LINEARIZATION OF GRAPHS

Question 1 (**)

The table below shows experimental data connecting two variables x and y.

x	1	2	3	4	5
y	12.0	14.4	17.3	20.7	27.0

It is assumed that x and y are related by an equation of the form

$$y = ab^x,$$

where a and b are non zero constants.

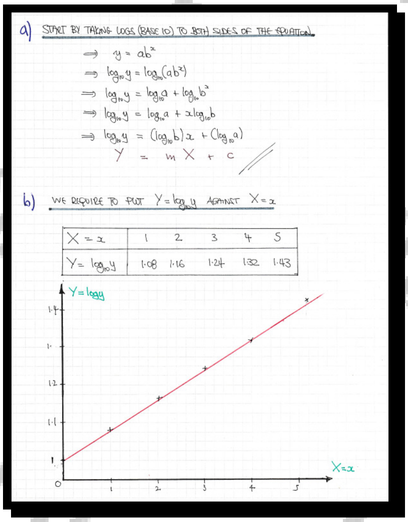
a) Find an equation of a straight line, in terms of well defined constants, in order to investigate the validity of this assumption.

b) Plot a suitable graph to show that the assumption of part **(a)** is valid.

c) Use the graph of part **(b)** to estimate, correct to 1 decimal place, the value of a and the value of b.

d) Estimate the value of y when $x = 2.5$.

$\boxed{\log y = x\log b + \log a}$, $\boxed{a \approx 9.5}$, $\boxed{b \approx 1.2}$, $\boxed{y \approx 15.0}$


a) START BY TAKING LOGS (BASE 10) TO BOTH SIDES OF THE EQUATION

$\Rightarrow y = ab^x$

$\Rightarrow \log_{10} y = \log_{10}(ab^x)$

$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^x$

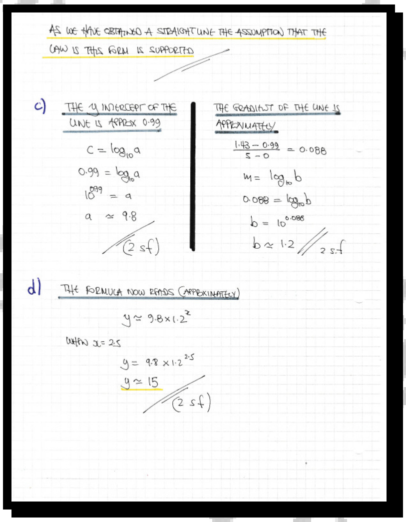
$\Rightarrow \log_{10} y = \log_{10} a + x\log_{10} b$

$\Rightarrow \log_{10} y = (\log_{10} b)x + (\log_{10} a)$

$Y = mX + c$

b) WE REQUIRE TO PLOT $Y = \log_{10} y$ AGAINST $X = x$

$X = x$	1	2	3	4	5
$Y = \log_{10} y$	1.08	1.16	1.24	1.32	1.43


AS WE HAVE OBTAINED A STRAIGHT LINE THE ASSUMPTION THAT THE LAW IS THIS FORM IS SUPPORTED

c) THE y INTERCEPT OF THE LINE IS APPROX 0.99

$c = \log_{10} a$

$0.99 = \log_{10} a$

$10^{0.99} = a$

$a \simeq 9.8$ (2 sf)

THE GRADIENT OF THE LINE IS APPROXIMATELY

$\frac{1.43 - 0.99}{5 - 0} = 0.088$

$m = \log_{10} b$

$0.088 = \log_{10} b$

$b = 10^{0.088}$

$b \simeq 1.2$ 2 s.f

d) THE FORMULA NOW READS (APPROXIMATELY)

$y \simeq 9.8 \times 1.2^x$

WHEN $x = 2.5$

$y = 9.8 \times 1.2^{2.5}$

$y \simeq 15$ (2 sf)

Question 2 (**)

The table below shows experimental data connecting two variables t and W.

t	1	3	4	7	8	10
W	2.0	4.0	6.5	19.0	34.0	65.0

It is assumed that t and W are related by an equation of the form

$$W = ab^t,$$

where a and b are non zero constants.

a) Find an equation of a straight line, in terms of well defined constants, in order to investigate the validity of this assumption.

b) Plot a suitable graph to show that the assumption of part **(a)** is valid.

c) Use the information from the graph to estimate, correct to 2 decimal places, the value of a and the value of b.

d) Estimate the value of W when $t = 20$.

[illegible], $\boxed{\log W = t\log b + \log a}$, $\boxed{a \approx 1.26}$, $\boxed{b \approx 1.48}$, $\boxed{W \approx 3200}$

a) LINEARIZING THE EXPONENTIAL GRAPH AS FOLLOWS

$\Rightarrow W = ab^t$

$\Rightarrow \log W = \log(ab^t)$

$\Rightarrow \log W = \log a + \log b^t$

$\Rightarrow \log W = \log a + t\log b$

$\Rightarrow \log W = (\log b)t + (\log a)$

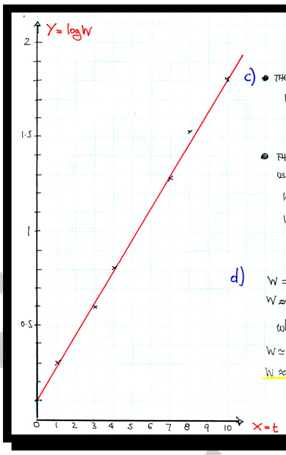
$\quad Y = mX + c$

b)

t	1	3	4	7	8	10
W	2	4	6.5	19	34	65
$X = t$	1	3	4	7	8	10
$Y = \log W$	0.30	0.60	0.81	1.28	1.53	1.81

PUTTING THESE VALUES ON AN ACCURATE DIAGRAM

(SEE NEXT PAGE)



c) • THE "y INTERCEPT" IS $\log a$

$\log_{10} a \approx 0.1$

$a \approx 10^{0.1}$

$a \approx 1.26$

• THE GRADIENT IS $\log b$

USING $(0, 0.1)$ & $(10, 1.8)$

$\log_{10} b \approx \frac{1.8 - 0.1}{10 - 0}$

$\log_{10} b \approx 0.17$

$b \approx 10^{0.17}$

$b \approx 1.48$

d) $W = ab^t$

$W \approx 1.26 \times (1.48)^t$

when $t = 20$

$W \approx 1.26 \times 1.48^{20}$

$W \approx 3200$

Question 3 (**)

The following table shows some experimental data.

x	5	10	15	20	25	30
y	1.7	4.5	11.0	26.0	70.0	160.0

It is assumed that the two variables x and y are related by the formula

$$y = ab^x,$$

where a and b are non zero constants.

a) Use a graphical method to show that the data is consistent with this assumption.

b) Find estimates for the values of a and b, correct to one decimal place.

c) Use the estimated values of a and b, to find an estimate for the value of y when $x = 60$.

$\boxed{}$, $\boxed{a \approx 0.7}$, $\boxed{b \approx 1.2}$, $\boxed{y \approx 39000}$

a) STARTING BY MANIPULATING THE FORMULA

$\Rightarrow y = ab^x$

$\Rightarrow \log_{10} y = \log_{10}(ab^x)$

$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^x$

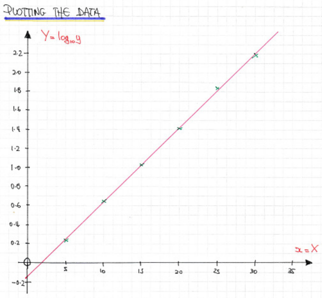
$\Rightarrow \log_{10} y = \log_{10} a + x\log_{10} b$

$\Rightarrow \underbrace{\log_{10} y}_{Y} = \underbrace{(\log_{10} b)}_{m}\underbrace{x}_{X} + \underbrace{\log_{10} a}_{c}$

PREPARE THE VALUES TO BY PLOTTED

$x = X$	5	10	15	20	25	30
y	1.7	4.5	11.0	26.0	70.0	160.0
$Y = \log_{10} y$	0.23	0.65	1.04	1.44	1.85	2.20

PLOTTING THE DATA



AS THE POINTS FORM A STRAIGHT LINE THE RELATIONSHIP IS INDEED OF THE FORM $y = ab^x$

b) NOW WE HAVE BY COMPARING/READING VALUES

- $\log_{10} a = c$
  $\log_{10} a = -0.16$
  $a = 10^{-0.16}$
  $a \approx 0.7$
- $\log_{10} b = m$
  $\log_{10} b = \frac{1.8 - 0.24}{25 - 5}$
  $\log_{10} b = 0.078$
  $b \approx 1.2$

c) USING $y = ab^x = 0.7 \times 1.2^x$ WITH $x = 60$ WE OBTAIN ≈ 39000

Question 4 (**)

The following table shows some experimental data.

t	2	4	6	8	10	12	14
P	20	64	110	180	260	320	420

It is assumed that the two variables t and P are related by the formula

$$P = at^b ,$$

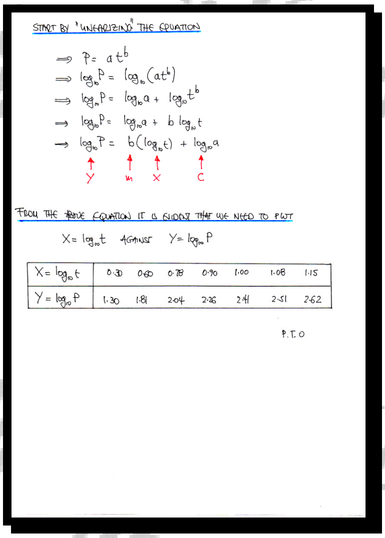
where a and b are non zero constants.

a) Use a graphical method to show that the data is consistent with this assumption.

b) Determine estimates for the value of a and the value of b, correct to one decimal place.

c) Use the estimated values of a and b, to find an estimate for the value of P when $t = 20$.

$\boxed{}$, $\boxed{a \approx 6.3}$, $\boxed{b \approx 1.6}$, $\boxed{P \approx 760}$



START BY "LINEARIZING" THE EQUATION

$\Rightarrow P = at^b$

$\Rightarrow \log_{10} P = \log_{10}(at^b)$

$\Rightarrow \log_{10} P = \log_{10} a + \log_{10} t^b$

$\Rightarrow \log_{10} P = \log_{10} a + b \log_{10} t$

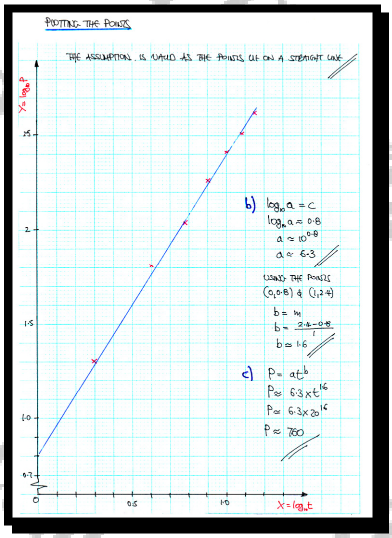
$\Rightarrow \underbrace{\log_{10} P}_{Y} = \underbrace{b}_{m}(\underbrace{\log_{10} t}_{X}) + \underbrace{\log_{10} a}_{C}$

FROM THE ABOVE EQUATION IT IS EVIDENT THAT WE NEED TO PLOT

$X = \log_{10} t$ AGAINST $Y = \log_{10} P$

$X = \log_{10} t$	0.30	0.60	0.78	0.90	1.00	1.08	1.15
$Y = \log_{10} P$	1.30	1.81	2.04	2.26	2.41	2.51	2.62

P.T.O



Question 5 (**)

The table below shows experimental data connecting two variables t and H.

t	5	10	20	40	50
H	4.1	8.5	18.0	42.0	50.0

It is assumed that t and H are related by an equation of the form

$$H = kt^n ,$$

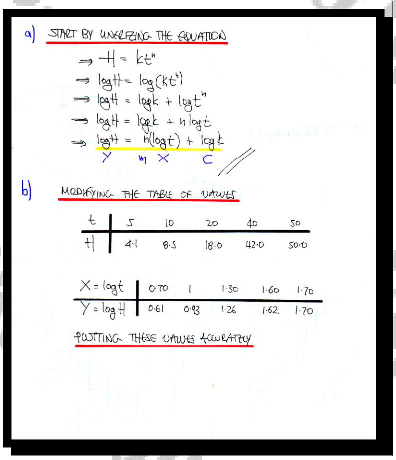
where k and n are non zero constants.

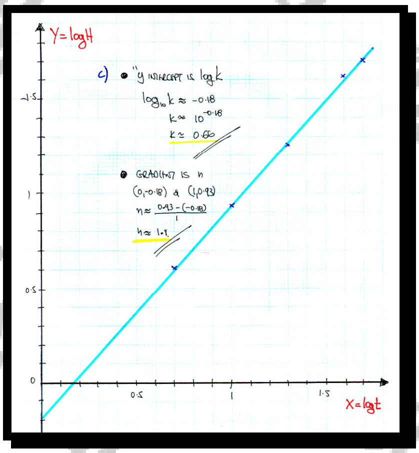
a) Find an equation of a straight line, in terms of well defined constants, in order to investigate the validity of this assumption.

b) Plot a suitable graph to show that the assumption of part (**a**) is valid.

c) Use the graph to estimate, correct to 2 significant figures, the value of k and the value of n.

$\boxed{}$, $\boxed{\log H = n \log t + \log k}$, $\boxed{k \approx 0.66}$, $\boxed{n \approx 1.1}$





Question 6 (+)**

The table below shows experimental data connecting two variables x and y.

$\boldsymbol{x}$	5	10	15	20	25
$\boldsymbol{y}$	57	73	96	135	175

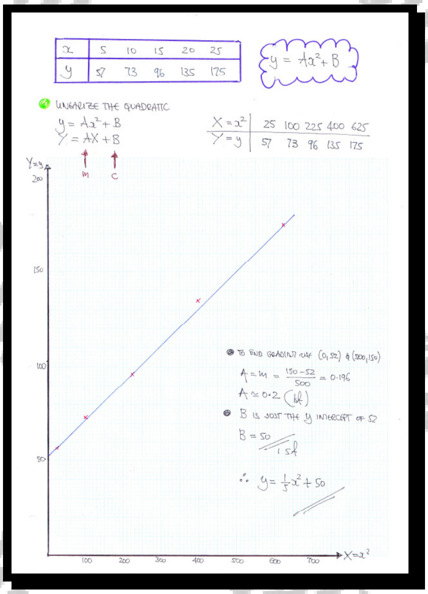
It is assumed that x and y are related by an equation of the form

$$y = Ax^2 + B,$$

where A and B are non zero constants.

By plotting accurately the equation of a suitable straight line, estimate correct to 1 significant figure the value of A and the value of B.

$\boxed{A \approx 0.2}$, $\boxed{B \approx 50}$



Question 7 (**+)

The table below shows experimental data connecting two variables x and y.

x	6	10	12	15	16
y	6	34	46	85	92

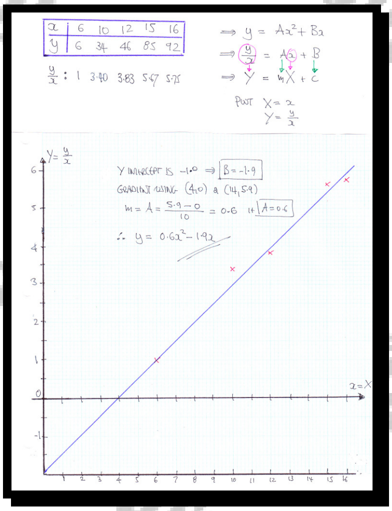
It is assumed that x and y are related by an equation of the form

$$y = Ax^2 + Bx,$$

where A and B are non zero constants.

By plotting accurately the equation of a suitable straight line, estimate correct to 1 decimal place the value of A and the value of B.

$\boxed{A \approx 0.6}$, $\boxed{B \approx -1.9}$



Question 8 (+)**

The table below shows experimental data connecting two variables x and y.

x	4	6	10	12	14
y	66	36	22	20	17

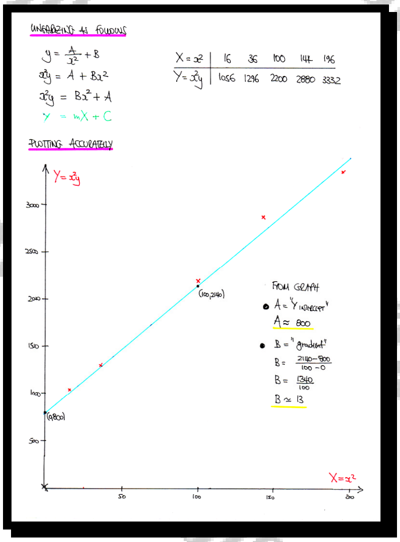
It is assumed that x and y are related by an equation of the form

$$y = \frac{A}{x^2} + B,$$

where A and B are non zero constants.

By plotting accurately the equation of a suitable straight line, estimate correct to 2 significant figures the value of A and the value of B.

$\boxed{}$, $\boxed{A \approx 800}$, $\boxed{B \approx 13}$



Question 9 (**+)

The variables x and y are thought to obey a law of the form

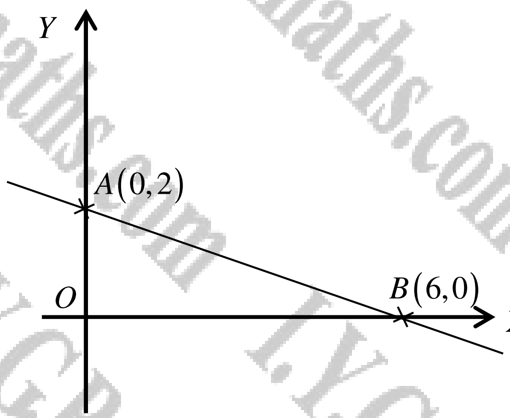
$$y = ax^n,$$

where a and n are non zero constants.

Let $X = \log_{10} x$ and $Y = \log_{10} y$.

a) Show there is a linear relationship between X and Y.

The figure below shows the graph of Y against X.



b) Determine the value of a and the value of n.

$\boxed{}$, $\boxed{n = -\frac{1}{3}}$, $\boxed{a = 100}$

a) "TAKING LOGS", BASE 10, FOR THE GIVEN EQUATION

$\Rightarrow y = ax^n$

$\Rightarrow \log_{10} y = \log_{10}(ax^n)$

$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} x^n$

$\Rightarrow \log_{10} y = \log_{10} a + n\log_{10} x$

$\Rightarrow \log_{10} y = n(\log_{10} x) + (\log_{10} a)$

$Y = m \cdot X + C$

$\therefore$ A LINEAR RELATIONSHIP INDEED

b) LOOKING AT THE y INTERCEPT, A(0,2)

$\Rightarrow \log_{10} a = 2$

$\Rightarrow a = 10^2$

$\Rightarrow a = 100$

LOOKING AT THE GRADIENT

$\Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = n$

$\Rightarrow \frac{0-2}{6-0} = n$

$\Rightarrow n = -\frac{1}{3}$

Question 10 (**+)

The variables x and y are thought to obey a law of the form

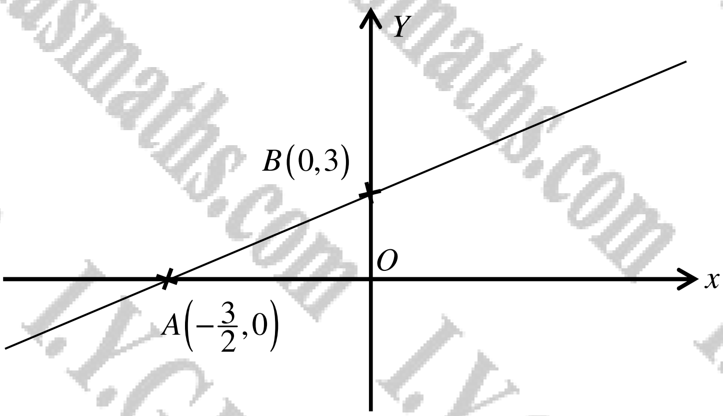
$$y = a \times k^x,$$

where a and k are positive constants.

Let $Y = \log_{10} y$.

a) Show there is a linear relationship between x and Y.

The figure below shows the graph of Y against x.



b) Determine the value of a and the value of k.

$\boxed{}$, $\boxed{a = 1000}$, $\boxed{k = 100}$

a) "TAKING LOGS", BASE 10, FOR THE EQUATION

$\Rightarrow y = a \times k^x$

$\Rightarrow \log_{10} y = \log_{10}(a \times k^x)$

$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} k^x$

$\Rightarrow \log_{10} y = \log_{10} a + x\log_{10} k$

$\Rightarrow \log_{10} y = (\log_{10} k)x + (\log_{10} a)$

$Y = m \ x \ c$

$\therefore$ A LINEAR RELATIONSHIP INDEED

b) LOOKING AT THE y INTERCEPT, $B(0,3)$

$\Rightarrow \log_{10} a = 3$

$\Rightarrow a = 10^3$

$\Rightarrow a = 1000$

LOOKING AT THE GRADIENT OF THE LINE THROUGH $A(-\frac{3}{2},0)$ & $(0,3)$

$\Rightarrow$ "$\frac{y_2 - y_1}{x_2 - x_1}$" $= \log_{10} k$

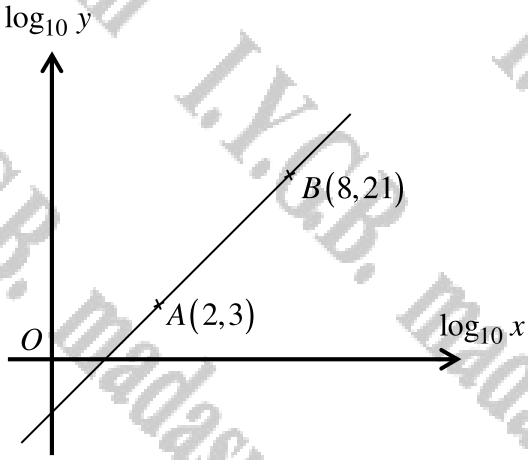
$\Rightarrow \frac{3-0}{0-(-\frac{3}{2})} = \log_{10} k$

$\Rightarrow 2 = \log_{10} k$

$\Rightarrow k = 10^2$

$\Rightarrow k = 100$

Question 11 (***)



The figure above shows a set of axes where $\log_{10} y$ is plotted against $\log_{10} x$.

A straight line passes through the points $A(2,3)$ and $B(8,21)$.

Express y in terms of x.

$\boxed{}$, $\boxed{y = \frac{1}{1000}x^3}$

LET $X = \log_{10} x$ & $Y = \log_{10} y$

$\Longrightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{21-3}{8-2} = \frac{18}{6} = 3$

$\Longrightarrow Y - Y_0 = m(X - X_0)$

$Y - 3 = 3(X - 2)$

$Y - 3 = 3X - 6$

$Y = 3X - 3$

REVERSING THE SUBSTITUTIONS

$\Rightarrow \log_{10} y = 3\log_{10} x - 3$

$\Rightarrow \log_{10} y = \log_{10} x^3 - 3$

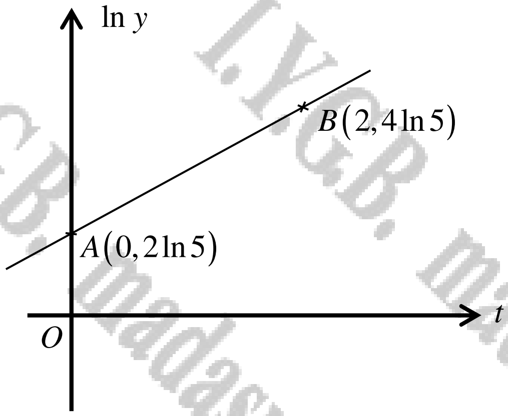
$\Rightarrow y = 10^{\log_{10} x^3 - 3}$

$\Rightarrow y = 10^{\log_{10} x^3} \times 10^{-3}$

$\Rightarrow y = x^3 \times \frac{1}{1000}$

$\Rightarrow y = \frac{x^3}{1000}$

Question 12 (***)

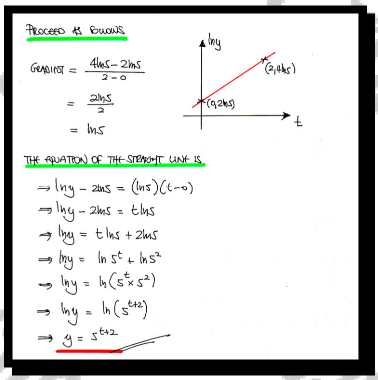


The figure above shows a set of axes where $\ln y$ is plotted against t.

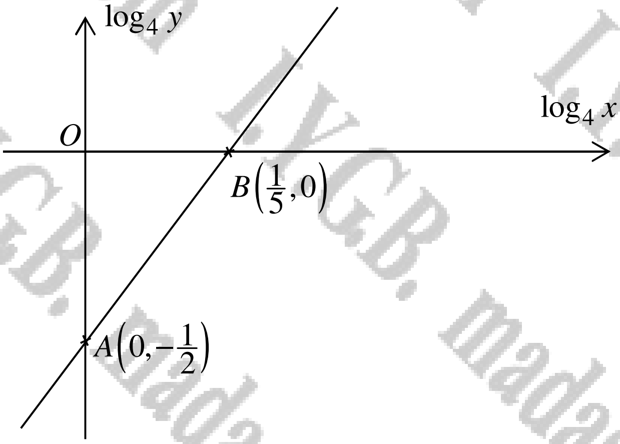
A straight line passes through the points $A(0, 2\ln 5)$ and $B(2, 4\ln 5)$.

Express y in terms of t.

$\boxed{}$, $\boxed{y = 5^{t+2}}$



Question 13 (***)



The figure above shows a set of axes where $\log_4 y$ is plotted against $\log_4 x$.

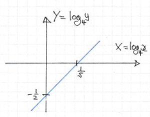
A straight line passes through the points $A\left(0,-\frac{1}{2}\right)$ and $B\left(\frac{1}{5},0\right)$.

Find a relationship, not involving logarithms between x and y.

$\boxed{}$, $\boxed{4y^2 = x^5}$

STARTING FROM THE GRAPH

$$m = \frac{\Delta y}{\Delta x} = \frac{0-\left(-\frac{1}{2}\right)}{\frac{1}{5}-0} = \frac{\frac{1}{2}}{\frac{1}{5}} = \frac{5}{2}$$



$$\therefore\ Y = \frac{5}{2}X - \frac{1}{2}$$

$$2Y = 5X - 1$$

$$2\log_4 y = 5\log_4 x - 1$$

$$\log_4 y^2 = \log_4 x^5 - \log_4 4$$

$$\log_4 y^2 = \log_4\left(\frac{x^5}{4}\right)$$

$$y^2 = \frac{x^5}{4}$$

$$4y^2 = x^5$$

Question 14 (***)

In each of the following equations x and y are variables, and A, B and k are non zero constants.

a) $y = Ax^2 + Bx$.

b) $y = \dfrac{A}{B+x}$.

c) $y = A\mathrm{e}^{kx} + x$.

d) $x^2\left(y^2 - A\right) = B$.

Express each of these equations in "straight line form" and state

- … the variables to be plotted in the x and y axis.
- … the gradient and the y intercept of the straight line.

$$\boxed{\begin{array}{l} X = x \\ Y = \dfrac{y}{x} \\ m = A \\ c = B \end{array}}, \quad \boxed{\begin{array}{l} X = x \\ Y = \dfrac{1}{y} \\ m = \dfrac{1}{A} \\ c = \dfrac{B}{A} \end{array}}, \quad \boxed{\begin{array}{l} X = x \\ Y = \ln\left(y - x\right) \\ m = k \\ c = \ln A \end{array}}, \quad \boxed{\begin{array}{l} X = \dfrac{1}{x^2} \\ Y = y^2 \\ m = B \\ c = A \end{array}}$$

a) $y = Ax^2 + Bx$

$\Rightarrow \dfrac{y}{x} = \underset{m}{A}x + \underset{c}{B}$

PLOT $X = x$

$Y = \dfrac{y}{x}$

b) $y = \dfrac{A}{B+x}$

$\Rightarrow \dfrac{1}{y} = \dfrac{x+B}{A}$

$\Rightarrow \dfrac{1}{y} = \underset{m}{\dfrac{1}{A}}x + \underset{c}{\dfrac{B}{A}}$

PLOT $Y = \dfrac{1}{y}$

$X = x$

c) $y = Ae^{kx} + x$

$\Rightarrow y - x = Ae^{kx}$

$\Rightarrow \ln(y-x) = \ln\left[Ae^{kx}\right]$

$\Rightarrow \ln(y-x) = \ln A + \ln e^{kx}$

$\Rightarrow \ln(y-x) = \ln A + kx$

$\Rightarrow \ln(y-x) = \underset{m}{k}x + \underset{c}{\ln A}$

PLOT $Y = \ln(y-x)$

$X = x$

d) $x^2(y^2 - A) = B$

$\Rightarrow y^2 - A = \dfrac{B}{x^2}$

$\Rightarrow y^2 = \dfrac{B^{m}}{x^2} + \underset{c}{A}$

LET $X = \dfrac{1}{x^2}$

$Y = y^2$

Question 15 (***)

The following table shows some experimental data.

x	2	4	6	7	10	12
y	66	36	34	30	34	34

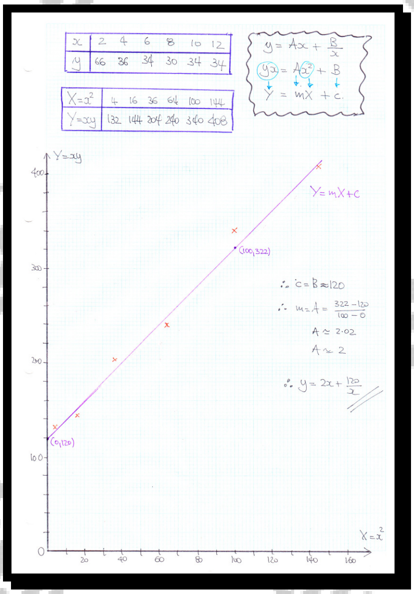
It is assumed that x and y are related by an equation of the form

$$y = Ax + \frac{B}{x},$$

where A and B are non zero constants.

By plotting accurately the equation of a suitable straight line estimate the value of A and the value of B.

$\boxed{A \approx 2}$, $\boxed{B \approx 120}$



Question 16 (***)

The following table shows some experimental data.

x	1	2	3	4	5	6	7
y	420	218	158	137	134	142	158

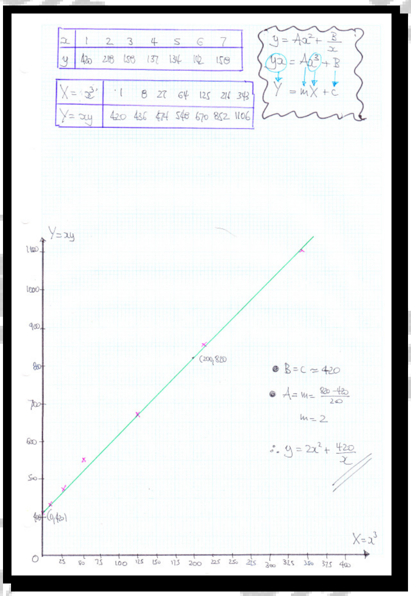
It is assumed that x and y are related by an equation of the form

$$y = Ax^2 + \frac{B}{x},$$

where A and B are non zero constants.

By plotting accurately the equation of a suitable straight line estimate the value of A and the value of B.

$\boxed{A \approx 2}$, $\boxed{B \approx 420}$



Question 17 (***)

The following table shows some experimental data.

x	2	4	6	7	10	12
y	1.6	3.2	4.2	5.0	5.6	6.2

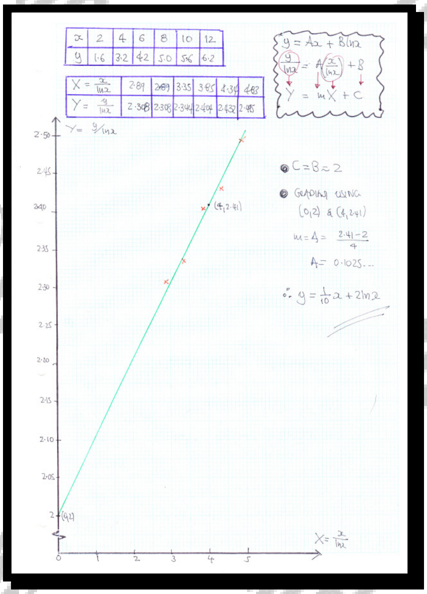
It is assumed that x and y are related by an equation of the form

$$y = Ax + B\ln x,$$

where A and B are non zero constants.

By plotting accurately the equation of a suitable straight line estimate the value of A and the value of B.

$\boxed{A \approx 0.10}$, $\boxed{B \approx 2.0}$



Question 18 (***)

A financial advisor wants to model the annual growth of a certain investment, based on the growth of this investment in the past seven years.

***n*, number of years**	1	2	3	4	5	6	7
***V*, in £1000**	44	48	55	63	67	75	82

He assumes the formula

$$V = 40\left(1+\frac{r}{100}\right)^n ,$$

where r represent the **constant** annual percentage growth and n represents the number of full years that elapsed since the start of the investment.

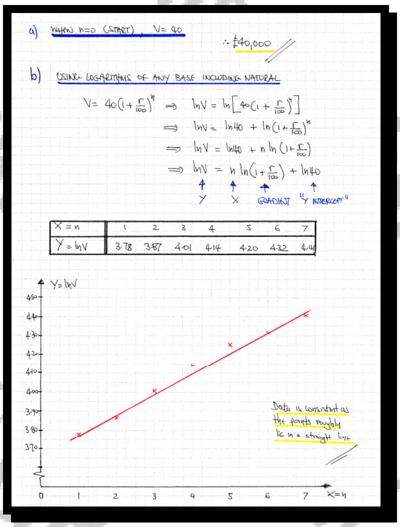
a) State the initial value of this investment.

b) Show that the data is consistent with his assumption by using a graphical method, involving logarithms.

c) Determine an estimate for the annual percentage growth of this investment, correct to two significant figures.

d) Estimate the value of this investment after 10 years, briefly commenting on the reliability of this estimate.

$\boxed{}$, $\boxed{£40,000}$, $\boxed{r \approx 11}$, $\boxed{£114,000}$



a) WHEN $n=0$ (START), $V=40$

$\therefore$ £40,000

b) USING LOGARITHMS OF ANY BASE INCLUDING NATURAL

$V = 40\left(1+\frac{r}{100}\right)^n \Rightarrow \ln V = \ln\left[40\left(1+\frac{r}{100}\right)^n\right]$

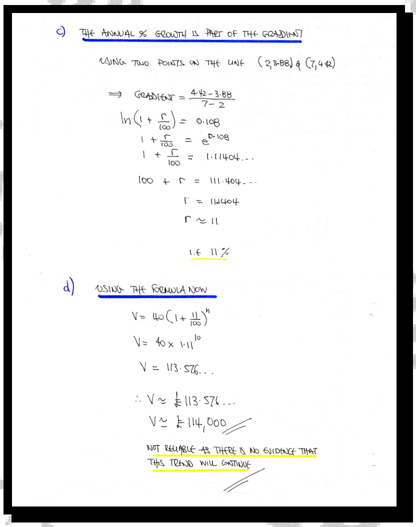
$\Rightarrow \ln V = \ln 40 + \ln\left(1+\frac{r}{100}\right)^n$

$\Rightarrow \ln V = \ln 40 + n\ln\left(1+\frac{r}{100}\right)$

$\Rightarrow \underbrace{\ln V}_{Y} = \underbrace{n}_{X}\underbrace{\ln\left(1+\frac{r}{100}\right)}_{\text{GRADIENT}} + \underbrace{\ln 40}_{\text{"Y INTERCEPT"}}$

$X=n$	1	2	3	4	5	6	7
$Y=\ln V$	3.78	3.87	4.01	4.14	4.20	4.32	4.41

Data is consistent as the points roughly lie in a straight line



c) THE ANNUAL % GROWTH IS PART OF THE GRADIENT

USING TWO POINTS ON THE LINE $(2, 3.88)$ & $(7, 4.42)$

$\Rightarrow$ GRADIENT $= \frac{4.42 - 3.88}{7-2}$

$\ln\left(1+\frac{r}{100}\right) = 0.108$

$1 + \frac{r}{100} = e^{0.108}$

$1 + \frac{r}{100} = 1.11404\ldots$

$100 + r = 111.404\ldots$

$r = 11.1404$

$r \approx 11$

i.e. 11%

d) USING THE FORMULA NOW

$V = 40\left(1+\frac{11}{100}\right)^n$

$V = 40 \times 1.11^{10}$

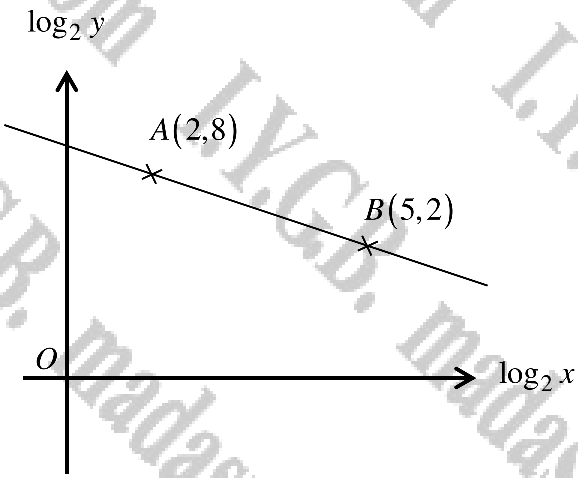
$V = 113.576\ldots$

$\therefore V \approx$ £113.576...

$V \approx$ £114,000

NOT RELIABLE AS THERE IS NO EVIDENCE THAT THIS TREND WILL CONTINUE

Question 19 (***+)

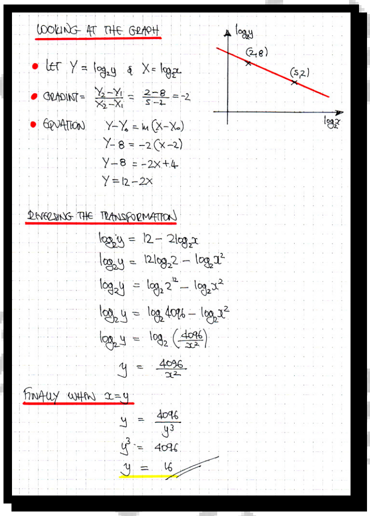


The figure above shows a set of axes where $\log_2 y$ is plotted against $\log_2 x$.

A straight line passes through the points $A(2,8)$ and $B(5,2)$.

Determine the value of y at the point where $y = x$.

$\boxed{y = 16}$



LOOKING AT THE GRAPH

- LET $Y = \log_2 y$ & $X = \log_2 x$
- GRADIENT $= \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{2-8}{5-2} = -2$
- EQUATION $Y - Y_0 = m(X - X_0)$

$Y - 8 = -2(X-2)$

$Y - 8 = -2X + 4$

$Y = 12 - 2X$

REVERSING THE TRANSFORMATION

$\log_2 y = 12 - 2\log_2 x$

$\log_2 y = 12\log_2 2 - \log_2 x^2$

$\log_2 y = \log_2 2^{12} - \log_2 x^2$

$\log_2 y = \log_2 4096 - \log_2 x^2$

$\log_2 y = \log_2\left(\frac{4096}{x^2}\right)$

$y = \frac{4096}{x^2}$

FINALLY WHEN $x = y$

$y = \frac{4096}{y^3}$

$y^3 = 4096$

$y = 16$

Question 20 (***+)

The following table shows some experimental data.

x	2	4	6	7	10	12
y	0.51	0.54	0.59	0.68	0.86	1.25

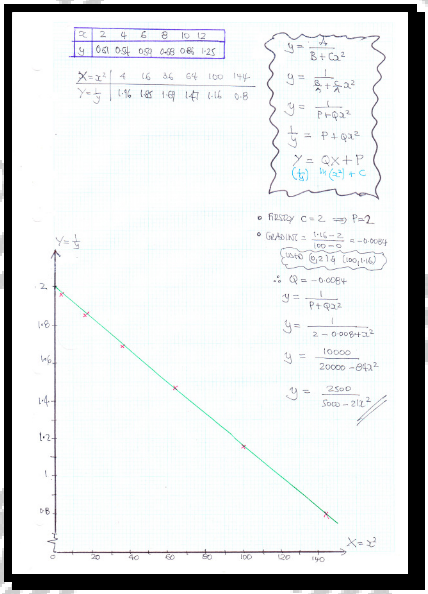
It is assumed that x and y are related by an equation of the form

$$y = \frac{A}{B + Cx^2},$$

where A, B and C are non zero constants.

By plotting accurately the equation of a suitable straight line, estimate correct to 2 significant figures the value of A, B and C.

$\boxed{A \approx 2500}$, $\boxed{B \approx 5000}$, $\boxed{C \approx -21}$



Question 21 (*****)

The table below shows experimental data connecting two variables x and y.

$\boldsymbol{t}$	5	10	15	30	70
$\boldsymbol{P}$	181	158	145	127	107

It is assumed that t and P are related by an equation of the form

$$P = A \times t^k ,$$

where A and k are non zero constants.

By linearizing the above equation, and using partial differentiation to obtain a line of least squares determine the value of A and the value of k.

$\boxed{A \approx 250}$, $\boxed{k \approx -0.2}$

t	5	10	15	30	70
P	181	158	145	127	107

$P = At^k$

$\ln P = \ln(At^k) = \ln A + \ln t^k$

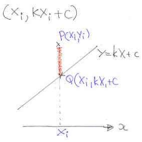
$\ln P = k\ln t + \ln A$

$Y = kX + C$

CONSIDER THE VERTICAL DISTANCE $|PQ|$ FROM THE POINT $P(X_i, Y_i)$, $i = 1,2,3,4,5$ & $Q(X_i, kX_i + C)$

$|PQ| = \sqrt{(Y_i - kX_i - c)^2}$

$|PQ|^2 = (Y_i - kX_i - c)^2$



LET T BE THE TOTAL OF SUCH SQUARED DISTANCES

$T = \sum_{i=1}^{5} (Y_i - kX_i - c)^2$

DIFFERENTIATE FOR MINIMIZING, NOTING X_i & Y_i ARE CONSTANTS

$\frac{\partial T}{\partial k} = \sum_{i=1}^{5} -2X_i(Y_i - kX_i - c)$

$\frac{\partial T}{\partial c} = \sum_{i=1}^{5} -2(Y_i - kX_i - c)$

SOLVE FOR ZERO

$$\Rightarrow \begin{cases} \cancel{-2}\sum_{i=1}^{5}\left[X_iY_i - kX_i^2 - cX_i\right] = 0 \\ \cancel{-2}\sum_{i=1}^{5}\left[Y_i - kX_i - c\right] = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \sum_{i=1}^{5} X_iY_i - k\sum_{i=1}^{5} X_i^2 - c\sum_{i=1}^{5} X_i = 0 & \times 5 \\ \sum_{i=1}^{5} Y_i - k\sum_{i=1}^{5} X_i - c\underbrace{\sum_{i=1}^{5} 1}_{=5} = 0 & \times \sum_{i=1}^{5} X_i \end{cases}$$

$$\Rightarrow \begin{cases} 5\sum_{i=1}^{5} X_iY_i - 5k\sum_{i=1}^{5} X_i^2 - 5c\sum_{i=1}^{5} X_i = 0 \\ \sum_{i=1}^{5} X_i \sum_{i=1}^{5} Y_i - k\sum_{i=1}^{5} X_i \sum_{i=1}^{5} X_i - 5c\sum_{i=1}^{5} X_i = 0 \end{cases}$$

SUBTRACT

$\Rightarrow 5\sum_{i=1}^{5} X_iY_i - \sum_{i=1}^{5} X_i \sum_{i=1}^{5} Y_i - 5k\sum_{i=1}^{5} X_i^2 + k\sum_{i=1}^{5} X_i X_i = 0$

$\Rightarrow 5\sum_{i=1}^{5} X_iY_i - \sum_{i=1}^{5} X_i \sum_{i=1}^{5} Y_i = k\left[5\sum_{i=1}^{5} X_i^2 - \sum_{i=1}^{5} X_i \sum_{i=1}^{5} X_i\right]$

$$\Rightarrow k = \frac{5\sum_{i=1}^{5} X_iY_i - \sum_{i=1}^{5} X_i \sum_{i=1}^{5} Y_i}{5\sum_{i=1}^{5} X_i^2 - \sum_{i=1}^{5} X_i \sum_{i=1}^{5} X_i}$$

& $5c = \sum_{i=1}^{5} Y_i - k\sum_{i=1}^{5} X_i$

$$c = \frac{1}{5}\sum_{i=1}^{5} Y_i - \frac{k}{5}\sum_{i=1}^{5} X_i$$

HENCE

$X = \ln t$	$\ln 5$	$\ln 10$	$\ln 15$	$\ln 30$	$\ln 70$
$Y = \ln P$	$\ln 181$	$\ln 158$	$\ln 145$	$\ln 127$	$\ln 107$

$\sum_{i=1}^{5} X = 14.270$ $\quad \sum_{i=1}^{5} X^2 = 44.844$

$\sum_{i=1}^{5} Y = 24.755$ $\quad \sum_{i=1}^{5} XY = 69.829$

$k = \frac{5 \times 69.829 - 14.270 \times 24.755}{5 \times 44.844 - 14.270 \times 14.270} = -0.1996\ldots \simeq \boxed{-0.2}$

$c = \frac{1}{5}(24.755) - \frac{-0.1996\ldots}{5} \times 14.270 = 5.5206\ldots$

$\therefore A = e^{5.5206\ldots}$

$A \simeq 249.79\ldots \simeq \boxed{250}$

$\therefore P = 250 \times t^{-0.2}$