SYSTEMATIC CURVE SKETCHING

Question 1 (**)

The curve C has equation

$$y = \frac{a}{x}, \quad x \neq 0,$$

where a is a positive constant.

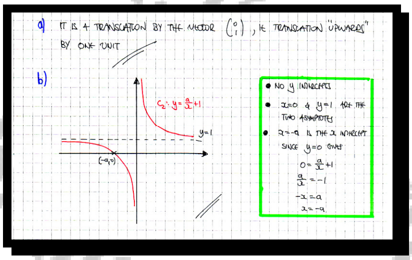
a) Describe geometrically the transformation that maps the graph of $y = \frac{a}{x}$ onto the graph of $y = \frac{a}{x} + 1$.

b) Sketch the graph of C.

The sketch must include the coordinates of …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of any asymptotes of the curve.

[ ] , translation "upwards" by 1 unit



Question 2 (**)

A curve C has equation

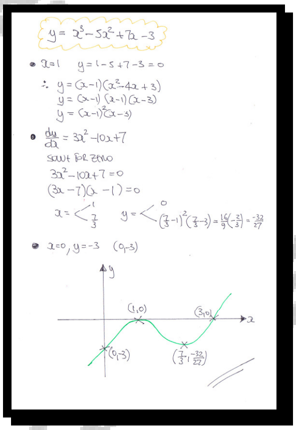
$$y = x^3 - 5x^2 + 7x - 3, \; x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.

graph



Question 3 (**)

A curve C has equation

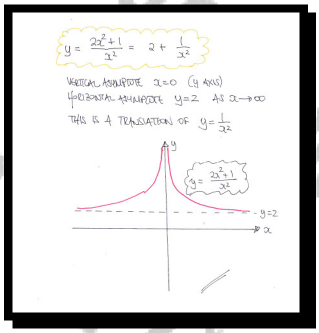
$$y = \frac{2x^2 + 1}{x^2}, \ x \in \mathbb{R}, \ x \neq 0.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 4 (***)

A curve C has equation

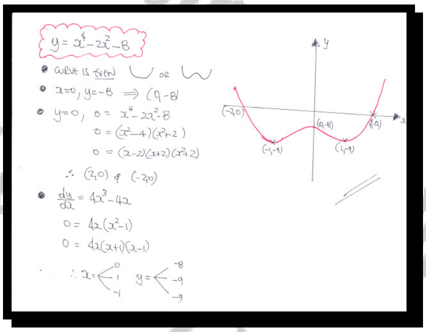
$$y = x^4 - 2x^2 - 8, \ x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.

graph



Question 5 (***)

A curve C has equation

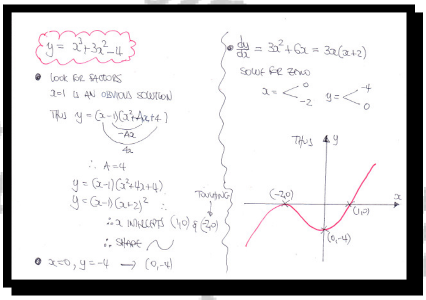
$$y = x^3 + 3x^2 - 4, \; x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.

graph



Question 6 (***)

A curve C has equation

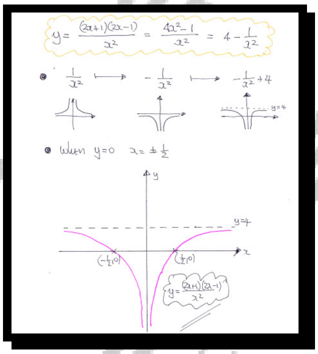
$$y = \frac{(2x+1)(2x-1)}{x^2}, \ x \in \mathbb{R}, \ x \neq 0.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 7 **(***)**

A curve C has equation

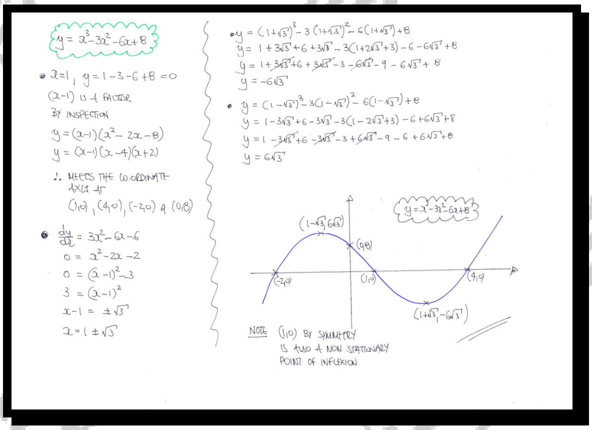
$$y = x^3 - 3x^2 - 6x + 8, \ x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.

graph



Question 8 (***+)

$$f(x) = \frac{4x-13}{x-3}, \; x \in \mathbb{R}, \; x \neq 3.$$

a) Show that the equation of $f(x)$ can be written as

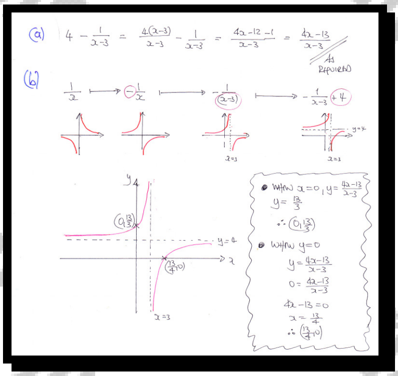
$$f(x) = 4 - \frac{1}{x-3}, \; x \in \mathbb{R}, \; x \neq 3.$$

b) Sketch the graph of $f(x)$.

The sketch must include …

- … the coordinates of the points where $f(x)$ meets the coordinate axes.
- … the equations of any asymptotes of the curve.

graph



Question 9 (***+)

The curve C has equation

$$y = \frac{2x+3}{x-2}, \; x \in \mathbb{R}, \; x \neq 2.$$

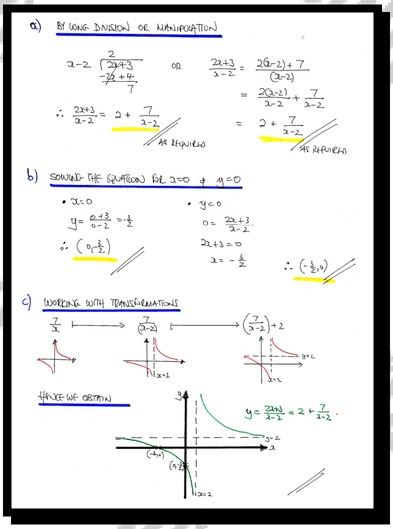
a) Show clearly that

$$\frac{2x+3}{x-2} \equiv 2 + \frac{7}{x-2}.$$

b) Find the coordinates of the points where C meets the coordinate axes.

c) Sketch the graph of C showing clearly the equations of any asymptotes.

$\boxed{}$, $\boxed{\left(0,-\frac{3}{2}\right),\left(-\frac{3}{2},0\right)}$



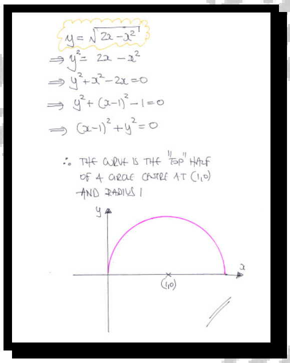
Question 10 (***+)

A curve C has equation

$$y = \sqrt{2x - x^2}, \quad x \in \mathbb{R}, \quad 0 \le x \le 2.$$

Sketch the graph of C.

graph



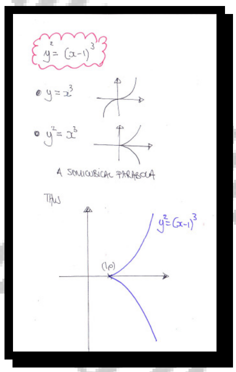
Question 11 (***+)

A curve C has equation

$$y^2 = (x-1)^3, \quad x \in \mathbb{R}, \quad x \ge 1.$$

Sketch the graph of C.

graph



Question 12 (***+)

$$f(x) = a - \frac{1}{b-x}, \ x \in \mathbb{R}, \ x \neq b,$$

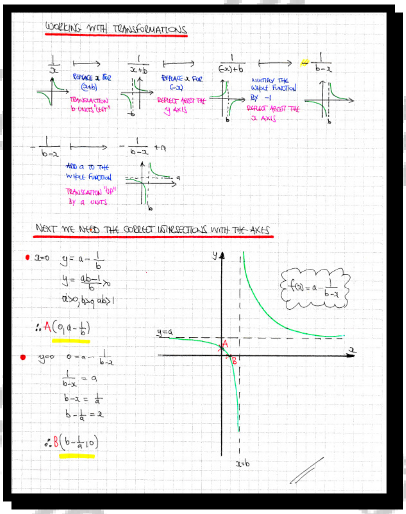
where a and b are positive constants such that $ab > 1$.

Sketch the graph of $f(x)$.

The sketch must include, in terms of a and b, …

- … the coordinates of the points where $f(x)$ meets the coordinate axes.
- … the equations of any asymptotes of the curve.

SY, graph



Question 13 **(***+)**

A curve C has equation

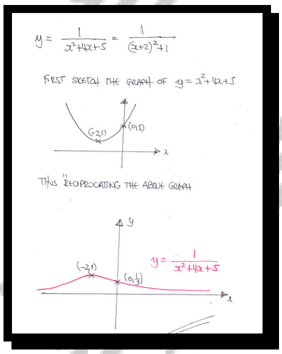
$$y = \frac{1}{x^2 + 4x + 5}, \quad x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 14 (***+)

$$f(x) = \frac{x-2}{x-3}, \ x \in \mathbb{R}, \ x \neq 3.$$

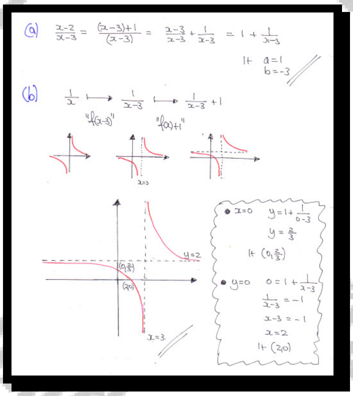
a) Express $f(x)$ in the form $f(x) = a + \frac{1}{x+b}$, where a and b are integers.

b) By considering a series of transformations which map the graph of $\frac{1}{x}$ onto the graph of $f(x)$, sketch the graph of $f(x)$.

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

$\boxed{a = 1}$, $\boxed{b = -3}$



Question 15 (***+)

A curve C has equation

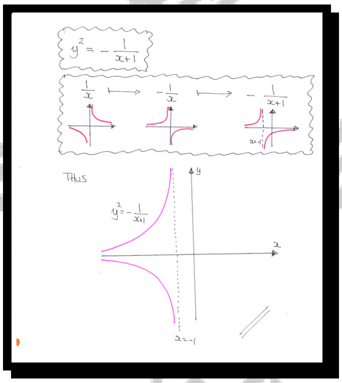
$$y^2 = -\frac{1}{x+1}, \ x \in \mathbb{R}, \ x \neq -1.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 16 (***+)

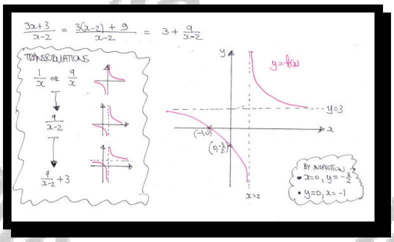
$$f(x)=\frac{3x+3}{x-2},\ x\in\mathbb{R},\ x\neq 2.$$

Sketch the graph of $f(x)$.

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

graph



Question 17 (***+)

A curve has equation $y = f(x)$ given by

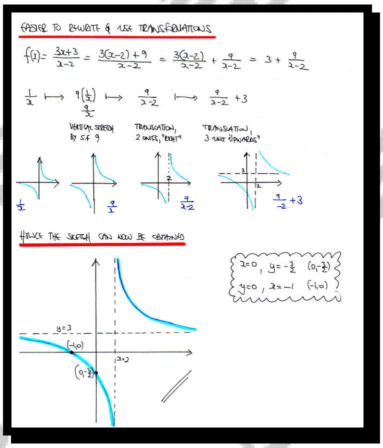
$$f(x) = \frac{3x-1}{x+2}, \quad x \in \mathbb{R}, \quad x \neq 2.$$

Sketch the graph of $f(x)$.

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve.

[ ] , [graph]



Question 18 (***+)

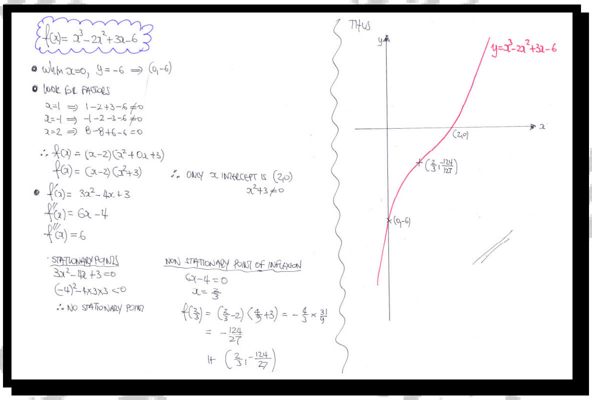
A curve C has equation

$$y = x^3 - 2x^2 + 3x - 6, \; x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,
- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.

graph



Question 19 (*+)**

A curve C has equation

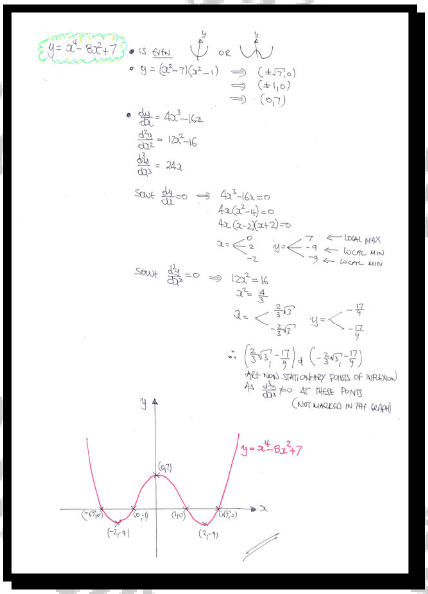
$$y = x^4 - 8x^2 + 7, \ x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.

graph



Question 20 (***+)

A curve C has equation

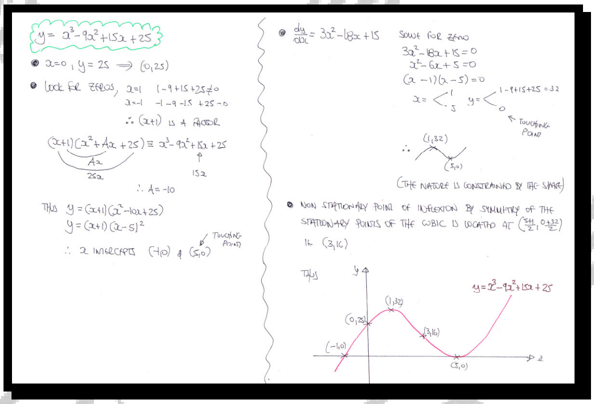
$$y = x^3 - 9x^2 + 15x + 25,\ x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.

graph



Question 21 (***+)

A curve C has equation

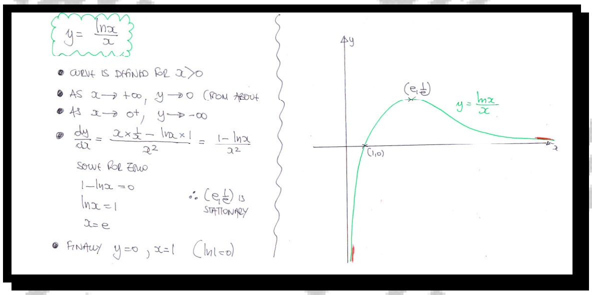
$$y = \frac{\ln x}{x}.$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 22 (***+)

A curve C has equation

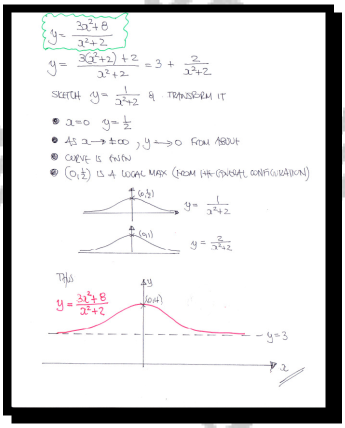
$$y = \frac{3x^2+8}{x^2+2}, \quad x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 23 (***+)

A curve C has equation

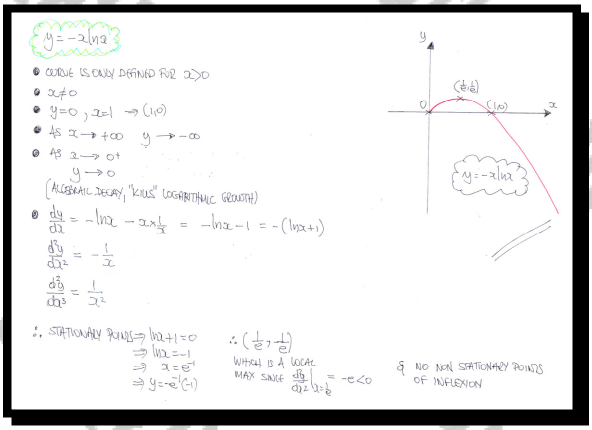
$$y = -x \ln x, \; x \in \mathbb{R}.$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

graph



Question 24 (***+)

A curve C has equation

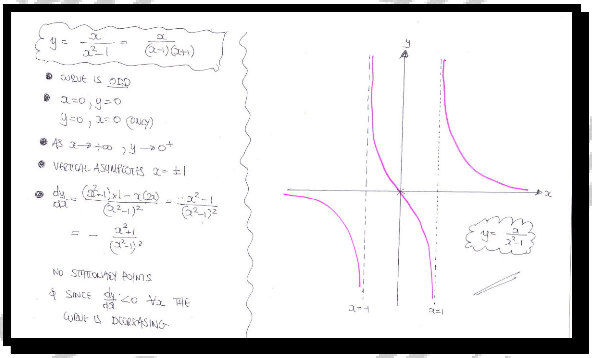
$$y = \frac{x}{x^2 - 1}, \ x \in \mathbb{R}.$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 25 (***+)

Sketch the parabola with equation

$$y^2 - 4y - 2x = 2 .$$

The sketch must include the …

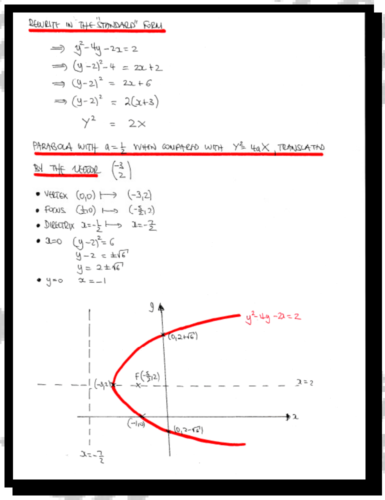
a) … coordinates of points of intersection with the coordinate axes.

b) … coordinates of the vertex of the parabola.

c) … coordinates of the focus of the parabola.

d) … equation of the directrix of the parabola.

graph



Question 26 (****)

A curve C has equation

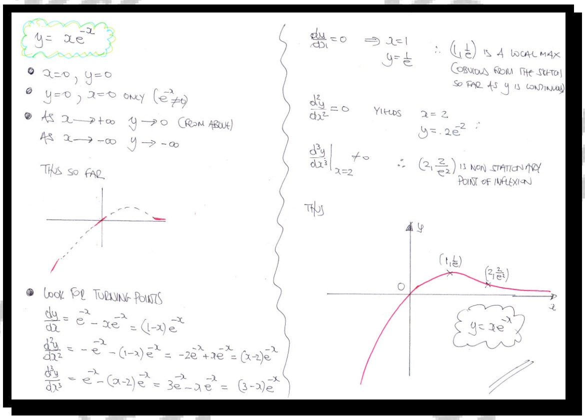
$$y = x\mathrm{e}^{-x}, \ x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

graph



Question 27 (****)

A curve C has equation

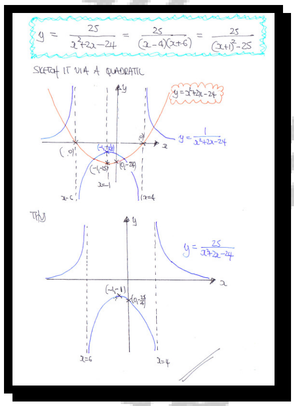
$$y = \frac{25}{x^2 + 2x - 24}, \quad x \in \mathbb{R}.$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 28 (****)

A curve C has equation

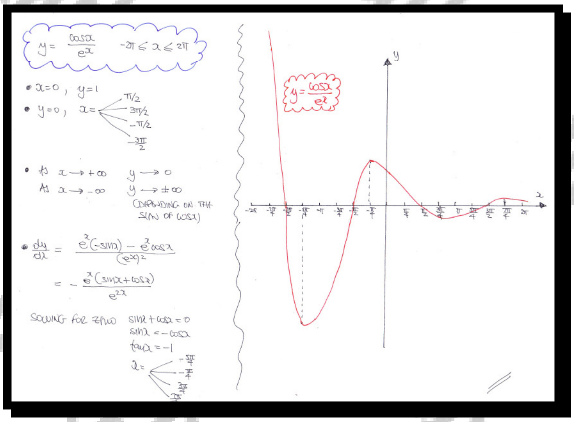
$$y = \frac{\cos x}{e^{x}}, \ -2\theta \le x \le 2\pi .$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the x coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 29 (****)

A curve C has equation

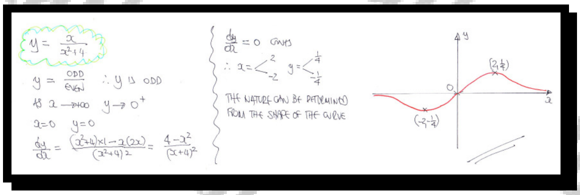
$$y = \frac{x}{x^2+4}, \; x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 30 (****)

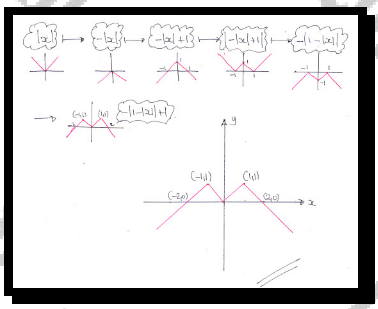
Sketch the graph of

$$y = 1 - \big|1 - |x|\big|, \; x \in \mathbb{R}.$$

The sketch must include the coordinates …

- … of any points where the graph meets the coordinate axes
- … of any cusps of the graph.

graph



Question 31 (****)

A curve C has equation

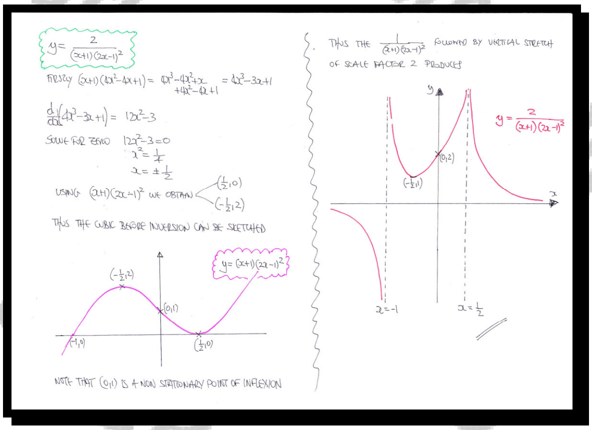
$$y = \frac{2}{(x+1)(2x-1)^2}, \quad x \in \mathbb{R}, \; x \neq -1, \; x \neq \frac{1}{2}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 32 (****)

A curve C has equation

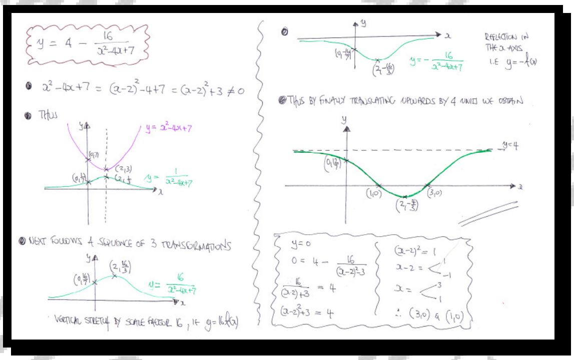
$$y = 4 - \frac{16}{x^2 - 4x + 7}, \quad x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 33 (****)

A curve C has equation

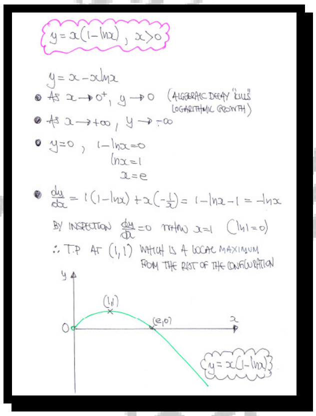
$$y = x(1 - \ln x), \ x \in \mathbb{R}, \ x > 0.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 34 (****)

A curve C has equation

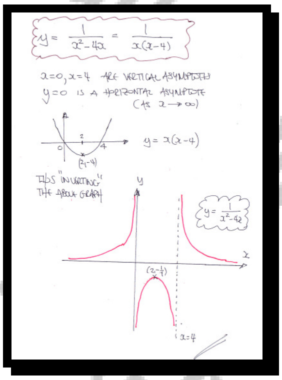
$$y = \frac{1}{x^2 - 4x}, \ x \in \mathbb{R}, \ x \neq 0.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 35 (****)

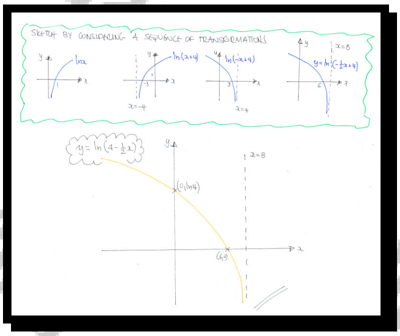
The function f is defined as

$$f : x \mapsto \ln\left(4 - \frac{1}{2}x\right),\ x \in \mathbb{R},\ x < 8.$$

Sketch the graph of f.

Indicate clearly any intersections with the axes and the equation of its asymptote.

graph



Question 36 (****)

A curve C has equation

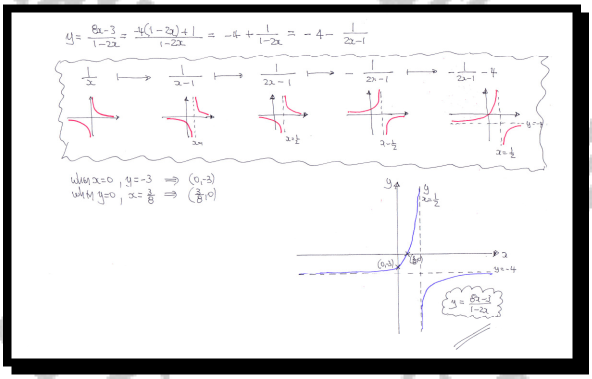
$$y = \frac{8x-3}{1-2x}, \; x \in \mathbb{R}, \; x \neq \frac{1}{2}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 37 (****)

A curve C has equation

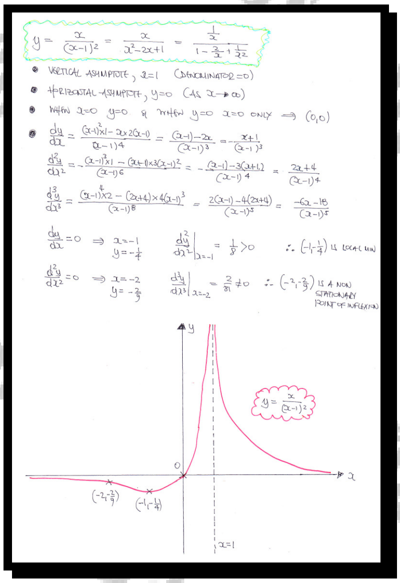
$$y = \frac{x}{(x-1)^2}, \quad x \in \mathbb{R}, \quad x \neq 1.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

graph



Question 38 (****)

A curve C has equation

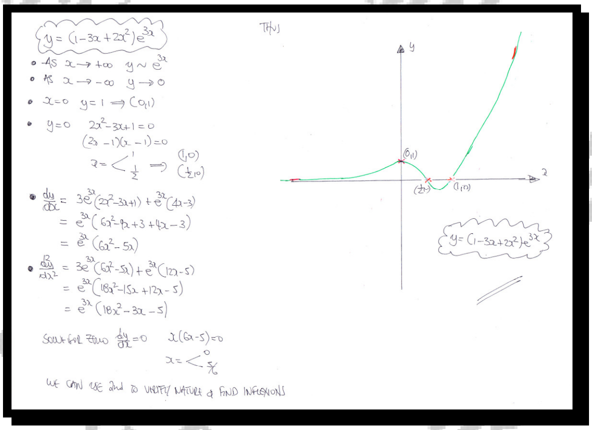
$$y = \left(1 - 3x + 2x^2\right)e^{3x}, \; x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 39 (****)

A curve C has equation

$$y = \frac{1}{x^3 - 9x^2 + 24x}, \quad x \in \mathbb{R}, \; x \neq 0.$$

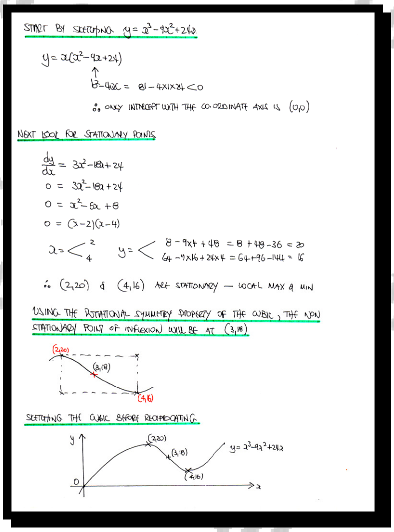
Sketch the graph of C.

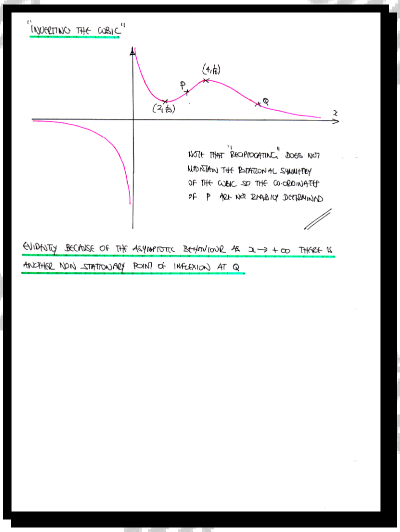
The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

You must further label any non stationary turning points, without explicitly giving their coordinates.

[ ], [graph]





Question 40 (****)

A curve C has equation

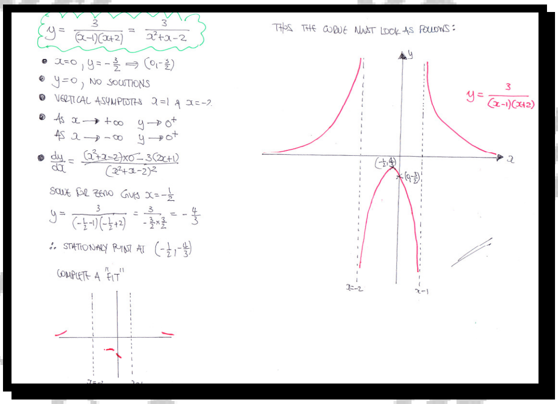
$$y = \frac{3}{(x-1)(x+2)}, \ x \in \mathbb{R}, \ x \neq -2, \ x \neq 1 .$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 41 (****)

A curve C has equation

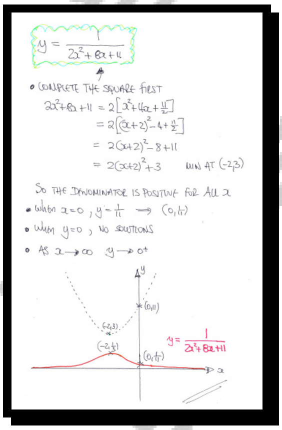
$$y = \frac{1}{2x^2 + 8x + 11}, \quad x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



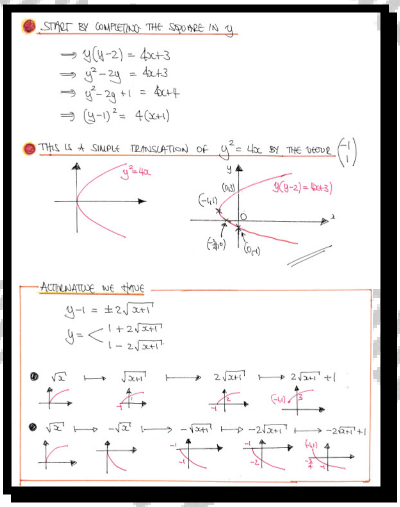
Question 42 (****)

Sketch the graph of the curve with equation

$$y(y-2) = 4x+3.$$

The sketch must include the coordinates of any intersections with the axes and the coordinates of the point where the tangent to the curve is parallel to the y axis.

[ ] , [graph]



Question 43 (****+)

A curve C has equation

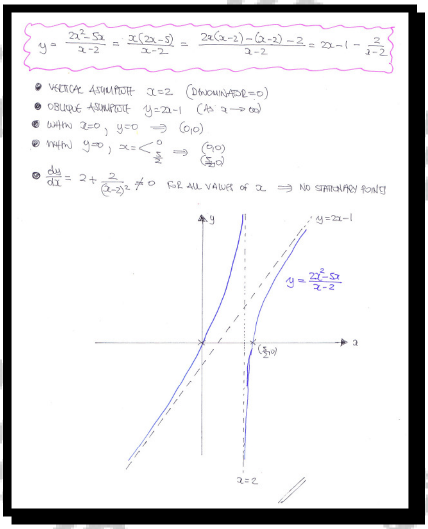
$$y = \frac{2x^2 - 5x}{x - 2}, \; x \in \mathbb{R}, \; x \neq 2.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 44 (****+)

A curve C has equation

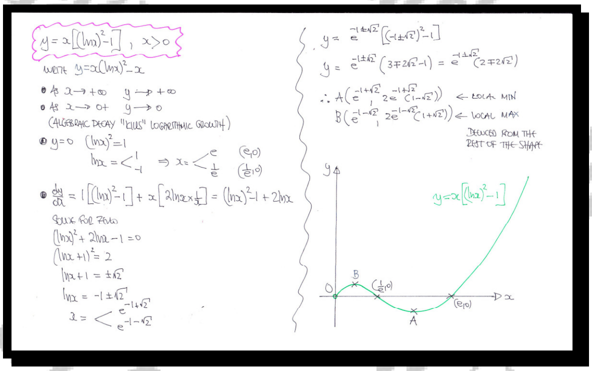
$$y = x\left[(\ln x)^2 - 1\right],\ x \in \mathbb{R},\ x > 0.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 45 (****+)

A curve C has equation

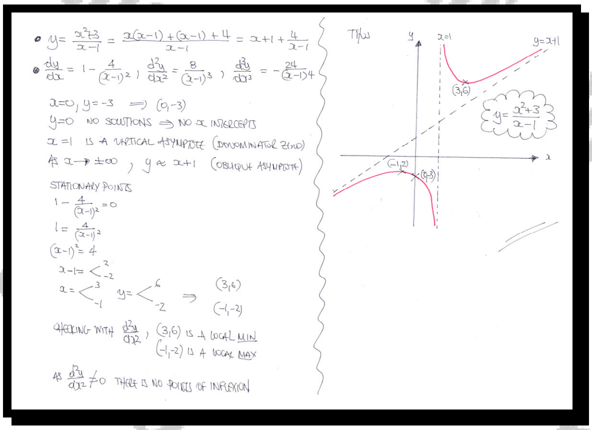
$$y = \frac{x^2+3}{x-1}, \quad x \in \mathbb{R}, \quad x \neq 1.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

graph



Question 46 (****+)

A curve C has equation $y = f(x)$

$$f(x) = x^3 - 14x^2 + 49x - 36,\ x \in \mathbb{R}.$$

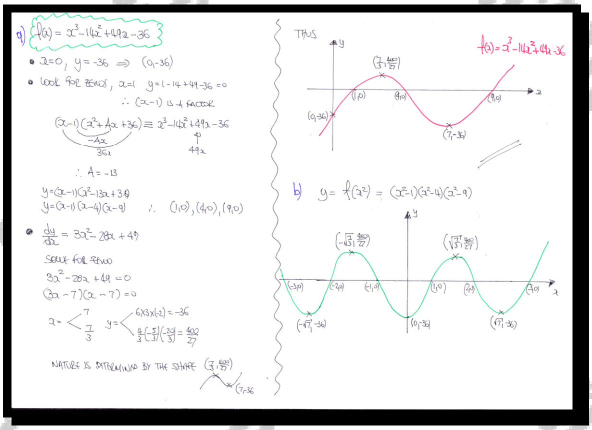
a) Sketch the graph of C.

b) Use the sketch of part (a) to deduce the graph of $y = f\left(x^2\right)$

Each of the sketches must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.

graph



Question 47 (****+)

A curve C has equation

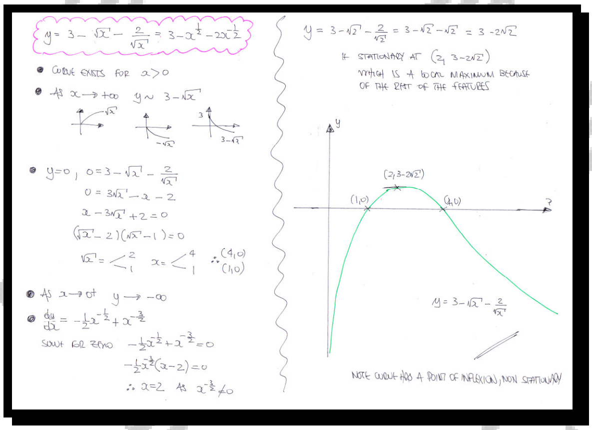
$$y = 3 - \sqrt{x} - \frac{2}{\sqrt{x}}.$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 48 (****+)

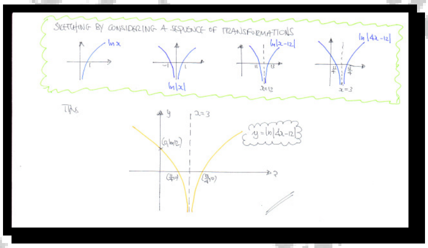
The function f is defined as

$$f : x \mapsto \ln|4x-12|,\ x \in \mathbb{R},\ x \neq 3 .$$

Sketch the graph of f.

Indicate clearly any intersections with the axes and the equation of its asymptote.

graph



Question 49 (****+)

A curve C has equation

$$y = 2x - 1 + 4e^{-2x},\ x \in \mathbb{R} .$$

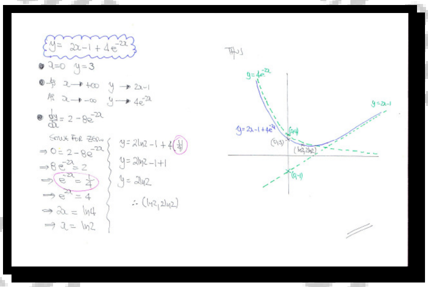
Sketch the graph of C, indicating clearly the behaviour for large positive and large negative values of x.

The graph must also include the exact coordinates, where appropriate, of …

… any points where the graph of C meets the coordinate axes.

… any turning points of C.

graph



Question 50 (****+)

A curve C has equation

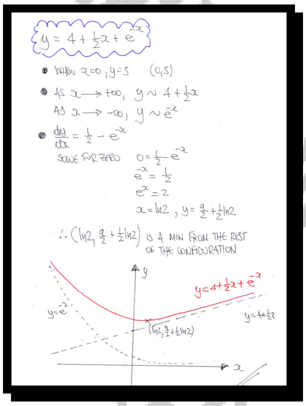
$$y = 4 + \frac{1}{2}x + e^{-x}, \ x \in \mathbb{R}.$$

Sketch the graph of C, indicating clearly the behaviour for large positive and large negative values of x.

The graph must also include the exact coordinates, where appropriate, …

- … of any points where the graph of C meets the coordinate axes.
- … of any turning points of C.

graph



Question 51 (****+)

A curve C has equation

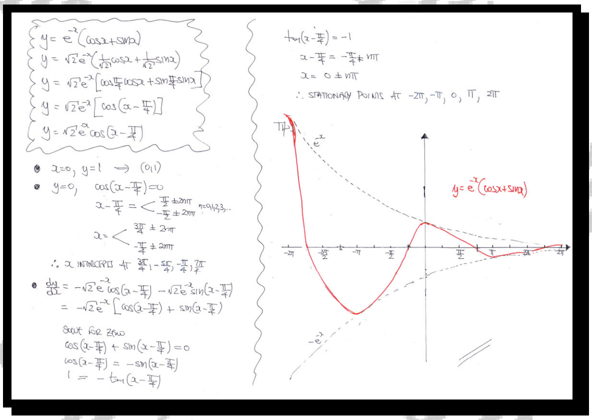
$$y = e^{-x}\left(\cos x + \sin x\right), \ x \in \mathbb{R}.$$

Sketch the graph of C, indicating clearly the behaviour for large positive and large negative values of x.

The graph must also include the exact coordinates, where appropriate, …

- … of any points where the graph of C meets the coordinate axes.
- … of any turning points of C.

graph



Question 52 (****+)

A curve C has equation

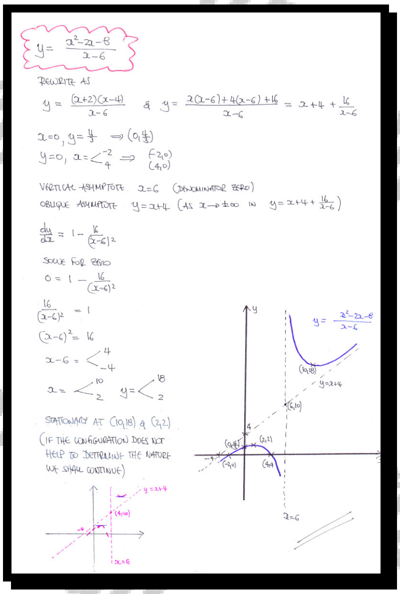
$$y=\frac{x^2-2x-8}{x-6},\ x\in\mathbb{R},\ x\neq 6.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 53 (****+)

A curve C has equation

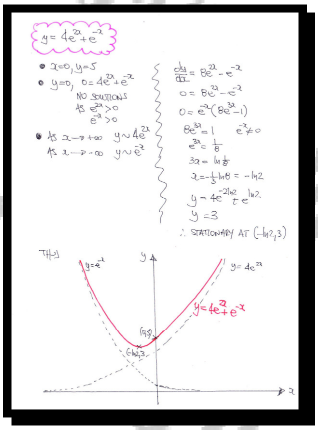
$$y = 4e^{2x} + e^{-x}, \ x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 54 (****+)

A curve C has equation

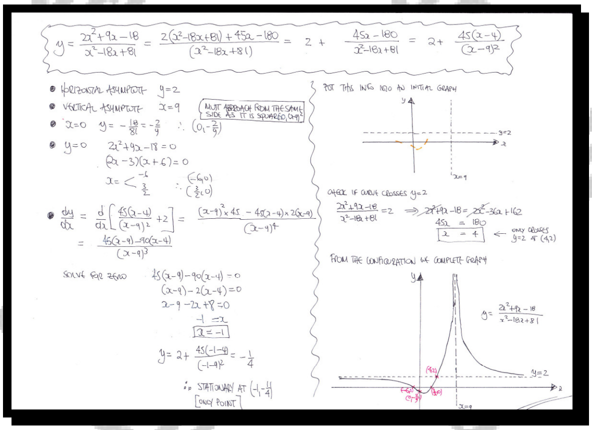
$$y = \frac{2x^2 + 9x - 18}{x^2 - 18x + 81}.$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



Question 55 (****+)

A curve C has equation

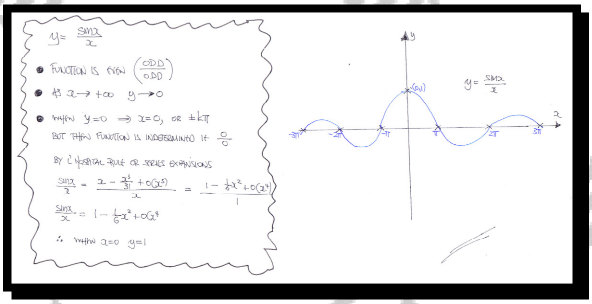
$$y = \frac{\sin x}{x}, \quad x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points, for $-3\pi \le x \le 3\pi$.

graph



Question 56 (****+)

A curve C has equation

$$y = 3\sinh x - 2\cosh x \,, \ \ x \in \mathbb{R} \,.$$

Sketch the graph of C.

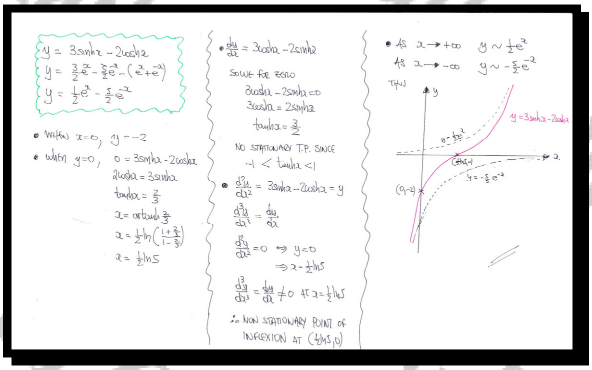
The sketch must include …

… the coordinates of any points where the graph of C meets the coordinates axes.

… the coordinates of any stationary or non stationary turning points.

… the behaviour of the curve for large positive and large negative values of x

graph



Question 57 (****+)

A curve C has equation

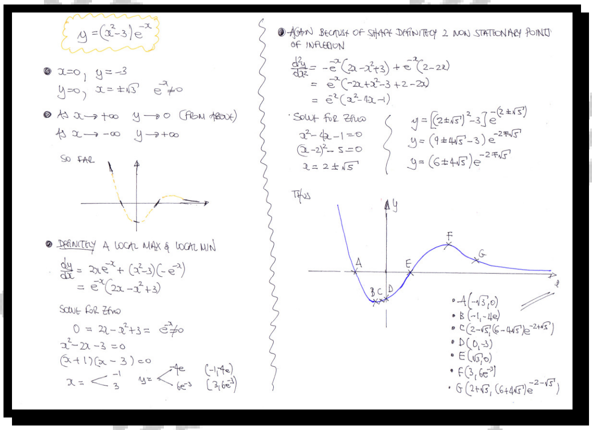
$$y = \left(x^2 - 3\right)e^{-x}, \; x \in \mathbb{R}.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

graph



Question 58 (****+)

A curve C has equation

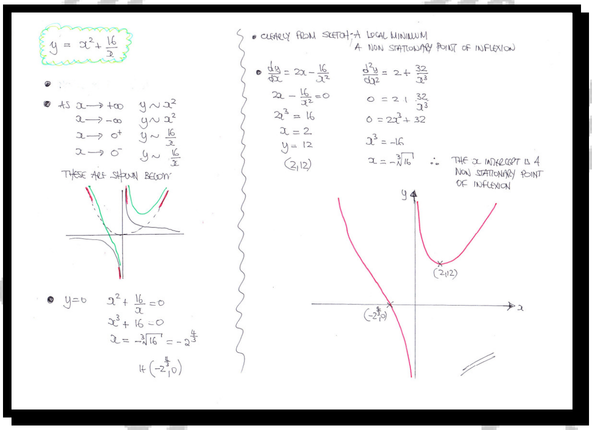
$$y = x^2 + \frac{16}{x}, \; x \in \mathbb{R}, \; x \neq 0.$$

Sketch the graph of C.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

graph



Question 59 (****+)

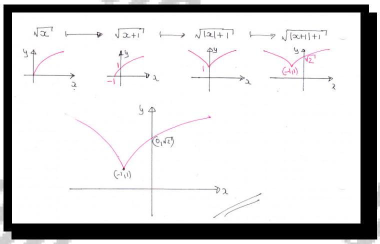
By considering the following sequence of transformations T_1, T_2 and T_3

$$\sqrt{x} \xrightarrow{T_1} \sqrt{x+1} \xrightarrow{T_2} \sqrt{|x|+1} \xrightarrow{T_3} \sqrt{|x+1|+1}$$

sketch the graph of $y = \sqrt{|x+1|+1}$.

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

graph



Question 60 (****+)

A curve C has equation

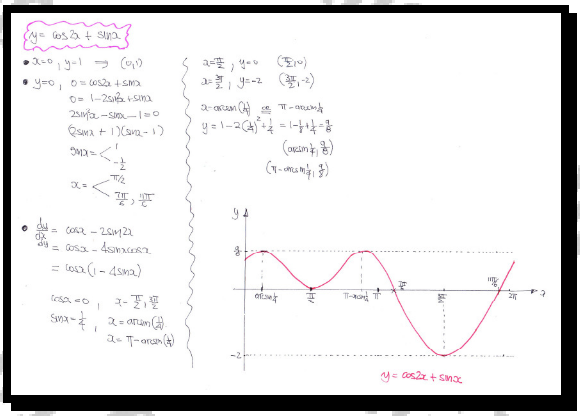
$$y = \cos 2x + \sin x, \ 0 \le x \le 2\pi.$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.

graph



Question 61 (****+)

A curve C has equation

$$y = x^4 - 32x + 15, \ x \in \mathbb{R}.$$

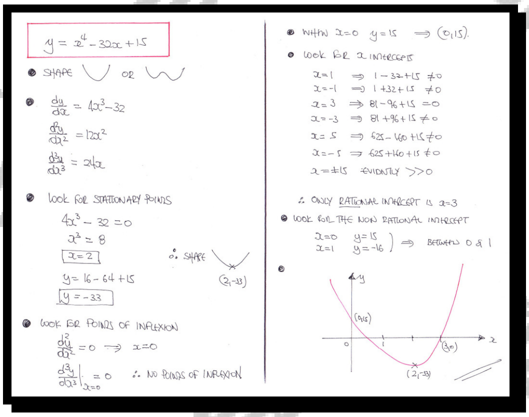
Sketch the graph of C.

The sketch must include

- the coordinates of any points where the graph of C meets the coordinate axes.

  If a coordinate is not rational indicate suitably on the axis an interval of consecutive integers in which the graph meets that particular coordinate axis.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

graph



Question 62 (****+)

A curve C has equation

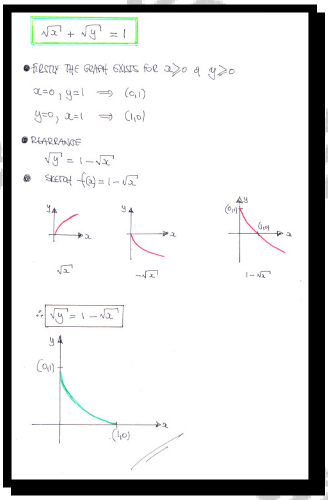
$$\sqrt{y}+\sqrt{x}=1.$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

[ ], [graph]



Question 63 (*****)

A curve C has equation

$$y^3 - y^2 = x, \ x \in \mathbb{R}, \ y \in \mathbb{R}.$$

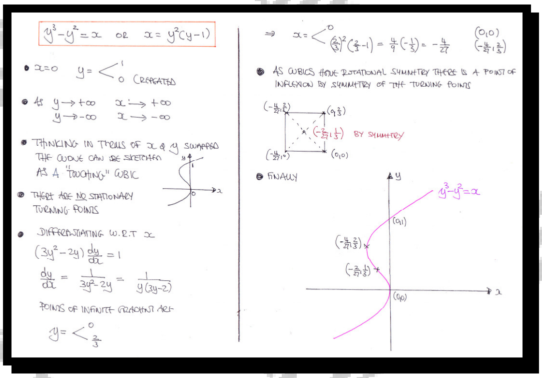
Sketch the graph of C.

The graph must include the coordinates …

… of any points where the graph of C meets the coordinate axes.

… of the three turning points of C, of which one is a point of inflection.

[ ] , [graph]



Question 64 (*****)

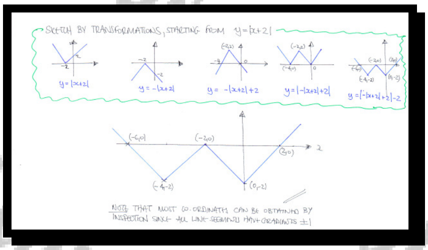
$$f(x) = \big|2 - |x+2|\big| - 2, \ x \in \mathbb{R}.$$

Sketch the graph of $f(x)$

The sketch must include the coordinates …

- … of any points where the graph meets the coordinate axes
- … of any cusps of the graph.

graph



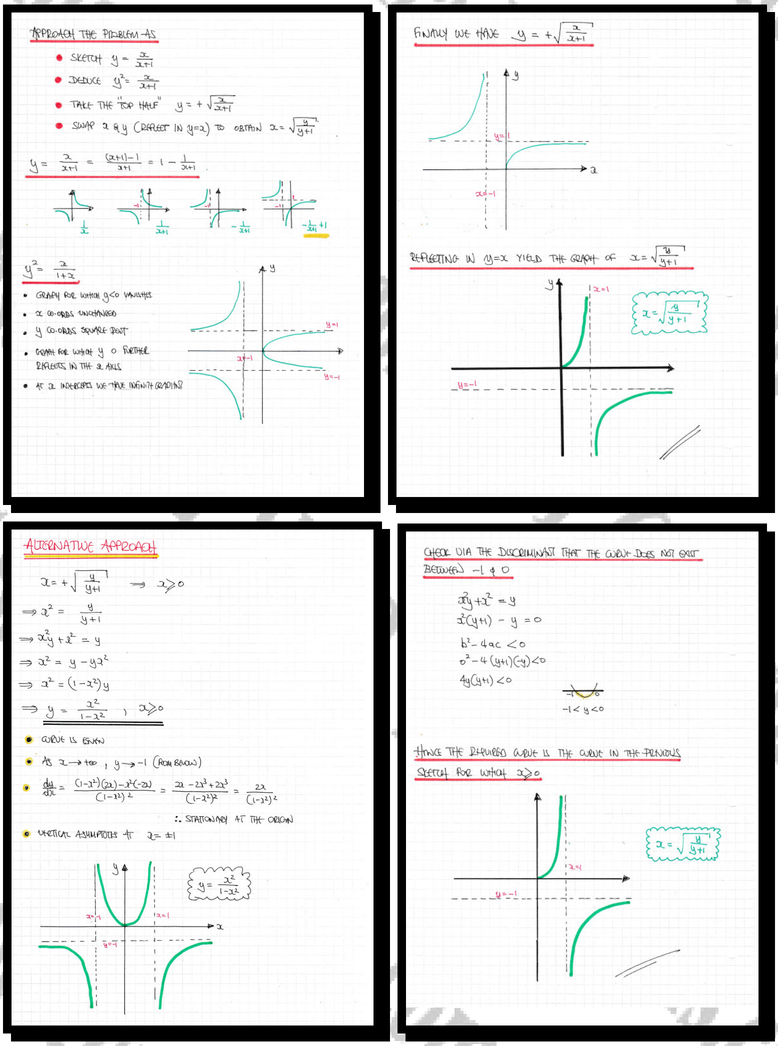
Question 65 (*****)

A curve C is defined in the largest real domain by the equation

$$x = \sqrt{\frac{y}{y+1}}.$$

Sketch a detailed graph of C, fully justifying its key features.

[ ], [graph]



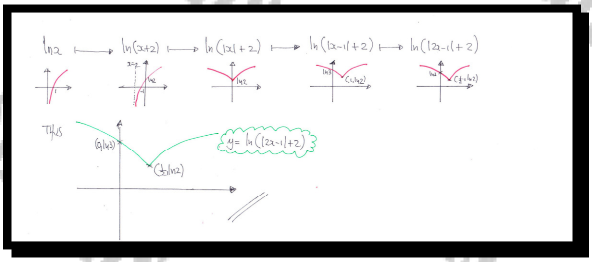
Question 66 (*****)

By considering a sequence of transformations, or otherwise, sketch the graph of

$$y = \ln\left(|2x-1|+2\right), \ x \in \mathbb{R}.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

graph



Question 67 (*****)

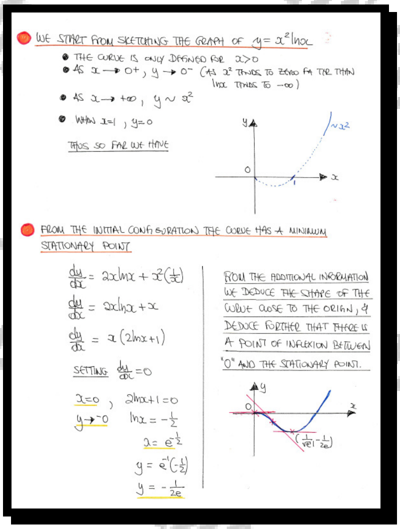
Sketch the graph of the curve with equation

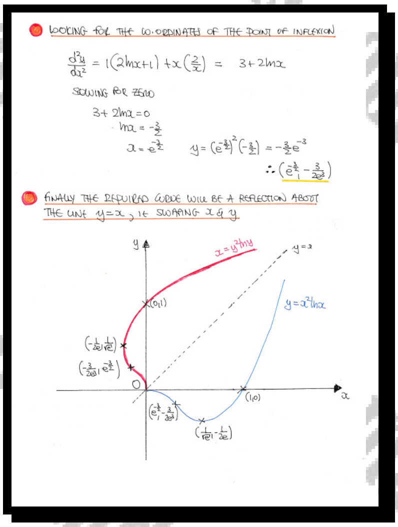
$$x = y^2 \ln y .$$

The sketch must include ...

- ... the coordinates of any intersections with the axes.
- ... the coordinates of any points where the tangent to the curve is parallel to the coordinate axes.
- ... the coordinates of any points of inflexion.

[illegible], graph





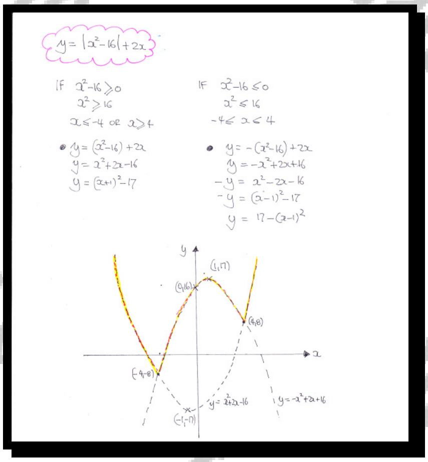
Question 68 (*****)

Sketch the graph of

$$y = \left| x^2 - 16 \right| + 2x \,, \; x \in \mathbb{R} \,.$$

The sketch must include the coordinates of any cusps or any stationary points

graph



Question 69 (*****)

A curve C is defined in the largest possible real number domain and has equation

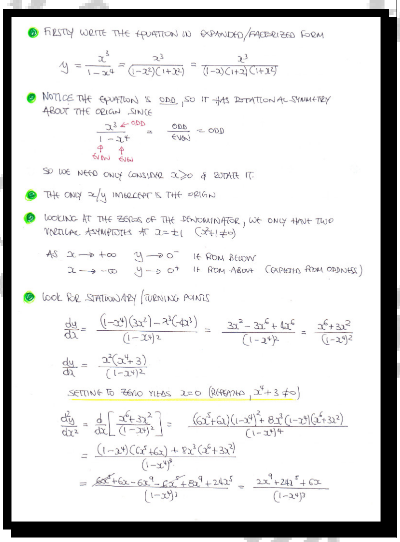
$$y = \frac{x^3}{1-x^4}.$$

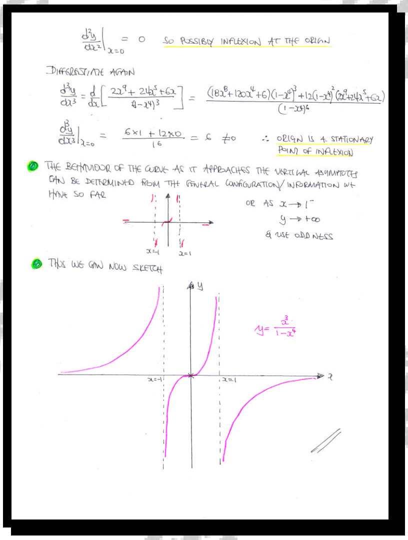
Sketch the graph of C.

The sketch must include

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the coordinates of any non stationary turning points.
- the equations of any asymptotes.

[ ] , [graph]





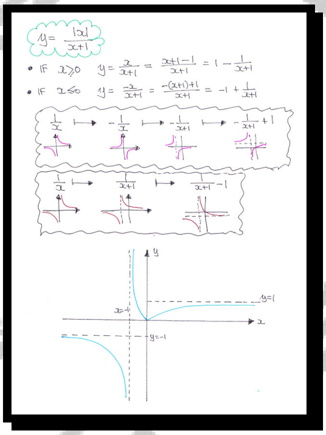
Question 70 (*****)

Sketch the graph of

$$y = \frac{|x|}{x+1}, \; x \in \mathbb{R}.$$

The sketch must include the equations of any asymptotes of the curve, and the coordinates of any points where the curve meets the coordinate axes.

graph



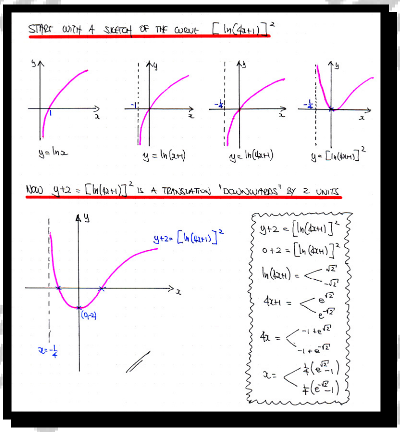
Question 71 (*****)

The curve C has equation

$$y+2=\left[\ln(4x+1)\right]^2, \quad x \in \mathbb{R}, \quad x \geq -\frac{1}{4}.$$

Sketch a detailed graph of C.

, graph



Question 72 (*****)

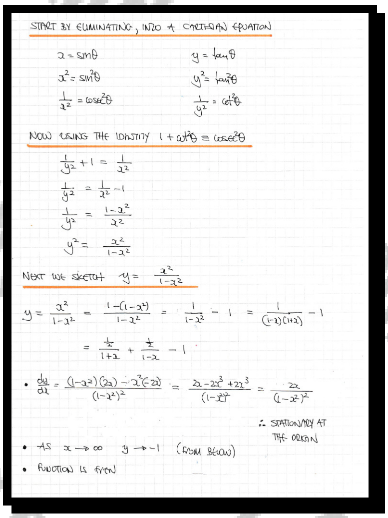
A curve C is defined in the largest real domain by the parametric equations

$$x = \sin\theta, \qquad y = \tan\theta.$$

Sketch a detailed graph of C, fully justifying its key features.

The sketch must include the range of values of θ, which produces each section of C.

[ ], [graph]



START BY ELIMINATING, INTO A CARTESIAN EQUATION

$x = \sin\theta$ $\quad$ $y = \tan\theta$

$x^2 = \sin^2\theta$ $\quad$ $y^2 = \tan^2\theta$

$\frac{1}{x^2} = \operatorname{cosec}^2\theta$ $\quad$ $\frac{1}{y^2} = \cot^2\theta$

NOW USING THE IDENTITY $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$

$\frac{1}{y^2} + 1 = \frac{1}{x^2}$

$\frac{1}{y^2} = \frac{1}{x^2} - 1$

$\frac{1}{y^2} = \frac{1-x^2}{x^2}$

$y^2 = \frac{x^2}{1-x^2}$

NEXT WE SKETCH $y = \frac{x^2}{1-x^2}$

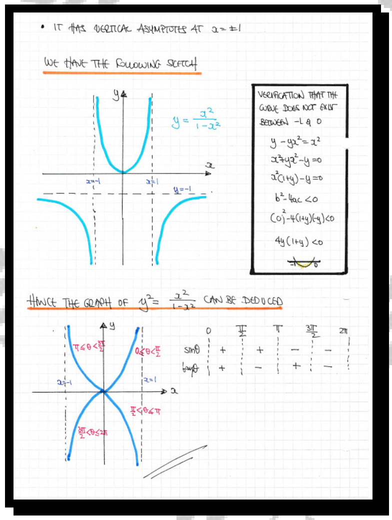
$y = \frac{x^2}{1-x^2} = \frac{1-(1-x^2)}{1-x^2} = \frac{1}{1-x^2} - 1 = \frac{1}{(1-x)(1+x)} - 1$

$= \frac{\frac{1}{2}}{1+x} + \frac{\frac{1}{2}}{1-x} - 1$

- $\frac{dy}{dx} = \frac{(1-x^2)(2x) - x^2(-2x)}{(1-x^2)^2} = \frac{2x - 2x^3 + 2x^3}{(1-x^2)^2} = \frac{2x}{(1-x^2)^2}$

$\therefore$ STATIONARY AT THE ORIGIN

- AS $x \to \infty$ $\quad y \to -1$ (FROM BELOW)
- FUNCTION IS EVEN



- IT HAS VERTICAL ASYMPTOTES AT $x = \pm 1$

WE HAVE THE FOLLOWING SKETCH

VERIFICATION THAT THE CURVE DOES NOT EXIST BETWEEN -1 & 0

$y - yx^2 = x^2$

$x^2 + yx^2 - y = 0$

$x^2(1+y) - y = 0$

$b^2 - 4ac < 0$

$(0)^2 - 4(1+y)(-y) < 0$

$4y(1+y) < 0$

HENCE THE GRAPH OF $y^2 = \frac{x^2}{1-x^2}$ CAN BE DEDUCED

	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
$\sin\theta$		+		+		−		−	
$\tan\theta$		+		−		+		−	

Question 73 (*****)

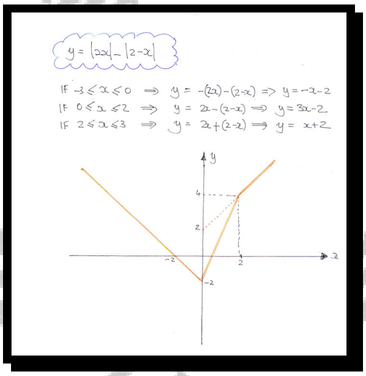
Sketch the graph of

$$y = |2x| - |2 - x|, \; x \in \mathbb{R}.$$

The sketch must include the coordinates of any points where the curve meets the coordinate axes.

[*No credit will be given to non analytical sketches based on plotting coordinates*]

graph



Question 74 (*****)

A curve C has equation

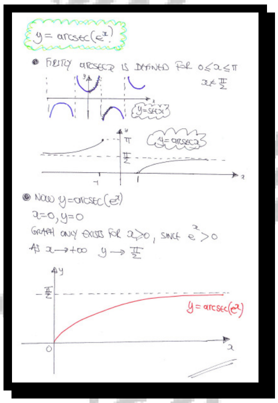
$$y = \operatorname{arcsec}\left(e^{x}\right).$$

Sketch the graph of C, for the largest possible domain.

The sketch must include, in exact form where appropriate,

- the coordinates of any points where the graph of C meets the coordinate axes.
- the coordinates of any stationary points.
- the equations of any asymptotes.

graph



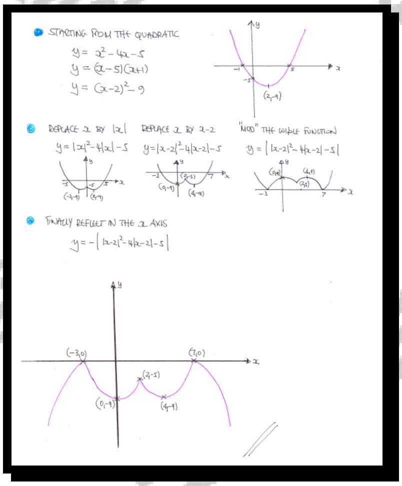
Question 75 (*****)

By considering a sequence of four transformations, or otherwise, sketch the graph of

$$y = -\left| |x-2|^2 - 4|x-2| - 5 \right|.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

$\boxed{}$, $\boxed{(-3,0),\ (7,0),\ (0,-9),\ (2,-5)}$



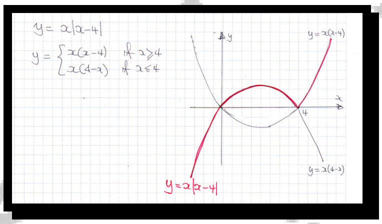
Question 76 (*****)

By considering the graphs of two separate curves, or otherwise, sketch the graph of

$$y = x|x-4|.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

$\boxed{\text{graph}}$



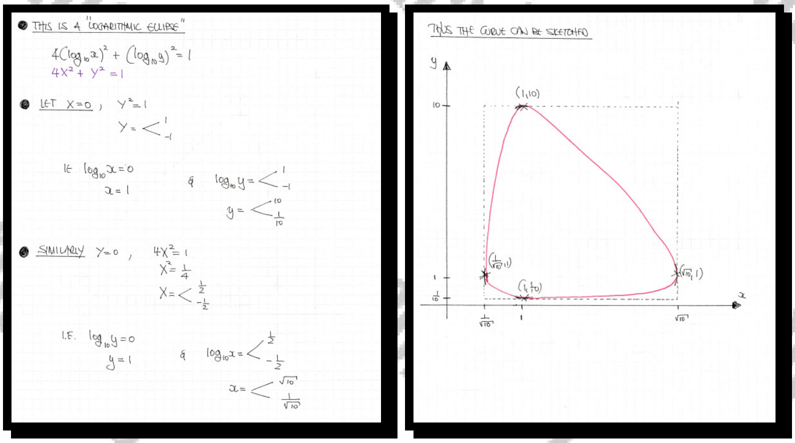
Question 77 (*****)

Sketch the graph of the curve with equation

$$4\left[\log_{10} x\right]^2 + \left[\log_{10} y\right]^2 = 1, \quad x > 0, \quad y > 0.$$

The sketch must include the coordinates of any points where the tangent to the curve is parallel to the coordinate axes.

[ ] , [graph]



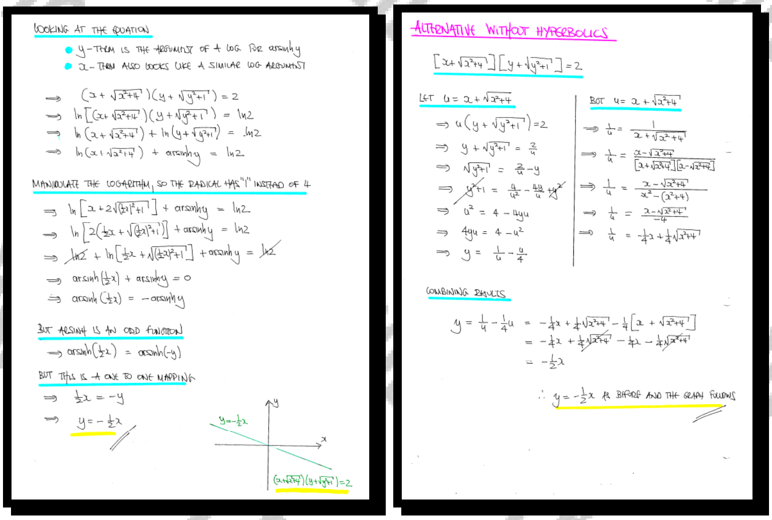
Question 78 (*****)

Sketch the graph of

$$\left[x+\sqrt{x^2+4}\right]\left[y+\sqrt{y^2+1}\right]=2, \quad x\in(-\infty,\infty), \ y\in(-\infty,\infty)$$

You must show a detailed method in this question

[ ] , [proof]



LOOKING AT THE EQUATION

- y – TERM IS THE ARGUMENT OF A LOG FOR arsinh y
- x – TERM ALSO LOOKS LIKE A SIMILAR LOG ARGUMENT

$\Rightarrow (x+\sqrt{x^2+4})(y+\sqrt{y^2+1})=2$

$\Rightarrow \ln\left[(x+\sqrt{x^2+4})(y+\sqrt{y^2+1})\right]=\ln 2$

$\Rightarrow \ln(x+\sqrt{x^2+4})+\ln(y+\sqrt{y^2+1})=\ln 2$

$\Rightarrow \ln(x+\sqrt{x^2+4})+\text{arsinh}\,y=\ln 2$

MANIPULATE THE LOGARITHM, SO THE RADICAL HAS "1" INSTEAD OF 4

$\Rightarrow \ln\left[x+2\sqrt{(\frac{1}{2}x)^2+1}\right]+\text{arsinh}\,y=\ln 2$

$\Rightarrow \ln\left[2\left(\frac{1}{2}x+\sqrt{(\frac{1}{2}x)^2+1}\right)\right]+\text{arsinh}\,y=\ln 2$

$\Rightarrow \ln 2+\ln\left[\frac{1}{2}x+\sqrt{(\frac{1}{2}x)^2+1}\right]+\text{arsinh}\,y=\ln 2$

$\Rightarrow \text{arsinh}(\frac{1}{2}x)+\text{arsinh}\,y=0$

$\Rightarrow \text{arsinh}(\frac{1}{2}x)=-\text{arsinh}\,y$

BUT ARSINH IS AN ODD FUNCTION

$\Rightarrow \text{arsinh}(\frac{1}{2}x)=\text{arsinh}(-y)$

BUT THIS IS A ONE TO ONE MAPPING

$\Rightarrow \frac{1}{2}x=-y$

$\Rightarrow y=-\frac{1}{2}x$

ALTERNATIVE WITHOUT HYPERBOLICS

$\left[x+\sqrt{x^2+4}\right]\left[y+\sqrt{y^2+1}\right]=2$

LET $u=x+\sqrt{x^2+4}$

$\Rightarrow u(y+\sqrt{y^2+1})=2$

$\Rightarrow y+\sqrt{y^2+1}=\frac{2}{u}$

$\Rightarrow \sqrt{y^2+1}=\frac{2}{u}-y$

$\Rightarrow y^2+1=\frac{4}{u^2}-\frac{4y}{u}+y^2$

$\Rightarrow u^2=4-4yu$

$\Rightarrow 4yu=4-u^2$

$\Rightarrow y=\frac{1}{u}-\frac{u}{4}$

BUT $u=x+\sqrt{x^2+4}$

$\Rightarrow \frac{1}{u}=\frac{1}{x+\sqrt{x^2+4}}$

$\Rightarrow \frac{1}{u}=\frac{x-\sqrt{x^2+4}}{[x+\sqrt{x^2+4}][x-\sqrt{x^2+4}]}$

$\Rightarrow \frac{1}{u}=\frac{x-\sqrt{x^2+4}}{x^2-(x^2+4)}$

$\Rightarrow \frac{1}{u}=\frac{x-\sqrt{x^2+4}}{-4}$

$\Rightarrow \frac{1}{u}=-\frac{1}{4}x+\frac{1}{4}\sqrt{x^2+4}$

COMBINING RESULTS

$y=\frac{1}{u}-\frac{1}{4}u=-\frac{1}{4}x+\frac{1}{4}\sqrt{x^2+4}-\frac{1}{4}\left[x+\sqrt{x^2+4}\right]$

$=-\frac{1}{4}x+\frac{1}{4}\sqrt{x^2+4}-\frac{1}{4}x-\frac{1}{4}\sqrt{x^2+4}$

$=-\frac{1}{2}x$

$\therefore y=-\frac{1}{2}x$ AS BEFORE AND THE GRAPH FOLLOWS

Question 79 (*****)

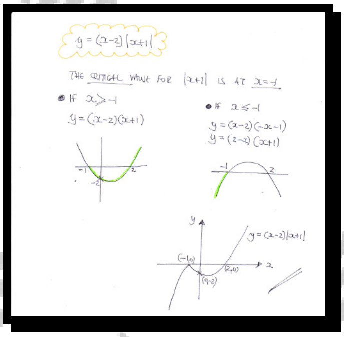
By considering the graphs of two separate curves, or otherwise, sketch the graph of

$$y = (x-2)|x+1|.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

[*No credit will be given to non analytical sketches based on plotting coordinates*]

graph



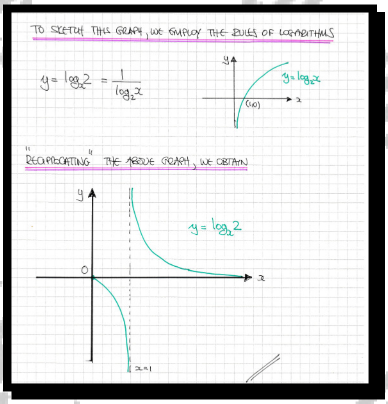
Question 80 (*****)

A curve C is defined in the largest real domain by the equation

$$y = \log_x 2.$$

Sketch a detailed graph of C.

graph



Question 81 (*****)

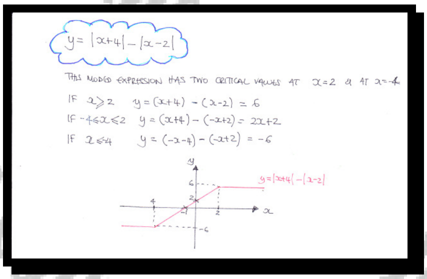
By considering the graphs of three separate lines, or otherwise, sketch the graph of

$$y = |x+4| - |x-2|$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

[*No credit will be given to non analytical sketches based on plotting coordinates*]

graph



Question 82 (*****)

A curve C is defined in the largest real domain by the equation

$$y = \frac{4x^2 - 25}{(2x-1)(x-2)(x+3)}.$$

a) Sketch the graph of C.

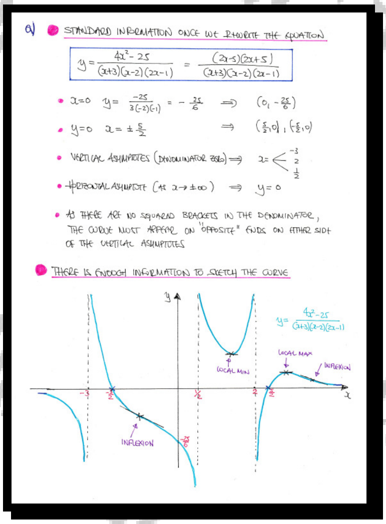
The sketch must include the equations of any asymptotes of C and the coordinates of any point where C meets the coordinate axes. Any turning points, including points of inflexion, must be clearly indicated but their coordinates need **not** be found.

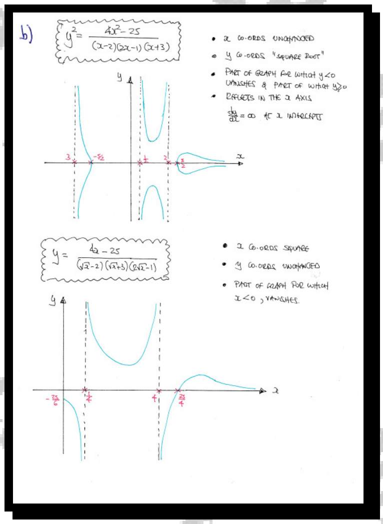
b) Hence sketch on separate set of axes the graph of ...

a) $$\dots\ y^2 = \frac{4x^2 - 25}{(2x-1)(x-2)(x+3)}.$$

b) $$\dots\ y = \frac{4x - 25}{\left(2\sqrt{x}-1\right)\left(\sqrt{x}-2\right)\left(\sqrt{x}+3\right)}.$$

[ ], [graph]





Question 83 (*****)

A curve is defined in the largest real domain by the equation

$$f(x) = -x^2 + 8x - 12.$$

Sketch on separate set of axes detailed graph of ...

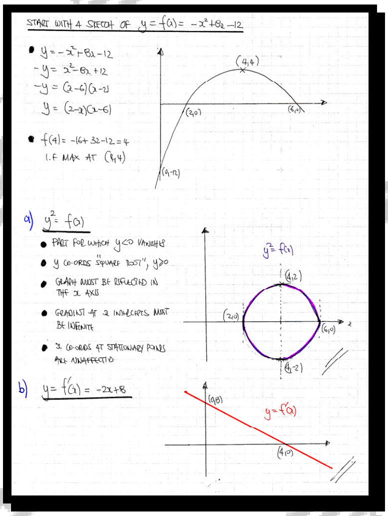
a) ... $y^2 = f(x)$.

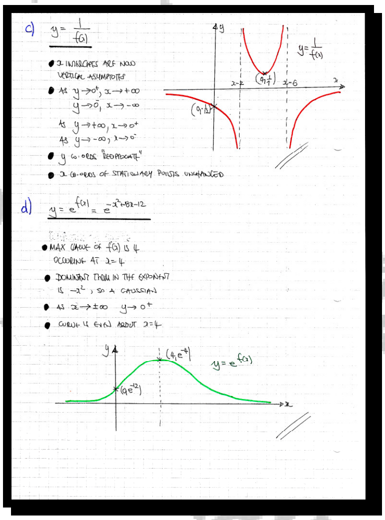
b) ... $y = f'(x)$.

c) ... $y = \dfrac{1}{f(x)}$.

d) ... $y^2 = e^{f(x)}$.

[ ] , [graph]



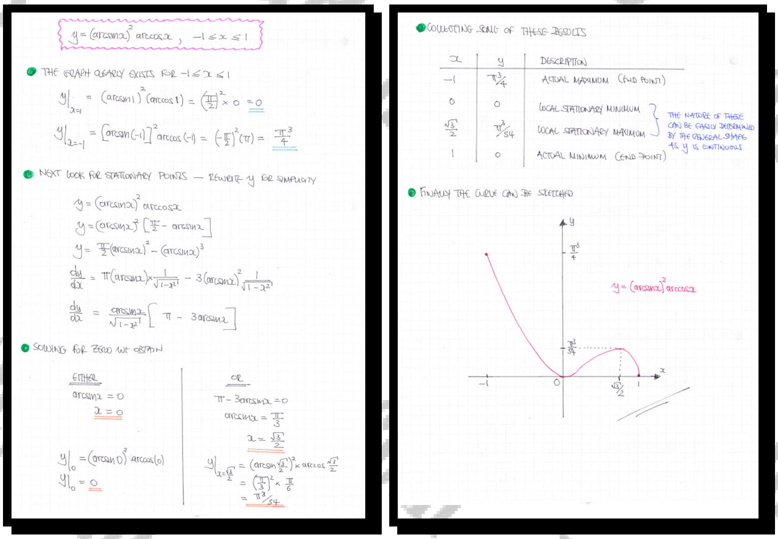


Question 84 (*****)

On a clearly labelled set of axes, draw a detailed sketch of the graph of

$$y=(\arcsin x)^2 \arccos x\,,\ -1 \le x \le 1\,.$$

graph



Question 85 (*****)

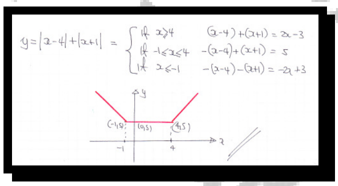
By considering the graphs of three separate lines, or otherwise, sketch the graph of

$$y=|x-4|+|x+1|$$

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusp of the curve.

[*No credit will be given to non analytical sketches based on plotting coordinates*]

graph



Question 86 (*****)

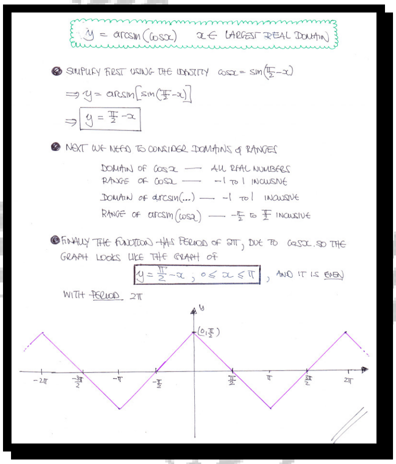
Sketch the graph of

$$f(x) = \arcsin(\cos x),$$

in the largest domain that the function is defined.

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusps of the curve.

[ ] , [graph]



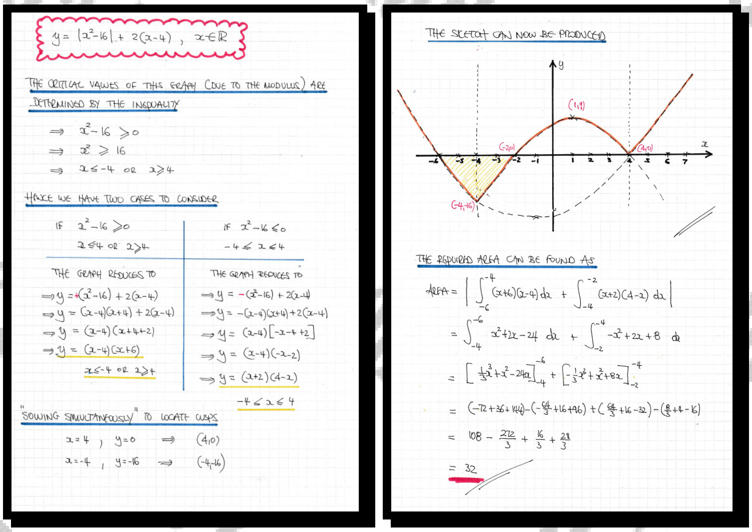
Question 87 (*****)

The curve C has equation

$$y = \left|x^2 - 16\right| + 2(x-4), \ x \in \mathbb{R}.$$

Sketch a detailed graph of C and hence show that the area of the finite region bounded by C and the x axis, for which $y<0$, is 32 square units.

[ ] , [proof]



$y = |x^2-16| + 2(x-4), \ x \in \mathbb{R}$

THE CRITICAL VALUES OF THIS GRAPH (DUE TO THE MODULUS) ARE DETERMINED BY THE INEQUALITY

$\Rightarrow x^2 - 16 \geq 0$

$\Rightarrow x^2 \geq 16$

$\Rightarrow x \leq -4$ OR $x \geq 4$

HENCE WE HAVE TWO CASES TO CONSIDER

IF $x^2-16 \geq 0$, $x \leq -4$ OR $x \geq 4$

THE GRAPH REDUCES TO

$\Rightarrow y = +(x^2-16) + 2(x-4)$

$\Rightarrow y = (x-4)(x+4) + 2(x-4)$

$\Rightarrow y = (x-4)(x+4+2)$

$\Rightarrow y = (x-4)(x+6)$, $x \leq -4$ OR $x \geq 4$

IF $x^2-16 \leq 0$, $-4 \leq x \leq 4$

THE GRAPH REDUCES TO

$\Rightarrow y = -(x^2-16) + 2(x-4)$

$\Rightarrow y = -(x-4)(x+4) + 2(x-4)$

$\Rightarrow y = (x-4)[-x-4+2]$

$\Rightarrow y = (x-4)(-x-2)$

$\Rightarrow y = (x+2)(4-x)$, $-4 \leq x \leq 4$

"SOLVING SIMULTANEOUSLY" TO LOCATE CUSPS

$x=4, \ y=0 \Rightarrow (4,0)$

$x=-4, \ y=-16 \Rightarrow (-4,-16)$

THE SKETCH CAN NOW BE PRODUCED

THE REQUIRED AREA CAN BE FOUND AS

$$\text{AREA} = \left| \int_{-6}^{-4} (x+6)(x-4)\,dx + \int_{-4}^{-2} (x+2)(4-x)\,dx \right|$$

$$= \int_{-4}^{-6} x^2+2x-24 \,dx + \int_{-2}^{-4} -x^2+2x+8 \,dx$$

$$= \left[\tfrac{1}{3}x^3 + x^2 - 24x\right]_{-4}^{-6} + \left[-\tfrac{1}{3}x^3 + x^2 + 8x\right]_{-2}^{-4}$$

$$= (-72+36+144) - \left(-\tfrac{64}{3}+16+96\right) + \left(\tfrac{64}{3}+16-32\right) - \left(\tfrac{8}{3}+4-16\right)$$

$$= 108 - \frac{272}{3} + \frac{16}{3} + \frac{28}{3}$$

$$= 32$$

Question 88 (*****)

A curve C is defined in the largest real domain by the equation

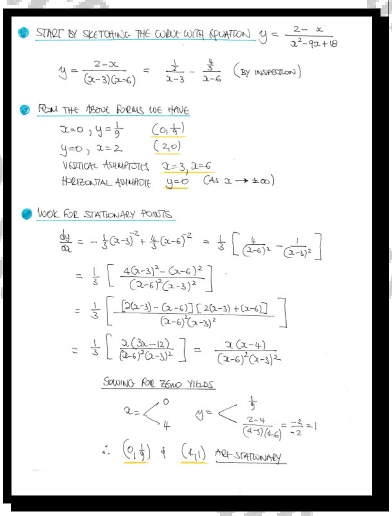
$$y = \frac{2-\sqrt{x}}{x-9\sqrt{x}+18}.$$

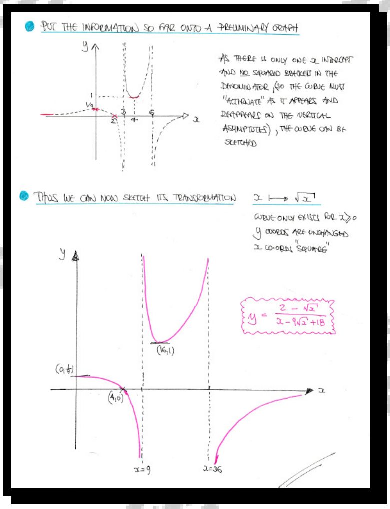
Sketch the graph of C.

The sketch must include

- ... the equations of any asymptotes of C.
- ... the coordinates of any point where C meets the coordinate axes.
- ... the coordinates any stationary points of C.

[ ], graph





Question 89 (*****)

A curve C is defined parametrically by

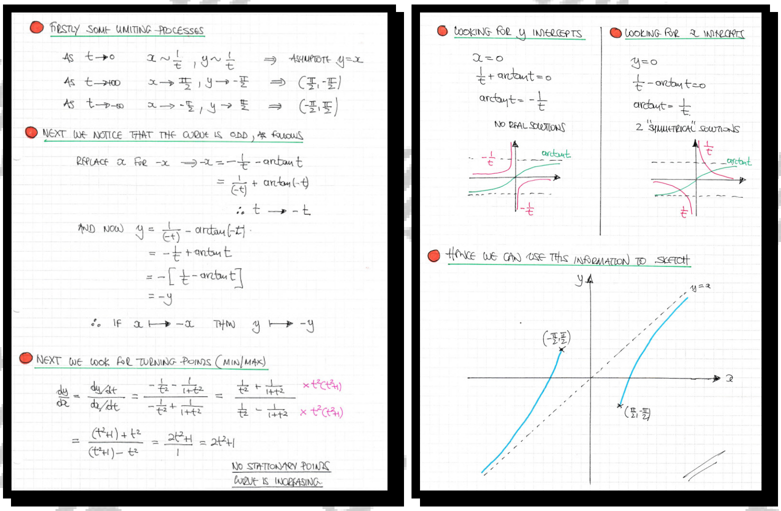
$$x = \frac{1}{t} + \arctan t, \quad y = \frac{1}{t} - \arctan t, \quad t \in \mathbb{R}, \ t \neq 0.$$

Sketch the graph of C.

Indicate the equations of any asymptotes, stationary points and any endpoints.

You need not mark the coordinates of any intersections with the axes.

[ ] , [graph]



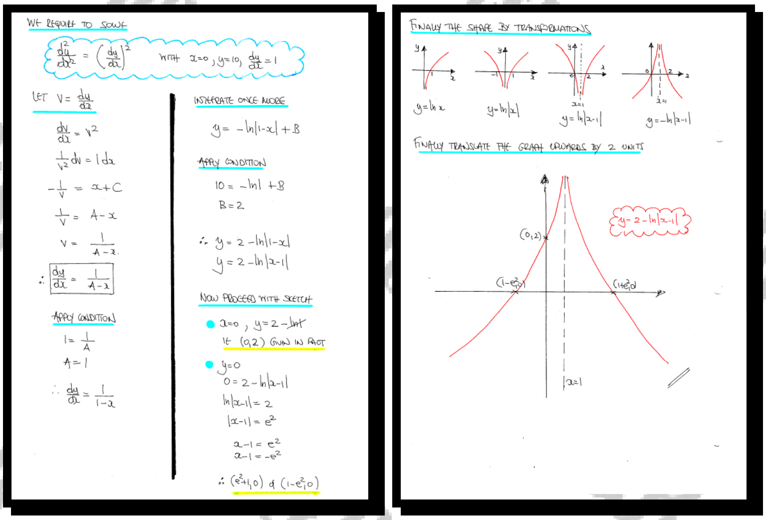
Question 90 (*****)

It is required to sketch the curve with equation $y = f(x)$, defined over the set of real numbers, in the greatest domain.

The curve has the property that at every point on the curve, the second derivative equals to the first derivative **squared**.

Showing all the relevant details, sketch the graph of $y = f(x)$, given further that the curve passes through the point $(0,2)$ and the gradient at that point is 1.

[ ], [graph]



Question 91 (*****)

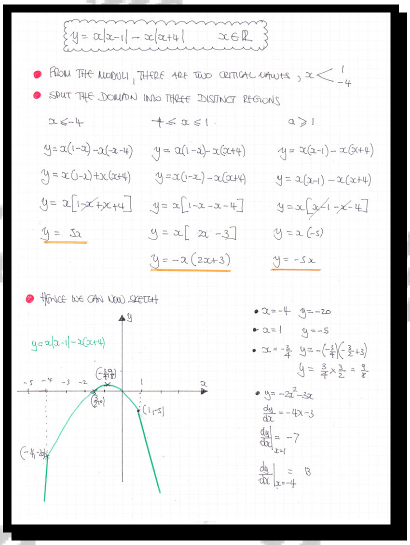
Sketch the graph of

$$y = x|x-1| - x|x+4|, \quad x \in \mathbb{R}.$$

Indicate the coordinates of any intersections with the axes, and the coordinates of any cusps of the curve.

[*No credit will be given to non analytical sketches based on plotting coordinates*]

[ ] , [graph]



Question 92 (*****)

A curve C has equation

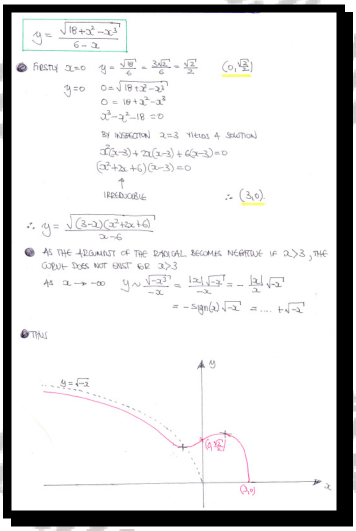
$$y = \frac{\sqrt{18 + x^2 - x^3}}{6 - x}.$$

It is given that C has two stationary points whose x coordinates have opposite signs.

Sketch the graph of C, for the largest possible domain.

- The sketch must include, in exact form where appropriate the coordinates of any points where the graph of C meets the coordinate axes the equations of any asymptotes.
- You need not find the coordinates of the stationary points of C.

SP-U , graph



Question 93 (*****)

Sketch the curve with equation

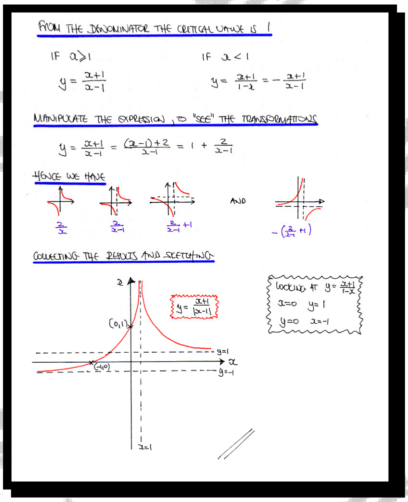
$$y = \frac{x+1}{|x-1|}, \; x \in \mathbb{R}, \; x \neq 1 .$$

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the asymptotes of the curve.

[*No credit will be given to non analytical sketches based on plotting coordinates*]

graph



Question 94 (*****)

Sketch in separate sets of axes detailed graphs of the following curves, fully justifying their key features.

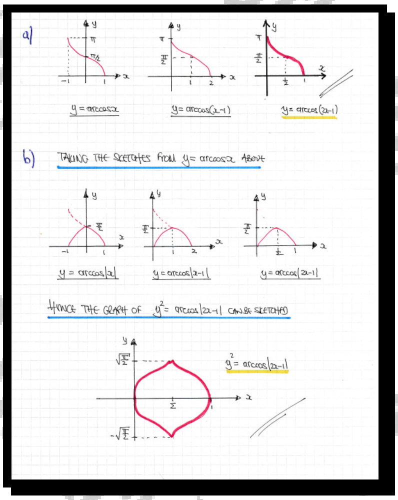
a) $y = \arccos(2x-1)$.

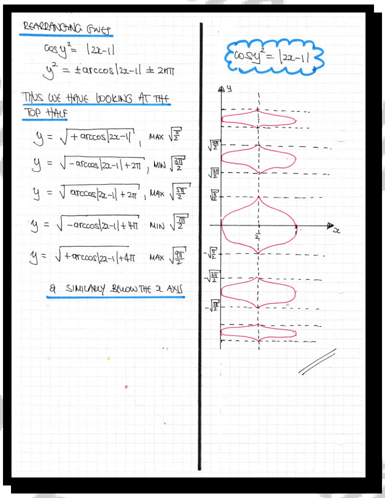
b) $y^2 = \arccos|2x-1|$.

c) $\cos y^2 = |2x-1|$

You may assume that each curve is defined in the largest real domain.

graph





Question 95 (*****)

Sketch the graph of the curve with equation

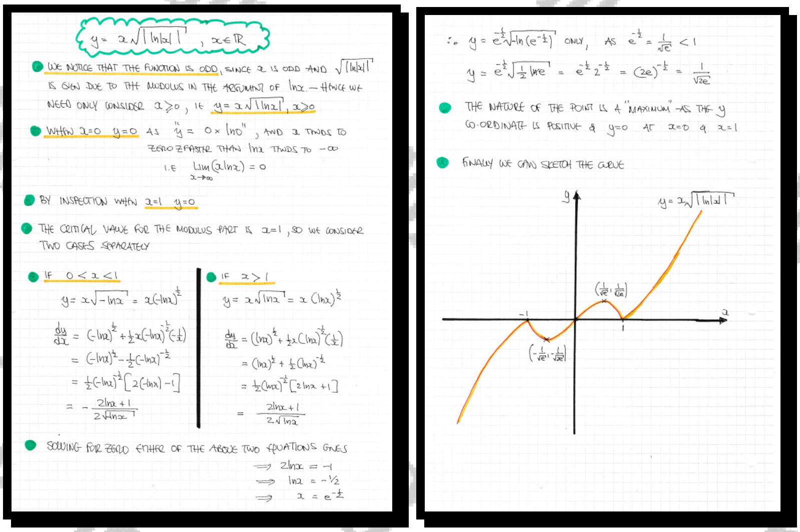
$$y = x\sqrt{\left|\ln|x|\right|}, \ x \in \mathbb{R}.$$

The sketch must include the coordinates of ...

... any points where the curve meets the coordinate axes.

... any stationary points.

[ ] , graph



Question 96 (*****)

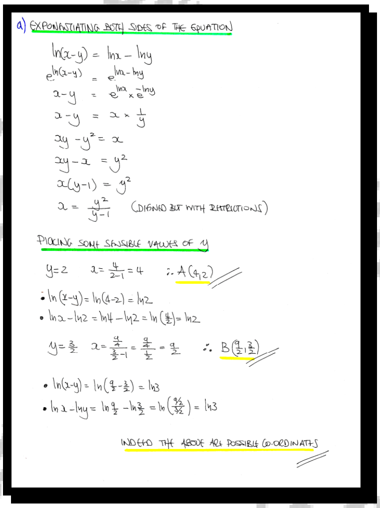
The distinct points A and B lie on the curve with equation

$$\ln(x-y) = \ln x + \ln y, \quad x \in (0,\infty), \quad y \in (0,\infty).$$

a) Determine possible coordinates for A and B, further verifying that these coordinates indeed satisfy the above given equation.

b) Sketch the curve, showing clearly all the relevant details.

$\boxed{A(4,2)}$, $\boxed{B\left(\frac{9}{2},\frac{3}{2}\right)}$


a) EXPONENTIATING BOTH SIDES OF THE EQUATION

$\ln(x-y) = \ln x - \ln y$

$e^{\ln(x-y)} = e^{\ln x - \ln y}$

$x - y = e^{\ln x} \times e^{-\ln y}$

$x - y = x \times \frac{1}{y}$

$xy - y^2 = x$

$xy - x = y^2$

$x(y-1) = y^2$

$x = \frac{y^2}{y-1}$ (DEFINED BUT WITH RESTRICTIONS)

PICKING SOME SENSIBLE VALUES OF y

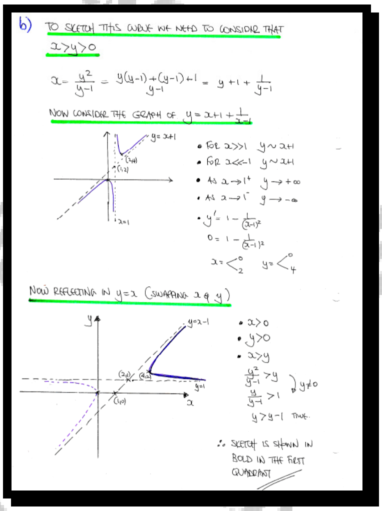
$y = 2 \quad x = \frac{4}{2-1} = 4 \quad \therefore A(4,2)$

- $\ln(x-y) = \ln(4-2) = \ln 2$
- $\ln x - \ln 2 = \ln 4 - \ln 2 = \ln\left(\frac{4}{2}\right) = \ln 2$

$y = \frac{3}{2} \quad x = \frac{\frac{9}{4}}{\frac{3}{2}-1} = \frac{\frac{9}{4}}{\frac{1}{2}} = \frac{9}{2} \quad \therefore B\left(\frac{9}{2},\frac{3}{2}\right)$

- $\ln(x-y) = \ln\left(\frac{9}{2}-\frac{3}{2}\right) = \ln 3$
- $\ln x - \ln y = \ln\frac{9}{2} - \ln\frac{3}{2} = \ln\left(\frac{9/2}{3/2}\right) = \ln 3$

INDEED THE ABOVE ARE POSSIBLE CO-ORDINATES



Question 97 (*****)

A curve C is defined in the largest real domain by the equation

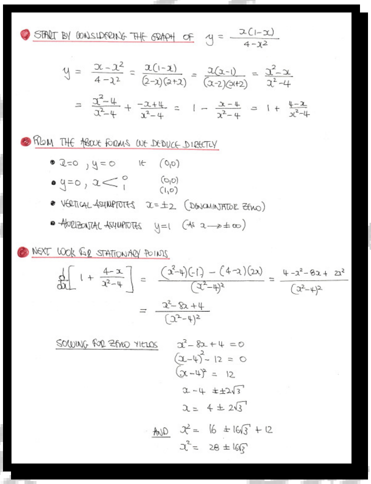
$$y = -\sqrt{\frac{x(1-x)}{4-x^2}}.$$

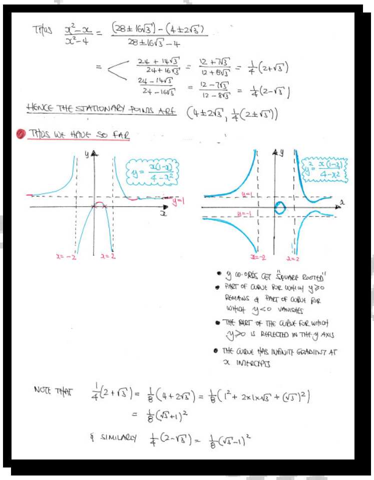
Sketch the graph of C.

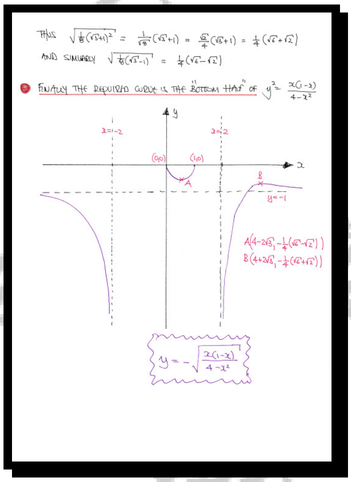
The sketch must include

- ... the equations of any asymptotes of C.
- ... the coordinates of any point where C meets the coordinate axes.
- ... the coordinates of the stationary points of C, giving the answer in the form $\left[2k + k\sqrt{3}, -\frac{1}{2k}\left(\sqrt{3k} + \sqrt{k}\right)\right]$, where k is a positive integer.

[ ], [graph]







Question 98 (*****)

The curve C has equation

$$y = A\ln|x| + Bx^2 + x, \ x \in \mathbb{R},$$

where A and B are non zero constants.

The curve has stationary points at $x = -1$ and at $x = 2$.

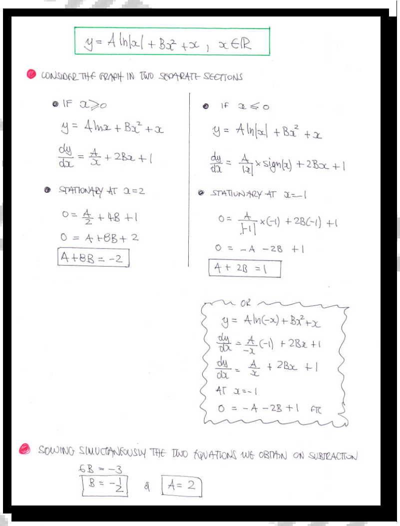
Sketch the graph of C.

The sketch must include …

- … the coordinates of all the stationary points.
- … the equations of the asymptotes of the curve.

You need not find any intercepts with the coordinate axes.

[ ], [graph]



$y = A\ln|x| + Bx^2 + x, \ x \in \mathbb{R}$

- CONSIDER THE GRAPH IN TWO SEPARATE SECTIONS

- IF $x > 0$

$y = A\ln x + Bx^2 + x$

$\frac{dy}{dx} = \frac{A}{x} + 2Bx + 1$

- STATIONARY AT $x = 2$

$0 = \frac{A}{2} + 4B + 1$

$0 = A + 8B + 2$

$A + 8B = -2$

- IF $x \leq 0$

$y = A\ln|x| + Bx^2 + x$

$\frac{dy}{dx} = \frac{A}{|x|} \times \text{sign}(x) + 2Bx + 1$

- STATIONARY AT $x = -1$

$0 = \frac{A}{|-1|} \times (-1) + 2B(-1) + 1$

$0 = -A - 2B + 1$

$A + 2B = 1$

OR

$y = A\ln(-x) + Bx^2 + x$

$\frac{dy}{dx} = \frac{A}{-x}(-1) + 2Bx + 1$

$\frac{dy}{dx} = \frac{A}{x} + 2Bx + 1$

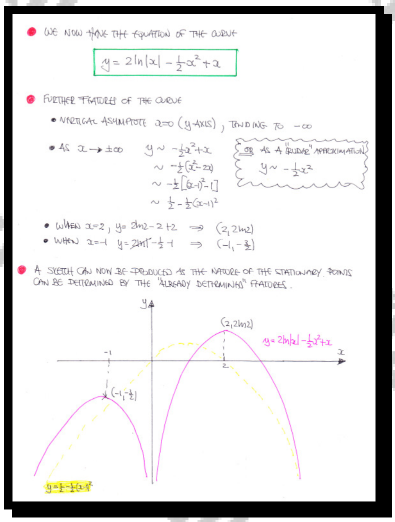
AT $x = -1$

$0 = -A - 2B + 1$ ETC

- SOLVING SIMULTANEOUSLY THE TWO EQUATIONS WE OBTAIN ON SUBTRACTION

$6B = -3$

$B = -\frac{1}{2}$ & $A = 2$



- WE NOW HAVE THE EQUATION OF THE CURVE

$y = 2\ln|x| - \frac{1}{2}x^2 + x$

- FURTHER FEATURES OF THE CURVE

- VERTICAL ASYMPTOTE $x = 0$ (y-AXIS), TENDING TO $-\infty$
- AS $x \to \pm\infty$ $\quad y \sim -\frac{1}{2}x^2 + x$

$\sim -\frac{1}{2}(x^2 - 2x)$

$\sim -\frac{1}{2}[(x-1)^2 - 1]$

$\sim \frac{1}{2} - \frac{1}{2}(x-1)^2$

OR AS A "RUDER" APPROXIMATION

$y \sim -\frac{1}{2}x^2$

- WHEN $x = 2$, $y = 2\ln 2 - 2 + 2 \Rightarrow (2, 2\ln 2)$
- WHEN $x = -1$ $\ y = 2\ln 1 - \frac{1}{2} - 1 \Rightarrow (-1, -\frac{3}{2})$

- A SKETCH CAN NOW BE PRODUCED AS THE NATURE OF THE STATIONARY POINTS CAN BE DETERMINED BY THE "ALREADY DETERMINED" FEATURES.

Question 99 (*****)

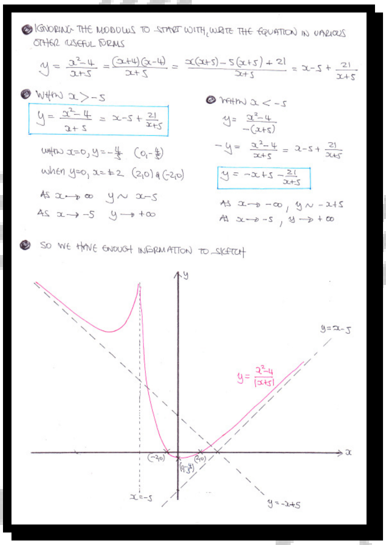
Sketch the curve with equation

$$y = \frac{x^2 - 4}{|x+5|}, \ x \in \mathbb{R}, \ x \neq -5.$$

The sketch must include …

- … the coordinates of all the points where the curve meets the coordinate axes.
- … the equations of the asymptotes of the curve.

[ ] , [graph]



Question 100 (*****)

A general curve C has equation

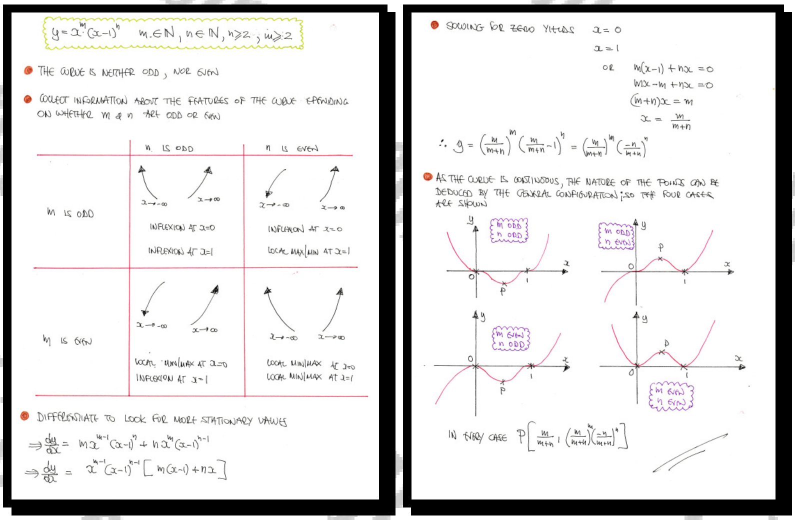
$$y = x^m (x-1)^n ,$$

where $x \in \mathbb{R}$, $m \in \mathbb{N}$, $m \geq 2$, $n \in \mathbb{N}$, $n \geq 2$.

Sketch in four separate of axes, the 4 separate shapes which C can take, $m \geq 2$.

The sketches must contain the coordinates of any stationary points.

[ ], [graph]



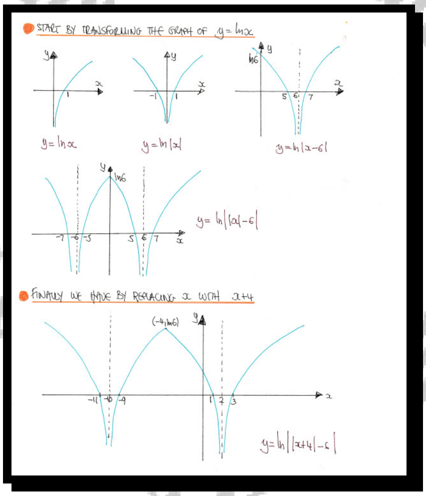
Question 101 (*****)

Sketch, in the largest real domain, the graph of

$$y = \ln\left|\left|x+4\right|-6\right|.$$

Indicate the coordinates of any intersections with the axes, the equations of any asymptotes and the coordinates of any cusps of the curve.

[ ] , [graph]



Question 102 (*****)

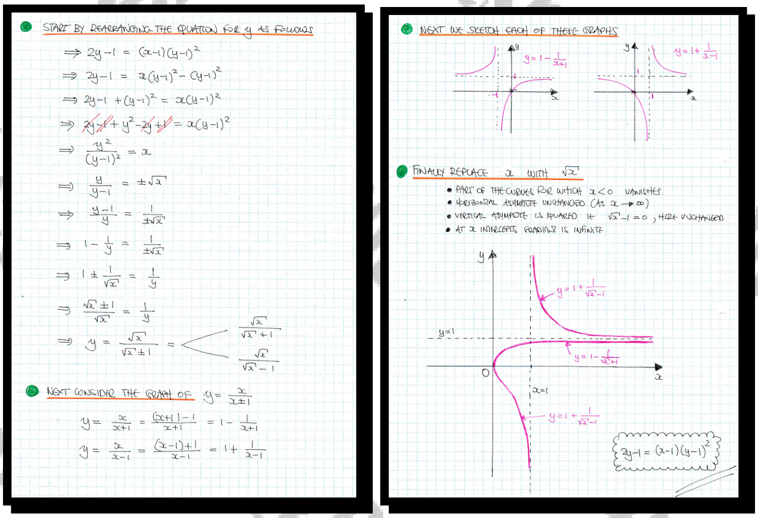
A curve C is defined, in the largest possible real domain, by the Cartesian equation

$$2y-1=(x-1)(y-1)^2.$$

By expressing the above equation in the form $y=f(x)$, sketch the graph of C.

Indicate the equations of any asymptotes, stationary points and any intersections with the coordinate axes.

[ ], [graph]



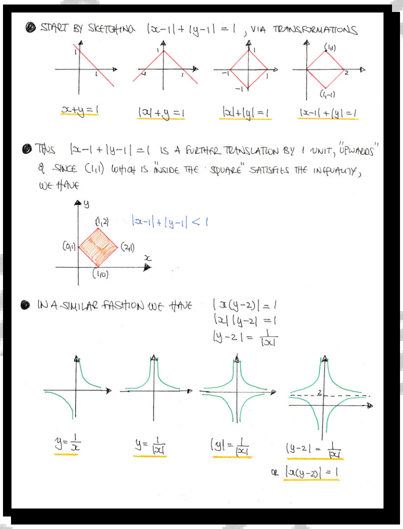
Question 103 (*****)

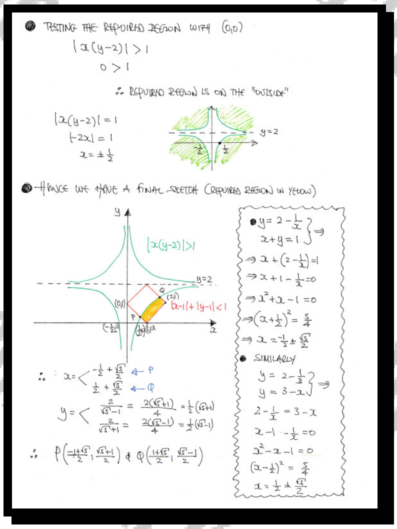
A finite region in the x-y plane is defined by the inequalities

$$|x-1|+|y-1|<1 \quad \text{and} \quad |x(y-2)|>1.$$

Sketch in detail this region, showing clearly any relevant coordinates.

graph





Question 104 (*****)

The curve C is defined in the greatest real domain by the equation

$$y = \frac{x}{(y-2)(y+1)(y-3)}.$$

a) Show that

$$\frac{dy}{dx} = \frac{1}{2(y-1)\left(ay^2+by+c\right)},$$

where a, b and c are integers to be found.

b) Determine the exact value of the gradient at the points on C, where $x = 40$.

c) Sketch the graph of C.

The sketch must include the coordinates of any points where C meets the coordinate axes, the coordinates of the points of infinite gradient. You must also find, with a full algebraic method, the line of symmetry of C.

[ ], $a = 2,\ b = -4,\ c = -3$, $\pm\frac{1}{78}$