KINEMATICS & DYNAMICS

(SIMPLE DIFFERENTIAL EQUATIONS)

Question 1 **(**)**

A toy of mass 0.5 kg, is placed on a slope.

The toy is set in motion down the slope with an initial speed 2 ms^{-1}.

The resultant force acting on the particle has magnitude $(4-v) \text{ N}$, where $v \text{ ms}^{-1}$ is the speed of the toy at time t s after it was set in motion.

By modelling the toy as a particle, determine the value of t when the speed of the toy reaches 3 ms^{-1}.

$$\boxed{t = \frac{1}{2}\ln 2 \approx 0.347 \text{ s}}$$

$$m\ddot{x} = (4-v)$$
$$\Rightarrow \frac{1}{2}\ddot{x} = 4-v$$
$$\Rightarrow \frac{1}{2}\frac{dv}{dt} = 4-v$$
$$\Rightarrow \frac{1}{4-v}\,dv = 2\,dt$$
$$\Rightarrow \int_{v=2}^{v=} \frac{1}{4-v} = \int_{t=0} 2\,dt$$
$$\Rightarrow \Big[-\ln|4-v|\Big]_2^3 = \Big[2t\Big]_0^t$$
$$\Rightarrow \Big[\ln|4-v|\Big]_2^3 = \Big[-2t\Big]_0^t$$
$$\Rightarrow \cancel{\ln 1} - \ln 2 = -2t$$
$$\Rightarrow t = \frac{1}{2}\ln 2$$
$$\Rightarrow t \approx 0.347$$

Question 2 (**)

A particle P is moving on the positive x axis with acceleration of magnitude

$$\frac{25}{x+1}\ \text{ms}^{-2},$$

acting away from the origin O.

When the displacement of P from O is x m, its speed is v ms^{-1}.

Given that P passes through O with speed 5 ms^{-1}, calculate the distance of P from O when its speed reaches 15 ms^{-1}.

$\boxed{d \approx 53.598\text{... m}}$

$$\boxed{\ddot{x} = \frac{25}{x+1}}$$

$$\Rightarrow v\frac{dv}{dx} = \frac{25}{x+1}$$

$$\Rightarrow v\,dv = \frac{25}{x+1}\,dx$$

$$\Rightarrow \int_{v=5}^{15} v\,dv = \int_{x=0}^{x} \frac{25}{x+1}\,dx$$

$$\Rightarrow \left[\tfrac{1}{2}v^2\right]_5^{15} = \left[25\ln|x+1|\right]_0^x$$

$$\Rightarrow \frac{225}{2} - \frac{25}{2} = 25\ln|x+1| - \cancel{25\ln 1}$$

$$\Rightarrow 100 = 25\ln|x+1|$$

$$\Rightarrow 4 = \ln|x+1|$$

$$\Rightarrow e^4 = x+1$$

$$\Rightarrow x = e^4 - 1 \simeq 53.60\text{m}$$

Question 3 (**)

A particle P, of mass 2 kg, is moving on the positive x axis.

At time t s, its displacement from the origin O is x m and its velocity is v ms^{-1}.

When $x = 1$, $v = 1$.

The resultant force acting on P has magnitude

$$\frac{2}{2x+1} \text{ N},$$

acting away from O.

a) Find an expression for v^2 in terms of x.

b) Find the distance of the particle from O when its speed is 2.739 ms^{-1}.

$$\boxed{v^2 = 1 + \ln\left(\frac{2x+1}{3}\right)}, \quad \boxed{d \approx 999\tfrac{1}{3} \text{ m}}$$

(a) $\ddot{x}$; 2 ; $\frac{2}{2x+1}$; $t=0$, $x=1$, $v=1$

$\Rightarrow m\ddot{x} = \frac{2}{2x+1}$

$\Rightarrow 2v\frac{dv}{dx} = \frac{2}{2x+1}$

$\Rightarrow 2v\,dv = \frac{2}{2x+1}\,dx$

$\Rightarrow \int_{v=1}^{v} 2v\,dv = \int_{x=1}^{x} \frac{2}{2x+1}\,dx$

$\Rightarrow \left[v^2\right]_1^v = \left[\ln(2x+1)\right]_1^x$

$\Rightarrow v^2 - 1 = \ln(2x+1) - \ln 3$

$\Rightarrow v^2 - 1 = \ln\left(\frac{2x+1}{3}\right)$

$\Rightarrow v^2 = 1 + \ln\left(\frac{2x+1}{3}\right)$

(b) when $v = 2.739$

$\Rightarrow 2.739^2 = 1 + \ln\left(\frac{2x+1}{3}\right)$

$\Rightarrow \ln\left(\frac{2x+1}{3}\right) = 6.502121$

$\Rightarrow \frac{2x+1}{3} = e^{6.502121}$

$\Rightarrow x \simeq 999.33\ldots \simeq 999\frac{1}{3}$ m

Question 4 **(**)**

A particle P is moving on the positive x axis with acceleration of magnitude

$$\frac{4}{5}\mathrm{e}^{-0.1x}\ \mathrm{ms}^{-2},$$

acting away from the origin O.

When the displacement of P from O is x m, its velocity is v ms^{-1}.

When $x=0$, $v=2$.

a) Find an expression for v^2 in terms of x.

b) Determine the exact value of x when $v=4$.

$\boxed{v^2 = 20-16\mathrm{e}^{-0.1x}}$, $\boxed{x = 20\ln 2}$

a) $\boxed{\ddot{x} = \frac{4}{5}e^{-0.1x}}$

$\Rightarrow v\frac{dv}{dx} = \frac{4}{5}e^{-0.1x}$

$\Rightarrow v\,dv = \frac{4}{5}e^{-0.1x}\,dx$

$\Rightarrow \int_{v=2}^{v} v\,dv = \int_{x=0}^{x} \frac{4}{5}e^{-0.1x}\,dx$

$\Rightarrow \left[\frac{1}{2}v^2\right]_2^v = \left[-8e^{-0.1x}\right]_0^x$

$\Rightarrow \frac{1}{2}v^2 - 2 = -8e^{-0.1x} + 8$

$\Rightarrow v^2 - 4 = -16e^{-0.1x} + 16$

$\Rightarrow v^2 = 20 - 16e^{-0.1x}$

b) when $v=4$

$\Rightarrow 4^2 = 20 - 16e^{-0.1x}$

$\Rightarrow 16e^{-0.1x} = 4$

$\Rightarrow e^{-0.1x} = \frac{1}{4}$

$\Rightarrow e^{0.1x} = 4$

$\Rightarrow 0.1x = \ln 4$

$\Rightarrow \frac{1}{10}x = 2\ln 2$

$\Rightarrow x = 20\ln 2$

Question 5 (**)

A particle P is moving on the x axis with acceleration of magnitude

$$2x - \frac{3}{2}x^2 \text{ ms}^{-2},$$

acting away from the origin O.

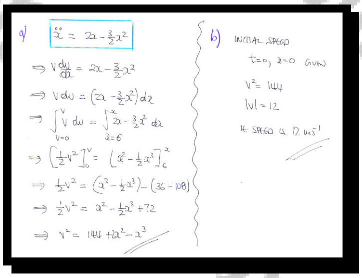
When the displacement of P from O is x m, its velocity is v ms^{-1}.

P passes through O when $t = 0$ and comes to instantaneous rest at $x = 6$.

a) Find an expression for v^2 in terms of x.

b) Determine the initial speed of P.

$\boxed{v^2 = 144 + 2x^2 - x^3}$, $\boxed{\text{speed} = 12 \text{ ms}^{-1}}$



Question 6 (**+)

A particle P of mass 2 kg, is moving on a horizontal x axis, in the positive x direction. The only force acting on P is F, which acts in the positive x direction.

The magnitude of F is

$$1+4\sqrt[3]{t},$$

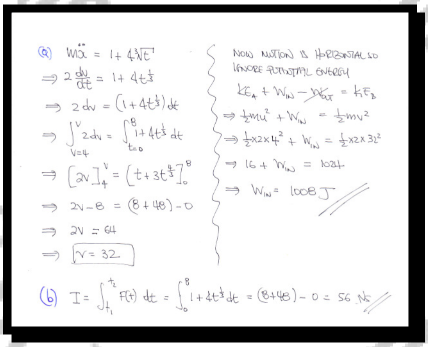
where t is the time in seconds since the particle had velocity 4 ms^{-1}.

For the interval $0 \le t \le 8$, determine …

a) … the work done by F.

b) … the impulse of F.

$\boxed{W_{\text{in}} = 1008 \text{ J}}$, $\boxed{I = 56 \text{ Ns}}$



(a) $m\ddot{x} = 1 + 4\sqrt[3]{t}$

$\Rightarrow 2\frac{dv}{dt} = 1 + 4t^{\frac{1}{3}}$

$\Rightarrow 2\,dv = (1+4t^{\frac{1}{3}})\,dt$

$\Rightarrow \int_{v=4}^{v} 2\,dv = \int_{t=0}^{8} 1 + 4t^{\frac{1}{3}}\,dt$

$\Rightarrow [2v]_4^v = [t + 3t^{\frac{4}{3}}]_0^8$

$\Rightarrow 2v - 8 = (8+48) - 0$

$\Rightarrow 2v = 64$

$\Rightarrow \boxed{v = 32}$

NOW MOTION IS HORIZONTAL SO IGNORE POTENTIAL ENERGY

$KE_A + W_{IN} - \cancel{W_{OUT}} = KE_B$

$\Rightarrow \frac{1}{2}mu^2 + W_{IN} = \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}\times 2\times 4^2 + W_{IN} = \frac{1}{2}\times 2\times 32^2$

$\Rightarrow 16 + W_{IN} = 1024$

$\Rightarrow W_{IN} = 1008 \text{ J}$

(b) $I = \int_{t_1}^{t_2} F(t)\,dt = \int_0^8 1 + 4t^{\frac{1}{3}}\,dt = (8+48) - 0 = 56 \text{ Ns}$

Question 7 (**+)

A particle P starts from rest and moves on the x axis with acceleration of magnitude

$$\frac{60}{(t+3)^2} \text{ N},$$

acting in the direction of x increasing.

a) Find an expression for the velocity of P, in terms of t.

b) Show that the distance covered by P in the first 6 s of its motion is

$$60(2 - \ln 3) \text{ m}.$$

$\boxed{}$, $\boxed{v = 20 - \frac{60}{t+3}}$

a) STANDARD METHODOLOGY USING CALCULUS

$\ddot{x} = \frac{60}{(t+3)^2}$ $\quad\quad t=0,\ v=\dot{x}=0,\ x=0$

ARBITRARY

$\Rightarrow \frac{dv}{dt} = \frac{60}{(t+3)^2}$

$\Rightarrow 1\,dv = \frac{60}{(t+3)^2}\,dt$

$\Rightarrow \int_{v=0}^{v} 1\,dv = \int_{t=0}^{t} 60(t+3)^{-2}\,dt$

$\Rightarrow [v]_0^v = \left[-60(t+3)^{-1}\right]_0^t$

$\Rightarrow v - 0 = \left[\frac{60}{t+3}\right]_t^0$

$\Rightarrow v = 20 - \frac{60}{t+3}$ //

b) CONTINUING BY WRITING $v = dx/dt$

$\Rightarrow \frac{dx}{dt} = 20 - \frac{60}{t+3}$

$\Rightarrow 1\,dx = \left(20 - \frac{60}{t+3}\right)dt$

$\Rightarrow \int_{x=0}^{x} 1\,dx = \int_{t=0}^{t=6} 20 - \frac{60}{t+3}\,dt$

$\Rightarrow [x]_0^x = \left[20t - 60\ln(t+3)\right]_0^6$

$\Rightarrow x - 0 = (120 - 60\ln 9) - (0 - 60\ln 3)$

$\Rightarrow x = 120 - 60\ln 9 + 60\ln 3$

$\Rightarrow x = 120 - 60[\ln 9 - \ln 3]$

$\Rightarrow x = 120 - 60\ln 3$

$\Rightarrow x = 60(2 - \ln 3)$ //

AS REQUIRED

Question 8 (**+)

A particle P of mass 2 kg starts from rest and moves on the x axis.

At time t s, the resultant force acting on P has magnitude

$$\frac{90}{(t+3)^2}\ \text{N},$$

acting in the direction of x increasing.

a) Find an expression for the velocity of P, in terms of t.

b) State the limiting value for the velocity of P.

c) Show that the distance covered by P in the first 6 s of its motion is

$$45(2-\ln 3)\ \text{m}.$$

$\boxed{v = 15 - \frac{45}{t+3}}$, $\boxed{v_{\text{max}} = 15\ \text{ms}^{-1}$}

(a)

$\ddot{x}$

$2 \longrightarrow \frac{90}{(t+3)^2}$

$t=0,\ v=0,\ x=0$ ARBITRARY

$\Rightarrow m\ddot{x} = \frac{90}{(t+3)^2}$

$\Rightarrow 2\frac{dv}{dt} = \frac{90}{(t+3)^2}$

$\Rightarrow 1\,dv = \frac{45}{(t+3)^2}\,dt$

$\Rightarrow \int_{v=0}^{v} 1\,dv = \int_{t=0}^{t} 45(t+3)^{-2}\,dt$

$\Rightarrow [v]_0^v = \left[-45(t+3)^{-1}\right]_0^t$

$\Rightarrow v = \left[\frac{45}{t+3}\right]_t^0$

$\Rightarrow v = 15 - \frac{45}{t+3}$ //

(b) $\therefore v_{\text{MAX}} = 15$ //

(As $t \to \infty$ $v \to 15$)

(c) $v = 15 - \frac{45}{t+3}$

$\Rightarrow \frac{dx}{dt} = 15 - \frac{45}{t+3}$

$\Rightarrow 1\,dx = \left(15 - \frac{45}{t+3}\right)dt$

$\Rightarrow \int_{x=0}^{x} 1\,dx = \int_{t=0}^{t=6} 15 - \frac{45}{t+3}\,dt$

$\Rightarrow [x]_0^x = \left[15t - 45\ln(t+3)\right]_0^6$

$\Rightarrow x = (90 - 45\ln 9) - (-45\ln 3)$

$\Rightarrow x = 90 + 45\ln 3 - 45\ln 9$

$\Rightarrow x = 90 + 45\ln\left(\frac{3}{9}\right)$

$\Rightarrow x = 90 + 45\ln\left(\frac{1}{3}\right)$

$\Rightarrow x = 90 - 45\ln 3$

$\Rightarrow x = 45(2 - \ln 3)$ //

As REQUIRED

Question 9 (***)

A particle P is moving on the x axis, starting from the origin O, and moving in the direction of x increasing with speed 8 ms^{-1}.

The acceleration of P is in the direction of x **decreasing** and has magnitude

$$\tfrac{3}{10}v^{\frac{1}{3}} \text{ ms}^{-2},$$

where v is the velocity of the particle at time t.

Find an expression for the displacement of P in terms of t.

$\boxed{}$, $\boxed{x = 64 - 2\left(4 - \frac{1}{5}t\right)^{\frac{5}{2}}}$

$\boxed{\ddot{x} = -\frac{3}{10}v^{\frac{1}{3}}, \quad t=0, \quad v=8, \quad x=0}$

STARTING BY OBTAINING AN EXPRESSION FOR $v = f(t)$

$\Rightarrow \frac{dv}{dt} = -\frac{3}{10}v^{\frac{1}{3}}$

$\Rightarrow 1\,dv = -\frac{3}{10}v^{\frac{1}{3}}\,dt$

$\Rightarrow \int_{v=8}^{v} v^{-\frac{1}{3}}\,dv = \int_{t=0}^{t} -\frac{3}{10}\,dt$

$\Rightarrow \left[\frac{3}{2}v^{\frac{2}{3}}\right]_8^v = \left[-\frac{3}{10}t\right]_0^t$

$\Rightarrow \frac{3}{2}v^{\frac{2}{3}} - \frac{3}{2}\times 4 = -\frac{3}{10}t - 0$

$\Rightarrow \frac{3}{2}v^{\frac{2}{3}} - 6 = -\frac{3}{10}t$

$\Rightarrow v^{\frac{2}{3}} - 4 = -\frac{2}{10}t$

$\Rightarrow v^{\frac{2}{3}} = 4 - \frac{1}{5}t$

$\Rightarrow \left(v^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}$

$\Rightarrow v = \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}$

FINALLY BY WRITING $v = \frac{dx}{dt}$ OBTAIN AN EXPRESSION FOR $x = g(t)$

$\Rightarrow \frac{dx}{dt} = \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}$

$\Rightarrow 1\,dx = \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}\,dt$

$\Rightarrow \int_{x=0}^{x} 1\,dx = \int_{t=0}^{t} \left(4 - \frac{1}{5}t\right)^{\frac{3}{2}}\,dt$

$\Rightarrow \left[x\right]_0^x = \left[-2\left(4 - \frac{1}{5}t\right)^{\frac{5}{2}}\right]_0^t$

$\Rightarrow x - 0 = \left[2\left(4 - \frac{1}{5}t\right)^{\frac{5}{2}}\right]_t^0$

$\Rightarrow x = 64 - 2\left(4 - \frac{1}{5}t\right)^{\frac{5}{2}}$

Question 10 **(***)**

A particle P, of mass $0.5\ \text{kg}$, is projected horizontally with speed $u\ \text{ms}^{-1}$ from a fixed origin O on a smooth horizontal plane. When P has been moving for t s, its speed is $v\ \text{ms}^{-1}$ and its displacement from O is x m.

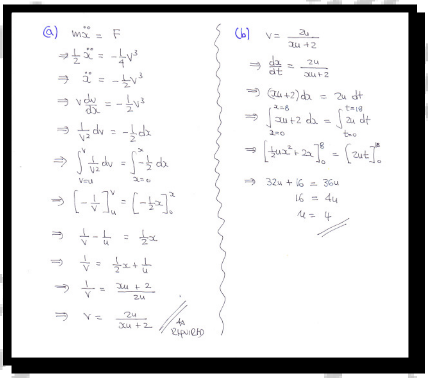
The only force acting on P is a resistive force of magnitude $\frac{1}{4}v^3$.

a) Show clearly that

$$v = \frac{2u}{ux+2}.$$

b) Given further that when $t = 18$, $x = 8$, determine the value of u.

$\boxed{u = 4}$



(a) $m\ddot{x} = F$

$\Rightarrow \frac{1}{2}\ddot{x} = -\frac{1}{4}v^3$

$\Rightarrow \ddot{x} = -\frac{1}{2}v^3$

$\Rightarrow v\frac{dv}{dx} = -\frac{1}{2}v^3$

$\Rightarrow \frac{1}{v^2}\,dv = -\frac{1}{2}\,dx$

$\Rightarrow \int_{v=u}^{v} \frac{1}{v^2}\,dv = \int_{x=0}^{x} -\frac{1}{2}\,dx$

$\Rightarrow \left[-\frac{1}{v}\right]_u^v = \left[-\frac{1}{2}x\right]_0^x$

$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{2}x$

$\Rightarrow \frac{1}{v} = \frac{1}{2}x + \frac{1}{u}$

$\Rightarrow \frac{1}{v} = \frac{xu+2}{2u}$

$\Rightarrow v = \frac{2u}{xu+2}$ // AS REQUIRED

(b) $v = \frac{2u}{xu+2}$

$\Rightarrow \frac{dx}{dt} = \frac{2u}{xu+2}$

$\Rightarrow (xu+2)\,dx = 2u\,dt$

$\Rightarrow \int_{x=0}^{x=8} xu+2\,dx = \int_{t=0}^{t=18} 2u\,dt$

$\Rightarrow \left[\frac{1}{2}ux^2+2x\right]_0^8 = \left[2ut\right]_0^{18}$

$\Rightarrow 32u + 16 = 36u$

$16 = 4u$

$u = 4$ //

Question 11 (***)

A particle of mass 2 kg is moving along the positive x axis under the action of a single force of magnitude F N, which acts along the x axis in the direction of x increasing.

When the particle is x m from the origin O, it is moving away from O with speed

$$\sqrt{8x^{\frac{3}{2}}+1} \text{ ms}^{-1}.$$

Find the value of F when the particle is 9 m away from O.

$\boxed{F = 27}$

$V = \left[6x^{\frac{3}{2}}+1\right]^{\frac{1}{2}}$

$\frac{dv}{dx} = \frac{1}{2}\left(6x^{\frac{3}{2}}+1\right)^{-\frac{1}{2}} \times 9x^{\frac{1}{2}}$

- $m\ddot{x} = F$

$\Rightarrow 2\ddot{x} = F$

$\Rightarrow 2v\frac{dv}{dx} = F$

$\Rightarrow 2\cancel{\left[6x^{\frac{3}{2}}+1\right]^{\frac{1}{2}}} \times \frac{1}{2}\cancel{\left[6x^{\frac{3}{2}}+1\right]^{-\frac{1}{2}}} \times 9x^{\frac{1}{2}} = F$

$\Rightarrow F = 9x^{\frac{1}{2}}$

Hence, when $x = 9$ $\quad F = 9 \times 9^{\frac{1}{2}}$

$F = 27\text{N}$

Question 12 (***)

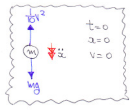
A particle P, of mass 2 kg, is released from rest and falls vertically.

When P has fallen a distance x m it has speed v ms^{-1}.

P is falling under the action of its weight and air resistance of magnitude $\frac{1}{10}v^2$ N.

Show that at the instant when P has fallen through a distance of 20 m its speed is approximately 13 ms^{-1}.

$\boxed{\text{proof}}$



$$m\ddot{x} = mg - \frac{1}{10}v^2$$
$$\Rightarrow 2\ddot{x} = 2g - \frac{1}{10}v^2$$
$$\Rightarrow 20\ddot{x} = 20g - v^2$$
$$\Rightarrow 20v\frac{dv}{dx} = 20g - v^2$$
$$\Rightarrow \frac{20v}{20g - v^2}\,dv = 1\,dx$$
$$\Rightarrow \int_{v=0}^{v} \frac{20v}{20g - v^2}\,dv = \int_{x=0}^{x=20} 1\,dx$$
$$\Rightarrow \int_0^v \frac{-2v}{20g - v^2}\,dv = \int_0^{20} -\frac{1}{10}\,dx$$
$$\Rightarrow \Big[\ln|20g - v^2|\Big]_0^v = \Big[-\frac{1}{10}x\Big]_0^{20}$$
$$\Rightarrow \ln|20g - v^2| - \ln(20g) = -2$$
$$\Rightarrow \ln\left(\frac{20g - v^2}{20g}\right) = -2$$
$$\Rightarrow \frac{20g - v^2}{20g} = e^{-2}$$
$$\Rightarrow 20g - v^2 = 20ge^{-2}$$
$$\Rightarrow v^2 = 20g - 20ge^{-2}$$
$$\Rightarrow v^2 \approx 169.4724\ldots$$
$$\Rightarrow v \approx 13.01$$
$$\Rightarrow v \approx 13 \quad \text{// AS REQUIRED}$$

Question 13 (***)

A particle of mass 0.25 kg, falls vertically from rest, and when the particle has been falling for t s, its speed is v ms^{-1}.

A resistive force of magnitude $\frac{1}{2}v$ is acting on the particle as it falls.

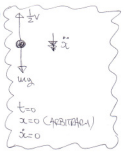
a) Show clearly that

$$v = 4.9\left(1 - e^{-2t}\right).$$

b) Calculate, correct to two decimal places, the distance the particle falls in the first 3 s of its motion.

$\boxed{d \approx 12.26 \text{ m}}$

(a)



$m\ddot{x} = mg - \frac{1}{2}v$

$\Rightarrow \frac{1}{4}\frac{dv}{dt} = \frac{1}{4}g - \frac{1}{2}v$

$\Rightarrow \frac{dv}{dt} = g - 2v$

$\Rightarrow \frac{1}{g-2v}\,dv = 1\,dt$

$\Rightarrow \int_{v=0}^{v} \frac{1}{g-2v} = \int_{t=0}^{t} 1\,dt$

$\Rightarrow \left[-\frac{1}{2}\ln|g-2v|\right]_0^v = \left[t\right]_0^t$

$\Rightarrow \left[\ln|g-2v|\right]_0^v = \left[-2t\right]_0^t$

$\Rightarrow \ln|g-2v| - \ln g = -2t$

$\Rightarrow \ln\left|\frac{g-2v}{g}\right| = -2t$

$\Rightarrow \frac{g-2v}{g} = e^{-2t}$

$\Rightarrow g - 2v = ge^{-2t}$

$\Rightarrow g - ge^{-2t} = 2v$

$\Rightarrow v = \frac{1}{2}g\left(1-e^{-2t}\right)$

$\Rightarrow v = 4.9\left(1-e^{-2t}\right)$ // AS REQUIRED

(b) $\frac{dx}{dt} = 4.9\left(1-e^{-2t}\right)$

$\Rightarrow \int_{x=0}^{x} 1\,dx = \int_{t=0}^{t=3} 4.9\left(1-e^{-2t}\right)dt$

$\Rightarrow \left[x\right]_0^x = \left[4.9\left(t+\frac{1}{2}e^{-2t}\right)\right]_0^3$

$\Rightarrow x = 4.9\left[\left(3+\frac{1}{2}e^{-6}\right)-\left(\frac{1}{2}\right)\right]$

$\Rightarrow x \simeq 12.26$ m //

Question 14 (***)

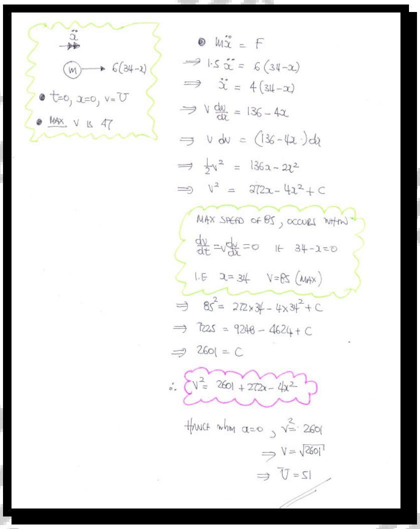
A particle P of mass 1.5 kg is moving along a horizontal x axis.

At time $t = 0$ s, P is passes through the origin O with speed U ms^{-1}, moving in the direction of x increasing.

At time t s, $|OP| = x$ and the resultant force F acting on P has magnitude $6(34 - x)$ N.

Given that for $t > 0$ the greatest speed of P is 85 ms^{-1}, determine the value of U.

$\boxed{U = 51}$


$\ddot{x}$

$m \longrightarrow 6(34-x)$

- $t=0,\ x=0,\ v=U$
- MAX v IS 47

- $m\ddot{x} = F$

$\Rightarrow 1.5\ddot{x} = 6(34-x)$

$\Rightarrow \ddot{x} = 4(34-x)$

$\Rightarrow v\frac{dv}{dx} = 136 - 4x$

$\Rightarrow v\,dv = (136-4x)\,dx$

$\Rightarrow \frac{1}{2}v^2 = 136x - 2x^2$

$\Rightarrow v^2 = 272x - 4x^2 + C$

MAX SPEED OF 85, OCCURS WHEN

$\frac{dv}{dt} = v\frac{dv}{dx} = 0$ IE $34 - x = 0$

I.E $x = 34$ $v = 85$ (MAX)

$\Rightarrow 85^2 = 272\times34 - 4\times34^2 + C$

$\Rightarrow 7225 = 9248 - 4624 + C$

$\Rightarrow 2601 = C$

$\therefore v^2 = 2601 + 272x - 4x^2$

HENCE WHEN $x = 0$, $v^2 = 2601$

$\Rightarrow v = \sqrt{2601}$

$\Rightarrow U = 51$

Question 15 (***+)

A particle P, of mass $\frac{1}{6}$ kg, is moving on a straight horizontal line under the action of the force

$$F = \left(\frac{1}{t}-1\right)\left(\frac{1}{t}+1\right) \text{ N},$$

where t s is the time the particle is in motion, $t > 0$.

The motion of P is resisted by a constant force of magnitude 4 N.

It is further given that when the velocity of P is 18 ms^{-1} its acceleration is 24 ms^{-2}.

Determine the values of t when the velocity of P is 10 ms^{-1}.

$\boxed{}$, $\boxed{t = 1, \ \frac{1}{5}}$

- STARTING WITH THE EQUATION OF MOTION

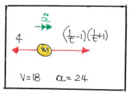
$\Rightarrow m\ddot{x} = \left(\frac{1}{t}-1\right)\left(\frac{1}{t}+1\right) - 4$

$\Rightarrow \frac{1}{6}\ddot{x} = \frac{1}{t^2} - 1 - 4$

$\Rightarrow \frac{1}{6}\ddot{x} = \frac{1}{t^2} - 5$

$\Rightarrow \ddot{x} = \frac{6}{t^2} - 30$

$\Rightarrow \frac{dv}{dt} = \frac{6}{t^2} - 30$



- NOW WHEN $v = 18$, $a = 24$ $\Rightarrow$ $24 = \frac{6}{t^2} - 30$

$54 = \frac{6}{t^2}$

$t^2 = \frac{1}{9}$

$t = +\frac{1}{3}$

- SEPARATING VARIABLES, SUBJECT TO THE CONDITION $t = \frac{1}{3}$, $v = 18$

$\Rightarrow 1 \, dv = \left(\frac{6}{t^2} - 30\right) dt$

$\Rightarrow \int_{v=18}^{10} 1 \, dv = \int_{t=\frac{1}{3}}^{t} \frac{6}{t^2} - 30 \, dt$

$\Rightarrow \left[v\right]_{18}^{10} = \left[-\frac{6}{t} - 30t\right]_{\frac{1}{3}}^{t}$

$\Rightarrow 10 - 18 = \left[\frac{6}{t} + 30t\right]_{t}^{\frac{1}{3}}$

$\Rightarrow -8 = (18 + 10t) - \left(\frac{6}{t} + 30t\right)$

$\Rightarrow -8 = 18 + 10 - \frac{6}{t} - 30t$

$\Rightarrow 0 = 36 - \frac{6}{t} - 30t$ $\quad \div 6$

$\Rightarrow 0 = 6 - \frac{1}{t} - 5t$

$\Rightarrow 0 = 6t - 1 - 5t^2$

$\Rightarrow 5t^2 - 6t + 1 = 0$

$\Rightarrow (5t - 1)(t - 1)$

$\Rightarrow t = \begin{cases} 1 \\ \frac{1}{5} \end{cases}$

Question 16 **(***+)**

A particle P, of mass 0.2 kg, is moving on the positive x axis under the action of a force F which is directed towards the origin O.

The magnitude of F is given by $\frac{k}{x^3}$, where k is the positive constant and x is the distance OP.

P has a speed of 10 ms^{-1}, away from the origin, when $OP = 1$ m and sometime later it has a speed of 1 ms^{-1}, away from the origin, when $OP = 10$ m.

a) Show that $k = 20$.

b) Calculate the time it took P to reduce its speed from 10 ms^{-1} to 1 ms^{-1}.

$\boxed{t = 4.95 \text{ s}}$

(a) $m\ddot{x} = -\frac{k}{x^3}$

$\Rightarrow \frac{1}{5}\ddot{x} = -\frac{k}{x^3}$

$\Rightarrow \frac{1}{5}v\frac{dv}{dx} = -\frac{k}{x^3}$

$\Rightarrow \int \frac{1}{5}v \, dv = \int -\frac{k}{x^3} \, dx$

$\Rightarrow \frac{1}{10}v^2 = \frac{k}{2x^2} + C$

$\Rightarrow \boxed{v^2 = \frac{5k}{x^2} + D}$

APPLY CONDITIONS:

$x=1, v=10 \Rightarrow 100 = \frac{5k}{1} + D$

$x=10, v=1 \Rightarrow 1 = \frac{5k}{100} + D$

$5k + D = 100$

$\frac{1}{20}k + D = 1$ $\Rightarrow$ SUBTRACT

$\frac{99}{20}k = 99$

$\frac{1}{20}k = 1$

$k = 20$

AS REQUIRED

(b) FIRSTLY $5k + D = 100$

$100 + D = 100$

$D = 0$

$\therefore v^2 = \frac{100}{x^2}$

$\Rightarrow v = \frac{10}{x}$

$\Rightarrow \frac{dx}{dt} = \frac{10}{x}$

$\Rightarrow x \, dx = 10 \, dt$

$\Rightarrow \int_{x=1}^{x=10} x \, dx = \int_{t=t_1}^{t=t_2} 10 \, dt$

$\Rightarrow \left[\frac{1}{2}x^2\right]_1^{10} = \left[10t\right]_{t_1}^{t_2}$

$\Rightarrow 50 - \frac{1}{2} = 10(t_2 - t_1)$

$\Rightarrow t_2 - t_1 = \frac{99}{20}$

$\therefore$ TIME 4.95 s

Question 17 (***+)

A particle P, of mass 0.5 kg, is moving on a straight horizontal line under the action of a constant force of magnitude 16 N. The motion of P is resisted by a force whose magnitude is proportional to the time t s, where t is measured from a given instant.

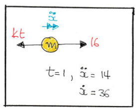
When $t = 1$, the velocity of P is 36 ms^{-1} and its acceleration is 14 ms^{-2}.

Determine the values of t when the velocity of P is 28 ms^{-1}.

$\boxed{}$, $\boxed{t = 3, \frac{5}{9}}$

WE HAVE "F = ma"

$$\Rightarrow 16 - kt = \frac{1}{2}\ddot{x}$$



APPLY A CONDITION & FIND k

$t=1$, $\ddot{x} = 14$

$16 - (k \times 1) = \frac{1}{2}(14)$

$16 - k = 7$

$k = 9$

$\therefore \frac{1}{2}\ddot{x} = 16 - 9t$

$\boxed{\ddot{x} = 32 - 18t}$

SOLVE THE O.D.E, BY SEPARATION OF VARIABLES AND SUBJECT TO THE CONDITIONS GIVEN

$$\Rightarrow \frac{dv}{dt} = 32 - 18t$$

$$\Rightarrow \int_{v=36}^{v=28} 1 \, dv = \int_{t=1}^{t} 32 - 18t \, dt$$

$$\Rightarrow \left[v \right]_{36}^{28} = \left[32t - 9t^2 \right]_1^t$$

$$\Rightarrow 28 - 36 = (32t - 9t^2) - (32 - 9)$$

$$\Rightarrow -8 = 32t - 9t^2 - 23$$

$$\Rightarrow 9t^2 - 32t + 15 = 0$$

$$\Rightarrow (t-3)(9t-5) = 0$$

$$\therefore t = \begin{cases} 3 \\ 5/9 \end{cases}$$

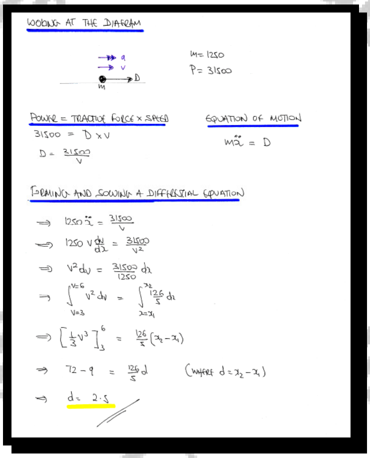
Question 18 (***+)

A car of mass 1250 kg is accelerating along a straight, horizontal road with the engine of the car producing a constant power of magnitude 31.5 kW .

The car is modelled as a particle with any other resistances to its motion ignored.

Find the distance covered by the car as its speed increases from 3 ms^{-1} to 6 ms^{-1}.

$\boxed{}$, $\boxed{d = 2.5 \text{ m}}$



LOOKING AT THE DIAGRAM

$m = 1250$

$P = 31500$

POWER = TRACTIVE FORCE × SPEED

$31500 = D \times v$

$D = \frac{31500}{v}$

EQUATION OF MOTION

$m\ddot{x} = D$

FORMING AND SOLVING A DIFFERENTIAL EQUATION

$\Rightarrow 1250\ddot{x} = \frac{31500}{v}$

$\Rightarrow 1250 v\frac{dv}{dx} = \frac{31500}{v^2}$

$\Rightarrow v^2\,dv = \frac{31500}{1250}\,dx$

$\Rightarrow \int_{v=3}^{v=6} v^2\,dv = \int_{x=x_1}^{x_2} \frac{126}{5}\,dx$

$\Rightarrow \left[\frac{1}{3}v^3\right]_3^6 = \frac{126}{5}(x_2 - x_1)$

$\Rightarrow 72 - 9 = \frac{126}{5}d$ (WHERE $d = x_2 - x_1$)

$\Rightarrow d = 2.5$

Question 19 (***+)

A car of mass 1600 kg moves along a straight horizontal road.

At time t s the resultant force acting on the car has magnitude

$$\frac{96000}{kt^2} \text{ N},$$

where k is a positive constant.

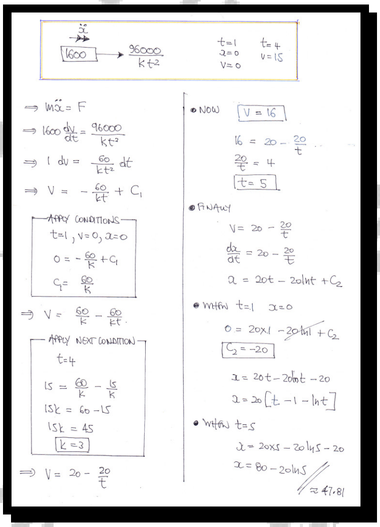
The resultant force acts in the direction of motion of the car.

At time t s , $t \geq 1$, the speed of the car is v ms^{-1} and the car is at a distance x m from a fixed point O, on the road.

When $t = 1$ the car is at rest at O and when $t = 4$ the speed of the car is 15 ms^{-1}.

Find the value of x when $v = 16$.

$$\boxed{x = 80 - 20\ln 5 \approx 47.81 \text{ m}}$$



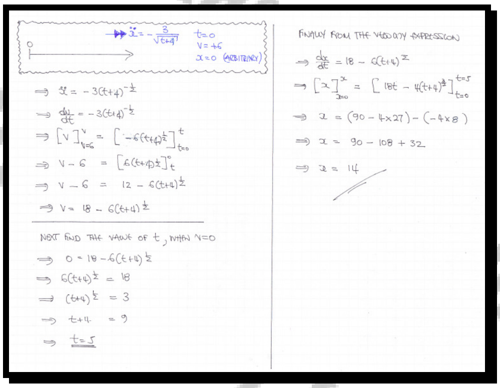
Question 20 (***+)

At time $t = 0$ s, a particle is at the origin O, and moving with speed 6 ms^{-1}, in the positive x direction.

For $t \geq 0$, the particle moves with acceleration of magnitude $-\dfrac{3}{\sqrt{t+4}}$, which is directed towards O.

Find the distance of the particle from O, when it comes to instantaneous rest.

$\boxed{x = 14 \text{ m}}$



$\ddot{x} = -\frac{3}{\sqrt{t+4}}$ $t=0$, $v=+6$, $x=0$ (ARBITRARY)

$\Rightarrow \ddot{x} = -3(t+4)^{-\frac{1}{2}}$

$\Rightarrow \frac{dv}{dt} = -3(t+4)^{-\frac{1}{2}}$

$\Rightarrow [v]_{v=6}^{v} = \left[-6(t+4)^{\frac{1}{2}}\right]_{t=0}^{t}$

$\Rightarrow v - 6 = \left[6(t+4)^{\frac{1}{2}}\right]_{t}^{0}$

$\Rightarrow v - 6 = 12 - 6(t+4)^{\frac{1}{2}}$

$\Rightarrow v = 18 - 6(t+4)^{\frac{1}{2}}$

NEXT FIND THE VALUE OF t, WHEN $v=0$

$\Rightarrow 0 = 18 - 6(t+4)^{\frac{1}{2}}$

$\Rightarrow 6(t+4)^{\frac{1}{2}} = 18$

$\Rightarrow (t+4)^{\frac{1}{2}} = 3$

$\Rightarrow t+4 = 9$

$\Rightarrow \underline{t=5}$

FINALLY FROM THE VELOCITY EXPRESSION

$\Rightarrow \frac{dx}{dt} = 18 - 6(t+4)^{\frac{1}{2}}$

$\Rightarrow [x]_{x=0}^{x} = \left[18t - 4(t+4)^{\frac{3}{2}}\right]_{t=0}^{t=5}$

$\Rightarrow x = (90 - 4\times 27) - (-4\times 8)$

$\Rightarrow x = 90 - 108 + 32$

$\Rightarrow x = 14$

Question 21 (***+)

A charged particle is accelerated in an electromagnetic field.

Its velocity, $v \text{ ms}^{-1}$, is given by

$$v = 2t\left(x+3\right),$$

where x m is the distance of the particle from a fixed origin O, at time t s.

Given that $x = 0$ when $t = 0$, determine the acceleration of the particle when $t = 2$.

$$\boxed{a = 54\mathrm{e}^4 \approx 399 \text{ ms}^{-2}}$$

- $V = 2t(x+3)$

$\Rightarrow \frac{dx}{dt} = 2t(x+3)$

$\Rightarrow \frac{1}{x+3}\,dx = 2t\,dt$

$\Rightarrow \int_{x=0}^{x} \frac{1}{x+3}\,dx = \int_{t=0}^{t} t^2$

$\Rightarrow \Big[\ln|x+3|\Big]_0^x = \Big[t^2\Big]_0^t$

$\Rightarrow \ln|x+3| - \ln 3 = t^2 - 0$

$\Rightarrow \ln\left|\frac{x+3}{3}\right| = t^2$

$\Rightarrow \frac{x+3}{3} = e^{t^2}$

$\Rightarrow \frac{x+3}{3} = 3e^{t^2}$

so

$\Rightarrow \boxed{x = -3 + 3e^{t^2}}$

Thus $V = 2t\left[(\cancel{-3} + 3e^{t^2}) \cancel{+3}\right]$

$\Rightarrow V = 2t\left[3e^{t^2}\right]$

$\Rightarrow V = 6te^{t^2}$

$\Rightarrow \frac{dv}{dt} = 6e^{t^2} + 12t^2e^{t^2}$

$\Rightarrow a = 6e^{t^2}\left[1 + 2t^2\right]$

Thus when $t = 2$

$a = 6 \times e^4 \times 9$

$a = 54e^2$

$\simeq 399 \text{ ms}^{-2}$

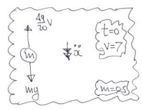
Question 22 (***+)

A particle P, of mass 0.5 kg, is projected vertically downwards, in a viscous fluid with an initial speed 7 ms^{-1}. When P has been moving for t s, its speed is $v \text{ ms}^{-1}$.

A resistive force of magnitude $\frac{49}{20}v$ is acting on P.

Calculate the approximate distance P covers in the first 0.5 s of its motion.

$\boxed{d \approx 1.93 \text{ m}}$



$m\ddot{x} = mg - \frac{49}{20}v$

$\frac{1}{2}\ddot{x} = \frac{1}{2}g - \frac{49}{20}v$

$10\ddot{x} = 98 - 49v$

$10\frac{dv}{dt} = 49(2-v)$

SEPARATE VARIABLES

$\frac{10}{2-v}\,dv = 49\,dt$

$\Rightarrow \frac{10}{v-2}\,dv = -4.9\,dt$ (INITIAL SPEED IS GREATER THAN 2)

$\Rightarrow \int_{v=7}^{v} \frac{10}{v-2}\,dv = \int_{t=0}^{t} -4.9\,dt$

$\Rightarrow 10\left[\ln|v-2|\right]_7^v = \left[-4.9t\right]_0^t$

$\Rightarrow \ln|v-2| - \ln 5 = -4.9t$

$\Rightarrow \ln\left|\frac{v-2}{5}\right| = -4.9t$

$\Rightarrow \frac{v-2}{5} = e^{-4.9t}$

$\Rightarrow v = 2 + 5e^{-4.9t}$

$\Rightarrow \frac{dx}{dt} = 2 + 5e^{-4.9t}$

$\Rightarrow \left[x\right]_{x=0}^{x} = \left[2t - \frac{5}{4.9}e^{-4.9t}\right]_{t=0}^{t=0.5}$

$\Rightarrow x = \left(1 - \frac{5}{4.9}e^{-2.45}\right) - \left(-\frac{5}{4.9}\right)$

$\Rightarrow x \approx 1.93 \text{ m}$

Question 23 (***+)

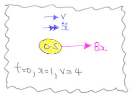
A particle P of mass 0.5 kg is moving in a straight line on a smooth horizontal plane.

At time $t = 0$ s, P is at a distance of 1 m from the origin O and moving away from O with speed 4 ms^{-1}.

The resultant force acting on P has magnitude $8x$ N away from O, where x is the distance OP.

Find a simplified expression for the speed of P at time t.

$$\boxed{x = e^{4t}}$$



$m\ddot{x} = 8x$

$\Rightarrow \frac{1}{2} v \frac{dv}{dx} = 8x$

$\Rightarrow \int_{v=4}^{v} v \, dv = \int_{x=1}^{x} 16x \, dx$

$\Rightarrow \left[\frac{1}{2}v^2\right]_4^v = \left[8x^2\right]_1^x$

$\Rightarrow \frac{1}{2}v^2 \cancel{-8} = 8x^2 \cancel{-8}$

$\Rightarrow v^2 = 16x^2$

$\Rightarrow v = 4x$

$\Rightarrow \frac{dx}{dt} = 4x$

$\Rightarrow \int_{x=1}^{x} \frac{1}{x} \, dx = \int_{t=0}^{t} 4 \, dt$

$\Rightarrow \left[\ln|x|\right]_1^x = \left[4t\right]_0^t$

$\Rightarrow \ln x - \cancel{\ln 1} = 4t$

$\Rightarrow x = e^{4t}$

Question 24 (***+)

A particle of mass 0.5 kg is moving along the positive x axis.

At time t s, where $t \geq 0$, the particle has displacement x m from the origin O, its velocity is v ms^{-1} and its acceleration is a ms^{-2}.

When $t = 0$, $x = 0$.

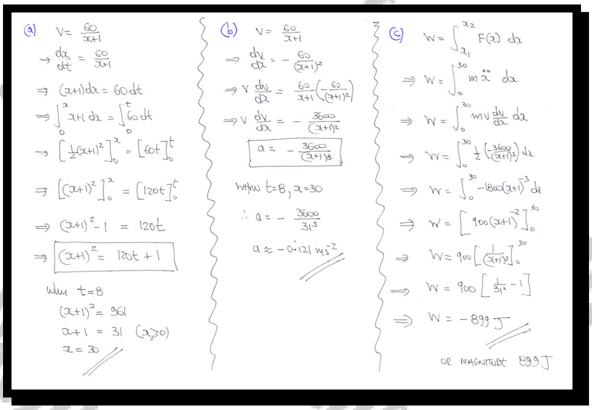
Given that $v = \dfrac{60}{x+1}$, $x \geq 0$, determine for the time interval $0 \leq t \leq 8$ …

a) … the displacement of the particle at the end of this time interval.

b) … the acceleration of the particle at the end of this time interval.

c) … the magnitude of the work done by the resultant force acting on this particle during this time interval.

$\boxed{x = 30 \text{ m}}$, $\boxed{a \approx -0.121 \text{ ms}^{-1}}$, $\boxed{|W| \approx 899 \text{ J}}$



Question 25 (***+)

A particle, of mass 0.5 kg, is moving on a straight line, under the action of a single force of magnitude

$$\left[\frac{25}{x^2}-\frac{50}{x^3}\right] \text{ N}, \ x>0,$$

where x is the distance of the particle from a fixed origin O.

The particle is released from the point where $x=1$, with speed 13 ms^{-1}, in the direction of x increasing.

It is further given that in moving the particle from $x=1$ to a point where $x=k$, $k>1$, the force does work of -4 J.

a) Determine the possible values of k.

b) Find the least speed of the particle in its consequent motion.

$\boxed{}$, $\boxed{k=5 \cup k=\frac{5}{4}}$, $\boxed{V_{\min}=12 \text{ ms}^{-1}}$

a) $F=\frac{25}{x^2}-\frac{50}{x^3}, \ x>0$

WORK BY F = −4

$\Rightarrow \int_1^k \frac{25}{x^2}-\frac{50}{x^3}\,dx=-4$

$\Rightarrow \left[-\frac{25}{x}+\frac{25}{x^2}\right]_1^k=-4$

$\Rightarrow \left(-\frac{25}{k}+\frac{25}{k^2}\right)-\cancel{(-25+25)}=-4 \quad \times k^2$

$\Rightarrow -25k+25=-4k^2$

$\Rightarrow 4k^2-25k+25=0$

$\Rightarrow (4k-5)(k-5)=0$

$\Rightarrow k = 5, \ \frac{5}{4}$

b) F = 0 YIELDS $\quad \frac{25}{x^2}-\frac{50}{x^3}=0$

$25x-50=0$

$x=2$

NEXT THE WORK FROM $x=1$ TO $x=2$

$W=\int_1^2 \frac{25}{x^2}-\frac{50}{x^3}\,dx=\left[-\frac{25}{x}+\frac{25}{x^2}\right]_1^2$

$=\left(-\frac{25}{2}+\frac{25}{4}\right)-\cancel{(-25+25)}=-\frac{25}{4}$ (IF WORK OUT)

BY ENERGIES

$\Rightarrow KE_{x=1} - W_{OUT} = KE_{x=2}$

$\Rightarrow \frac{1}{2}mu^2-\frac{25}{4}=\frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}(0.5)\times 13^2-\frac{25}{4}=\frac{1}{2}(0.5)v^2$

$\Rightarrow \frac{169}{4}-\frac{25}{4}=\frac{1}{4}v^2$

$\Rightarrow 169-25=v^2$

$\Rightarrow v^2=144$

$\Rightarrow |v|=12 \text{ ms}^{-1}$

Question 26 (****)

A particle P is moving on the x axis, starting from rest at the origin O.

The acceleration of P is in the direction of x increasing and has magnitude

$$\frac{0.5}{v+3}\ \text{ms}^{-2},$$

where v is the subsequent velocity of the particle.

Find the distance P covers in the first 7 seconds of its motion.

$\boxed{}$, $\boxed{d = \frac{11}{3}}$

$$\boxed{\ddot{x} = \frac{0.5}{v+3} \qquad t=0,\ x=0,\ \dot{x}=v=0}$$

START FROM OBTAINING A RELATIONSHIP BETWEEN v & t

$\Rightarrow \frac{dv}{dt} = \frac{0.5}{v+3}$

$\Rightarrow (v+3)\,dv = \frac{1}{2}dt$

$\Rightarrow \int_{v=0}^{v} v+3\ dv = \int_{t=0}^{t} \frac{1}{2}\ dt$

$\Rightarrow \left[\frac{1}{2}v^2+3v\right]_0^v = \left[\frac{1}{2}t\right]_0^t$

$\Rightarrow \frac{1}{2}v^2+3v = \frac{1}{2}t$

$\Rightarrow v^2+6v = t$

COMPLETING THE SQUARE TO COMPLETE THE REARRANGEMENT

$\Rightarrow v^2+6v+9 = t+9$

$\Rightarrow (v+3)^2 = t+9$

$\Rightarrow v+3 = \pm\sqrt{t+9}$

$\Rightarrow v = -3+\sqrt{t+9} \qquad v \geqslant 0$ FOR $t \geqslant 0$

$\Rightarrow \frac{dx}{dt} = -3+(t+9)^{\frac{1}{2}}$

$\Rightarrow \int_{x=0}^{x} 1\ dx = \int_{t=0}^{7} -3+(t+9)^{\frac{1}{2}}$

$\Rightarrow [x]_0^x = \left[-3t+\frac{2}{3}(t+9)^{\frac{3}{2}}\right]_0^7$

$\Rightarrow x = \left(-21+\frac{2}{3}\times 64\right)-\left(0+\frac{2}{3}\times 27\right)$

$\Rightarrow x = -21+\frac{128}{3}-18$

$\Rightarrow x = \frac{128}{3}-39 = \frac{128}{3}-\frac{117}{3} = \frac{11}{3}$

ALTERNATIVE

$$\boxed{\text{AFTER USING } \frac{dv}{dt} = \frac{0.5}{v+3} \text{ & OBTAINING } v^2+6v=t}$$

$\Rightarrow v\frac{dv}{dx} = \frac{0.5}{v+3}$

$\Rightarrow (v^2+3v)\,dv = 0.5\,dx$

$\Rightarrow \int_{v=0}^{v} v^2+3v\ dv = \int_{x=0}^{x} 0.5\,dx$

$\Rightarrow \left[\frac{1}{3}v^3+\frac{3}{2}v^2\right]_0^v = \left[\frac{1}{2}x\right]_0^x$

$\Rightarrow \frac{1}{3}v^3+\frac{3}{2}v^2 = \frac{1}{2}x$

$\Rightarrow x = \frac{2}{3}v^3+3v^2$

NOW USING $t=7$

$\Rightarrow v^2+6v = 7$

$\Rightarrow v^2+6v-7 = 0$

$\Rightarrow (v-1)(v+7) = 0$

$\Rightarrow v = 1$ ~~-7~~

HENCE WE FINALLY HAVE

$x = \frac{2}{3}v^3+3v^2$

$x = \frac{2}{3}+3$

$x = \frac{11}{3}$ AS BEFORE

Question 27 (****)

A particle is moving along the positive x axis.

At time t s, the particle is at a distance of x m from the origin O and is moving away from O with speed v ms^{-1}.

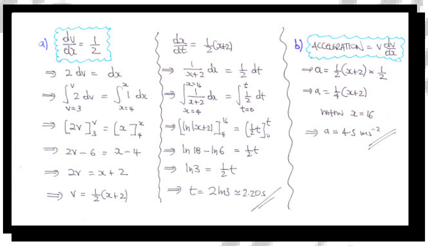
The particle is moving in such a way so that the rate of change of change of its speed with respect to the distance covered is $0.5\ \text{s}^{-1}$.

It is further given that $x = 4$ and $v = 3$ when $t = 0$.

a) Find the value of t when $x = 16$.

b) Determine the acceleration of the particle when $x = 16$.

$\boxed{t = 2\ln 3 \approx 2.20 \text{ s}}$, $\boxed{a = 4.5 \text{ ms}^{-2}}$


a) $\boxed{\dfrac{dv}{dx} = \dfrac{1}{2}}$

$\Rightarrow 2\,dv = dx$

$\Rightarrow \int_{v=3}^{v} 2\,dv = \int_{x=4}^{x} 1\,dx$

$\Rightarrow [2v]_3^v = [x]_4^x$

$\Rightarrow 2v - 6 = x - 4$

$\Rightarrow 2v = x + 2$

$\Rightarrow v = \frac{1}{2}(x+2)$

$\frac{dx}{dt} = \frac{1}{2}(x+2)$

$\Rightarrow \frac{1}{x+2}\,dx = \frac{1}{2}\,dt$

$\Rightarrow \int_{x=4}^{x=16} \frac{1}{x+2}\,dx = \int_{t=0}^{t} \frac{1}{2}\,dt$

$\Rightarrow [\ln|x+2|]_4^{16} = [\frac{1}{2}t]_0^t$

$\Rightarrow \ln 18 - \ln 6 = \frac{1}{2}t$

$\Rightarrow \ln 3 = \frac{1}{2}t$

$\Rightarrow t = 2\ln 3 \simeq 2.20\text{s}$

b) $\boxed{\text{ACCELERATION} = v\frac{dv}{dx}}$

$\Rightarrow a = \frac{1}{2}(x+2) \times \frac{1}{2}$

$\Rightarrow a = \frac{1}{4}(x+2)$

WHEN $x = 16$

$\Rightarrow a = 4.5 \text{ ms}^{-2}$