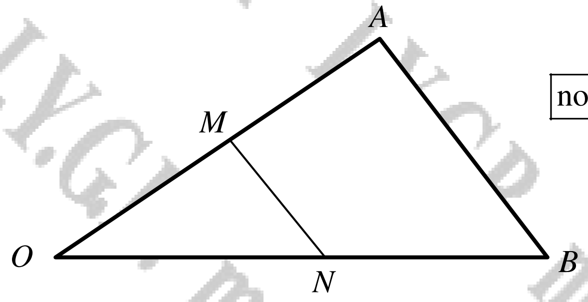
# VECTOR PRACTICE

## Part A

# INTRODUCING VECTOR ALGEBRA AND GEOMETRY

**Question 1** (**)

The figure below shows the triangle $OAB$.



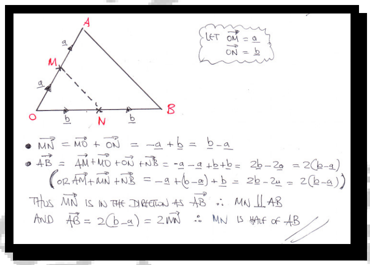

not drawn accurately

The point $M$ is the midpoint of $OA$ and the point $N$ is the midpoint of $OB$.

Let $\overrightarrow{OM} = \mathbf{a}$ and $\overrightarrow{ON} = \mathbf{b}$.

By finding simplified expressions for $\overrightarrow{MN}$ and $\overrightarrow{AB}$, in terms of $\mathbf{a}$ and $\mathbf{b}$, show that $MN$ is parallel to $AB$, and half its length.

proof



**Question 2 (**+)**

$OABC$ is a square.

The point $M$ is the midpoint of $AB$ and the point $N$ is the midpoint of $MC$.

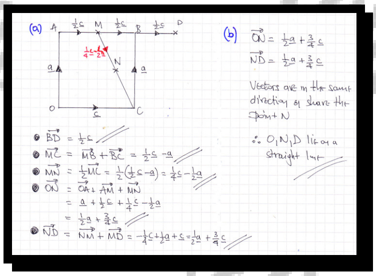
The point $D$ is such so that $\overrightarrow{AD} = \frac{3}{2}\overrightarrow{AB}$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

**a)** Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{BD}$, $\overrightarrow{MC}$, $\overrightarrow{MN}$, $\overrightarrow{ON}$ and $\overrightarrow{ND}$.

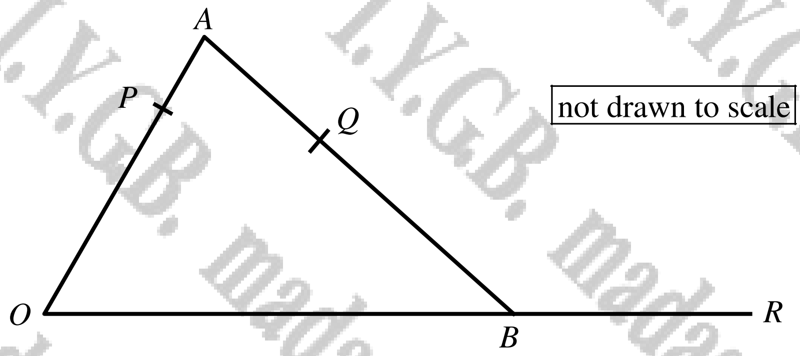
**b)** Deduce, showing your reasoning, that $O$, $N$ and $D$ are collinear.

$\boxed{\overrightarrow{BD} = \frac{1}{2}\mathbf{c}}$, $\boxed{\overrightarrow{MC} = \frac{1}{2}\mathbf{c} - \mathbf{a}}$, $\boxed{\overrightarrow{MN} = \frac{1}{4}\mathbf{c} - \frac{1}{2}\mathbf{a}}$, $\boxed{\overrightarrow{ON} = \frac{1}{2}\mathbf{a} + \frac{3}{4}\mathbf{c}}$, $\boxed{\overrightarrow{ND} = \frac{1}{2}\mathbf{a} + \frac{3}{4}\mathbf{c}}$



**Question 3** (***)

The figure below shows a triangle $OAB$.



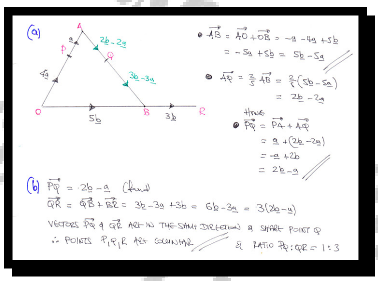

- The point $P$ lies on $OA$ so that $OP : PA = 4:1$.
- The point $Q$ lies on $AB$ so that $AQ : QB = 2:3$
- The side $OB$ is extended to the point $R$ so that $OB : BR = 5:3$.

Let $\overrightarrow{PA} = \mathbf{a}$ and $\overrightarrow{OB} = 5\mathbf{b}$.

**a)** Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{AQ}$ and $\overrightarrow{PQ}$.

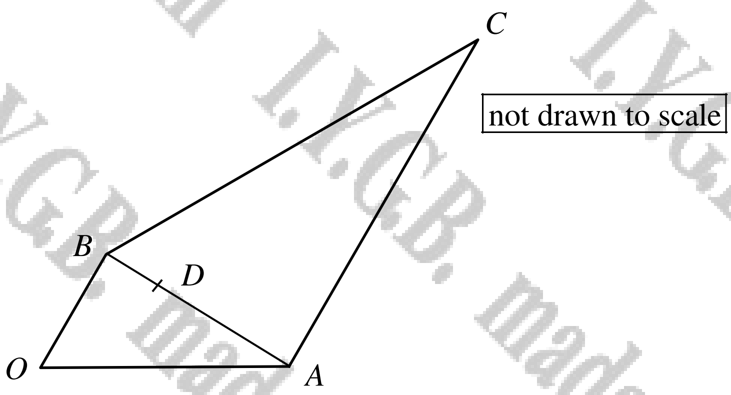
**b)** Deduce, showing your reasoning, that $P$, $Q$ and $R$ are collinear and state the ratio of $PQ : QR$.

$\boxed{\overrightarrow{AB} = 5\mathbf{b} - 5\mathbf{a}}$, $\boxed{\overrightarrow{AQ} = 2\mathbf{b} - 2\mathbf{a}}$, $\boxed{\overrightarrow{PQ} = 2\mathbf{b} - \mathbf{a}}$, $\boxed{PQ : QR = 1:3}$



**Question 4** (***)

The figure below shows a trapezium $OBCA$ where $OB$ is parallel to $AC$.



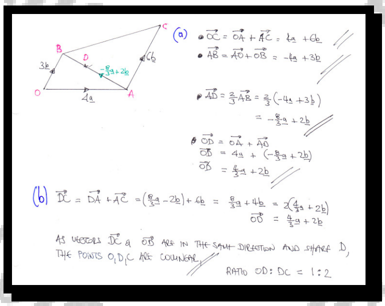

The point $D$ lies on $BA$ so that $BD : DA = 1 : 2$.

Let $\overrightarrow{OA} = 4\mathbf{a}$, $\overrightarrow{OB} = 3\mathbf{b}$ and $\overrightarrow{AC} = 6\mathbf{b}$.

**a)** Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{OC}$, $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{OD}$.

**b)** Deduce, showing your reasoning, that $O, D$ and $C$ are collinear and state the ratio of $OD : DC$.

$\boxed{\overrightarrow{OC} = 4\mathbf{a} + 6\mathbf{b}}$, $\boxed{\overrightarrow{AB} = -4\mathbf{a} + 3\mathbf{b}}$, $\boxed{\overrightarrow{AD} = -\frac{8}{3}\mathbf{a} + 2\mathbf{b}}$, $\boxed{\overrightarrow{OD} = \frac{4}{3}\mathbf{a} + 2\mathbf{b}}$, $\boxed{OD : DC = 1 : 2}$


(a) • $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 4\underline{a} + 6\underline{b}$

• $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -4\underline{a} + 3\underline{b}$

• $\overrightarrow{AD} = \frac{2}{3}\overrightarrow{AB} = \frac{2}{3}(-4\underline{a} + 3\underline{b}) = -\frac{8}{3}\underline{a} + 2\underline{b}$

• $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$\overrightarrow{OD} = 4\underline{a} + (-\frac{8}{3}\underline{a} + 2\underline{b})$

$\overrightarrow{OD} = \frac{4}{3}\underline{a} + 2\underline{b}$

(b) $\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AC} = (\frac{8}{3}\underline{a} - 2\underline{b}) + 6\underline{b} = \frac{8}{3}\underline{a} + 4\underline{b} = 2(\frac{4}{3}\underline{a} + 2\underline{b})$

$\overrightarrow{OD} = \frac{4}{3}\underline{a} + 2\underline{b}$

AS VECTORS $\overrightarrow{DC}$ & $\overrightarrow{OD}$ ARE IN THE SAME DIRECTION AND SHARE D, THE POINTS O, D, C ARE COLLINEAR.

RATIO OD : DC = 1 : 2

**Question 5** (***)

$OABC$ is a parallelogram and the point $M$ is the midpoint of $AB$.

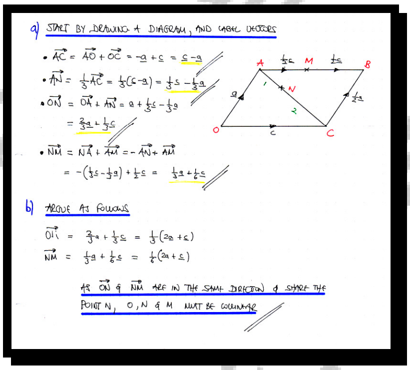
The point $N$ lies on the diagonal $AC$ so that $AN : NC = 1:2$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

**a)** Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{c}$, for each of the vectors $\overrightarrow{AC}$, $\overrightarrow{AN}$, $\overrightarrow{ON}$ and $\overrightarrow{NM}$.

**b)** Deduce, showing your reasoning, that $O$, $N$ and $M$ are collinear.

$\boxed{\overrightarrow{AC} = \mathbf{c} - \mathbf{a}}$, $\boxed{\overrightarrow{AN} = \frac{1}{3}\mathbf{c} - \frac{1}{3}\mathbf{a}}$, $\boxed{\overrightarrow{ON} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}}$, $\boxed{\overrightarrow{NM} = \frac{1}{3}\mathbf{a} + \frac{1}{6}\mathbf{c}}$


a) START BY DRAWING A DIAGRAM, AND LABEL VECTORS

- $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\underline{a} + \underline{c} = \underline{c} - \underline{a}$
- $\overrightarrow{AN} = \frac{1}{3}\overrightarrow{AC} = \frac{1}{3}(\underline{c} - \underline{a}) = \frac{1}{3}\underline{c} - \frac{1}{3}\underline{a}$
- $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN} = \underline{a} + \frac{1}{3}\underline{c} - \frac{1}{3}\underline{a} = \frac{2}{3}\underline{a} + \frac{1}{3}\underline{c}$
- $\overrightarrow{NM} = \overrightarrow{NA} + \overrightarrow{AM} = -\overrightarrow{AN} + \overrightarrow{AM} = -\left(\frac{1}{3}\underline{c} - \frac{1}{3}\underline{a}\right) + \frac{1}{2}\underline{c} = \frac{1}{3}\underline{a} + \frac{1}{6}\underline{c}$



b) ARGUE AS FOLLOWS

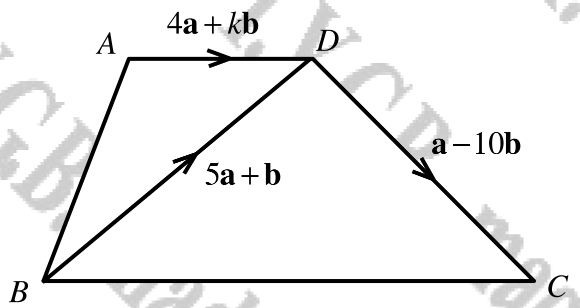
$\overrightarrow{ON} = \frac{2}{3}\underline{a} + \frac{1}{3}\underline{c} = \frac{1}{3}(2\underline{a} + \underline{c})$

$\overrightarrow{NM} = \frac{1}{3}\underline{a} + \frac{1}{6}\underline{c} = \frac{1}{6}(2\underline{a} + \underline{c})$

AS $\overrightarrow{ON}$ & $\overrightarrow{NM}$ ARE IN THE SAME DIRECTION & SHARE THE POINT N, O, N & M MUST BE COLLINEAR

**Question 6** (***+)

The figure below shows a trapezium $OBCA$ where $AD$ is parallel to $BC$.



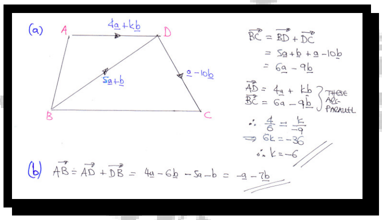

The following information is given for this trapezium.

$\overrightarrow{BD} = 5\mathbf{a}+\mathbf{b}$, $\overrightarrow{DC} = \mathbf{a}-10\mathbf{b}$ and $\overrightarrow{AD} = 4\mathbf{a}+k\mathbf{b}$, where $k$ is an integer.

**a)** Find the value of $k$.

**b)** Find a simplified expression for $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$\boxed{k=-6}$, $\boxed{\overrightarrow{AB} = -\mathbf{a}-7\mathbf{b}}$



**Question 7 (***+)**

$OAB$ is a triangle with the point $P$ being the midpoint of $OB$ and the point $Q$ being the midpoint of $AB$.

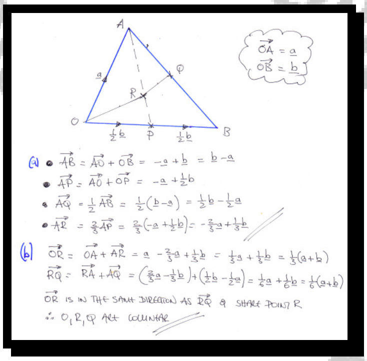
The point $R$ is such so that $\overrightarrow{AR} = \frac{2}{3}\overrightarrow{AP}$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

**a)** Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{AP}$, $\overrightarrow{AQ}$ and $\overrightarrow{AR}$.

**b)** By finding simplified expressions, in terms $\mathbf{a}$ and $\mathbf{b}$, for two more suitable vectors, show that the points $O$, $R$ and $Q$ are collinear.

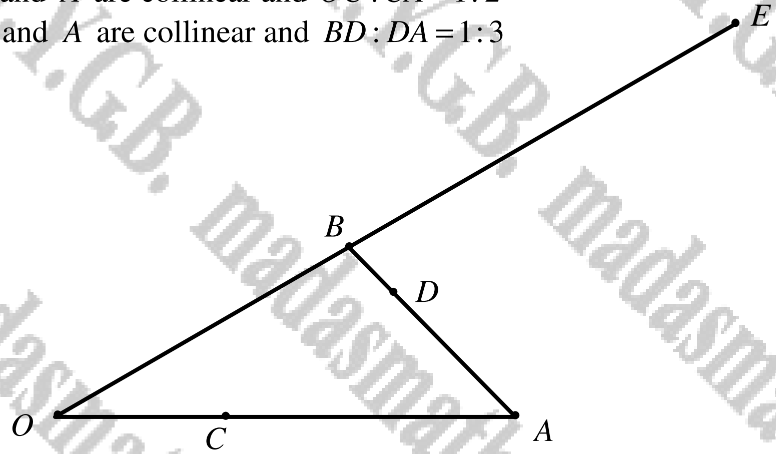
$\boxed{\overrightarrow{AB} = \mathbf{b} - \mathbf{a}}$, $\boxed{\overrightarrow{AP} = \frac{1}{2}\mathbf{b} - \mathbf{a}}$, $\boxed{\overrightarrow{AQ} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}}$, $\boxed{\overrightarrow{AR} = \frac{1}{3}\mathbf{b} - \frac{2}{3}\mathbf{a}}$



**Question 8** (***+)

The figure below shows the points $O$, $C$, $A$, $D$, $B$ and $E$, which are related as follows.

- $O$, $B$ and $E$ are collinear and $OB : BE = 1 : 2$
- $O$, $C$ and $A$ are collinear and $OC : CA = 1 : 2$
- $B$, $D$ and $A$ are collinear and $BD : DA = 1 : 3$




Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

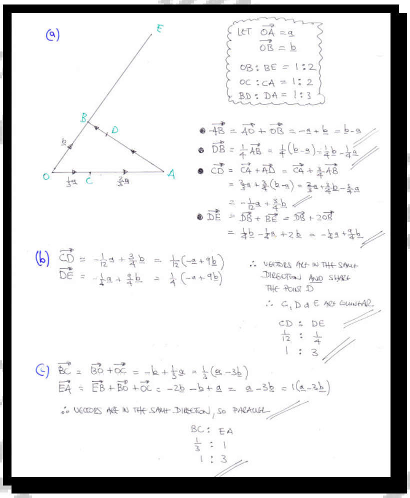
**a)** Find simplified expressions, in terms of $\mathbf{a}$ and $\mathbf{b}$, for each of the vectors $\overrightarrow{AB}$, $\overrightarrow{DB}$, $\overrightarrow{CD}$ and $\overrightarrow{DE}$.

**b)** Show that the points $C$, $D$ and $E$ are collinear, and find the ratio $CD : DE$.

**c)** Show further that $BC$ is parallel to $EA$, and find the ratio $BC : EA$.

$\boxed{\overrightarrow{AB} = \mathbf{b} - \mathbf{a}}$, $\boxed{\overrightarrow{DB} = \frac{1}{4}\mathbf{b} - \frac{1}{4}\mathbf{a}}$, $\boxed{\overrightarrow{CD} = -\frac{1}{12}\mathbf{a} + \frac{3}{4}\mathbf{b}}$, $\boxed{\overrightarrow{DE} = -\frac{1}{4}\mathbf{a} + \frac{9}{4}\mathbf{b}}$,

$\boxed{CD : DE = 1 : 3}$, $\boxed{BC : EA = 1 : 3}$


(a)

LET $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$

OB : BE = 1 : 2
OC : CA = 1 : 2
BD : DA = 1 : 3

- $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\underline{a} + \underline{b} = \underline{b} - \underline{a}$
- $\overrightarrow{DB} = \frac{1}{4}\overrightarrow{AB} = \frac{1}{4}(\underline{b} - \underline{a}) = \frac{1}{4}\underline{b} - \frac{1}{4}\underline{a}$
- $\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD} = \overrightarrow{CA} + \frac{3}{4}\overrightarrow{AB}$
  $= \frac{2}{3}\underline{a} + \frac{3}{4}(\underline{b} - \underline{a}) = \frac{2}{3}\underline{a} + \frac{3}{4}\underline{b} - \frac{3}{4}\underline{a}$
  $= -\frac{1}{12}\underline{a} + \frac{3}{4}\underline{b}$
- $\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE} = \overrightarrow{DB} + 2\overrightarrow{OB}$
  $= \frac{1}{4}\underline{b} - \frac{1}{4}\underline{a} + 2\underline{b} = -\frac{1}{4}\underline{a} + \frac{9}{4}\underline{b}$

(b) $\overrightarrow{CD} = -\frac{1}{12}\underline{a} + \frac{3}{4}\underline{b} = \frac{1}{12}(-\underline{a} + 9\underline{b})$

$\overrightarrow{DE} = -\frac{1}{4}\underline{a} + \frac{9}{4}\underline{b} = \frac{1}{4}(-\underline{a} + 9\underline{b})$

∴ VECTORS ARE IN THE SAME DIRECTION AND SHARE THE POINT D

∴ C, D & E ARE COLLINEAR

CD : DE
$\frac{1}{12}$ : $\frac{1}{4}$
1 : 3

(c) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -\underline{b} + \frac{1}{3}\underline{a} = \frac{1}{3}(\underline{a} - 3\underline{b})$

$\overrightarrow{EA} = \overrightarrow{EB} + \overrightarrow{BO} + \overrightarrow{OC} = -2\underline{b} - \underline{b} + \underline{a} = \underline{a} - 3\underline{b} = 1(\underline{a} - 3\underline{b})$

∴ VECTORS ARE IN THE SAME DIRECTION, SO PARALLEL

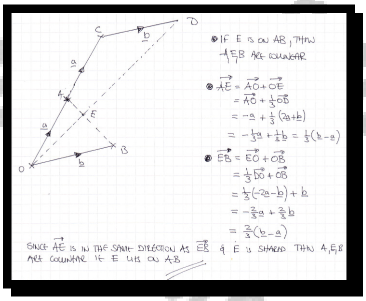
BC : EA
$\frac{1}{3}$ : 1
1 : 3

**Question 9** (****)

Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = 2\mathbf{a}$ and $\overrightarrow{OD} = 2\mathbf{a} + \mathbf{b}$.

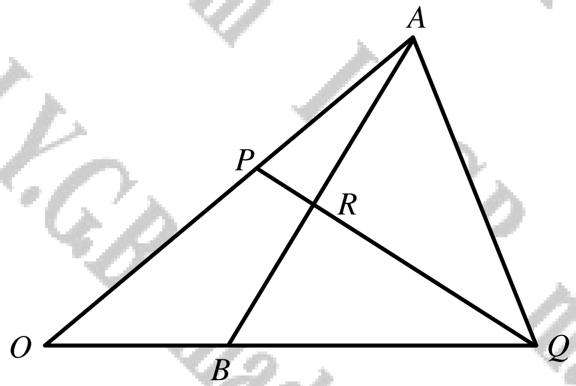
If $\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OD}$ prove that the point $E$ lies on the straight line $AB$.

$\boxed{\text{proof}}$



**Question 10** (****+)

The figure below shows a triangle $OAQ$.




not drawn to scale

- The point $P$ lies on $OA$ so that $OP : OA = 3:5$.
- The point $B$ lies on $OQ$ so that $OB : BQ = 1:2$.

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

**a)** Given that $\overrightarrow{AR} = h\overrightarrow{AB}$, where $h$ is a scalar parameter with $0 < h < 1$, show that

$$\overrightarrow{OR} = (1-h)\mathbf{a} + h\mathbf{b}.$$

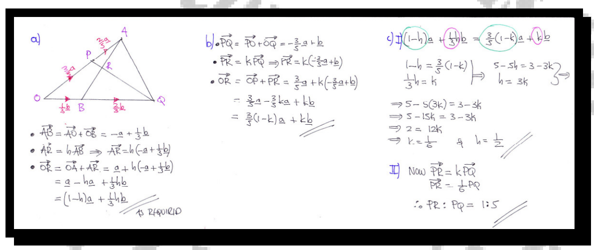
**b)** Given further that $\overrightarrow{PR} = k\overrightarrow{PQ}$, where $k$ is a scalar parameter with $0 < k < 1$, find a similar expression for $\overrightarrow{OR}$ in terms of $k$, $\mathbf{a}$, $\mathbf{b}$.

**c)** Determine …

**i.** … the value of $k$ and the value of $h$.

**ii.** … the ratio of $\overrightarrow{PR} : \overrightarrow{PQ}$.

$\boxed{\overrightarrow{OR} = \frac{3}{5}(1-k)\mathbf{a} + k\mathbf{b}}$, $\boxed{k = \frac{1}{6}}$, $\boxed{h = \frac{1}{2}}$, $\boxed{PR : PQ = 1:5}$



**Question 11** (*****)

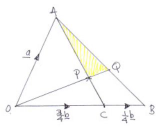
$OAB$ is a triangle and $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

- The point $C$ lies on $OB$ so that $OC : CB = 3:1$.
- The point $P$ lies on $AC$ so that $AP : PC = 2:1$.
- The point $Q$ lies on $AB$ so that $O$, $P$ and $Q$ are collinear.

Let $\overrightarrow{OQ} = m\overrightarrow{OP}$ and $\overrightarrow{AQ} = n\overrightarrow{AB}$

Find the value of $m$ and the value of $n$, and hence write down the ratio $AQ : QB$.

$\boxed{m = \frac{6}{5},\ n = \frac{3}{5}}$, $\boxed{AQ : QB = 3:2}$




- $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\underline{a} + \frac{3}{4}\underline{b}$
- $\overrightarrow{AP} = \frac{2}{3}\overrightarrow{AC} = \frac{2}{3}\left(-\underline{a} + \frac{3}{4}\underline{b}\right) = -\frac{2}{3}\underline{a} + \frac{1}{2}\underline{b}$
- $\overrightarrow{PC} = \frac{1}{3}\overrightarrow{AC} = \frac{1}{3}\left(-\underline{a} + \frac{3}{4}\underline{b}\right) = -\frac{1}{3}\underline{a} + \frac{1}{4}\underline{b}$
- $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\underline{a} + \underline{b} = \underline{b} - \underline{a}$

- NOW LOOK AT $\widehat{APQ}$

$\Rightarrow \overrightarrow{AP} + \overrightarrow{PQ} = \overrightarrow{AQ}$

$\Rightarrow \overrightarrow{AP} + \left(\overrightarrow{PO} + \overrightarrow{OQ}\right) = n\overrightarrow{AB}$

$\Rightarrow \overrightarrow{AP} - \overrightarrow{OP} + m\overrightarrow{OP} = n\overrightarrow{AB}$

$\Rightarrow \overrightarrow{AP} + (m-1)\overrightarrow{OP} = n\overrightarrow{AB}$

$\Rightarrow \overrightarrow{AP} + (m-1)\left(\overrightarrow{OA} + \overrightarrow{AP}\right) = n\overrightarrow{AB}$

$\Rightarrow \overrightarrow{AP} + (m-1)\overrightarrow{OA} + (m-1)\overrightarrow{AP} = n\overrightarrow{AB}$

$\Rightarrow m\overrightarrow{AP} + (m-1)\overrightarrow{OA} = n\overrightarrow{AB}$

$\Rightarrow m\left(-\frac{2}{3}\underline{a} + \frac{1}{2}\underline{b}\right) + (m-1)\underline{a} = n(\underline{b} - \underline{a})$

$\Rightarrow \left(-\frac{2}{3}m + m - 1\right)\underline{a} + \frac{1}{2}m\underline{b} = n\underline{b} - n\underline{a}$

$\Rightarrow \left(\frac{1}{3}m - 1\right)\underline{a} + \frac{1}{2}m\underline{b} = n\underline{b} - n\underline{a}$

HENCE

$\frac{1}{2}m = n$ & $\frac{1}{3}m - 1 = -n$

$\boxed{m = 2n}$

$m - 3 = -3n$

$2n - 3 = -3n$

$5n = 3$

$n = \frac{3}{5}$

$m = \frac{6}{5}$

AND FINALLY

$\overrightarrow{AQ} = \frac{3}{5}\overrightarrow{AB}$

$\overrightarrow{AQ} : \overrightarrow{QB} = 3:2$

**Question 12**

Find the value of $\lambda$ and $\mu$, given that the vectors $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

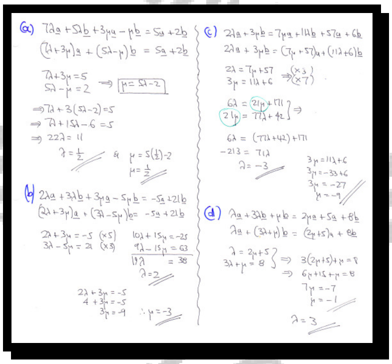
**a)** $7\lambda\mathbf{a}+5\lambda\mathbf{b}+3\mu\mathbf{a}-\mu\mathbf{b}=5\mathbf{a}+2\mathbf{b}$

**b)** $2\lambda\mathbf{a}+3\lambda\mathbf{b}+3\mu\mathbf{a}-5\mu\mathbf{b}=-5\mathbf{a}+21\mathbf{b}$

**c)** $2\lambda\mathbf{a}+3\mu\mathbf{b}=7\mu\mathbf{a}+11\lambda\mathbf{b}+57\mathbf{a}+6\mathbf{b}$

**d)** $\lambda\mathbf{a}+3\lambda\mathbf{b}+\mu\mathbf{b}=2\mu\mathbf{a}+5\mathbf{a}+8\mathbf{b}$

$\boxed{\lambda=\frac{1}{2},\mu=\frac{1}{2}}$, $\boxed{\lambda=2,\mu=-3}$, $\boxed{\lambda=-3,\mu=-9}$, $\boxed{\lambda=3,\mu=-1}$



(a) $7\lambda\underline{a}+5\lambda\underline{b}+3\mu\underline{a}-\mu\underline{b}=5\underline{a}+2\underline{b}$

$(7\lambda+3\mu)\underline{a}+(5\lambda-\mu)\underline{b}=5\underline{a}+2\underline{b}$

$7\lambda+3\mu=5$
$5\lambda-\mu=2 \Rightarrow \boxed{\mu=5\lambda-2}$

$\Rightarrow 7\lambda+3(5\lambda-2)=5$
$\Rightarrow 7\lambda+15\lambda-6=5$
$\Rightarrow 22\lambda=11$
$\lambda=\frac{1}{2}$ & $\mu=5(\frac{1}{2})-2$
$\mu=\frac{1}{2}$

(b) $2\lambda\underline{a}+3\lambda\underline{b}+3\mu\underline{a}-5\mu\underline{b}=-5\underline{a}+21\underline{b}$

$(2\lambda+3\mu)\underline{a}+(3\lambda-5\mu)\underline{b}=-5\underline{a}+21\underline{b}$

$2\lambda+3\mu=-5 \ (\times 5)$ $\quad 10\lambda+15\mu=-25$
$3\lambda-5\mu=21 \ (\times 3)$ $\quad 9\lambda-15\mu=63$
$19\lambda=38$
$\lambda=2$

$2\lambda+3\mu=-5$
$4+3\mu=-5$
$3\mu=-9 \quad \therefore \mu=-3$

(c) $2\lambda\underline{a}+3\mu\underline{b}=7\mu\underline{a}+11\lambda\underline{b}+57\underline{a}+6\underline{b}$

$2\lambda\underline{a}+3\mu\underline{b}=(7\mu+57)\underline{a}+(11\lambda+6)\underline{b}$

$2\lambda=7\mu+57$
$3\mu=11\lambda+6 \Rightarrow \begin{matrix}(\times 3)\\(\times 7)\end{matrix}$

$6\lambda=21\mu+171$
$21\mu=77\lambda+42$ $\Rightarrow$

$6\lambda=(77\lambda+42)+171$
$-213=71\lambda$
$\lambda=-3$

$3\mu=11\lambda+6$
$3\mu=-33+6$
$3\mu=-27$
$\mu=-9$

(d) $\lambda\underline{a}+3\lambda\underline{b}+\mu\underline{b}=2\mu\underline{a}+5\underline{a}+8\underline{b}$

$\lambda\underline{a}+(3\lambda+\mu)\underline{b}=(2\mu+5)\underline{a}+8\underline{b}$

$\lambda=2\mu+5$
$3\lambda+\mu=8$ $\Rightarrow 3(2\mu+5)+\mu=8$
$\Rightarrow 6\mu+15+\mu=8$
$7\mu=-7$
$\mu=-1$

$\lambda=3$

# VECTOR COMPONENTS

# AND

# 3D-COORDINATES

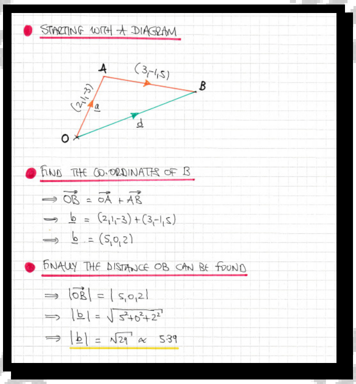
**Question 1**

Relative to a fixed origin $O$, the point $A$ has coordinates $(2,1,-3)$.

The point $B$ is such so that $\overrightarrow{AB} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$.

Determine the distance of $B$ from $O$.

$\boxed{\phantom{xxxx}}$, $\boxed{|OB| = \sqrt{29}}$



STARTING WITH A DIAGRAM



FIND THE CO-ORDINATES OF B

$\Rightarrow \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$\Rightarrow \underline{b} = (2,1,-3) + (3,-1,5)$

$\Rightarrow \underline{b} = (5,0,2)$

FINALLY THE DISTANCE OB CAN BE FOUND

$\Rightarrow |\overrightarrow{OB}| = |5,0,2|$

$\Rightarrow |\underline{b}| = \sqrt{5^2+0^2+2^2}$

$\Rightarrow |\underline{b}| = \sqrt{29} \approx 5.39$

**Question 2**

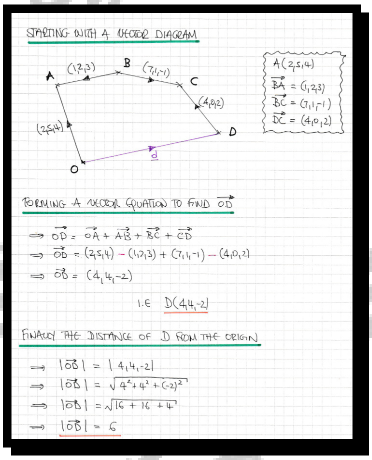
Relative to a fixed origin $O$, the point $A$ has coordinates $(2,5,4)$.

The points $B$, $C$ and $D$ are such so that

$$\overrightarrow{BA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{BC} = 7\mathbf{i} + \mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{DC} = 4\mathbf{i} + 2\mathbf{k}.$$

Determine the distance of $D$ from the origin.

$\boxed{\phantom{xxxx}}$, $\boxed{|OD| = 6}$




FORMING A VECTOR EQUATION TO FIND $\overrightarrow{OD}$

$\Rightarrow \overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$

$\Rightarrow \overrightarrow{OD} = (2,5,4) - (1,2,3) + (7,1,-1) - (4,0,2)$

$\Rightarrow \overrightarrow{OD} = (4,4,-2)$

I.E $D(4,4,-2)$

FINALLY THE DISTANCE OF D FROM THE ORIGIN

$\Rightarrow |\overrightarrow{OD}| = |4,4,-2|$

$\Rightarrow |\overrightarrow{OD}| = \sqrt{4^2 + 4^2 + (-2)^2}$

$\Rightarrow |\overrightarrow{OD}| = \sqrt{16 + 16 + 4}$

$\Rightarrow |\overrightarrow{OD}| = 6$

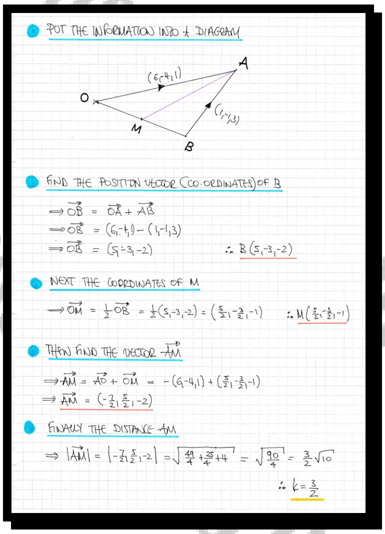
**Question 3**

Relative to a fixed origin $O$, the point $A$ has coordinates $(6,-4,1)$.

The point $B$ is such so that $\overrightarrow{BA} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

If the point $M$ is the midpoint of $OB$, show that $\left|\overrightarrow{AM}\right| = k\sqrt{10}$, where $k$ is a rational constant to be found.

$\boxed{\phantom{xxxxx}}$, $\boxed{k = \frac{3}{2}}$




PUT THE INFORMATION INTO A DIAGRAM

FIND THE POSITION VECTOR (CO-ORDINATES) OF B

$\Rightarrow \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$\Rightarrow \overrightarrow{OB} = (6,-4,1) - (1,-1,3)$

$\Rightarrow \overrightarrow{OB} = (5,-3,-2)$ $\quad \therefore B(5,-3,-2)$

NEXT THE COORDINATES OF M

$\Rightarrow \overrightarrow{OM} = \frac{1}{2}\overrightarrow{OB} = \frac{1}{2}(5,-3,-2) = \left(\frac{5}{2},-\frac{3}{2},-1\right)$ $\quad \therefore M\left(\frac{5}{2},-\frac{3}{2},-1\right)$

THEN FIND THE VECTOR $\overrightarrow{AM}$

$\Rightarrow \overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM} = -(6,-4,1) + \left(\frac{5}{2},-\frac{3}{2},-1\right)$

$\Rightarrow \overrightarrow{AM} = \left(-\frac{7}{2},\frac{5}{2},-2\right)$

FINALLY THE DISTANCE AM

$\Rightarrow |\overrightarrow{AM}| = \left|-\frac{7}{2},\frac{5}{2},-2\right| = \sqrt{\frac{49}{4}+\frac{25}{4}+4} = \sqrt{\frac{90}{4}} = \frac{3}{2}\sqrt{10}$

$\therefore k = \frac{3}{2}$

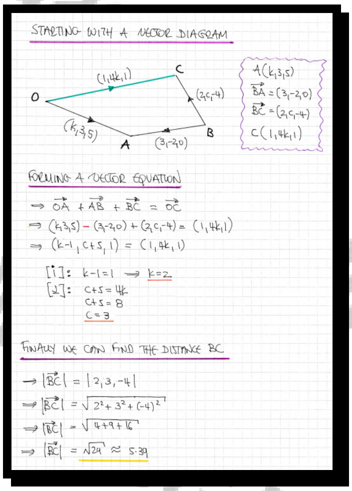
**Question 4**

Relative to a fixed origin $O$, the point $A$ has coordinates $(k,3,5)$, where $k$ is a scalar constant.

The points $B$ and $C$ are such so that $\overrightarrow{BA} = 3\mathbf{i} - 2\mathbf{j}$ and $\overrightarrow{BC} = 2\mathbf{i} + c\mathbf{j} - 4\mathbf{k}$, where $c$ is a scalar constant.

If the coordinates of $C$ are $(1,4k,1)$, determine the distance $BC$.

$\boxed{\phantom{xxxx}}$, $\boxed{|BC| = \sqrt{29}}$



STARTING WITH A VECTOR DIAGRAM



$A(k,3,5)$
$\overrightarrow{BA} = (3,-2,0)$
$\overrightarrow{BC} = (2,c,-4)$
$C(1,4k,1)$

FORMING A VECTOR EQUATION

$\Rightarrow \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{OC}$

$\Rightarrow (k,3,5) - (3,-2,0) + (2,c,-4) = (1,4k,1)$

$\Rightarrow (k-1, c+5, 1) = (1,4k,1)$

$[\mathbf{i}]: \ k-1=1 \Rightarrow k=2$

$[\mathbf{j}]: \ c+5 = 4k$
$c+5 = 8$
$c = 3$

FINALLY WE CAN FIND THE DISTANCE BC

$\rightarrow |\overrightarrow{BC}| = |2,3,-4|$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{2^2+3^2+(-4)^2}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{4+9+16}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{29} \approx 5.39$

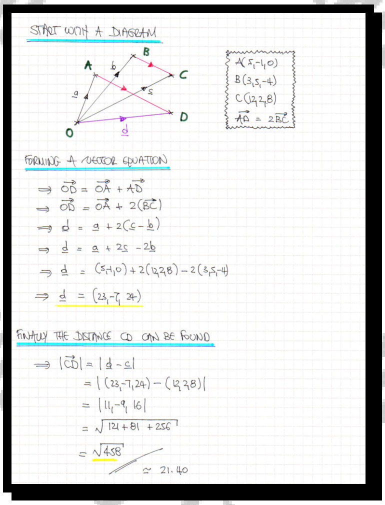
**Question 5**

The points $A(5,-1,0)$, $B(3,5,-4)$, $C(12,2,8)$ are referred relative to a fixed origin $O$.

The point $D$ is such so that $\overrightarrow{AD} = 2\overrightarrow{BC}$.

Determine the distance $CD$.

$$\boxed{|CD| = \sqrt{458} \approx 21.40}$$

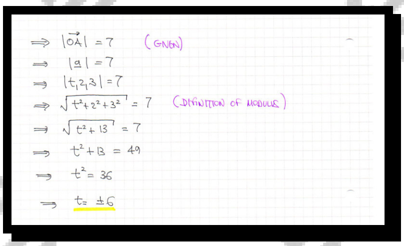



**Question 6**

The point $A(t,2,3)$, where $t$ is a constant, is referred relative to a fixed origin $O$.

Given that $|\overrightarrow{OA}| = 7$, find the possible values of $t$.

$$\boxed{t = \pm 6}$$

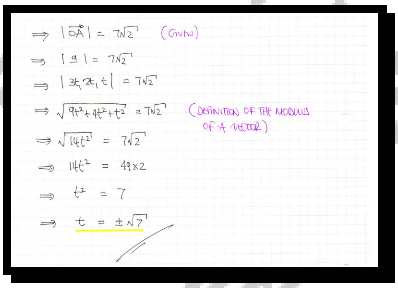


**Question 7**

The point $A(3t, 2t, t)$, where $t$ is a constant, is referred relative to a fixed origin $O$.

Given that $|\overline{OA}| = 7\sqrt{2}$, find the possible values of $t$.

$\boxed{t = \pm\sqrt{7}}$

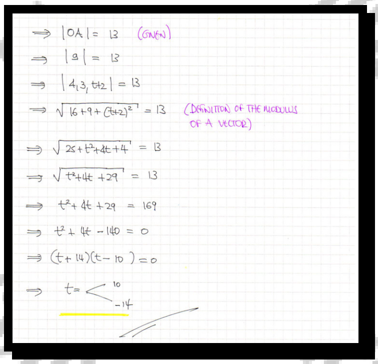



**Question 8**

The point $A(4, 3, t+2)$, where $t$ is a constant, is referred relative to a fixed origin $O$.

Given that $|\overline{OA}| = 13$, find the possible values of $t$.

$\boxed{t = 10, -14}$

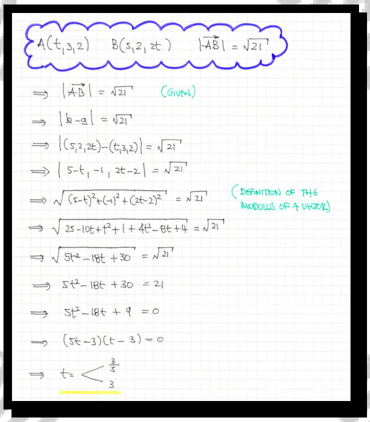


**Question 9**

The points $A(t,3,2)$ and $B(5,2,2t)$, where $t$ is a scalar constant, are referred relative to a fixed origin $O$.

Given that $\left|\overrightarrow{AB}\right| = \sqrt{21}$, find the possible values of $t$.

$$\boxed{t = 3,\ t = \frac{3}{5}}$$



$A(t,3,2) \quad B(5,2,2t) \quad |\overrightarrow{AB}| = \sqrt{21}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{21}$ (GIVEN)

$\Rightarrow |\underline{b} - \underline{a}| = \sqrt{21}$

$\Rightarrow |(5,2,2t)-(t,3,2)| = \sqrt{21}$

$\Rightarrow |5-t, -1, 2t-2| = \sqrt{21}$

$\Rightarrow \sqrt{(5-t)^2+(-1)^2+(2t-2)^2} = \sqrt{21}$ (DEFINITION OF THE MODULUS OF A VECTOR)

$\Rightarrow \sqrt{25-10t+t^2+1+4t^2-8t+4} = \sqrt{21}$

$\Rightarrow \sqrt{5t^2-18t+30} = \sqrt{21}$

$\Rightarrow 5t^2-18t+30 = 21$

$\Rightarrow 5t^2-18t+9 = 0$

$\Rightarrow (5t-3)(t-3) = 0$

$\Rightarrow t = \frac{3}{5},\ 3$

**Question 10**

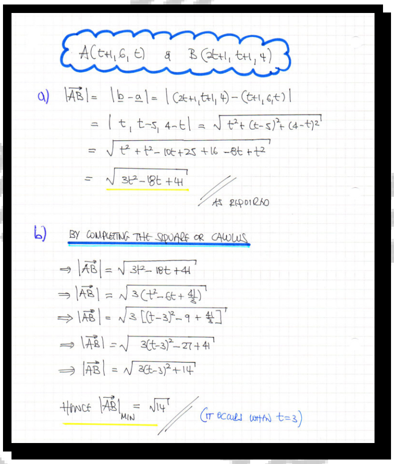
The variable points $A(t+1,6,t)$ and $B(2t+1,t+1,4)$, where $t$ is a scalar variable, are referred relative to a fixed origin $O$.

**a)** Show that

$$\left|\overrightarrow{AB}\right| = \sqrt{3t^2 - 18t + 41}.$$

**b)** Hence find the shortest distance between $A$ and $B$, as $t$ varies.

$$\boxed{\left|\overrightarrow{AB}\right|_{\text{min}} = \sqrt{14}}$$



$A(t+1,6,t)$ & $B(2t+1,t+1,4)$

a) $|\overrightarrow{AB}| = |\underline{b}-\underline{a}| = |(2t+1,t+1,4)-(t+1,6,t)|$

$= |t, t-5, 4-t| = \sqrt{t^2+(t-5)^2+(4-t)^2}$

$= \sqrt{t^2+t^2-10t+25+16-8t+t^2}$

$= \sqrt{3t^2-18t+41}$ // AS REQUIRED

b) BY COMPLETING THE SQUARE OR CALCULUS

$\Rightarrow |\overrightarrow{AB}| = \sqrt{3t^2-18t+41}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{3(t^2-6t+\frac{41}{3})}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{3[(t-3)^2-9+\frac{41}{3}]}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{3(t-3)^2-27+41}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{3(t-3)^2+14}$

HENCE $|\overrightarrow{AB}|_{MIN} = \sqrt{14}$ // (IT OCCURS WHEN $t=3$)

**Question 11**

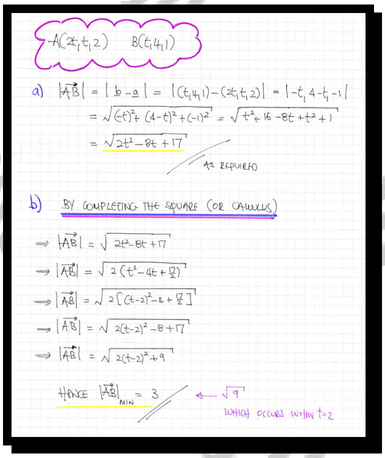
The variable points $A(2t, t, 2)$ and $B(t, 4, 1)$, where $t$ is a scalar variable, are referred relative to a fixed origin $O$.

**a)** Show that

$$|\overrightarrow{AB}| = \sqrt{2t^2 - 8t + 17}.$$

**b)** Hence find the shortest distance between $A$ and $B$, as $t$ varies.

$$\boxed{|\overrightarrow{AB}|_{\min} = 3}$$



$A(2t, t, 2) \quad B(t, 4, 1)$

a) $|\overrightarrow{AB}| = |\underline{b} - \underline{a}| = |(t, 4, 1) - (2t, t, 2)| = |-t, 4-t, -1|$

$= \sqrt{(-t)^2 + (4-t)^2 + (-1)^2} = \sqrt{t^2 + 16 - 8t + t^2 + 1}$

$= \sqrt{2t^2 - 8t + 17}$

AS REQUIRED

b) BY COMPLETING THE SQUARE (OR CALCULUS)

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2t^2 - 8t + 17}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2(t^2 - 4t + \frac{17}{2})}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2[(t-2)^2 - 4 + \frac{17}{2}]}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2(t-2)^2 - 8 + 17}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{2(t-2)^2 + 9}$

HENCE $|\overrightarrow{AB}|_{\text{MIN}} = 3 \leftarrow \sqrt{9}$

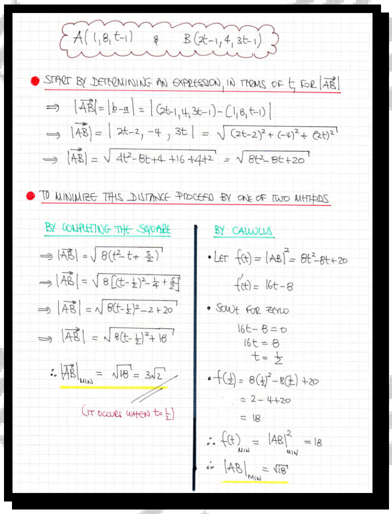
WHICH OCCURS WHEN $t=2$

## Question 12

The variable points $A(1,8,t-1)$ and $B(2t-1,4,3t-1)$, where $t$ is a scalar variable, are referred relative to a fixed origin $O$.

Find the shortest distance between $A$ and $B$, as $t$ varies.

$$\boxed{\left|\overrightarrow{AB}\right|_{\min} = \sqrt{18}}$$

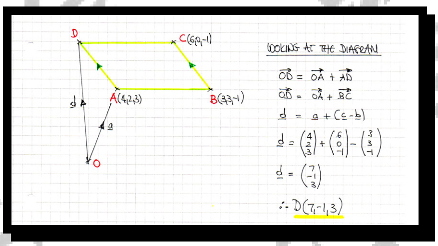



## Question 13

The points $A(4,2,3)$, $B(3,3,-1)$ and $C(6,0,-1)$ are referred with respect to a fixed origin $O$.

If $A$, $B$, $C$ and the point $D$ form the parallelogram $ABCD$, use vector algebra to find the coordinates of $D$.

$$\boxed{D(7,-1,3)}$$

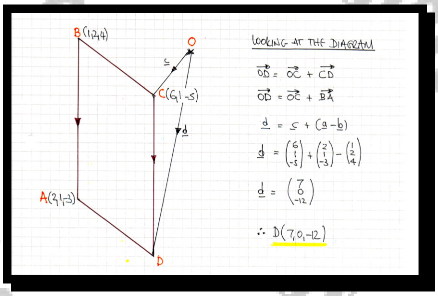


## Question 14

The points $A(2,1,-3)$, $B(1,2,4)$ and $C(6,1,-5)$ are referred with respect to a fixed origin $O$.

If $A$, $B$, $C$ and the point $D$ form the parallelogram $ABCD$, use vector algebra to find the coordinates of $D$.

$\boxed{D(7,0,-12)}$

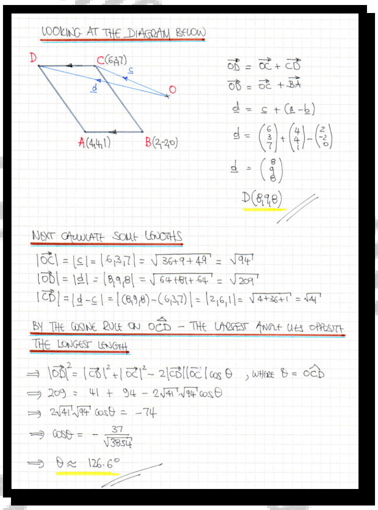



## Question 15

The points $A(4,4,1)$, $B(2,-2,0)$ and $C(6,3,7)$ are referred with respect to a fixed origin $O$.

If $A$, $B$, $C$ and the point $D$ form the parallelogram $ABCD$, use vector algebra to find the coordinates of $D$ and hence calculate the angle $OCD$.

$\boxed{D(8,9,8)}$, $\boxed{\measuredangle OCD \approx 126.6°}$

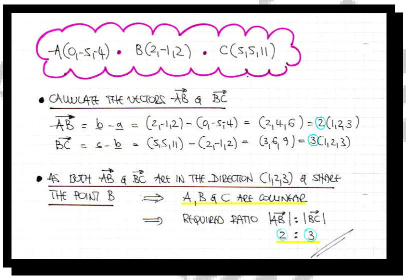


## Question 16

The points $A(0,-5,-4)$, $B(2,-1,2)$ and $C(5,5,11)$ are referred with respect to a fixed origin $O$.

Show that $A$, $B$ and $C$ are collinear and find the ratio $AB:BC$.

$\boxed{2:3}$

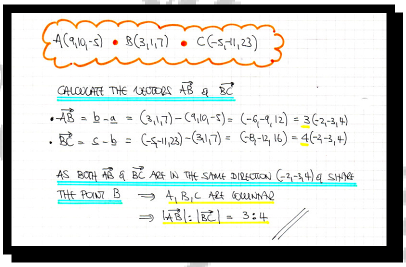



## Question 17

The points $A(9,10,-5)$, $B(3,1,7)$ and $C(-5,-11,23)$ are referred with respect to a fixed origin $O$.

Show that $A$, $B$ and $C$ are collinear and find the ratio $AB:BC$.

$\boxed{3:4}$



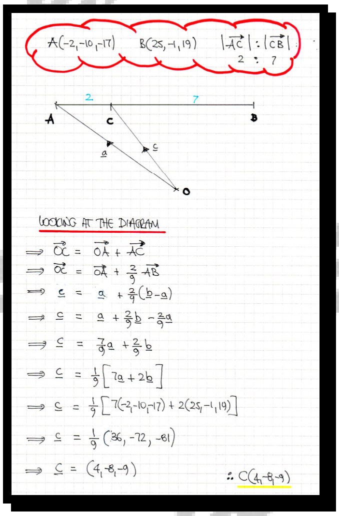
**Question 18**

The points $A(-2,-10,-17)$ and $B(25,-1,19)$ are referred with respect to a fixed origin $O$.

The point $C$ is such so that $ACB$ forms a straight line.

Given further that $\dfrac{|\overrightarrow{AC}|}{|\overrightarrow{CB}|} = \dfrac{2}{7}$, determine the coordinates of $C$.

$\boxed{C(4,-8,-9)}$



**Question 19**

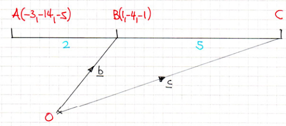
The points $A(-3,-14,-5)$ and $B(1,-4,-1)$ are referred relative to a fixed origin $O$.

The point $C$ is such so that $ABC$ forms a straight line.

Given further that $\dfrac{|\overrightarrow{AB}|}{|\overrightarrow{BC}|}=\dfrac{2}{5}$, determine the coordinates of $C$.

$\boxed{C(11,21,9)}$

PUT THE INFORMATION IN A DIAGRAM




$\Rightarrow \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

$\Rightarrow \overrightarrow{OC} = \overrightarrow{OB} + \frac{5}{2}\overrightarrow{AB}$

$\Rightarrow \underline{c} = \underline{b} + \frac{5}{2}(\underline{b}-\underline{a})$

$\Rightarrow \underline{c} = \underline{b} + \frac{5}{2}\underline{b} - \frac{5}{2}\underline{a}$

$\Rightarrow \underline{c} = \frac{7}{2}\underline{b} - \frac{5}{2}\underline{a}$

$\Rightarrow \underline{c} = \frac{1}{2}(7\underline{b} - 5\underline{a})$

$\Rightarrow \underline{c} = \frac{1}{2}[7(1,-4,-1) - 5(-3,-14,-5)]$

$\Rightarrow \underline{c} = \frac{1}{2}(22,42,18)$

$\Rightarrow \underline{c} = (11,21,9)$

## Question 20

The points $A(2,-1,4)$, $B(0,-5,10)$, $C(3,1,3)$ and $D(6,7,-8)$ are referred relative to a fixed origin $O$.

**a)** Use vector algebra to show that three of the above four points are collinear.

A triangle is drawn using three of the above four points as its vertices.

**b)** Given further that the triangle has the largest possible area, determine, in exact surd form, the length of its shortest side.

$\boxed{\sqrt{94}}$

a) $A(2,-1,4)$ $B(0,-5,10)$ $C(3,1,3)$ $D(6,7,-8)$

- PICK A POINT AT RANDOM AND CALCULATE ALL OTHER VECTORS TO THE OTHER 3 POINTS

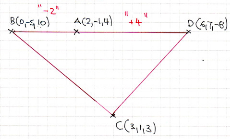
$\overrightarrow{AB} = \underline{b} - \underline{a} = (0,-5,10) - (2,-1,4) = (-2,-4,6) = 2(-1,-2,3)$
$\overrightarrow{AC} = \underline{c} - \underline{a} = (3,1,3) - (2,-1,4) = (1,2,-1) = 1(1,2,-1)$
$\overrightarrow{AD} = \underline{d} - \underline{a} = (6,7,-8) - (2,-1,4) = (4,8,-12) = 4(1,2,-3)$

- HENCE WE HAVE $\overrightarrow{AB}$ & $\overrightarrow{AD}$ IN "PARALLEL CONFIGURATION"

$\overrightarrow{AB} = 2(-1,-2,3) = -2\,\underline{(1,2,-3)}$
$\overrightarrow{AD} = 4\,\underline{(1,2,-3)}$

$\therefore$ A, B & D ARE COLLINEAR

b) - DRAWING A DIAGRAM




- THE LENGTH OF BD IS $6|1,2,-3|$ (OR COMPUTE $|\underline{d}-\underline{b}|$)

$\Rightarrow 6\sqrt{1+4+9} = 6\sqrt{14}$

- ALSO WE HAVE

• $|\overrightarrow{BC}| = |\underline{c} - \underline{b}| = |(3,1,3) - (0,-5,10)| = |3,6,-7|$
$= \sqrt{9+36+49} = \sqrt{94}$

• $|\overrightarrow{DC}| = |\underline{c} - \underline{d}| = |(3,1,3) - (6,7,-8)| = |-3,-6,11|$
$= \sqrt{9+36+121} = \sqrt{166}$

$\therefore$ THE SHORTEST SIDE OF THE TRIANGLE WHICH HAS THE LARGEST AREA IS $\sqrt{94}$

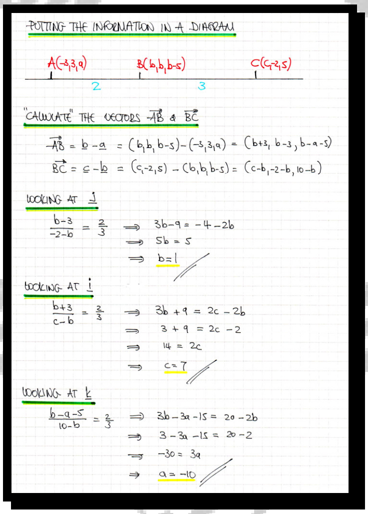
**Question 21**

The points $A(-3,3,a)$, $B(b,b,b-5)$ and $C(c,-2,5)$, where $a$, $b$ and $c$ are scalar constants, are referred relative to a fixed origin $O$.

Ii is further given that $A$, $B$ and $C$ are collinear and the ratio $\left|\overrightarrow{AB}\right| : \left|\overrightarrow{BC}\right| = 2:3$.

Use vector algebra to find the value of $a$, the value of $b$ and the value of $c$.

$\boxed{[a,b,c]=[-10,1,7]}$



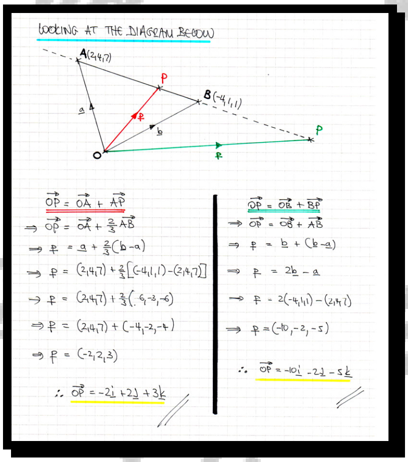
**Question 22**

With respect to a fixed origin, the points $A$ and $B$ have position vectors $2\mathbf{i}+4\mathbf{j}+7\mathbf{k}$ and $-4\mathbf{i}+\mathbf{j}+\mathbf{k}$, respectively.

The point $P$ lies on the straight line through $A$ and $B$.

Find the possible position vectors of $P$ if $\left|\overrightarrow{AP}\right| = 2\left|\overrightarrow{PB}\right|$.

$$\boxed{\overrightarrow{OP} = \mathbf{p} = -2\mathbf{i}+2\mathbf{j}+3\mathbf{k}}, \quad \boxed{\overrightarrow{OP} = \mathbf{p} = -10\mathbf{i}-2\mathbf{j}-5\mathbf{k}}$$




$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$$
$$\Rightarrow \overrightarrow{OP} = \overrightarrow{OA} + \tfrac{2}{3}\overrightarrow{AB}$$
$$\Rightarrow \underline{p} = \underline{a} + \tfrac{2}{3}(\underline{b}-\underline{a})$$
$$\Rightarrow \underline{p} = (2,4,7) + \tfrac{2}{3}[(-4,1,1)-(2,4,7)]$$
$$\Rightarrow \underline{p} = (2,4,7) + \tfrac{2}{3}(-6,-3,-6)$$
$$\Rightarrow \underline{p} = (2,4,7) + (-4,-2,-4)$$
$$\Rightarrow \underline{p} = (-2,2,3)$$
$$\therefore \overrightarrow{OP} = -2\underline{i}+2\underline{j}+3\underline{k}$$

$$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$$
$$\Rightarrow \overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{AB}$$
$$\Rightarrow \underline{p} = \underline{b} + (\underline{b}-\underline{a})$$
$$\Rightarrow \underline{p} = 2\underline{b} - \underline{a}$$
$$\Rightarrow \underline{p} = 2(-4,1,1)-(2,4,7)$$
$$\Rightarrow \underline{p} = (-10,-2,-5)$$
$$\therefore \overrightarrow{OP} = -10\underline{i}-2\underline{j}-5\underline{k}$$

**Question 23**

With respect to a fixed origin, the points $A$ and $B$ have position vectors $10\mathbf{i}+9\mathbf{j}-6\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}+10\mathbf{k}$, respectively.

The position vector of the point $C$ has $\mathbf{i}$ component equal to $2$.

The distance of $C$ from both $A$ and $B$ is $12$ units.

Show that one of the two possible position vectors of $C$ is $2\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ and determine the other.

$$\boxed{\mathbf{c}=2\mathbf{i}+\frac{61}{25}\mathbf{j}+\frac{2}{25}\mathbf{k}}$$

$A(10,9,-6) \quad B(6,-3,10) \quad C(2,y,z)$

START BY FINDING $\vec{AC}$ & $\vec{BC}$

$\vec{AC} = \underline{c}-\underline{a} = (2,y,z)-(10,9,-6) = (-8,y-9,z+6)$

$\vec{BC} = \underline{c}-\underline{b} = (2,y,z)-(6,-3,10) = (-4,y+3,z-10)$

NEXT SET SIMPLIFIED EXPRESSIONS FOR EACH OF THE MODULI

$\Rightarrow |-8,y-9,z+6| = 12$

$\Rightarrow \sqrt{64+(y-9)^2+(z+6)^2} = 12$

$\Rightarrow 64+(y-9)^2+(z+6)^2 = 144$

$\Rightarrow (y-9)^2+(z+6)^2 = 80$

$\Rightarrow y^2-18y+81+z^2+12z+36 = 80$

$\Rightarrow y^2+z^2-18y+12z = -37$

$\Rightarrow |-4,y+3,z-10| = 12$

$\Rightarrow \sqrt{16+(y+3)^2+(z-10)^2} = 12$

$\Rightarrow 16+(y+3)^2+(z-10)^2 = 144$

$\Rightarrow (y+3)^2+(z-10)^2 = 128$

$\Rightarrow y^2+6y+9+z^2-20z+100 = 128$

$\Rightarrow y^2+z^2+6y-20z = 19$

SOLVING SIMULTANEOUSLY BY SUBTRACTING THE EQUATIONS

$$\Rightarrow \left\{\begin{array}{l} \cancel{y^2}+\cancel{z^2}+6y-20z=19 \\ \cancel{y^2}+\cancel{z^2}-18y+12z=-37 \end{array}\right\} \Rightarrow 24y-32z=56$$

$\Rightarrow 3y-4z=7$

$\Rightarrow \underline{3y=4z+7}$

TAKE ONE OF THE EQUATIONS SUCH AS

$\Rightarrow y^2+z^2+6y-20z=19$

$\Rightarrow 9y^2+9z^2+54y-180z=171 \quad \times 9$

$\Rightarrow (3y)^2+9z^2+18(3y)-180z=171$

$\Rightarrow (4z+7)^2+9z^2+18(4z+7)-180z=171$

$\Rightarrow 16z^2+56z+49+9z^2+72z+126-180z-171=0$

$\Rightarrow 25z^2-52z+4=0$

$\Rightarrow (z-2)(25z-2)=0$

$$\Rightarrow z = \left\langle \begin{array}{l} 2 \\ \frac{2}{25} \end{array}\right.$$

FINALLY USING $3y=4z+7$

- IF $z=2$

  $3y=15$

  $\underline{\underline{y=5}}$

- IF $z=\frac{2}{25}$

  $3y=\frac{8}{25}+7$

  $3y=\frac{183}{25}$

  $\underline{\underline{y=\frac{61}{25}}}$

$\therefore (2,5,2)$ & $\left(2,\frac{61}{25},\frac{2}{25}\right)$

# ANGLES AND VECTORS

**Question 1**

Find the angle between each pair of vectors.

a) $4\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $8\mathbf{i}+2\mathbf{j}-3\mathbf{k}$

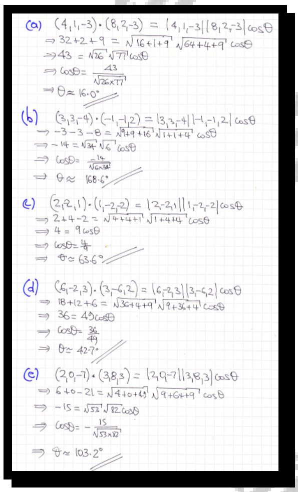
b) $3\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ and $-\mathbf{i}-\mathbf{j}+2\mathbf{k}$

c) $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $\mathbf{i}-2\mathbf{j}-2\mathbf{k}$

d) $6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $3\mathbf{i}-6\mathbf{j}+2\mathbf{k}$

e) $2\mathbf{i}-7\mathbf{k}$ and $3\mathbf{i}+8\mathbf{j}+3\mathbf{k}$

$\boxed{16.0°}$, $\boxed{168.6°}$, $\boxed{63.6°}$, $\boxed{42.7°}$, $\boxed{103.2°}$


(a) $(4,1,-3)\cdot(8,2,-3) = |4,1,-3||8,2,-3|\cos\theta$
$\Rightarrow 32+2+9 = \sqrt{16+1+9}\sqrt{64+4+9}\cos\theta$
$\Rightarrow 43 = \sqrt{26}\sqrt{77}\cos\theta$
$\Rightarrow \cos\theta = \frac{43}{\sqrt{26\times 77}}$
$\Rightarrow \theta \approx 16.0°$

(b) $(3,3,-4)\cdot(-1,-1,2) = |3,3,-4||-1,-1,2|\cos\theta$
$\Rightarrow -3-3-8 = \sqrt{9+9+16}\sqrt{1+1+4}\cos\theta$
$\Rightarrow -14 = \sqrt{34}\sqrt{6}\cos\theta$
$\Rightarrow \cos\theta = \frac{-14}{\sqrt{6\times 34}}$
$\Rightarrow \theta \approx 168.6°$

(c) $(2,2,1)\cdot(1,-2,-2) = |2,-2,1||1,-2,-2|\cos\theta$
$\Rightarrow 2+4-2 = \sqrt{4+4+1}\sqrt{1+4+4}\cos\theta$
$\Rightarrow 4 = 9\cos\theta$
$\Rightarrow \cos\theta = \frac{4}{9}$
$\Rightarrow \theta \simeq 63.6°$

(d) $(6,-2,3)\cdot(3,-6,2) = |6,-2,3||3,-6,2|\cos\theta$
$\Rightarrow 18+12+6 = \sqrt{36+4+9}\sqrt{9+36+4}\cos\theta$
$\Rightarrow 36 = 49\cos\theta$
$\Rightarrow \cos\theta = \frac{36}{49}$
$\Rightarrow \theta \simeq 42.7°$

(e) $(2,0,-7)\cdot(3,8,3) = |2,0,-7||3,8,3|\cos\theta$
$\Rightarrow 6+0-21 = \sqrt{4+0+49}\sqrt{9+64+9}\cos\theta$
$\Rightarrow -15 = \sqrt{53}\sqrt{82}\cos\theta$
$\Rightarrow \cos\theta = -\frac{15}{\sqrt{53\times 82}}$
$\Rightarrow \theta \approx 103.2°$

## Question 2

Find the angle between each pair of vectors.

**a)** $3\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$

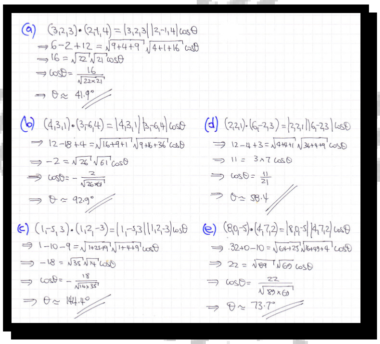
**b)** $4\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$

**c)** $\mathbf{i}-5\mathbf{j}+3\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}-3\mathbf{k}$

**d)** $2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$

**e)** $8\mathbf{i}-5\mathbf{k}$ and $4\mathbf{i}+7\mathbf{j}+2\mathbf{k}$

$\boxed{41.9°}$, $\boxed{92.9°}$, $\boxed{144.4°}$, $\boxed{58.4°}$, $\boxed{73.7°}$



**Question 3**

Find the angle between each pair of vectors.

**a)** $2\mathbf{i}+4\mathbf{j}+6\mathbf{k}$ and $4\mathbf{i}-\mathbf{j}-\mathbf{k}$

**b)** $4\mathbf{i}+2\mathbf{j}-7\mathbf{k}$ and $\mathbf{i}-\mathbf{j}-5\mathbf{k}$

**c)** $2\mathbf{i}-6\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+5\mathbf{j}-\mathbf{k}$

**d)** $3\mathbf{i}+3\mathbf{j}-\mathbf{k}$ and $2\mathbf{i}+\mathbf{j}-\mathbf{k}$

**e)** $3\mathbf{i}-\mathbf{j}-5\mathbf{k}$ and $\mathbf{i}+\mathbf{j}+2\mathbf{k}$

$\boxed{93.6°}$, $\boxed{31.0°}$, $\boxed{150.6°}$, $\boxed{20.5°}$, $\boxed{123.5°}$

(a) $(2,4,6)\cdot(4,-1,-1)=|2,4,6||4,-1,-1|\cos\theta$
$\Rightarrow (2\times4)+4(-1)+6(-1)=\sqrt{4+16+36}\sqrt{16+1+1}\cos\theta$
$\Rightarrow 8-4-6=\sqrt{56\times18}\cos\theta$
$\Rightarrow \cos\theta=-\frac{2}{\sqrt{56\times18}}$
$\Rightarrow \theta=93.6°$

(b) $(4,2,-7)\cdot(1,-1,-5)=|4,2,-7||1,-1,-5|\cos\theta$
$\Rightarrow 4\times1+2(-1)-7(-5)=\sqrt{16+4+49}\sqrt{1+1+25}\cos\theta$
$\Rightarrow 4-2+35=\sqrt{69}\sqrt{27}\cos\theta$
$\Rightarrow 37=\sqrt{69\times27}\cos\theta$
$\Rightarrow \cos\theta=\frac{37}{\sqrt{69\times27}}$
$\Rightarrow \theta=31.0°$

(c) $(2,-6,1)\cdot(1,5,-1)=|2,-6,1||1,5,-1|\cos\theta$
$\Rightarrow (2\times1)-6\times5+1(-1)=\sqrt{4+36+1}\sqrt{1+25+1}\cos\theta$
$\Rightarrow 2-30-1=\sqrt{41\times27}\cos\theta$
$\Rightarrow -29=\sqrt{41\times27}\cos\theta$
$\Rightarrow \cos\theta=-\frac{29}{\sqrt{41\times27}}$
$\Rightarrow \theta\simeq150.6°$

(d) $(3,3,-1)\cdot(2,1,-1)=|3,3,-1||2,1,-1|\cos\theta$
$\Rightarrow (3\times2)+(3\times1)-1(-1)=\sqrt{9+9+1}\sqrt{4+1+1}\cos\theta$
$\Rightarrow 6+3+1=\sqrt{19}\sqrt{6}\cos\theta$
$\Rightarrow \cos\theta=\frac{10}{\sqrt{19\times6}}$
$\Rightarrow \theta=20.5°$

(e) $(3,-1,-5)\cdot(1,1,2)=|3,-1,-5||1,1,2|\cos\theta$
$\Rightarrow (3\times1)-(1\times1)-5\times2=\sqrt{9+1+25}\sqrt{1+1+4}\cos\theta$
$\Rightarrow 3-1-10=\sqrt{35}\sqrt{6}\cos\theta$
$\Rightarrow -8=\sqrt{35\times6}\cos\theta$
$\Rightarrow \cos\theta=-\frac{8}{\sqrt{6\times35}}$
$\Rightarrow \theta=123.5°$

**Question 4**

Find the angle between each pair of vectors.

**a)** $\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $3\mathbf{i}-\mathbf{j}+\mathbf{k}$

**b)** $\mathbf{i}-2\mathbf{j}+\mathbf{k}$ and $2\mathbf{i}+\mathbf{j}-3\mathbf{k}$

**c)** $3\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$

**d)** $\mathbf{i}-\mathbf{j}-2\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}-\mathbf{k}$

**e)** $3\mathbf{i}-4\mathbf{j}-12\mathbf{k}$ and $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$

$\boxed{84.8°}$, $\boxed{109.1°}$, $\boxed{25.2°}$, $\boxed{80.4°}$, $\boxed{75.1°}$

(a) $(1,3,1)\cdot(3,-1,1) = |1,3,1||3,-1,1|\cos\theta$
$\Rightarrow \cancel{3}-\cancel{3}+1 = \sqrt{1+9+1}\sqrt{9+1+1}\cos\theta$
$\Rightarrow 1 = \sqrt{11}\sqrt{11}\cos\theta$
$\Rightarrow 1 = 11\cos\theta$
$\Rightarrow \cos\theta = \frac{1}{11}$
$\Rightarrow \theta \approx 84.8°$

(b) $(1,-2,1)\cdot(2,1,-3) = |1,-2,1||2,1,-3|\cos\theta$
$\Rightarrow \cancel{2}-\cancel{2}-3 = \sqrt{1+4+1}\sqrt{4+1+9}\cos\theta$
$\Rightarrow -3 = \sqrt{6}\sqrt{14}\cos\theta$
$\Rightarrow \cos\theta = -\frac{3}{\sqrt{6\times14}}$
$\Rightarrow \theta \approx 109.1°$

(c) $(3,6,-2)\cdot(1,2,-2) = |3,6,-2||1,2,-2|\cos\theta$
$\Rightarrow 3+12+4 = \sqrt{9+36+4}\sqrt{1+4+4}\cos\theta$
$\Rightarrow 19 = 7\times3\cos\theta$
$\Rightarrow \cos\theta = \frac{19}{21}$
$\Rightarrow \theta \approx 25.2°$

(d) $(1,-1,-2)\cdot(1,2,-1) = |1,-1,-2||1,2,-1|\cos\theta$
$\Rightarrow 1-\cancel{2}+\cancel{2} = \sqrt{1+1+4}\sqrt{1+4+1}\cos\theta$
$\Rightarrow 1 = 6\cos\theta$
$\Rightarrow \cos\theta = \frac{1}{6}$
$\Rightarrow \theta \approx 80.4°$

(e) $(3,-4,-12)\cdot(2,2,-1) = |3,-4,-12||2,2,-1|\cos\theta$
$\Rightarrow 6-8+12 = \sqrt{9+16+144}\sqrt{4+4+1}\cos\theta$
$\Rightarrow 10 = 13\times3\cos\theta$
$\Rightarrow \cos\theta = \frac{10}{39}$
$\Rightarrow \theta = 75.1°$

**Question 5**

Find the angle $\measuredangle CAB$ for each set of the coordinates given.

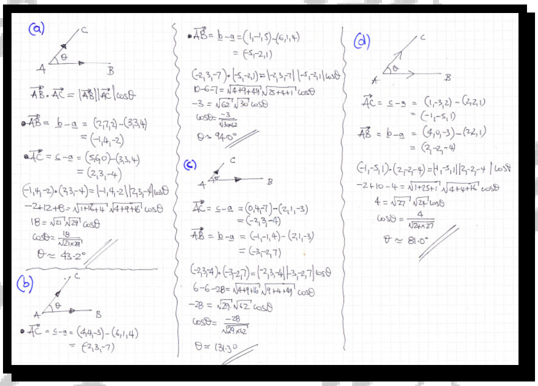
**a)** $A(3,3,4)$, $B(2,7,2)$, $C(5,6,0)$

**b)** $A(6,1,4)$, $B(1,-1,5)$, $C(4,4,-3)$

**c)** $A(2,1,-3)$, $B(-1,-1,4)$, $C(0,4,-7)$

**d)** $A(2,2,1)$, $B(4,0,-3)$, $C(1,-3,2)$

$\boxed{43.2°}$, $\boxed{94.0°}$, $\boxed{131.3°}$, $\boxed{81.0°}$



**Question 6**

Find the angle $\measuredangle CAB$ for each set of the coordinates given.

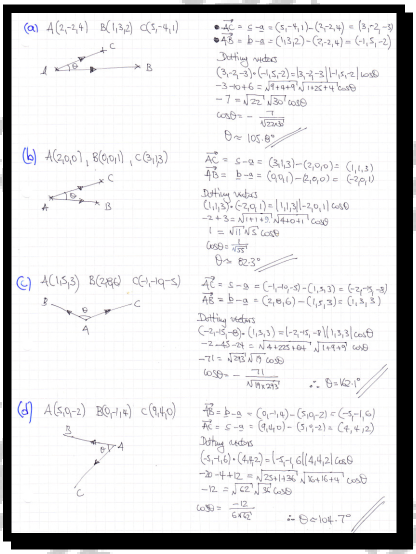
**a)** $A(2,-2,4)$, $B(1,3,2)$, $C(5,-4,1)$

**b)** $A(2,0,0)$, $B(0,0,1)$, $C(3,1,3)$

**c)** $A(1,5,3)$, $B(2,8,6)$, $C(-1,-10,-5)$

**d)** $A(5,0,-2)$, $B(0,-1,4)$, $C(9,4,0)$

$\boxed{105.8°}$, $\boxed{82.3°}$, $\boxed{162.1°}$, $\boxed{104.7°}$



(a) $A(2,-2,4)$ $B(1,3,2)$ $C(5,-4,1)$

- $\overrightarrow{AC} = \underline{c} - \underline{a} = (5,-4,1)-(2,-2,4) = (3,-2,-3)$
- $\overrightarrow{AB} = \underline{b} - \underline{a} = (1,3,2)-(2,-2,4) = (-1,5,-2)$

Dotting vectors

$(3,-2,-3)\cdot(-1,5,-2) = |3,-2,-3||-1,5,-2|\cos\theta$

$-3-10+6 = \sqrt{9+4+9}\sqrt{1+25+4}\cos\theta$

$-7 = \sqrt{22}\sqrt{30}\cos\theta$

$\cos\theta = -\frac{7}{\sqrt{22\times30}}$

$\theta \approx 105.8°$

(b) $A(2,0,0)$, $B(0,0,1)$, $C(3,1,3)$

$\overrightarrow{AC} = \underline{c} - \underline{a} = (3,1,3)-(2,0,0) = (1,1,3)$

$\overrightarrow{AB} = \underline{b} - \underline{a} = (0,0,1)-(2,0,0) = (-2,0,1)$

Dotting vectors

$(1,1,3)\cdot(-2,0,1) = |1,1,3||-2,0,1|\cos\theta$

$-2+3 = \sqrt{1+1+9}\sqrt{4+0+1}\cos\theta$

$1 = \sqrt{11}\sqrt{5}\cos\theta$

$\cos\theta = \frac{1}{\sqrt{55}}$

$\theta \simeq 82.3°$

(c) $A(1,5,3)$ $B(2,8,6)$ $C(-1,-10,-5)$

$\overrightarrow{AC} = \underline{c} - \underline{a} = (-1,-10,-5)-(1,5,3) = (-2,-15,-8)$

$\overrightarrow{AB} = \underline{b} - \underline{a} = (2,8,6)-(1,5,3) = (1,3,3)$

Dotting vectors

$(-2,-15,-8)\cdot(1,3,3) = |-2,-15,-8||1,3,3|\cos\theta$

$-2-45-24 = \sqrt{4+225+64}\sqrt{1+9+9}\cos\theta$

$-71 = \sqrt{293}\sqrt{19}\cos\theta$

$\cos\theta = -\frac{71}{\sqrt{19\times293}}$ $\therefore \theta = 162.1°$

(d) $A(5,0,-2)$ $B(0,-1,4)$ $C(9,4,0)$

$\overrightarrow{AB} = \underline{b} - \underline{a} = (0,-1,4)-(5,0,-2) = (-5,-1,6)$

$\overrightarrow{AC} = \underline{c} - \underline{a} = (9,4,0)-(5,0,-2) = (4,4,2)$

Dotting vectors

$(-5,-1,6)\cdot(4,4,2) = |-5,-1,6||4,4,2|\cos\theta$

$-20-4+12 = \sqrt{25+1+36}\sqrt{16+16+4}\cos\theta$

$-12 = \sqrt{62}\sqrt{36}\cos\theta$

$\cos\theta = \frac{-12}{6\sqrt{62}}$ $\therefore \theta \simeq 104.7°$

**Question 7**

The vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular, and $\lambda$ is a scalar constant.

Find in each case the possible value(s) of $\lambda$.

**a)** $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + \lambda\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} + \lambda\mathbf{j} - 5\mathbf{k}$

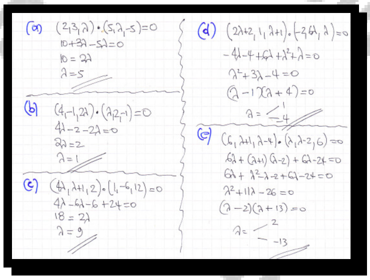
**b)** $\mathbf{a} = 4\mathbf{i} - \mathbf{j} + 2\lambda\mathbf{k}$ and $\mathbf{b} = \lambda\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

**c)** $\mathbf{a} = 4\lambda\mathbf{i} + (\lambda+1)\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 6\mathbf{j} + 12\mathbf{k}$

**d)** $\mathbf{a} = (2\lambda+2)\mathbf{i} + \mathbf{j} + (\lambda+1)\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 6\lambda\mathbf{j} + \lambda\mathbf{k}$

**e)** $\mathbf{a} = 6\mathbf{i} + (\lambda+1)\mathbf{j} + (\lambda-4)\mathbf{k}$ and $\mathbf{b} = \lambda\mathbf{i} + (\lambda-2)\mathbf{j} + 6\mathbf{k}$

$\boxed{\lambda_a = 5}$, $\boxed{\lambda_b = 1}$, $\boxed{\lambda_c = 9}$, $\boxed{\lambda_d = 1, -4}$, $\boxed{\lambda_e = 2, -13}$



(a) $(2,3,\lambda)\cdot(5,\lambda,-5)=0$
$10+3\lambda-5\lambda=0$
$10=2\lambda$
$\lambda=5$

(b) $(4,-1,2\lambda)\cdot(\lambda,2,-1)=0$
$4\lambda-2-2\lambda=0$
$2\lambda=2$
$\lambda=1$

(c) $(4\lambda,\lambda+1,2)\cdot(1,-6,12)=0$
$4\lambda-6\lambda-6+24=0$
$18=2\lambda$
$\lambda=9$

(d) $(2\lambda+2,1,\lambda+1)\cdot(-2,6\lambda,\lambda)=0$
$-4\lambda-4+6\lambda+\lambda^2+\lambda=0$
$\lambda^2+3\lambda-4=0$
$(\lambda-1)(\lambda+4)=0$
$\lambda = 1, -4$

(e) $(6,\lambda+1,\lambda-4)\cdot(\lambda,\lambda-2,6)=0$
$6\lambda+(\lambda+1)(\lambda-2)+6\lambda-24=0$
$6\lambda+\lambda^2-\lambda-2+6\lambda-24=0$
$\lambda^2+11\lambda-26=0$
$(\lambda-2)(\lambda+13)=0$
$\lambda = 2, -13$

**Question 8**

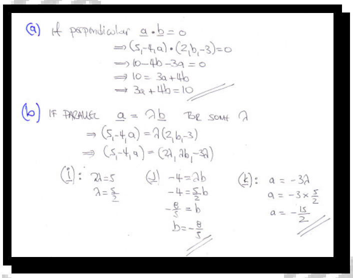
The vectors **a** and **b** are given by

$$\mathbf{a} = 5\mathbf{i} - 4\mathbf{j} + a\mathbf{k}\ ,\ \ \mathbf{b} = 2\mathbf{i} + b\mathbf{j} - 3\mathbf{k}\ .$$

**a)** If **a** and **b** are perpendicular find a relationship between $a$ and $b$.

**b)** If instead **a** and **b** are parallel find the value of $a$ and the value of $b$.

$\boxed{3a + 4b = 10}$, $\boxed{a = -\frac{8}{5}}$, $\boxed{b = -\frac{15}{2}}$


(a) if perpendicular $\underline{a} \cdot \underline{b} = 0$

$\Rightarrow (5, -4, a) \cdot (2, b, -3) = 0$

$\Rightarrow 10 - 4b - 3a = 0$

$\Rightarrow 10 = 3a + 4b$

$\Rightarrow 3a + 4b = 10$

(b) IF PARALLEL $\underline{a} = \lambda \underline{b}$ FOR SOME $\lambda$

$\Rightarrow (5, -4, a) = \lambda(2, b, -3)$

$\Rightarrow (5, -4, a) = (2\lambda, \lambda b, -3\lambda)$

(i): $2\lambda = 5$, $\lambda = \frac{5}{2}$

(j) $-4 = \lambda b$, $-4 = \frac{5}{2}b$, $-\frac{8}{5} = b$, $b = -\frac{8}{5}$

(k): $a = -3\lambda$, $a = -3 \times \frac{5}{2}$, $a = -\frac{15}{2}$

**Question 9**

Find a vector with **integer** components which is perpendicular to **both** the vectors given below.

(Do not use the cross product)

**a)** $2\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ and $\mathbf{i}+\mathbf{j}-3\mathbf{k}$

**b)** $6\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $-\mathbf{i}-3\mathbf{j}+\mathbf{k}$

**c)** $7\mathbf{i}-2\mathbf{j}-\mathbf{k}$ and $6\mathbf{i}+\mathbf{j}+5\mathbf{k}$

**d)** $\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ and $4\mathbf{i}+3\mathbf{j}+5\mathbf{k}$

**e)** $8\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}+2\mathbf{k}$

$\boxed{\mathbf{i}+2\mathbf{j}+\mathbf{k}}$, $\boxed{5\mathbf{i}-8\mathbf{j}-19\mathbf{k}}$, $\boxed{9\mathbf{i}+41\mathbf{j}-19\mathbf{k}}$, $\boxed{-16\mathbf{i}+3\mathbf{j}+11\mathbf{k}}$, $\boxed{\mathbf{i}-22\mathbf{j}-36\mathbf{k}}$

(a) LET REQUIRED VECTOR BE $(x,y,z)$

THUS $(x,y,z)\cdot(2,-3,4)=0$, $(x,y,z)\cdot(1,1,-3)=0$ $\Rightarrow$ $2x-3y+4z=0$, $x+y-3z=0$

$\boxed{\text{LET } z=1}$ $\quad 2x-3y+4=0$, $x+y-3=0$ $\Rightarrow$ $2x-3y=-4$, $x+y=3$ $\Rightarrow$

$2x-3y=-4$, $(\times 3)\ 3x+3y=9$ $\xrightarrow{\text{ADD}}$ $5x=5$; $x=1$, $y=2$, $z=1$

$\therefore$ REQUIRED VECTOR IS $\underline{i}+2\underline{j}+\underline{k}$

(b) LET REQUIRED VECTOR BE $(x,y,z)$

THUS $(x,y,z)\cdot(6,-1,2)=6x-y+2z=0$
$(x,y,z)\cdot(-1,-3,1)=-x-3y+z=0$

$\boxed{\text{LET } x=1}$ $\quad 6-y+2z=0$, $-1-3y+z=0$ $\Rightarrow$ $-y+2z=-6$, $-3y+z=1$ $\Rightarrow$

$\times(-3)\ 3y-6z=18$, $-3y+z=1$ Add $-5z=19$, $\boxed{z=-\frac{19}{5}}$

$\therefore -3y-\frac{19}{5}=1$
$-3y=\frac{24}{5}$
$\boxed{y=-\frac{8}{5}}$

i.e. REQUIRED VECTOR $\left(1,-\frac{8}{5},-\frac{19}{5}\right)$
SCALE IT BY 5 $\quad(5,-8,-19)$
i.e. $5\underline{i}-8\underline{j}-19\underline{k}$

(c) LET THE REQUIRED VECTOR BE $(x,y,z)$

$(x,y,z)\cdot(7,-2,-1)=0$, $(x,y,z)\cdot(6,1,5)=0$ $\Rightarrow$ $7x-2y-z=0$, $6x+y+5z=0$ LET $\boxed{x=1}$

$7-2y-z=0$ $(\times 1)$, $6+y+5z=0$ $(\times 2)$ $\Rightarrow$ $-2y-z=-7$, $2y+10z=-12$ Add $9z=-19$, $\boxed{z=-\frac{19}{9}}$

THUS $6+y+5z=0$
$y=-6-5z$
$y=-6-5\left(-\frac{19}{9}\right)$
$y=-6+\frac{95}{9}$
$\boxed{y=\frac{41}{9}}$

$\therefore$ VECTOR IS $\left(1,\frac{41}{9},-\frac{19}{9}\right)$
SCALE IT TO $(9,41,-19)$
$\therefore 9\underline{i}+41\underline{j}-19\underline{k}$

(d) LET THE REQUIRED VECTOR BE $(x,y,z)$

$(x,y,z)\cdot(1,-2,2)=0$, $(x,y,z)\cdot(4,3,5)=0$ $\Rightarrow$ $x-2y+2z=0$, $4x+3y+5z=0$ $\boxed{\text{LET } z=1}$

$x-2y+2=0$ $(\times 3)$, $4x+3y+5=0$ $(\times 2)$ $\Rightarrow$ $3x-6y+6=0$, $8x+6y+10=0$ Add

$11x+16=0$
$\boxed{x=-\frac{16}{11}}$

$x-2y+2=0$
$-\frac{16}{11}-2y+2=0$
$\frac{6}{11}=2y$
$\boxed{y=\frac{3}{11}}$

$\therefore \left(-\frac{16}{11},\frac{3}{11},1\right)$
SCALE THE VECTOR TO $(-16,3,11)$
i.e. $-16\underline{i}+3\underline{j}+11\underline{k}$

(e) LET THE REQUIRED VECTOR BE $(x,y,z)$

$(x,y,z)\cdot(8,2,-1)=0$, $(x,y,z)\cdot(6,-3,2)=0$ $\Rightarrow$ $8x+2y-z=0$, $6x-3y+2z=0$ LET $\boxed{x=1}$

$8+2y-z=0$ $(\times 2)$, $6-3y+2z=0$ $(\times 1)$ $\Rightarrow$ $16+4y-2z=0$, $6-3y+2z=0$ Add $22+y=0$, $\boxed{y=-22}$

THUS $8+2y-z=0$
$8-44-z=0$
$\boxed{z=-36}$

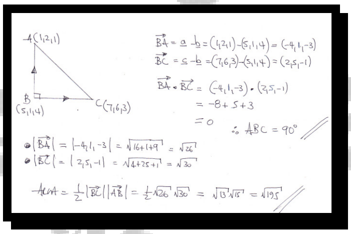
$\therefore (1,-22,-36)$
i.e. $\underline{i}-22\underline{j}-36\underline{k}$

**Question 10**

The points $A$, $B$ and $C$ have coordinates $(1,2,1)$, $(5,1,4)$ and $(7,6,3)$, respectively.

Show that $\measuredangle ABC = 90°$ and hence find the exact area of the triangle $ABC$.

$\boxed{\text{area} = \sqrt{195}}$




$\vec{BA} = \underline{a} - \underline{b} = (1,2,1) - (5,1,4) = (-4,1,-3)$
$\vec{BC} = \underline{c} - \underline{b} = (7,6,3) - (5,1,4) = (2,5,-1)$

$\vec{BA} \cdot \vec{BC} = (-4,1,-3) \cdot (2,5,-1)$
$= -8 + 5 + 3$
$= 0$ $\therefore \measuredangle ABC = 90°$

- $|\vec{BA}| = |-4,1,-3| = \sqrt{16+1+9} = \sqrt{26}$
- $|\vec{BC}| = |2,5,-1| = \sqrt{4+25+1} = \sqrt{30}$

$\text{Area} = \frac{1}{2}|\vec{BC}||\vec{AB}| = \frac{1}{2}\sqrt{26}\sqrt{30} = \sqrt{13}\sqrt{15} = \sqrt{195}$

# THE EQUATION OF A LINE

## Question 1

**a)** Find a vector equation of the straight line $l$, that passes through the point $A(7,-1,2)$ and is in the direction $-2\mathbf{i}+3\mathbf{j}+\mathbf{k}$.

**b)** If $B(p,q,6)$ lies on $l$ find the value of $p$ and the value of $q$.

$\boxed{\mathbf{r}=7\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})}$, $\boxed{p=-1}$, $\boxed{q=11}$

(a) $r$ = (Fixed point) + $\lambda$(direction vector)
$r=(7,-1,2)+\lambda(-2,3,1)$
$(x,y,z)=(7-2\lambda, 3\lambda-1, \lambda+2)$

(b) $(p,q,6)=(7-2\lambda, 3\lambda-1, \lambda+2)$
(k): $\lambda+2=6 \Rightarrow \lambda=4$
$\therefore p=7-2\times4=-1$
$\therefore q=3\times4-1=11$

## Question 2

**a)** Find a vector equation of the straight line $l$, that passes through the point $A(2,-6,-4)$ and is parallel to the vector $\mathbf{i}+2\mathbf{j}+5\mathbf{k}$.

**b)** If $B(p,0,q)$ lies on $l$ find the value of $p$ and the value of $q$.

$\boxed{\mathbf{r}=2\mathbf{i}-6\mathbf{j}-4\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+5\mathbf{k})}$, $\boxed{p=5}$, $\boxed{q=11}$

(a) $r$ = (FIXED POINT) + $\lambda$(DIRECTION VECTOR)
$r=(2,-6,-4)+\lambda(1,2,5)$
$(x,y,z)=(\lambda+2, 2\lambda-6, 5\lambda-4)$

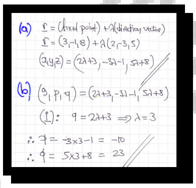
(b) BY INSPECTING $j$ $\lambda=3$ at B
$\therefore p=\lambda+2=3+2=5$
$q=5\lambda-4=5\times3-4=11$

**Question 3**

**a)** Find a vector equation of the straight line $l$, that passes through the point $A(3,-1,8)$ and is in the direction $2\mathbf{i}-3\mathbf{j}+5\mathbf{k}$.

**b)** If $B(9,p,q)$ lies on $l$ find the value of $p$ and the value of $q$.

$\boxed{\mathbf{r}=3\mathbf{i}-\mathbf{j}+8\mathbf{k}+\lambda(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})}$, $\boxed{p=-10}$, $\boxed{q=23}$



(a) $\underline{r} = (\text{fixed point}) + \lambda(\text{direction vector})$

$\underline{r} = (3,-1,8) + \lambda(2,-3,5)$

$(x,y,z) = (2\lambda+3, -3\lambda-1, 5\lambda+8)$

(b) $(9,p,q) = (2\lambda+3, -3\lambda-1, 5\lambda+8)$

$(\underline{i})$: $9 = 2\lambda+3 \Rightarrow \lambda = 3$

$\therefore p = -3\times3-1 = -10$

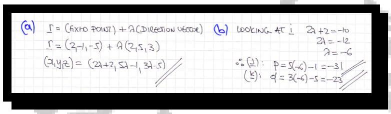
$\therefore q = 5\times3+8 = 23$

**Question 4**

**a)** Find a vector equation of the straight line $l$ that passes through the point $A(2,-1,-5)$ and is in the direction $2\mathbf{i}+5\mathbf{j}+3\mathbf{k}$.

**b)** If $B(-10,p,q)$ lies on $l$ find the values o $p$ and the value of $q$.

$\boxed{\mathbf{r}=2\mathbf{i}-\mathbf{j}-5\mathbf{k}+\lambda(2\mathbf{i}+5\mathbf{j}+3\mathbf{k})}$, $\boxed{p=-31}$, $\boxed{q=-23}$



(a) $\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (2,-1,-5) + \lambda(2,5,3)$

$(x,y,z) = (2\lambda+2, 5\lambda-1, 3\lambda-5)$

(b) LOOKING AT $\underline{i}$ $\quad 2\lambda+2=-10$

$2\lambda = -12$

$\lambda = -6$

$\therefore (\underline{j})$: $p = 5(-6)-1 = -31$

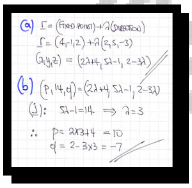
$(\underline{k})$: $q = 3(-6)-5 = -23$

**Question 5**

**a)** Determine a vector equation of the straight line $l$ that passes through the point $A(4,-1,2)$ and is in the direction $2\mathbf{i}+5\mathbf{j}-3\mathbf{k}$.

**b)** If $B(p,14,q)$ lies on $l$ find the value of $p$ and the value of $q$.

$\boxed{\mathbf{r}=4\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}+5\mathbf{j}-3\mathbf{k})}$, $\boxed{p=10}$, $\boxed{q=-7}$

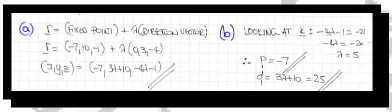

(a) $\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION})$
$\underline{r} = (4,-1,2) + \lambda(2,5,-3)$
$(x,y,z) = (2\lambda+4,\ 5\lambda-1,\ 2-3\lambda)$
(b) $(p,14,q) = (2\lambda+4,\ 5\lambda-1,\ 2-3\lambda)$
$(\underline{j})$: $5\lambda-1=14 \Rightarrow \lambda=3$
$\therefore$ $p = 2\times3+4 = 10$
$q = 2-3\times3 = -7$

**Question 6**

**a)** Determine a vector equation of the straight line $l$ that passes through the point $A(-7,10,-1)$ and is parallel to the vector $3\mathbf{j}-4\mathbf{k}$.

**b)** If $B(p,q,-21)$ lies on $l$ find the value of $p$ and the value of $q$.

$\boxed{\mathbf{r}=-7\mathbf{i}+10\mathbf{j}-\mathbf{k}+\lambda(3\mathbf{j}-4\mathbf{k})}$, $\boxed{p=-7}$, $\boxed{q=25}$

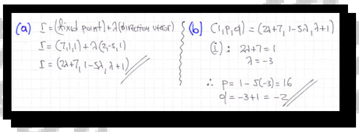

(a) $\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$
$\underline{r} = (-7,10,-1) + \lambda(0,3,-4)$
$(x,y,z) = (-7,\ 3\lambda+10,\ -4\lambda-1)$
(b) LOOKING AT $\underline{k}$: $-4\lambda-1=-21$
$-4\lambda=-20$
$\lambda=5$
$\therefore$ $p=-7$
$q = 3\lambda+10 = 25$

**Question 7**

**a)** Determine a vector equation of the straight line $l$ that passes through the point $A(7,1,1)$ and is parallel to the vector $2\mathbf{i}-5\mathbf{j}+\mathbf{k}$.

**b)** If $B(1,p,q)$ lies on $l$ find the value of $p$ and the value of $q$.

$\boxed{\mathbf{r}=7\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}-5\mathbf{j}+\mathbf{k})}$, $\boxed{p=16}$, $\boxed{q=-2}$

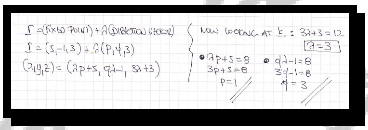



**Question 8**

The straight line $l$ passes through the point $A(5,-1,3)$ and is parallel to the vector $p\mathbf{i}+q\mathbf{j}+3\mathbf{k}$.

If $B(8,8,12)$ lies on $l$ find the value of $p$ and the value of $q$.

$\boxed{p=1}$, $\boxed{q=3}$

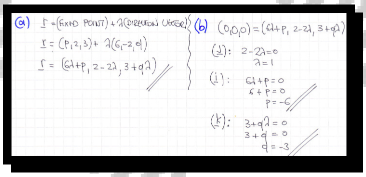


**Question 9**

**a)** Determine a vector equation of the straight line $l$ that passes through the point $A(p,2,3)$ and is in the direction $6\mathbf{i}-2\mathbf{j}+q\mathbf{k}$, where $p$ and $q$ are scalar constants.

**b)** If $l$ passes through the origin, find the value of $p$ and the value of $q$.

$\boxed{\mathbf{r} = p\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(6\mathbf{i}-2\mathbf{j}+q\mathbf{k})}$, $\boxed{p=-6}$, $\boxed{q=-3}$

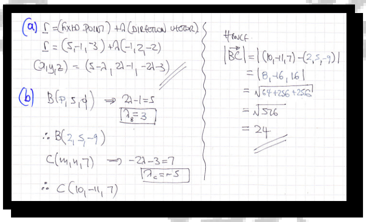



**Question 10**

**a)** Determine a vector equation of the straight line $l$ that passes through the point $A(5,-1,-3)$ and is parallel to the vector $-\mathbf{i}+2\mathbf{j}-2\mathbf{k}$.

**b)** If both the points $B(p,5,q)$ and $C(m,n,7)$ lie on $l$, find the distance $BC$.

$\boxed{\mathbf{r} = 5\mathbf{i}-\mathbf{j}-3\mathbf{k}+\lambda(-\mathbf{i}+2\mathbf{j}-2\mathbf{k})}$, $\boxed{|BC| = 24}$

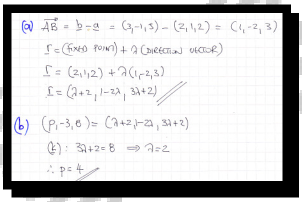


**Question 11**

**a)** Determine a vector equation of the straight line $l$ that passes through the points $A(2,1,2)$ and $B(3,-1,5)$.

**b)** Given that $P(p,-3,8)$ lies on $l$ find the value of $p$.

$$\boxed{\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})}, \ \boxed{p = 4}$$



(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (3,-1,5) - (2,1,2) = (1,-2,3)$

$\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (2,1,2) + \lambda(1,-2,3)$

$\underline{r} = (\lambda+2, 1-2\lambda, 3\lambda+2)$

(b) $(p,-3,8) = (\lambda+2, 1-2\lambda, 3\lambda+2)$

$(\underline{k}): 3\lambda+2=8 \Rightarrow \lambda=2$

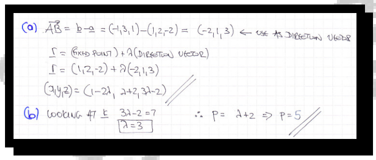
$\therefore p=4$

**Question 12**

**a)** Find a vector equation of the straight line $l$ that passes through the points $A(1,2,-2)$ and $B(-1,3,1)$.

**b)** Given that $P(-5,p,7)$ lies on $l$ find the value of $p$.

$$\boxed{\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k})}, \ \boxed{p = 5}$$



(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (-1,3,1) - (1,2,-2) = (-2,1,3)$ ← USE AS DIRECTION VECTOR

$\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (1,2,-2) + \lambda(-2,1,3)$

$(x,y,z) = (1-2\lambda, \lambda+2, 3\lambda-2)$

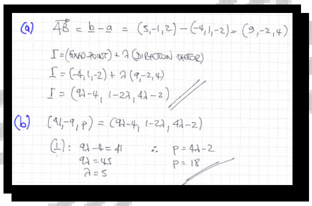
(b) LOOKING AT $\underline{k}$ $3\lambda-2=7$ $\boxed{\lambda=3}$ $\therefore p = \lambda+2 \Rightarrow p=5$

**Question 13**

**a)** Determine a vector equation of the straight line $l$ that passes through the points $A(-4,1,-2)$ and $B(5,-1,2)$.

**b)** Given that $P(41,-9,p)$ lies on $l$ find the value of $p$.

$\boxed{\mathbf{r} = -4\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \lambda(9\mathbf{i} - 2\mathbf{j} + 4\mathbf{k})}$, $\boxed{p = 18}$


(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (5,-1,2) - (-4,1,-2) = (9,-2,4)$

$\underline{r} = $ (FIXED POINT) $+ \lambda$ (DIRECTION VECTOR)

$\underline{r} = (-4,1,-2) + \lambda(9,-2,4)$

$\underline{r} = (9\lambda-4,\ 1-2\lambda,\ 4\lambda-2)$

(b) $(41,-9,p) = (9\lambda-4,\ 1-2\lambda,\ 4\lambda-2)$

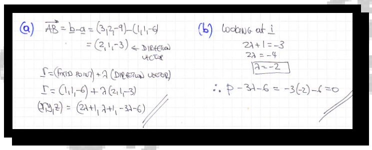
$\underline{i}$: $9\lambda - 4 = 41$, $9\lambda = 45$, $\lambda = 5$ $\therefore$ $p = 4\lambda - 2$, $p = 18$

**Question 14**

**a)** Find a vector equation of the straight line $l$ that passes through the points $A(1,1,-6)$ and $B(3,2,-9)$.

**b)** Given that $P(-3,-1,p)$ lies on $l$ find the value of $p$.

$\boxed{\mathbf{r} = \mathbf{i} + \mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})}$, $\boxed{p = 0}$


(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (3,2,-9) - (1,1,-6) = (2,1,-3)$ ← DIRECTION VECTOR

$\underline{r} = $ (FIXED POINT) $+ \lambda$ (DIRECTION VECTOR)

$\underline{r} = (1,1,-6) + \lambda(2,1,-3)$

$(x,y,z) = (2\lambda+1,\ \lambda+1,\ -3\lambda-6)$

(b) LOOKING AT $\underline{i}$

$2\lambda + 1 = -3$, $2\lambda = -4$, $\boxed{\lambda = -2}$

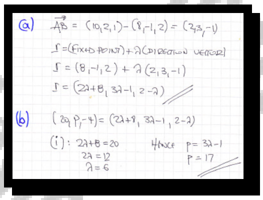
$\therefore p = -3\lambda - 6 = -3(-2) - 6 = 0$

**Question 15**

**a)** Determine a vector equation of the straight line $l$ that passes through the points $A(8,-1,2)$ and $B(10,2,1)$.

**b)** Given that $P(20, p, -4)$ lies on $l$ find the value of $p$.

$\boxed{\mathbf{r} = 8\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})}$, $\boxed{p = 17}$



(a) $\overrightarrow{AB} = (10,2,1) - (8,-1,2) = (2,3,-1)$

$\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (8,-1,2) + \lambda(2,3,-1)$

$\underline{r} = (2\lambda+8, 3\lambda-1, 2-\lambda)$

(b) $(20, p, -4) = (2\lambda+8, 3\lambda-1, 2-\lambda)$

(i): $2\lambda+8 = 20$
$2\lambda = 12$
$\lambda = 6$

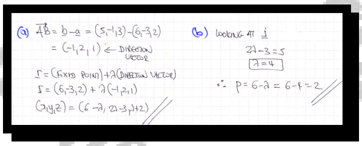
HENCE $p = 3\lambda - 1$
$p = 17$

**Question 16**

**a)** Find a vector equation of the straight line $l$ that passes through the points $A(6,-3,2)$ and $B(5,-1,3)$.

**b)** Given that $P(p,5,6)$ lies on $l$ find the value of $p$.

$\boxed{\mathbf{r} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})}$, $\boxed{p = 2}$



(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (5,-1,3) - (6,-3,2)$
$= (-1,2,1) \leftarrow$ DIRECTION VECTOR

$\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (6,-3,2) + \lambda(-1,2,1)$

$(x,y,z) = (6-\lambda, 2\lambda-3, \lambda+2)$

(b) LOOKING AT $\underline{j}$

$2\lambda - 3 = 5$
$\boxed{\lambda = 4}$

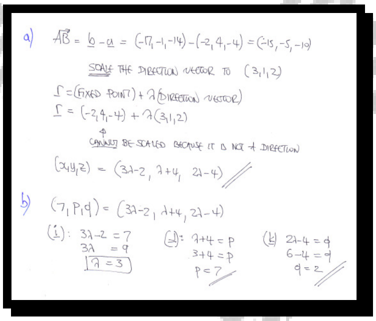
$\therefore p = 6 - \lambda = 6 - 4 = 2$

## Question 17

**a)** Determine a vector equation of the straight line $l$ that passes through the points $A(-2,4,-4)$ and $B(-17,-1,-14)$.

**b)** Given that $P(7,p,q)$ lies on $l$ find the value of $p$ and the value of $q$.

$$\boxed{\mathbf{r} = -2\mathbf{i}+4\mathbf{j}-4\mathbf{k}+\lambda(3\mathbf{i}+2\mathbf{j}+\mathbf{k})}, \ \boxed{p=7}, \ \boxed{q=2}$$



a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (-17,-1,-14)-(-2,4,-4) = (-15,-5,-10)$

SCALE THE DIRECTION VECTOR TO $(3,1,2)$

$\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (-2,4,-4) + \lambda(3,1,2)$

CANNOT BE SCALED BECAUSE IT IS NOT A DIRECTION

$(x,y,z) = (3\lambda-2, \lambda+4, 2\lambda-4)$

b) $(7,p,q) = (3\lambda-2, \lambda+4, 2\lambda-4)$

$(\underline{i})$: $3\lambda-2=7$, $3\lambda=9$, $\boxed{\lambda=3}$

$(\underline{j})$: $\lambda+4=p$, $3+4=p$, $p=7$

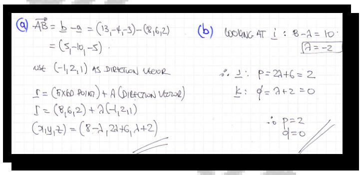
$(\underline{k})$: $2\lambda-4=q$, $6-4=q$, $q=2$

## Question 18

**a)** Find a vector equation of the straight line $l$ that passes through the points $A(8,6,2)$ and $B(13,-4,-3)$.

**b)** Given that $C(10,p,q)$ lies on $l$ find the value of $p$ and the value of $q$.

$$\boxed{\mathbf{r} = 8\mathbf{i}+6\mathbf{j}+2\mathbf{k}+\lambda(-\mathbf{i}+2\mathbf{j}+\mathbf{k})}, \ \boxed{p=2}, \ \boxed{q=0}$$



(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (13,-4,-3)-(8,6,2) = (5,-10,-5)$

USE $(-1,2,1)$ AS DIRECTION VECTOR

$\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (8,6,2) + \lambda(-1,2,1)$

$(x,y,z) = (8-\lambda, 2\lambda+6, \lambda+2)$

(b) LOOKING AT $\underline{i}$: $8-\lambda = 10$, $\boxed{\lambda=-2}$

$\therefore$ $\underline{j}$: $p = 2\lambda+6 = 2$

$\underline{k}$: $q = \lambda+2 = 0$

$\therefore$ $p=2$, $q=0$

**Question 19**

**a)** Determine a vector equation of the straight line $l$ that passes through the points $A(6,5,1)$ and $B(4,4,-1)$.

**b)** Given the point $C(p,q,q)$ lies on $l$ find the value of $p$ and the value of $q$.

$$\boxed{\mathbf{r} = 6\mathbf{i}+5\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}+2\mathbf{k})}, \ \boxed{p=14}, \ \boxed{q=9}$$

(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (4,4,-1)-(6,5,1) = (-2,-1,-2)$

Take $(2,1,2)$ as the direction vector

$\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (6,5,1) + \lambda(2,1,2)$

$\underline{r} = (2\lambda+6, \lambda+5, 2\lambda+1)$

(b) $(p,q,q) = (2\lambda+6, \lambda+5, 2\lambda+1)$

$p = 2\lambda+6$

$\left.\begin{array}{l} q = \lambda+5 \\ q = 2\lambda+1 \end{array}\right\} \Rightarrow \begin{array}{c} \lambda+5 = 2\lambda+1 \\ 4 = \lambda \end{array}$

$\therefore \ p = 2\times4+6 \Rightarrow p = 14$

$q = 4+5 \Rightarrow q = 9$

**Question 20**

Given that the points $A(4,6,-2)$, $B(9,1,3)$ and $C(1,p,q)$ lie on a straight line find a vector equation for the straight line and hence find the value of $p$ and the value of $q$.

$$\boxed{\mathbf{r} = 4\mathbf{i}+6\mathbf{j}-2\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+\mathbf{k})}, \ \boxed{p=9}, \ \boxed{q=-5}$$

(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (9,1,3)-(4,6,-2) = (5,-5,5)$ use $(1,-1,1)$ as direction

$\underline{r} = (\text{FIXED POINT}) + \lambda(\text{DIRECTION VECTOR})$

$\underline{r} = (4,6,-2) + \lambda(1,-1,1)$

$(x,y,z) = (\lambda+4, 6-\lambda, \lambda-2)$

(b) LOOKING AT $\underline{i}$: $\lambda+4=1$

$\boxed{\lambda=-3}$

$\therefore \ p = 6-\lambda$, $q = \lambda-2$

$\therefore \ p = 9$, $q = -5$

**Question 21**

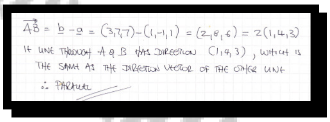
Show that the straight line with vector equation

$$\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}),$$

where $\lambda$ is a scalar parameter

and the straight line through the points $A(1,-1,1)$ and $B(3,7,7)$ are parallel.

$\boxed{\text{proof}}$


$\overrightarrow{AB} = \underline{b} - \underline{a} = (3,7,7) - (1,-1,1) = (2,8,6) = 2(1,4,3)$

IF LINE THROUGH A & B HAS DIRECTION (1,4,3), WHICH IS THE SAME AS THE DIRECTION VECTOR OF THE OTHER LINE

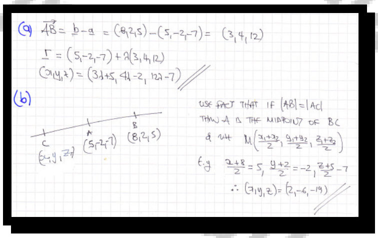
$\therefore$ PARALLEL

**Question 22**

**a)** Find a vector equation of the straight line $l$ that passes through the points $A(5,-2,-7)$ and $B(8,2,5)$.

**b)** Find the coordinates of the point $C$ which also lies on $l$ with $|AB| = |AC|$,

$\boxed{\mathbf{r} = 5\mathbf{i} - 2\mathbf{j} - 7\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k})}$, $\boxed{C(2,-6,-19)}$


(a) $\overrightarrow{AB} = \underline{b} - \underline{a} = (8,2,5) - (5,-2,-7) = (3,4,12)$

$\underline{r} = (5,-2,-7) + \lambda(3,4,12)$

$(x,y,z) = (3\lambda+5, 4\lambda-2, 12\lambda-7)$

(b)



USE FACT THAT IF $|AB| = |AC|$ THEN A IS THE MIDPOINT OF BC & USE $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

E.g. $\frac{x+8}{2} = 5$, $\frac{y+2}{2} = -2$, $\frac{z+5}{2} = -7$

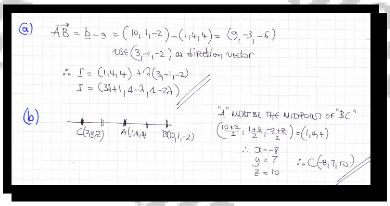
$\therefore (x,y,z) = (2,-6,-19)$

## Question 23

**a)** Find a vector equation of the straight line $l$ through the points $A(1,4,4)$ and $B(10,1,-2)$.

**b)** Find the coordinates of the point $C$, given that it lies on $l$ so that $|AB| = |AC|$.

$\boxed{\mathbf{r} = \mathbf{i} + 4\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} - 2\mathbf{k})}$, $\boxed{B(-8,7,10)}$



(a) $\vec{AB} = \underline{b} - \underline{a} = (10,1,-2) - (1,4,4) = (9,-3,-6)$

USE $(3,-1,-2)$ as direction vector

$\therefore \underline{r} = (1,4,4) + \lambda(3,-1,-2)$

$\underline{r} = (3\lambda+1, 4-\lambda, 4-2\lambda)$

(b) $C(x,y,z)$ $A(1,4,4)$ $B(10,1,-2)$

"A" MUST BE THE MIDPOINT OF "BC"

$\left(\frac{10+x}{2}, \frac{1+y}{2}, \frac{-2+z}{2}\right) = (1,4,4)$

$\therefore x = -8$, $y = 7$, $z = 10$ $\therefore C(-8,7,10)$

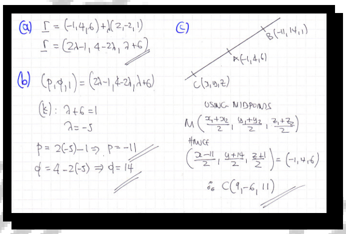
## Question 24

**a)** Find a vector equation of the straight line $l$ that passes through the point $A(-1,4,6)$ and is parallel to the vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

**b)** Given that $B(p,q,1)$ lies on $l$, find the value of $p$ and the value of $q$.

**c)** Find the coordinates of the point $C$, given that it lies on $l$ with $|AB| = |AC|$.

$\boxed{\mathbf{r} = -\mathbf{i} + 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})}$, $\boxed{p = -11}$, $\boxed{q = 14}$, $\boxed{C(9,-6,11)}$



(a) $\underline{r} = (-1,4,6) + \lambda(2,-2,1)$

$\underline{r} = (2\lambda-1, 4-2\lambda, \lambda+6)$

(b) $(p,q,1) = (2\lambda-1, 4-2\lambda, \lambda+6)$

$(\underline{k})$: $\lambda + 6 = 1$

$\lambda = -5$

$p = 2(-5) - 1 \Rightarrow p = -11$

$q = 4 - 2(-5) \Rightarrow q = 14$

(c) $B(-11,14,1)$ $A(-1,4,6)$ $C(x,y,z)$

USING MIDPOINTS

$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

HENCE

$\left(\frac{x-11}{2}, \frac{y+14}{2}, \frac{z+1}{2}\right) = (-1,4,6)$

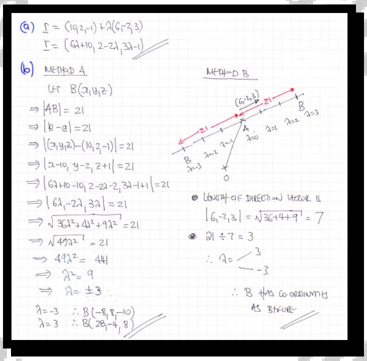
$\therefore C(9,-6,11)$

**Question 25**

**a)** Find a vector equation of the straight line $l$ that passes through the point $A(10,2,-1)$ and is parallel to the vector $6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$.

**b)** Find the two possible sets of coordinates of the point $B$ given that it lies on $l$ and that $|AB|=21$ units.

$\boxed{\mathbf{r}=10\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(6\mathbf{i}-2\mathbf{j}+3\mathbf{k})}$, $\boxed{B(28,-4,8) \text{ or } B(-8,8,-10)}$

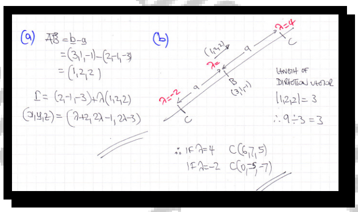


**Question 26**

**a)** Find a vector equation of the straight line $l$ which passes through the points $A(2,-1,-3)$ and $B(3,1,-1)$.

**b)** Find the two possible sets of coordinates of the point $C$ given that it also lies on $l$ and that $|BC|=9$ units.

$\boxed{\mathbf{r}=2\mathbf{i}-\mathbf{j}-3\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})}$, $\boxed{C(6,7,5) \text{ or } C(0,-5,-7)}$



**Question 27**

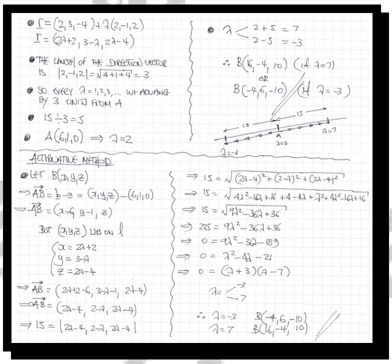
The point $A(6,1,0)$ lies on the straight line $l$ with equation

$$\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}),$$

where $\lambda$ is a scalar parameter.

If the point $B$ also lies on $l$ so that the distance $AB$ is 15 units find the possible coordinates of the point $B$.

$$\boxed{B(16,-4,10) \text{ or } B(-4,6,-10)}$$



**Question 28**

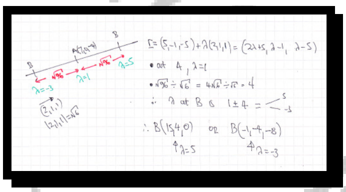
The point $A(7,0,-4)$ lies on the straight line $l$ with equation

$$\mathbf{r} = 5\mathbf{i} - \mathbf{j} - 5\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k}),$$

where $\lambda$ is a scalar parameter.

If the point $B$ also lies on $l$ so that $|AB| = \sqrt{96}$, find the possible coordinates of $B$.

$$\boxed{B(15,4,0) \text{ or } B(-1,-4,-8)}$$



## Question 29

For each of the pairs of the straight lines shown below,

**i.** … prove that they intersect.

**ii.** … find the coordinates of their point of intersection.

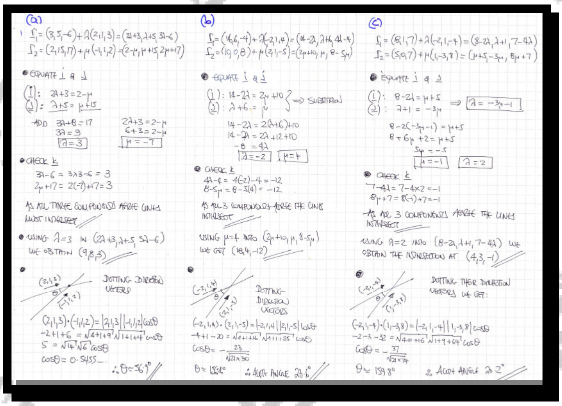
**iii.** … calculate the <u>acute</u> angle between them.

**a)**
$$\mathbf{r}_1 = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k} + \lambda\left(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}\right)$$
$$\mathbf{r}_2 = 2\mathbf{i} + 15\mathbf{j} + 17\mathbf{k} + \mu\left(-\mathbf{i} + \mathbf{j} + 2\mathbf{k}\right)$$

**b)**
$$\mathbf{r}_1 = 14\mathbf{i} + 6\mathbf{j} - 4\mathbf{k} + \lambda\left(-2\mathbf{i} + \mathbf{j} + 4\mathbf{k}\right)$$
$$\mathbf{r}_2 = 10\mathbf{i} + 8\mathbf{k} + \mu\left(2\mathbf{i} + \mathbf{j} - 5\mathbf{k}\right)$$

**c)**
$$\mathbf{r}_1 = 8\mathbf{i} + \mathbf{j} + 7\mathbf{k} + \lambda\left(-2\mathbf{i} + \mathbf{j} - 4\mathbf{k}\right)$$
$$\mathbf{r}_2 = 5\mathbf{i} + 7\mathbf{k} + \mu\left(\mathbf{i} - 3\mathbf{j} + 8\mathbf{k}\right)$$

$\boxed{(9,8,3)\ \&\ 56.9°}$, $\boxed{(18,4,-12)\ \&\ 23.6°}$, $\boxed{(4,3,-1)\ \&\ 20.2°}$



**Question 30**

For each of the pairs of the straight lines shown below,

**i.** … prove that they intersect.

**ii.** … find the coordinates of their point of intersection.

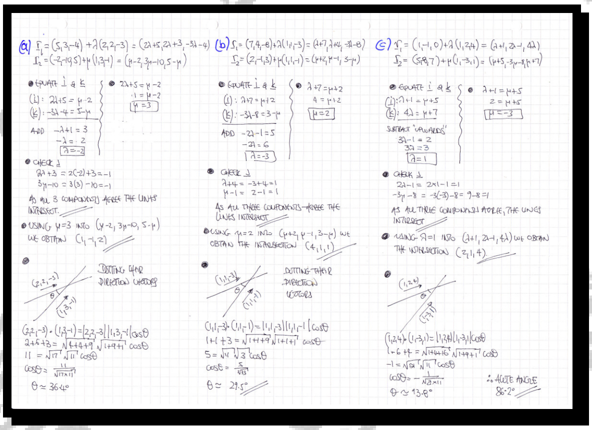
**iii.** … calculate the <u>acute</u> angle between them.

**a)**
$$\mathbf{r}_1 = 5\mathbf{i}+3\mathbf{j}-4\mathbf{k}+\lambda(2\mathbf{i}+2\mathbf{j}-3\mathbf{k})$$
$$\mathbf{r}_2 = -2\mathbf{i}-10\mathbf{j}+5\mathbf{k}+\mu(\mathbf{i}+3\mathbf{j}-\mathbf{k})$$

**b)**
$$\mathbf{r}_1 = 7\mathbf{i}+4\mathbf{j}-8\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}-3\mathbf{k})$$
$$\mathbf{r}_2 = 2\mathbf{i}-\mathbf{j}+3\mathbf{k}+\mu(\mathbf{i}+\mathbf{j}-\mathbf{k})$$

**c)**
$$\mathbf{r}_1 = \mathbf{i}-\mathbf{j}+\lambda(\mathbf{i}+2\mathbf{j}+4\mathbf{k})$$
$$\mathbf{r}_2 = 5\mathbf{i}-8\mathbf{j}+7\mathbf{k}+\mu(\mathbf{i}-3\mathbf{j}+\mathbf{k})$$

$\boxed{(1,-1,2) \ \& \ 36.4°}$, $\boxed{(4,1,1) \ \& \ 29.5°}$, $\boxed{(2,1,4) \ \& \ 86.2°}$



**Question 31**

For each of the pairs of the straight lines shown below,

**i.** … prove that they intersect.

**ii.** … find the coordinates of their point of intersection.

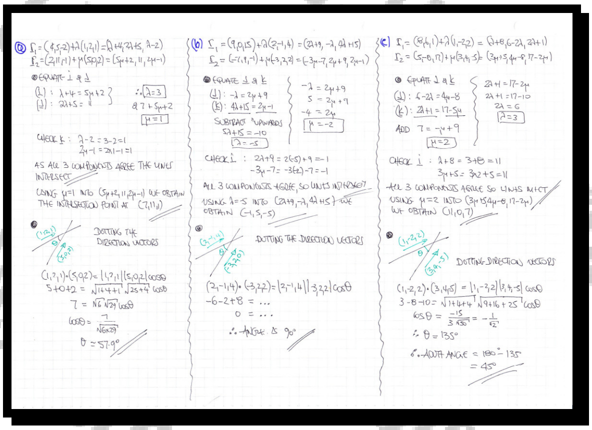
**iii.** … calculate the <u>acute</u> angle between them.

**a)**
$$\mathbf{r}_1 = 4\mathbf{i}+5\mathbf{j}-2\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+\mathbf{k})$$
$$\mathbf{r}_2 = 2\mathbf{i}+11\mathbf{j}-\mathbf{k}+\mu(5\mathbf{i}+2\mathbf{k})$$

**b)**
$$\mathbf{r}_1 = 9\mathbf{i}+15\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+4\mathbf{k})$$
$$\mathbf{r}_2 = -7\mathbf{i}+9\mathbf{j}-\mathbf{k}+\mu(-3\mathbf{i}+2\mathbf{j}+2\mathbf{k})$$

**c)**
$$\mathbf{r}_1 = 8\mathbf{i}+6\mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+2\mathbf{k})$$
$$\mathbf{r}_2 = 5\mathbf{i}-8\mathbf{j}+17\mathbf{k}+\mu(3\mathbf{i}+4\mathbf{j}-5\mathbf{k})$$

$\boxed{(7,11,1)\ \&\ 57.9°}$, $\boxed{(-1,5,-5)\ \&\ 90°}$, $\boxed{(11,0,7)\ \&\ 45°}$



**Question 32**

For each of the pairs of the straight lines shown below,

**i.** … prove that they intersect.

**ii.** … find the coordinates of their point of intersection.

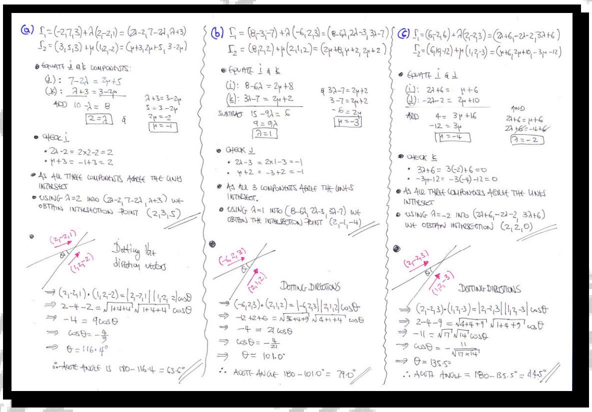
**iii.** … calculate the <u>acute</u> angle between them.

**a)**
$$\mathbf{r}_1 = -2\mathbf{i} + 7\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$
$$\mathbf{r}_2 = 3\mathbf{i} + 5\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$$

**b)**
$$\mathbf{r}_1 = 8\mathbf{i} - 3\mathbf{j} - 7\mathbf{k} + \lambda(-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$$
$$\mathbf{r}_2 = 8\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

**c)**
$$\mathbf{r}_1 = 6\mathbf{i} - 2\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$$
$$\mathbf{r}_2 = 6\mathbf{i} + 10\mathbf{j} - 12\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$\boxed{(2,3,5)\ \&\ 63.6°}$, $\boxed{(2,-1,-4)\ \&\ 79.0°}$, $\boxed{(2,2,0)\ \&\ 44.5°}$

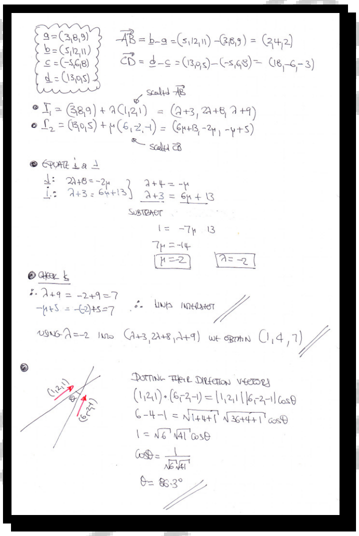


**Question 33**

Prove that the straight line through $A(3,8,9)$ and $B(5,12,11)$ intersects with the straight line through $C(-5,6,8)$ and $D(13,0,5)$.

Find the point of intersection and the <u>acute</u> angle between the two straight lines.

$\boxed{(1,4,7)}$, $\boxed{86.3°}$



$\underline{a}=(3,8,9)$
$\underline{b}=(5,12,11)$
$\underline{c}=(-5,6,8)$
$\underline{d}=(13,0,5)$

$\overrightarrow{AB}=\underline{b}-\underline{a}=(5,12,11)-(3,8,9)=(2,4,2)$

$\overrightarrow{CD}=\underline{d}-\underline{c}=(13,0,5)-(-5,6,8)=(18,-6,-3)$

scaled $\overrightarrow{AB}$

- $\underline{r}_1=(3,8,9)+\lambda(1,2,1)=(\lambda+3,\,2\lambda+8,\,\lambda+9)$
- $\underline{r}_2=(13,0,5)+\mu(6,-2,-1)=(6\mu+13,\,-2\mu,\,-\mu+5)$

scaled $\overrightarrow{CD}$

- EQUATE $\underline{i}$ & $\underline{j}$

$\underline{j}$: $2\lambda+8=-2\mu$
$\underline{i}$: $\lambda+3=6\mu+13$

$\lambda+4=-\mu$
$\lambda+3=6\mu+13$

SUBTRACT

$1=-7\mu-13$

$7\mu=-14$

$\boxed{\mu=-2}$ $\boxed{\lambda=-2}$

- CHECK $\underline{k}$

$\therefore\ \lambda+9=-2+9=7$
$-\mu+5=-(-2)+5=7$ $\therefore$ LINES INTERSECT

USING $\lambda=-2$ INTO $(\lambda+3,2\lambda+8,\lambda+9)$ WE OBTAIN $(1,4,7)$

- 

$(1,2,1)$ $\theta$ $(6,-2,-1)$

DOTTING THEIR DIRECTION VECTORS

$(1,2,1)\cdot(6,-2,-1)=|1,2,1||6,-2,-1|\cos\theta$

$6-4-1=\sqrt{1+4+1}\sqrt{36+4+1}\cos\theta$

$1=\sqrt{6}\sqrt{41}\cos\theta$

$\cos\theta=\frac{1}{\sqrt{6}\sqrt{41}}$

$\theta=86.3°$

# SHORTEST DISTANCES INVOLVING LINES

**Question 1**

The straight line $l$ has vector equation

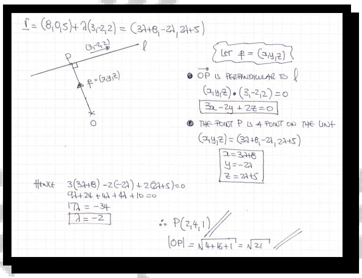
$$\mathbf{r} = 8\mathbf{i} + 5\mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}),$$

where $\lambda$ is a scalar parameter.

The point $P$ lies on $l$ so that $OP$ is perpendicular to $l$, where $O$ is the origin.

Find the coordinates of $P$ and the distance $OP$.

$\boxed{P(2,4,1)}$, $\boxed{|OP| = \sqrt{21}}$



**Question 2**

The straight line $l$ has vector equation

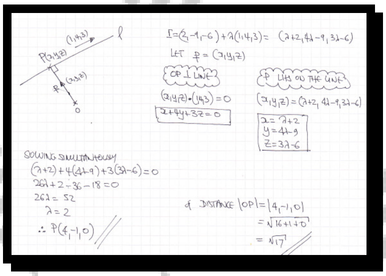
$$\mathbf{r} = 2\mathbf{i} - 9\mathbf{j} - 6\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$$

where $\lambda$ is a scalar parameter.

The point $P$ lies on $l$ so that $OP$ is perpendicular to $l$, where $O$ is the origin.

Find the coordinates of $P$ and the distance $OP$.

$\boxed{P(4,-1,0)}$, $\boxed{|OP| = \sqrt{17}}$



**Question 3**

The straight line $l$ has vector equation

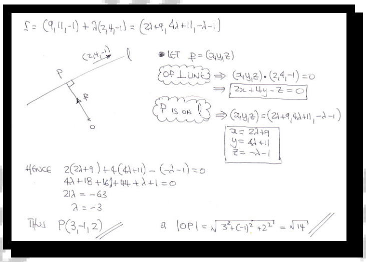
$$\mathbf{r} = 9\mathbf{i} + 11\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + 4\mathbf{j} - \mathbf{k})$$

where $\lambda$ is a scalar parameter.

The point $P$ lies on $l$ so that $OP$ is perpendicular to $l$, where $O$ is the origin.

Find the coordinates of $P$ and the distance $OP$.

$\boxed{P(3,-1,2)}$, $\boxed{|OP| = \sqrt{14}}$


$\mathbf{r} = (9,11,-1) + \lambda(2,4,-1) = (2\lambda+9, 4\lambda+11, -\lambda-1)$

• LET $\mathbf{p} = (x,y,z)$

OP ⊥ LINE $\Rightarrow (x,y,z)\cdot(2,4,-1) = 0$

$\Rightarrow \boxed{2x + 4y - z = 0}$

P IS ON $l$ $\Rightarrow (x,y,z) = (2\lambda+9, 4\lambda+11, -\lambda-1)$

$\boxed{\begin{array}{l} x = 2\lambda+9 \\ y = 4\lambda+11 \\ z = -\lambda-1 \end{array}}$

HENCE $2(2\lambda+9) + 4(4\lambda+11) - (-\lambda-1) = 0$

$4\lambda + 18 + 16\lambda + 44 + \lambda + 1 = 0$

$21\lambda = -63$

$\lambda = -3$

THUS $P(3,-1,2)$ & $|OP| = \sqrt{3^2 + (-1)^2 + 2^2} = \sqrt{14}$

**Question 4**

The straight line $l$ has vector equation

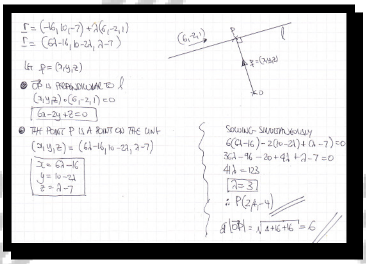
$$\mathbf{r} = -16\mathbf{i} + 10\mathbf{j} - 7\mathbf{k} + \lambda(6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

where $\lambda$ is a scalar parameter.

The point $P$ lies on $l$ so that $OP$ is perpendicular to $l$, where $O$ is the origin.

Find the coordinates of $P$ and the distance $OP$.

$\boxed{P(2,4,-4)}$, $\boxed{|OP| = 6}$



**Question 5**

The straight line $l$ has vector equation

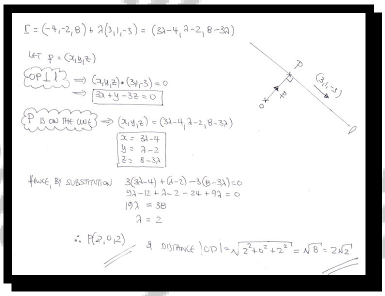
$$\mathbf{r} = -4\mathbf{i} - 2\mathbf{j} + 8\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - 3\mathbf{k})$$

where $\lambda$ is a scalar parameter.

The point $P$ lies on $l$ so that $OP$ is perpendicular to $l$, where $O$ is the origin.

Find the coordinates of $P$ and the distance $OP$.

$\boxed{P(2,0,2)}$, $\boxed{|OP| = 2\sqrt{2}}$



$\underline{r} = (-4,-2,8) + \lambda(3,1,-3) = (3\lambda-4, \lambda-2, 8-3\lambda)$

LET $P = (x,y,z)$

OP ⊥ l $\Rightarrow (x,y,z)\cdot(3,1,-3) = 0$
$\Rightarrow \boxed{3x + y - 3z = 0}$

P IS ON THE LINE $\Rightarrow (x,y,z) = (3\lambda-4, \lambda-2, 8-3\lambda)$

$x = 3\lambda - 4$
$y = \lambda - 2$
$z = 8 - 3\lambda$

HENCE, BY SUBSTITUTION $3(3\lambda-4) + (\lambda-2) - 3(8-3\lambda) = 0$
$9\lambda - 12 + \lambda - 2 - 24 + 9\lambda = 0$
$19\lambda = 38$
$\lambda = 2$

$\therefore P(2,0,2)$ & DISTANCE $|OP| = \sqrt{2^2+0^2+2^2} = \sqrt{8} = 2\sqrt{2}$

**Question 6**

The straight line $l$ has vector equation

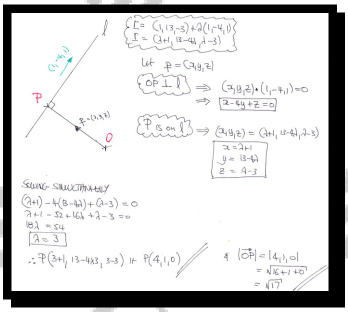
$$\mathbf{r} = \mathbf{i} + 13\mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} - 4\mathbf{j} + \mathbf{k})$$

where $\lambda$ is a scalar parameter.

The point $P$ lies on $l$ so that $OP$ is perpendicular to $l$, where $O$ is the origin.

Find the coordinates of $P$ and the distance $OP$.

$\boxed{P(4,1,0)}$, $\boxed{|OP| = \sqrt{17}}$



**Question 7**

The straight line $l$ has vector equation

$$\mathbf{r} = 4\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$$

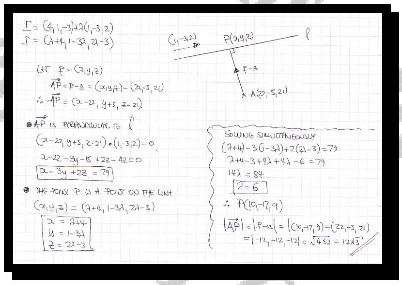
where $\lambda$ is a scalar parameter.

The point $A$ has coordinates $(22, -5, 21)$.

The point $P$ lies on $l$ so that $AP$ is perpendicular to $l$.

Find the coordinates of $P$ and the distance $AP$.

$\boxed{P(10, -17, 9)}$, $\boxed{|AP| = 12\sqrt{3}}$



$\underline{r} = (4,1,-3) + \lambda(1,-3,2)$
$\underline{r} = (\lambda+4, 1-3\lambda, 2\lambda-3)$

LET $\underline{p} = (x,y,z)$
$\overrightarrow{AP} = \underline{p} - \underline{a} = (x,y,z) - (22,-5,21)$
$\therefore \overrightarrow{AP} = (x-22, y+5, z-21)$

- $\overrightarrow{AP}$ IS PERPENDICULAR TO $l$

$(x-22, y+5, z-21) \cdot (1,-3,2) = 0$
$x - 22 - 3y - 15 + 2z - 42 = 0$
$\boxed{x - 3y + 2z = 79}$

- THE POINT $P$ IS A POINT ON THE LINE

$(x,y,z) = (\lambda+4, 1-3\lambda, 2\lambda-3)$
$\boxed{\begin{array}{l} x = \lambda+4 \\ y = 1-3\lambda \\ z = 2\lambda-3 \end{array}}$

SOLVING SIMULTANEOUSLY
$(\lambda+4) - 3(1-3\lambda) + 2(2\lambda-3) = 79$
$\lambda + 4 - 3 + 9\lambda + 4\lambda - 6 = 79$
$14\lambda = 84$
$\boxed{\lambda = 6}$
$\therefore P(10,-17,9)$

$|\overrightarrow{AP}| = |\underline{p} - \underline{a}| = |(10,-17,9) - (22,-5,21)|$
$= |-12,-12,-12| = \sqrt{432} = 12\sqrt{3}$

**Question 8**

The straight line $l$ has vector equation

$$\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{k}),$$

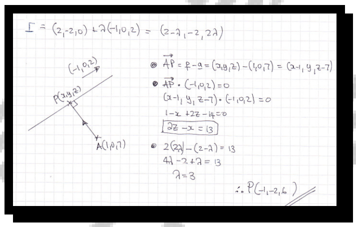
where $\lambda$ is a scalar parameter.

The point $A$ has coordinates $(1,0,7)$.

The point $P$ lies on $l$ so that $AP$ is perpendicular to $l$.

Find the coordinates of $P$.

$\boxed{P(-1,-2,6)}$



## Question 9

The straight line $l$ has vector equation

$$\mathbf{r} = 17\mathbf{i} + 6\mathbf{j} + 47\mathbf{k} + \lambda(\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}),$$

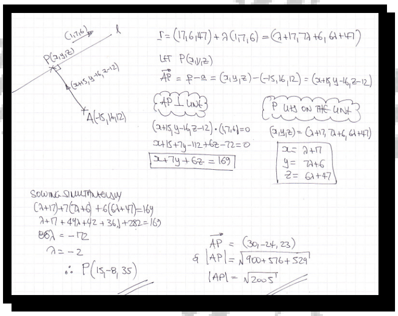
where $\lambda$ is a scalar parameter.

The point $A$ has coordinates $(-15,16,12)$.

The point $P$ lies on $l$ so that $AP$ is perpendicular to $l$.

Find the coordinates of $P$ and the distance $AP$.

$\boxed{P(15,-8,35)}$, $\boxed{|AP| = \sqrt{2005}}$



**Question 10**

The parallel lines $l_1$ and $l_2$ have respective vector equations

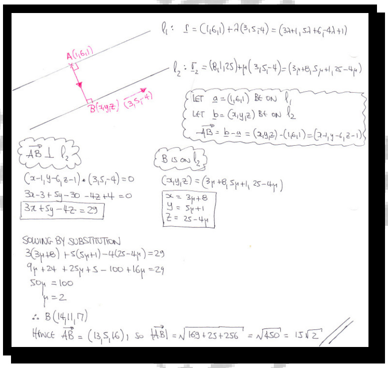
$$\mathbf{r}_1 = \mathbf{i} + 6\mathbf{j} + \mathbf{k} + \lambda(3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k})$$

$$\mathbf{r}_2 = 8\mathbf{i} + \mathbf{j} + 25\mathbf{k} + \mu(3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k})$$

where $\lambda$ and $\mu$ are scalar parameters.

Find the distance between $l_1$ and $l_2$.

$\boxed{15\sqrt{2}}$



**Question 11**

The parallel lines $l_1$ and $l_2$ have respective vector equations

$$\mathbf{r}_1 = 8\mathbf{i} + 3\mathbf{j} + \lambda(2\mathbf{i} - \mathbf{k})$$

$$\mathbf{r}_2 = -\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} - \mathbf{k})$$

where $\lambda$ and $\mu$ are scalar parameters.

Find the distance between $l_1$ and $l_2$.

$\boxed{3}$