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INTEGRATION

BY TRIGONOMETRIC IDENTITIES

(WITHOUT ANSWERS)

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Question 1

Carry out the following integrations:

1. $\int 3\sin^2 x \, dx$

2. $\int 4\cos^2 x \, dx$

3. $\int 3\sin x \cos x \, dx$

4. $\int (2 - 3\sin x)^2 \, dx$

5. $\int (1 - \cos 2x)^2 \, dx$

6. $\int 2\tan^2 x \, dx$

7. $\int 5\cot^2 x \, dx$

8. $\int (2\tan x - \cot x)^2 \, dx$

9. $\int \frac{4\sin x}{\cos^2 x} \, dx$

10. $\int \frac{\cos x}{3\sin^2 x} \, dx$

Question 2

Integrate:

1. $\int (2 + \sin x)^2 dx$

2. $\int \sin x (1 + \sec^2 x) dx$

3. $\int (1 - 2 \cos x)^2 dx$

4. $\int \frac{1}{\cos^2 x \tan^2 x} dx$

5. $\int 2 + 2 \tan^2 x dx$

6. $\int \frac{1 + \cos x}{\sin^2 x} dx$

7. $\int \frac{(1 + \cos x)^2}{\sin^2 x} dx$

8. $\int 4 \cos^2 x dx$

9. $\int 3 \cot^2 x dx$

10. $\int (2 \cos x - 3 \sin x)^2 dx$

Question 3

Integrate:

1. $\int \sin 2x \operatorname{cosec} x \, dx$

2. $\int \frac{1 + \sin x}{\cos^2 x} \, dx$

3. $\int \tan^2 x \, dx$

4. $\int \frac{(1 + \sin x)^2}{\cos^2 x} \, dx$

5. $\int \frac{\cos^2 x}{1 + \sin x} \, dx$

6. $\int \frac{1}{1 + \cos x} \, dx$

7. $\int \frac{(1 + 2 \cos x)^2}{3 \sin^2 x} \, dx$

8. $\int \sin x \sin 3x \, dx$

9. $\int \sin^2 2x \, dx$

10. $\int 2 \cos 3x \sin x \, dx$

Question 4

Integrate:

1. $\int \frac{\cos 2x}{1 - \cos^2 2x} dx$

2. $\int \cot^2 3x dx$

3. $\int \sin 2x \sec x dx$

4. $\int \frac{1}{\sin x \cos^2 x} dx$

5. $\int \frac{1}{\sec x - 1} dx$

6. $\int 1 - \cot^2 x dx$

7. $\int (2 \cos x - 3)^2 dx$

8. $\int (3 \sin x - \cos x)^2 dx$

9. $\int \frac{1}{\cos x \sin^2 x} dx$

10. $\int \sin^2 x \sec^2 x dx$

Question 5

Integrate:

1. $\int \sin 3x \cos 2x \, dx$

2. $\int \frac{1}{\sin x \cos x} \, dx$

3. $\int \frac{1}{1 - \sin x} \, dx$

4. $\int \sin^2 2x \, dx$

5. $\int \frac{\cos 2x}{\cos^2 x} \, dx$

6. $\int \cos^2 x \sin^2 x \, dx$

7. $\int (\sin x + 2 \cos x)^2 \, dx$

8. $\int \frac{1}{\sin^2 x \cos^2 x} \, dx$

9. $\int \sqrt{\sin^2 x + (\cos x - 1)^2} \, dx$

10. $\int \frac{1 - \cos x}{1 + \cos x} \, dx$

Question 6

Integrate:

1. $\int \frac{1+\sin x}{1-\sin x} dx$

Question 7

Carry out the following integrations:

$$1. \int_0^{\frac{\pi}{2}} 4 \sin^2 x \, dx = \pi$$

$$2. \int_0^{\frac{\pi}{6}} 24 \cos^2 x \, dx = \pi + 3$$

$$3. \int_0^{\frac{\pi}{6}} 8 \sin x \cos x \, dx = 1$$

$$4. \int_0^{\frac{\pi}{2}} (1 - \sin x)^2 \, dx = \frac{3\pi}{2} - 4$$

$$5. \int_0^{\frac{\pi}{6}} (1 - \cos 3x)^2 \, dx = \frac{\pi}{4} - \frac{2}{3}$$

$$6. \int_0^{\frac{\pi}{4}} 4 \tan^2 x \, dx = 4 - \pi$$

$$7. \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (3 \cot x + \tan x)^2 \, dx = \frac{2}{3} (10\sqrt{3} - \pi)$$

$$8. \int_0^{\frac{\pi}{4}} (\sec x + 4 \cos x)^2 \, dx = 4\pi + 5$$

$$9. \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x} \, dx = 1$$

$$10. \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} \, dx = \sqrt{2} - 1$$

Question 8

Carry out the following integrations:

$$1. \int_0^{\frac{\pi}{4}} \cos^2 x \, dx = \frac{1}{8}(\pi + 2)$$

$$2. \int_0^{\frac{\pi}{2}} \sin^2 x \, dx = \frac{\pi}{4}$$

$$3. \int_0^{\frac{\pi}{2}} (2 \sin x - 3 \cos x)^2 \, dx = \frac{1}{4}(13\pi - 24)$$

$$4. \int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos x)^2 \, dx = 4\pi + 3\sqrt{3}$$

$$5. \int_0^{\frac{\pi}{4}} \tan^2 x \, dx = \frac{1}{4}(4 - \pi)$$

$$6. \int_0^{\frac{\pi}{6}} \sin x \sin 3x \, dx = \frac{\sqrt{3}}{16}$$

$$7. \int_0^{\frac{\pi}{3}} \frac{1}{1 - \sin x} \, dx = 1 + \sqrt{3}$$

$$8. \int_0^{\frac{\pi}{2}} \left(1 + \tan \frac{x}{2}\right)^2 \, dx = 2 + \ln 4$$

$$9. \int_0^{\frac{\pi}{2}} \cos^3 x \, dx = \frac{2}{3}$$

$$10. \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cot^2 2x \, dx = \frac{1}{2} - \frac{\sqrt{3}}{6} - \frac{\pi}{24}$$

Question 9

Carry out the following integrations:

1. $\int_0^{\frac{\pi}{12}} 6\sin^2 \theta \, d\theta = \frac{1}{4}(\pi - 3)$

2. $\int_0^{\frac{\pi}{6}} \sin^3 \theta \, d\theta = \frac{5}{24}$

3. $\int_0^{\frac{\pi}{12}} 10\sin 8\theta \cos 2\theta \, d\theta = \frac{1}{12}(16 + 3\sqrt{3})$

4. $\int_0^{\frac{\pi}{4}} (\cos x + \sec x)^2 \, dx = \frac{5}{8}(\pi + 2)$

5. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x + \cot x)^2 \, dx = \frac{1}{8}(26 - \pi - 4\sqrt{2})$