2m+11}{m-2} - \frac{2m+3}{m-2}\right)^2 + \left(\frac{15m}{m-2} - \frac{7m}{m-2}\right)^2 = 64$
$\Rightarrow \frac{64}{(m-2)^2} + \frac{64m^2}{(m-2)^2} = 64$
$\Rightarrow \frac{1}{(m-2)^2} + \frac{m^2}{(m-2)^2} = 1$
$\Rightarrow 1 + m^2 = (m-2)^2$
$\Rightarrow 1 + m^2 = m^2 - 4m + 4$
$\Rightarrow 4m = 3$
$\Rightarrow m = \frac{3}{4}$

$\therefore$ THE REQUIRED LINES ARE $x = 2$ & $y = \frac{3}{4}x - \frac{3}{2}$

Question 12 (*****)

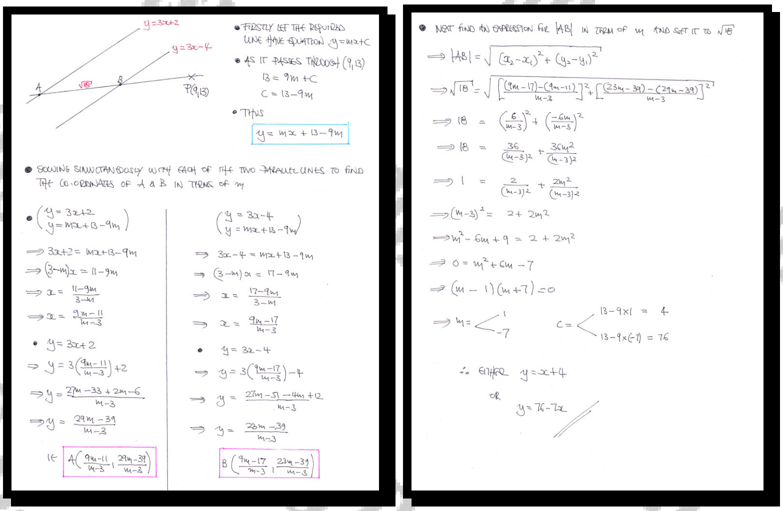
The straight parallel lines l_1 and l_2 have respective equations

$$y = 3x + 2 \qquad \text{and} \qquad y = 3x - 4 .$$

The straight line l_3, passing through the point $P(9,13)$, intersects l_1 and l_2 at the points A and B respectively.

Given that $|AB| = \sqrt{18}$ determine the possible equation of l_3.

$\boxed{y = x + 4}$, $\boxed{y = 76 - 7x}$



• FIRSTLY LET THE REQUIRED LINE HAVE EQUATION $y = mx + c$

• AS IT PASSES THROUGH $(9,13)$

$13 = 9m + c$

$c = 13 - 9m$

• THUS

$\boxed{y = mx + 13 - 9m}$

• SOLVING SIMULTANEOUSLY WITH EACH OF THE TWO PARALLEL LINES TO FIND THE CO-ORDINATES OF A & B IN TERMS OF m

• $\begin{pmatrix} y = 3x + 2 \\ y = mx + 13 - 9m \end{pmatrix}$

$\Rightarrow 3x + 2 = mx + 13 - 9m$

$\Rightarrow (3 - m)x = 11 - 9m$

$\Rightarrow x = \frac{11 - 9m}{3 - m}$

$\Rightarrow x = \frac{9m - 11}{m - 3}$

• $y = 3x + 2$

$\Rightarrow y = 3\left(\frac{9m - 11}{m - 3}\right) + 2$

$\Rightarrow y = \frac{27m - 33 + 2m - 6}{m - 3}$

$\Rightarrow y = \frac{29m - 39}{m - 3}$

ie $\boxed{A\left(\frac{9m - 11}{m - 3}, \frac{29m - 39}{m - 3}\right)}$

$\begin{pmatrix} y = 3x - 4 \\ y = mx + 13 - 9m \end{pmatrix}$

$\Rightarrow 3x - 4 = mx + 13 - 9m$

$\Rightarrow (3 - m)x = 17 - 9m$

$\Rightarrow x = \frac{17 - 9m}{3 - m}$

$\Rightarrow x = \frac{9m - 17}{m - 3}$

• $y = 3x - 4$

$\Rightarrow y = 3\left(\frac{9m - 17}{m - 3}\right) - 4$

$\Rightarrow y = \frac{27m - 51 - 4m + 12}{m - 3}$

$\Rightarrow y = \frac{23m - 39}{m - 3}$

$\boxed{B\left(\frac{9m - 17}{m - 3}, \frac{23m - 39}{m - 3}\right)}$

• NEXT FIND AN EXPRESSION FOR $|AB|$ IN TERM OF m AND SET IT TO $\sqrt{18}$

$\Rightarrow |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{18} = \sqrt{\left[\frac{(9m - 17) - (9m - 11)}{m - 3}\right]^2 + \left[\frac{(23m - 39) - (29m - 39)}{m - 3}\right]^2}$

$\Rightarrow 18 = \left(\frac{6}{m - 3}\right)^2 + \left(\frac{-6m}{m - 3}\right)^2$

$\Rightarrow 18 = \frac{36}{(m - 3)^2} + \frac{36m^2}{(m - 3)^2}$

$\Rightarrow 1 = \frac{2}{(m - 3)^2} + \frac{2m^2}{(m - 3)^2}$

$\Rightarrow (m - 3)^2 = 2 + 2m^2$

$\Rightarrow m^2 - 6m + 9 = 2 + 2m^2$

$\Rightarrow 0 = m^2 + 6m - 7$

$\Rightarrow (m - 1)(m + 7) = 0$

$\Rightarrow m = 1$ or -7 $\qquad c = 13 - 9 \times 1 = 4$ or $13 - 9 \times (-7) = 76$

$\therefore$ EITHER $y = x + 4$

OR

$y = 76 - 7x$

Question 13 (*****)

The straight parallel lines l_1 and l_2 have respective equations

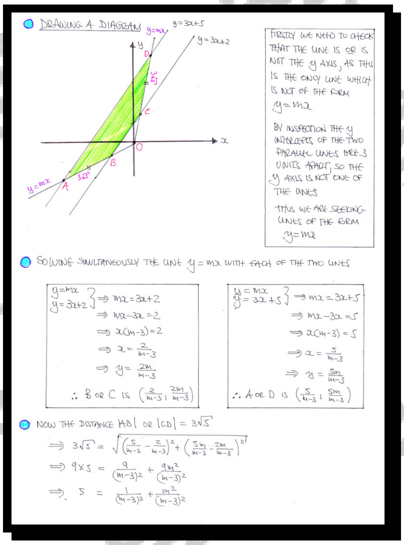
$$y = 3x + 5 \quad \text{and} \quad y = 3x + 2 .$$

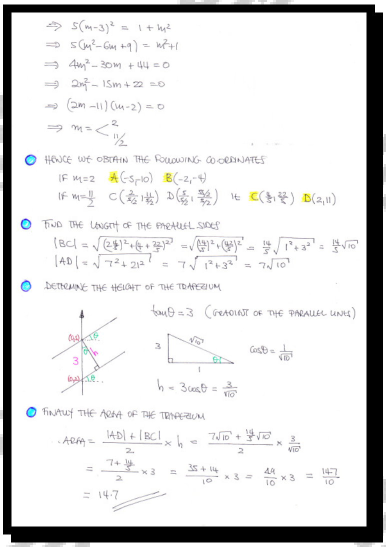
Two more straight lines, both passing through the origin O, intersect l_1 and l_2 forming a trapezium $ABCD$.

The trapezium $ABCD$ is isosceles with $|AB| = |CD| = 3\sqrt{5}$.

Determine the area of this trapezium.

$\boxed{\text{area} = 14.7}$





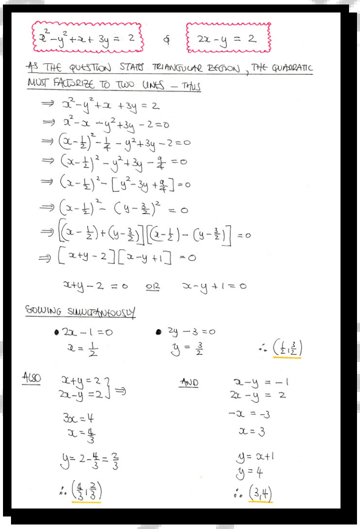
Question 14 (*****)

Show that the area of the triangular region bounded by

$$x^2 - y^2 + x + 3y = 2 \qquad \text{and} \qquad 2x - y = 2,$$

is $2\frac{1}{12}$ square units.

[ ], [proof]



$x^2 - y^2 + x + 3y = 2$ & $2x - y = 2$

AS THE QUESTION STATES TRIANGULAR REGION, THE QUADRATIC MUST FACTORIZE TO TWO LINES – THUS

$\Rightarrow x^2 - y^2 + x + 3y = 2$

$\Rightarrow x^2 - x - y^2 + 3y - 2 = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - y^2 + 3y - 2 = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 - y^2 + 3y - \frac{9}{4} = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 - \left[y^2 - 3y + \frac{9}{4}\right] = 0$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 - \left(y - \frac{3}{2}\right)^2 = 0$

$\Rightarrow \left[\left(x - \frac{1}{2}\right) + \left(y - \frac{3}{2}\right)\right]\left[\left(x - \frac{1}{2}\right) - \left(y - \frac{3}{2}\right)\right] = 0$

$\Rightarrow [x + y - 2][x - y + 1] = 0$

$x + y - 2 = 0$ OR $x - y + 1 = 0$

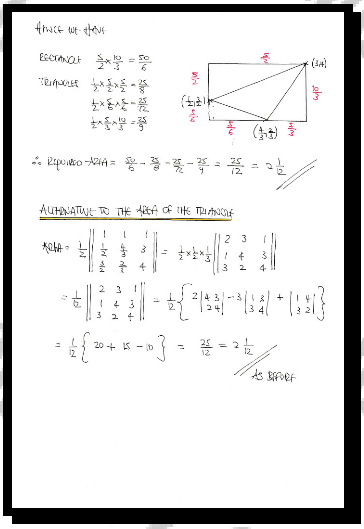
SOLVING SIMULTANEOUSLY

- $2x - 1 = 0$, $x = \frac{1}{2}$
- $2y - 3 = 0$, $y = \frac{3}{2}$

$\therefore \left(\frac{1}{2}, \frac{3}{2}\right)$

ALSO $\left.\begin{array}{l} x + y = 2 \\ 2x - y = 2 \end{array}\right\} \Rightarrow$ $3x = 4$, $x = \frac{4}{3}$, $y = 2 - \frac{4}{3} = \frac{2}{3}$ $\therefore \left(\frac{4}{3}, \frac{2}{3}\right)$

AND $x - y = -1$, $2x - y = 2$, $-x = -3$, $x = 3$, $y = x + 1$, $y = 4$ $\therefore (3, 4)$



HENCE WE HAVE

RECTANGLE $\frac{5}{2} \times \frac{10}{3} = \frac{50}{6}$

TRIANGLES $\frac{1}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{25}{8}$

$\frac{1}{2} \times \frac{5}{6} \times \frac{5}{6} = \frac{25}{72}$

$\frac{1}{2} \times \frac{5}{3} \times \frac{10}{3} = \frac{25}{9}$

$\therefore$ REQUIRED AREA $= \frac{50}{6} - \frac{25}{8} - \frac{25}{72} - \frac{25}{9} = \frac{25}{12} = 2\frac{1}{12}$

ALTERNATIVE TO THE AREA OF THE TRIANGLE

$$\text{AREA} = \frac{1}{2}\left\|\begin{matrix} 1 & 1 & 1 \\ \frac{1}{2} & \frac{4}{3} & 3 \\ \frac{3}{2} & \frac{2}{3} & 4 \end{matrix}\right\| = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}\left\|\begin{matrix} 2 & 3 & 1 \\ 1 & 4 & 3 \\ 3 & 2 & 4 \end{matrix}\right\|$$

$$= \frac{1}{12}\left\|\begin{matrix} 2 & 3 & 1 \\ 1 & 4 & 3 \\ 3 & 2 & 4 \end{matrix}\right\| = \frac{1}{12}\left\{2\begin{vmatrix} 4 & 3 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix}\right\}$$

$$= \frac{1}{12}\{20 + 15 - 10\} = \frac{25}{12} = 2\frac{1}{12}$$

AS BEFORE

Question 15 (*****)

A family of straight lines has equation

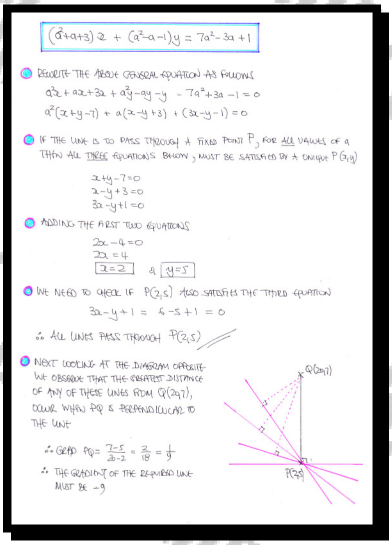
$$\left(a^2+a+3\right)x+\left(a^2-a-3\right)y=7a^2-3a+1,$$

where a is a parameter.

The point Q has coordinates $(20,7)$.

Show that this family of lines passes through a fixed point P for all values of a, and hence determine the equation of a straight line from this family of straight lines, which is furthest away from Q.

$P(2,5)$, $y=23-9x$



$(a^2+a+3)x+(a^2-a-1)y=7a^2-3a+1$

- REWRITE THE ABOVE GENERAL EQUATION AS FOLLOWS

$a^2x+ax+3x+a^2y-ay-y-7a^2+3a-1=0$

$a^2(x+y-7)+a(x-y+3)+(3x-y-1)=0$

- IF THE LINE IS TO PASS THROUGH A FIXED POINT P, FOR ALL VALUES OF a THEN ALL THREE EQUATIONS BELOW, MUST BE SATISFIED BY A UNIQUE $P(x,y)$

$x+y-7=0$

$x-y+3=0$

$3x-y+1=0$

- ADDING THE FIRST TWO EQUATIONS

$2x-4=0$

$2x=4$

$\boxed{x=2}$ & $\boxed{y=5}$

- WE NEED TO CHECK IF $P(2,5)$ ALSO SATISFIES THE THIRD EQUATION

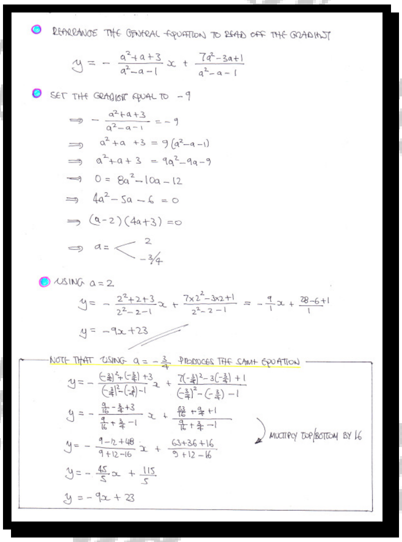
$3x-y+1=6-5+1=0$

$\therefore$ ALL LINES PASS THROUGH $P(2,5)$

- NEXT LOOKING AT THE DIAGRAM OPPOSITE WE OBSERVE THAT THE GREATEST DISTANCE OF ANY OF THESE LINES FROM $Q(20,7)$, OCCUR WHEN PQ IS PERPENDICULAR TO THE LINE

$\therefore$ GRAD $PQ=\frac{7-5}{20-2}=\frac{2}{18}=\frac{1}{9}$

$\therefore$ THE GRADIENT OF THE REQUIRED LINE MUST BE -9



- REARRANGE THE GENERAL EQUATION TO READ OFF THE GRADIENT

$y=-\frac{a^2+a+3}{a^2-a-1}x+\frac{7a^2-3a+1}{a^2-a-1}$

- SET THE GRADIENT EQUAL TO -9

$\Rightarrow -\frac{a^2+a+3}{a^2-a-1}=-9$

$\Rightarrow a^2+a+3=9(a^2-a-1)$

$\Rightarrow a^2+a+3=9a^2-9a-9$

$\Rightarrow 0=8a^2-10a-12$

$\Rightarrow 4a^2-5a-6=0$

$\Rightarrow (a-2)(4a+3)=0$

$\Rightarrow a=2$ or $-3/4$

- USING $a=2$

$y=-\frac{2^2+2+3}{2^2-2-1}x+\frac{7\times2^2-3\times2+1}{2^2-2-1}=-\frac{9}{1}x+\frac{28-6+1}{1}$

$y=-9x+23$

NOTE THAT USING $a=-\frac{3}{4}$ PRODUCES THE SAME EQUATION

$y=-\frac{(-\frac{3}{4})^2+(-\frac{3}{4})+3}{(-\frac{3}{4})^2-(-\frac{3}{4})-1}x+\frac{7(-\frac{3}{4})^2-3(-\frac{3}{4})+1}{(-\frac{3}{4})^2-(-\frac{3}{4})-1}$

$y=-\frac{\frac{9}{16}-\frac{3}{4}+3}{\frac{9}{16}+\frac{3}{4}-1}x+\frac{\frac{63}{16}+\frac{9}{4}+1}{\frac{9}{16}+\frac{3}{4}-1}$

MULTIPLY TOP/BOTTOM BY 16

$y=-\frac{9-12+48}{9+12-16}x+\frac{63+36+16}{9+12-16}$

$y=-\frac{45}{5}x+\frac{115}{5}$

$y=-9x+23$

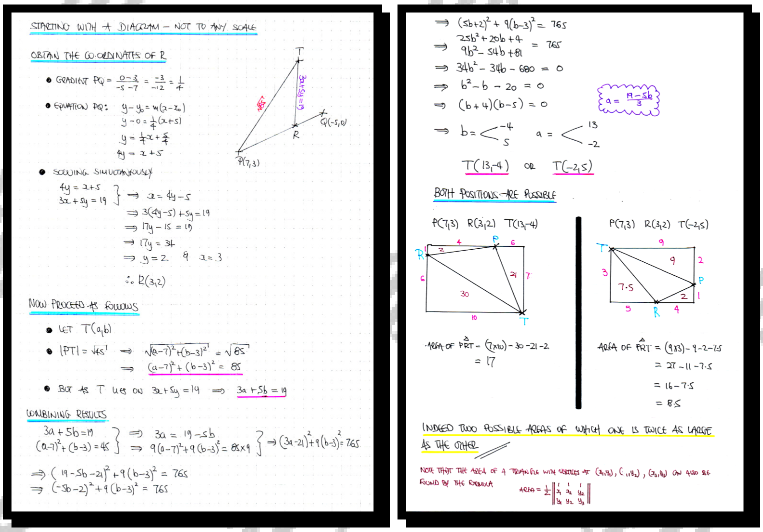
Question 16 (*****)

The points P and Q have coordinates $(7,3)$ and $(-5,0)$, respectively.

The straight line segment RT, with equation $3x+5y=19$, intersects the straight line segment PQ at the point R.

Given further that the length of PT is $\sqrt{85}$, show that the area of the triangle PTR can take two values, one being twice as large as the other.

[ ] , [proof]


STARTING WITH A DIAGRAM – NOT TO ANY SCALE

OBTAIN THE CO-ORDINATES OF R

- GRADIENT PQ $= \frac{0-3}{-5-7} = \frac{-3}{-12} = \frac{1}{4}$
- EQUATION PQ: $y-y_0 = m(x-x_0)$
  $y-0 = \frac{1}{4}(x+5)$
  $y = \frac{1}{4}x + \frac{5}{4}$
  $4y = x+5$
- SOLVING SIMULTANEOUSLY
  $\left.\begin{array}{l}4y = x+5\\ 3x+5y=19\end{array}\right\} \Rightarrow x = 4y-5$
  $\Rightarrow 3(4y-5)+5y = 19$
  $\Rightarrow 17y - 15 = 19$
  $\Rightarrow 17y = 34$
  $\Rightarrow y = 2$ & $x = 3$
  $\therefore R(3,2)$

NOW PROCEED AS FOLLOWS

- LET $T(a,b)$
- $|PT| = \sqrt{45} \Rightarrow \sqrt{(a-7)^2+(b-3)^2} = \sqrt{85}$
  $\Rightarrow (a-7)^2+(b-3)^2 = 85$
- BUT AS T LIES ON $3x+5y=19 \Rightarrow 3a+5b=19$

COMBINING RESULTS

$\left.\begin{array}{l}3a+5b=19\\ (a-7)^2+(b-3)=45\end{array}\right\} \Rightarrow \left.\begin{array}{l}3a = 19-5b\\ 9(a-7)^2+9(b-3)^2 = 85\times 9\end{array}\right\} \Rightarrow (3a-21)^2+9(b-3)^2 = 765$

$\Rightarrow (19-5b-21)^2+9(b-3)^2 = 765$
$\Rightarrow (-5b-2)^2+9(b-3)^2 = 765$
$\Rightarrow (5b+2)^2+9(b-3)^2 = 765$
$\Rightarrow \begin{array}{l}25b^2+20b+4\\ 9b^2-54b+81\end{array} = 765$
$\Rightarrow 34b^2-34b-680 = 0$
$\Rightarrow b^2-b-20 = 0$
$\Rightarrow (b+4)(b-5) = 0$
$\Rightarrow b = -4, 5 \quad a = 13, -2$ $\quad a = \frac{19-5b}{3}$

$T(13,-4)$ OR $T(-2,5)$

BOTH POSITIONS ARE POSSIBLE

$P(7,3)$ $R(3,2)$ $T(13,-4)$

AREA OF $\hat{PRT} = (7\times10)-30-21-2$
$= 17$

$P(7,3)$ $R(3,2)$ $T(-2,5)$

AREA OF $\hat{PRT} = (9\times3)-9-2-7.5$
$= 27-11-7.5$
$= 16-7.5$
$= 8.5$

INDEED TWO POSSIBLE AREAS OF WHICH ONE IS TWICE AS LARGE AS THE OTHER

NOTE THAT THE AREA OF A TRIANGLE WITH VERTICES AT (x_1,y_1), (x_2,y_2), (x_3,y_3) CAN ALSO BE FOUND BY THE FORMULA

AREA $= \frac{1}{2}\left\|\begin{array}{ccc}1 & 1 & 1\\ x_1 & x_2 & x_3\\ y_1 & y_2 & y_3\end{array}\right\|$

Question 17 (*****)

Two parallel straight lines, L_1 and L_2, have respective equations

$$y = 2x + 5 \qquad \text{and} \qquad y = 2x - 1 \ .$$

L_1 and L_2, are tangents to a circle centred at the point C .

A third line L_3 is perpendicular to L_1 and L_2, and meets the circle in two distinct points, A and B.

Given that L_3 passes through the point $(9,0)$, find, in exact simplified surd form, the coordinates of C.

$\boxed{}$, $\boxed{C\left[\frac{1}{10}\left(5+\sqrt{61}\right), \frac{1}{5}\left(15+\sqrt{61}\right)\right]}$