PARAMETRIC EQUATIONS

ALGEBRAIC ELIMINATIONS

Question 1

Find a Cartesian equation for each of the following parametric relationships.

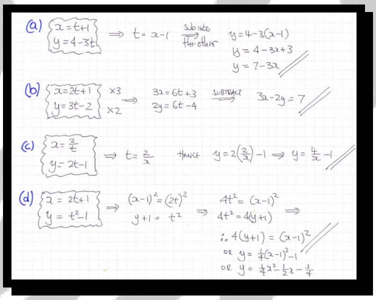
a) $x = t + 1, \quad y = 4 - 3t, \qquad t \in \mathbb{R}$

b) $x = 2t + 1, \quad y = 3t - 2, \qquad t \in \mathbb{R}$

c) $x = \dfrac{2}{t}, \quad y = 2t - 1, \qquad t \in \mathbb{R}, \quad t \neq 0$

d) $x = 2t + 1, \quad y = t^2 - 1, \qquad t \in \mathbb{R}$

$$\boxed{y = 7 - 3x}, \quad \boxed{3x - 2y = 7}, \quad \boxed{y = \frac{4}{x} - 1}, \quad \boxed{(x-1)^2 = 4(y+1)}$$



Question 2

Find a Cartesian equation for each of the following parametric relationships.

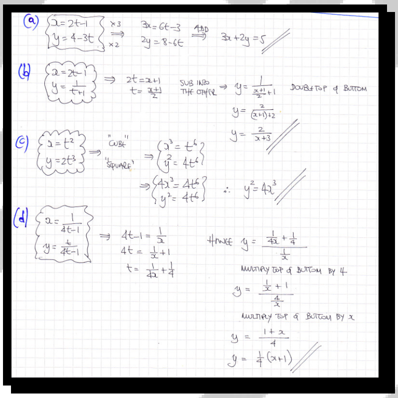
a) $x = 2t - 1, \quad y = 4 - 3t, \qquad t \in \mathbb{R}$

b) $x = 2t - 1, \quad y = \dfrac{1}{t+1}, \qquad t \in \mathbb{R}, \quad t \neq -1$

c) $x = t^2, \quad y = 2t^3, \qquad t \in \mathbb{R}$

d) $x = \dfrac{1}{4t-1}, \quad y = \dfrac{t}{4t-1}, \qquad t \in \mathbb{R}, \quad t \neq \dfrac{1}{4}$

$\boxed{3x + 2y = 5}, \quad \boxed{y = \dfrac{2}{x+3}}, \quad \boxed{y^2 = 4x^3}, \quad \boxed{y = \frac{1}{4}(x+1)}$



Question 3

Find a Cartesian equation for each of the following parametric relationships.

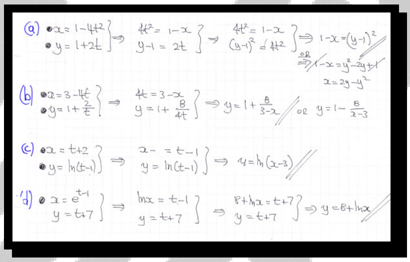
a) $x = 1 - 4t^2, \quad y = 1 + 2t, \qquad t \in \mathbb{R}$

b) $x = 3 - 4t, \quad y = 1 + \dfrac{2}{t}, \qquad t \in \mathbb{R} \quad t \neq 0$

c) $x = t + 2, \quad y = \ln(t-1), \qquad t \in \mathbb{R} \quad t > 1$

d) $x = e^{t-1}, \quad y = t + 7, \qquad t \in \mathbb{R}$

$\boxed{x = 2y - y^2}, \ \boxed{y = 1 - \dfrac{8}{x-3}}, \ \boxed{y = \ln(x-3)}, \ \boxed{y = 8 + \ln x}$



Question 4

Find a Cartesian equation for each of the following parametric expressions.

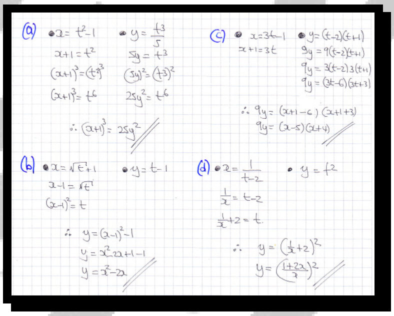
a) $x = t^2 - 1, \quad y = \dfrac{t^3}{5}, \qquad t \in \mathbb{R}$

b) $x = \sqrt{t} + 1, \quad y = t - 1, \qquad t \in \mathbb{R},\ t \geq 0$

c) $x = 3t - 1, \quad y = (t-2)(t+1), \qquad t \in \mathbb{R}$

d) $x = \dfrac{1}{t-2}, \quad y = t^2, \qquad t \in \mathbb{R},\ t \neq 2$

$\boxed{25y^2 = (x+1)^3}$, $\boxed{y = x^2 - 2x}$, $\boxed{9y = (x-5)(x+4)}$, $\boxed{y = \left(\dfrac{2x+1}{x}\right)^2}$



Question 5

Find a Cartesian equation for each of the following parametric equations.

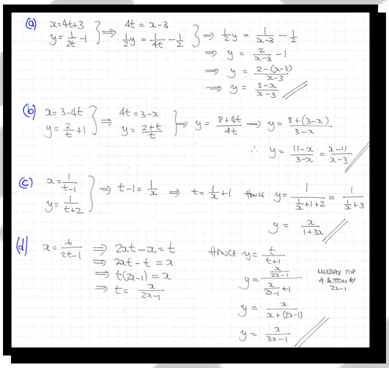
a) $x = 4t + 3, \quad y = \dfrac{1}{2t} - 1, \qquad t \in \mathbb{R},\ t \neq 0$

b) $x = 3 - 4t, \quad y = \dfrac{2}{t} + 1, \qquad t \in \mathbb{R},\ t \neq 0$

c) $x = \dfrac{1}{t-1}, \quad y = \dfrac{1}{t+2}, \qquad t \in \mathbb{R},\ t \neq 1, -2$

d) $x = \dfrac{t}{2t-1}, \quad y = \dfrac{t}{t+1}, \qquad t \in \mathbb{R},\ t \neq -1, \dfrac{1}{2}$

$\boxed{y = \dfrac{2}{x-3} - 1}$, $\boxed{y = \dfrac{x-11}{x-3}}$, $\boxed{y = \dfrac{x}{1+3x}}$, $\boxed{y = \dfrac{x}{3x-1}}$



(a) $x = 4t+3$, $y = \frac{1}{2t} - 1$ $\Rightarrow$ $4t = x-3$, $\frac{1}{2}y = \frac{1}{4t} - \frac{1}{2}$ $\Rightarrow$ $\frac{1}{2}y = \frac{1}{x-3} - \frac{1}{2}$

$\Rightarrow y = \frac{2}{x-3} - 1$

$\Rightarrow y = \frac{2-(x-3)}{x-3}$

$\Rightarrow y = \frac{5-x}{x-3}$

(b) $x = 3-4t$, $y = \frac{2}{t}+1$ $\Rightarrow$ $4t = 3-x$, $y = \frac{2+t}{t}$ $\Rightarrow y = \frac{8+4t}{4t} \Rightarrow y = \frac{8+(3-x)}{3-x}$

$\therefore y = \frac{11-x}{3-x} = \frac{x-11}{x-3}$

(c) $x = \frac{1}{t-1}$, $y = \frac{1}{t+2}$ $\Rightarrow t-1 = \frac{1}{x} \Rightarrow t = \frac{1}{x}+1$ hence $y = \frac{1}{\frac{1}{x}+1+2} = \frac{1}{\frac{1}{x}+3}$

$y = \frac{x}{1+3x}$

(d) $x = \frac{t}{2t-1} \Rightarrow 2xt - x = t$

$\Rightarrow 2xt - t = x$

$\Rightarrow t(2x-1) = x$

$\Rightarrow t = \frac{x}{2x-1}$

Hence $y = \frac{t}{t+1}$

$y = \frac{\frac{x}{2x-1}}{\frac{x}{2x-1}+1}$ MULTIPLY TOP & BOTTOM BY $2x-1$

$y = \frac{x}{x+(2x-1)}$

$y = \frac{x}{3x-1}$

Question 6

Find a Cartesian equation for each of the following parametric expressions.

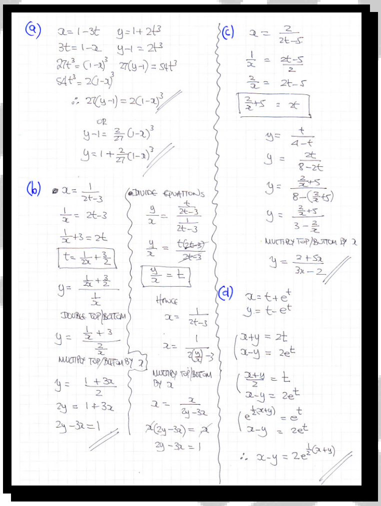
a) $x = 1 - 3t, \quad y = 1 + 2t^3, \qquad t \in \mathbb{R}$

b) $x = \dfrac{1}{2t-3}, \quad y = \dfrac{t}{2t-3}, \qquad t \in \mathbb{R},\ t \neq \dfrac{3}{2}$

c) $x = \dfrac{2}{2t-5}, \quad y = \dfrac{t}{4-t}, \qquad t \in \mathbb{R},\ t \neq 4,\ t \neq \dfrac{5}{2}$

d) $x = t + e^t, \quad y = t - e^t, \qquad t \in \mathbb{R}$

$\boxed{y = 1 + \dfrac{2}{27}(1-x)^3}, \ \boxed{2y - 3x = 1}, \ \boxed{y = \dfrac{5x+2}{3x-2}}, \ \boxed{x - y = 2e^{\frac{1}{2}(x+y)}}$



Question 7

Find a Cartesian equation for each of the following parametric expressions.

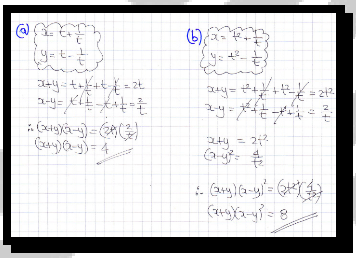
a) $x = t + \dfrac{1}{t}, \quad y = t - \dfrac{1}{t}, \qquad t \in \mathbb{R}, \ t \neq 0$

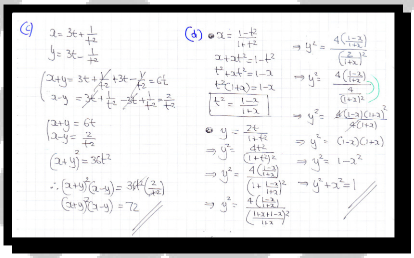
b) $x = t^2 + \dfrac{1}{t}, \quad y = t^2 - \dfrac{1}{t}, \qquad t \in \mathbb{R}, \quad t \neq 0$

c) $x = 3t + \dfrac{1}{t^2}, \quad y = 3t - \dfrac{1}{t^2}, \qquad t \in \mathbb{R}, \ t \neq 0$

d) $x = \dfrac{1-t^2}{1+t^2}, \quad y = \dfrac{2t}{1+t^2}, \qquad t \in \mathbb{R}$

$\boxed{x^2 - y^2 = 4}, \quad \boxed{(x+y)(x-y)^2 = 8}, \quad \boxed{(x-y)(x+y)^2 = 72}, \quad \boxed{x^2 + y^2 = 1}$





Question 8

Find a Cartesian equation for each of the following parametric equations.

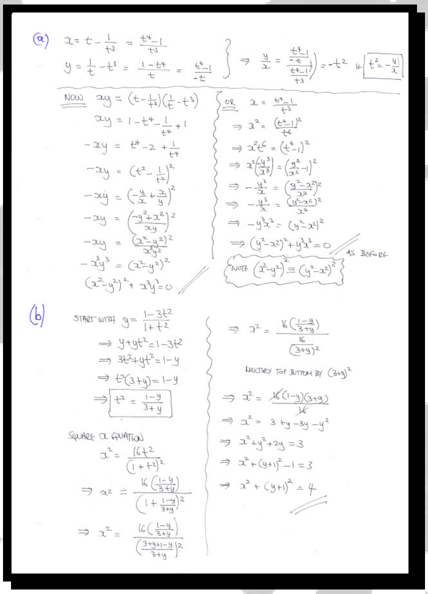
a) $x = t - \dfrac{1}{t^3}, \quad y = \dfrac{1}{t} - t^3, \qquad t \in \mathbb{R},\ t \neq 0$

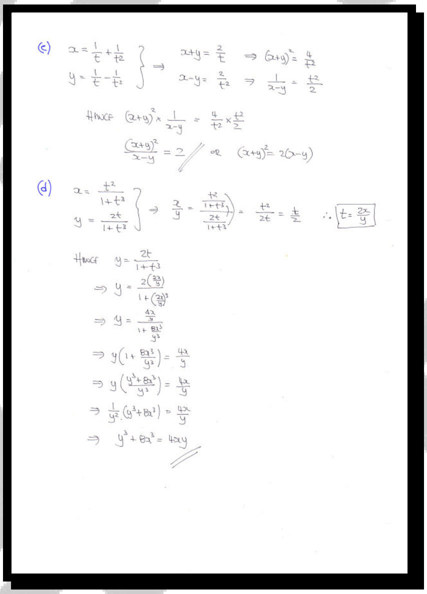
b) $x = \dfrac{4t}{1+t^2}, \quad y = \dfrac{1-3t^2}{1+t^2}, \qquad t \in \mathbb{R}$

c) $x = \dfrac{1}{t} + \dfrac{1}{t^2}, \quad y = \dfrac{1}{t} - \dfrac{1}{t^2}, \qquad t \in \mathbb{R}, t \neq 0$

d) $x = \dfrac{t^2}{1+t^3}, \quad y = \dfrac{2t}{1+t^3}, \qquad t \in \mathbb{R}, t \neq -1$

$\boxed{\left(y^2 - x^2\right)^2 + x^3y^3 = 0}$, $\boxed{x^2 + \left(y+1\right)^2 = 4}$, $\boxed{\left(x+y\right)^2 = 2\left(x-y\right)}$, $\boxed{y^3 + 8x^3 = 4xy}$





TRIGONOMETRIC ELIMINATIONS

Question 1

Find a Cartesian equation for each of the following parametric relationships.

a) $x = 5\cos t, \quad y = 5\sin t, \quad 0 \le t < 2\pi$

b) $x = 1 + 2\cos\theta, \quad y = 2 + \sin\theta, \quad 0 \le \theta < 2\pi$

c) $x = 2\tan\theta, \quad y = \cos\theta, \quad 0 \le \theta < 2\pi$

d) $x = \cos t, \quad y = \operatorname{cosec} t, \quad 0 \le t < 2\pi$

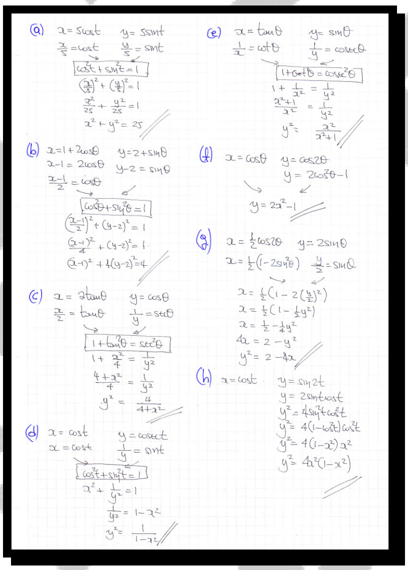
e) $x = \tan\theta, \quad y = \sin\theta, \quad 0 \le \theta < 2\pi$

f) $x = \cos\theta, \quad y = \cos 2\theta, \quad 0 \le \theta < 2\pi$

g) $x = \frac{1}{2}\cos 2\theta, \quad y = 2\sin\theta, \quad 0 \le \theta < 2\pi$

h) $x = \cos t, \quad y = \sin 2t, \quad 0 \le t < 2\pi$

$\boxed{x^2 + y^2 = 25}$, $\boxed{(x-1)^2 + 4(y-2)^2 = 4}$, $\boxed{y^2 = \frac{4}{4+x^2}}$, $\boxed{y^2 = \frac{1}{1-x^2}}$, $\boxed{y^2 = \frac{x^2}{1+x^2}}$,
$\boxed{y = 2x^2 - 1}$, $\boxed{y^2 = 2 - 4x}$, $\boxed{y^2 = 4x^2\left(1 - x^2\right)}$



Question 2

Find a Cartesian equation for each of the following parametric relationships.

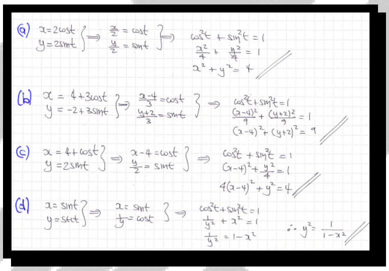
a) $x = 2\cos t, \quad y = 2\sin t, \quad 0 \le t < 2\pi$

b) $x = 4 + 3\cos t, \quad y = -2 + 3\sin t, \quad 0 \le t < 2\pi$

c) $x = 4 + \cos t, \quad y = 2\sin t, \quad 0 \le t < 2\pi$

d) $x = \sin t, \quad y = \sec t, \quad 0 \le t < 2\pi$

$\boxed{x^2 + y^2 = 4}$, $\boxed{(x-4)^2 + (y+2)^2 = 9}$, $\boxed{4(x-4)^2 + y^2 = 4}$, $\boxed{y^2 = \frac{1}{1-x^2}}$



(a) $x = 2\cos t$, $y = 2\sin t$ $\Rightarrow$ $\frac{x}{2} = \cos t$, $\frac{y}{2} = \sin t$ $\Rightarrow$ $\cos^2 t + \sin^2 t = 1$
$\frac{x^2}{4} + \frac{y^2}{4} = 1$
$x^2 + y^2 = 4$

(b) $x = 4 + 3\cos t$, $y = -2 + 3\sin t$ $\Rightarrow$ $\frac{x-4}{3} = \cos t$, $\frac{y+2}{3} = \sin t$ $\Rightarrow$ $\cos^2 t + \sin^2 t = 1$
$\frac{(x-4)^2}{9} + \frac{(y+2)^2}{9} = 1$
$(x-4)^2 + (y+2)^2 = 9$

(c) $x = 4 + \cos t$, $y = 2\sin t$ $\Rightarrow$ $x - 4 = \cos t$, $\frac{y}{2} = \sin t$ $\Rightarrow$ $\cos^2 t + \sin^2 t = 1$
$(x-4)^2 + \frac{y^2}{4} = 1$
$4(x-4)^2 + y^2 = 4$

(d) $x = \sin t$, $y = \sec t$ $\Rightarrow$ $x = \sin t$, $\frac{1}{y} = \cos t$ $\Rightarrow$ $\cos^2 t + \sin^2 t = 1$
$\frac{1}{y^2} + x^2 = 1$
$\frac{1}{y^2} = 1 - x^2$
$\therefore y^2 = \frac{1}{1-x^2}$

Question 3

Eliminate θ to obtain a Cartesian equation for the following parametric equations.

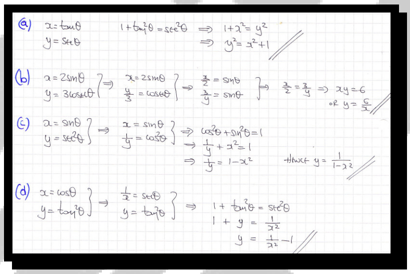
a) $x = \tan\theta$, $y = \sec\theta$, $0 \le \theta < 2\pi$

b) $x = 2\sin\theta$, $y = 3\operatorname{cosec}\theta$, $0 \le \theta < 2\pi$

c) $x = \sin\theta$, $y = \sec^2\theta$, $0 \le \theta < 2\pi$

d) $x = \cos\theta$, $y = \tan^2\theta$, $0 \le \theta < 2\pi$

$\boxed{y^2 = x^2 + 1}$, $\boxed{y = \frac{6}{x}}$, $\boxed{y = \frac{1}{1-x^2}}$, $\boxed{y = \frac{1}{x^2} - 1}$



Question 4

Eliminate the parameter θ to obtain a Cartesian equation for each of the following parametric equations.

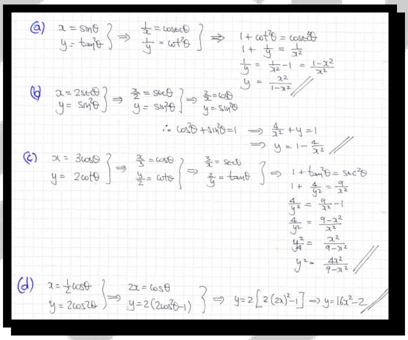
a) $x = \sin\theta,\ \ y = \tan^2\theta, \qquad 0 \le \theta < 2\pi$

b) $x = 2\sec\theta,\ \ y = \sin^2\theta, \qquad 0 \le \theta < 2\pi$

c) $x = 3\cos\theta,\ \ y = 2\cot\theta, \qquad 0 \le \theta < 2\pi$

d) $x = \frac{1}{2}\cos\theta,\ \ y = 2\cos 2\theta, \qquad 0 \le \theta < 2\pi$

$\boxed{y = \frac{x^2}{1-x^2}}$, $\boxed{y = 1 - \frac{4}{x^2}}$, $\boxed{y^2 = \frac{4x^2}{9-x^2}}$, $\boxed{y = 2\left(8x^2 - 1\right)}$



Question 5

Find a Cartesian equation for each of the following parametric relationships.

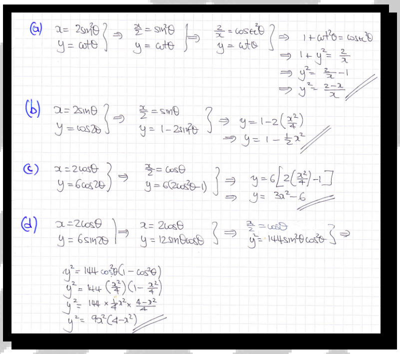
a) $x = 2\sin^2\theta, \quad y = \cot\theta, \qquad 0 \le \theta < 2\pi$

b) $x = 2\sin\theta, \quad y = \cos 2\theta, \qquad 0 \le \theta < 2\pi$

c) $x = 2\cos\theta, \quad y = 6\cos 2\theta, \qquad 0 \le \theta < 2\pi$

d) $x = 2\cos\theta, \quad y = 6\sin 2\theta, \qquad 0 \le \theta < 2\pi$

$\boxed{y^2 = \dfrac{2-x}{x}}$, $\boxed{y = 1 - \frac{1}{2}x^2}$, $\boxed{y = 3x^2 - 6}$, $\boxed{y^2 = 9x^2\left(4 - x^2\right)}$


(a) $x = 2\sin^2\theta$, $y = \cot\theta$ $\Rightarrow$ $\frac{x}{2} = \sin^2\theta$, $y = \cot\theta$ $\Rightarrow$ $\frac{2}{x} = \operatorname{cosec}^2\theta$, $y = \cot\theta$ $\Rightarrow$ $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\Rightarrow 1 + y^2 = \frac{2}{x}$
$\Rightarrow y^2 = \frac{2}{x} - 1$
$\Rightarrow y^2 = \frac{2-x}{x}$

(b) $x = 2\sin\theta$, $y = \cos 2\theta$ $\Rightarrow$ $\frac{x}{2} = \sin\theta$, $y = 1 - 2\sin^2\theta$ $\Rightarrow$ $y = 1 - 2\left(\frac{x^2}{4}\right)$
$\Rightarrow y = 1 - \frac{1}{2}x^2$

(c) $x = 2\cos\theta$, $y = 6\cos 2\theta$ $\Rightarrow$ $\frac{x}{2} = \cos\theta$, $y = 6(2\cos^2\theta - 1)$ $\Rightarrow$ $y = 6\left[2\left(\frac{x^2}{4}\right) - 1\right]$
$\Rightarrow y = 3x^2 - 6$

(d) $x = 2\cos\theta$, $y = 6\sin 2\theta$ $\Rightarrow$ $x = 2\cos\theta$, $y = 12\sin\theta\cos\theta$ $\Rightarrow$ $\frac{x}{2} = \cos\theta$, $y^2 = 144\sin^2\theta\cos^2\theta$ $\Rightarrow$

$y^2 = 144\cos^2\theta(1 - \cos^2\theta)$
$y^2 = 144\left(\frac{x^2}{4}\right)\left(1 - \frac{x^2}{4}\right)$
$y^2 = 144 \times \frac{1}{4}x^2 \times \frac{4-x^2}{4}$
$y^2 = 9x^2(4-x^2)$

Question 6

Eliminate the parameter θ to obtain a Cartesian equation for each of the following parametric equations.

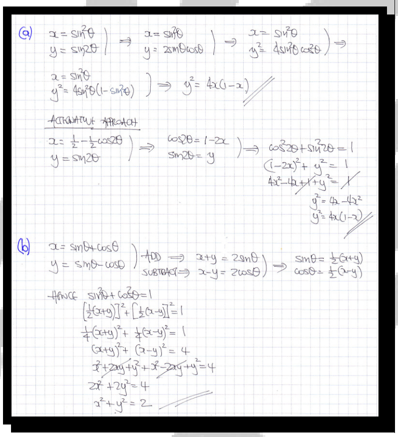
a) $x = \sin^2\theta, \quad y = \sin 2\theta, \qquad 0 \le \theta < 2\pi$

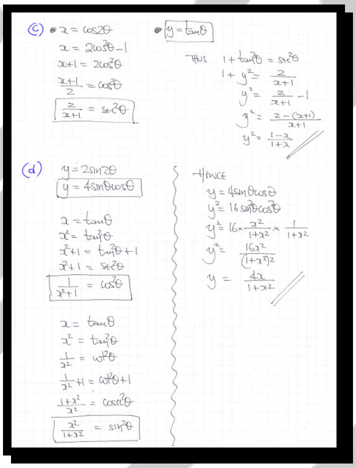
b) $x = \sin\theta + \cos\theta, \quad y = \sin\theta - \cos\theta, \qquad 0 \le \theta < 2\pi$

c) $x = \cos 2\theta, \quad y = \tan\theta, \qquad 0 \le \theta < 2\pi$

d) $x = \tan\theta, \quad y = 2\sin 2\theta, \qquad 0 \le \theta < 2\pi$

$$\boxed{y^2 = 4x(1-x)}, \quad \boxed{x^2 + y^2 = 2}, \quad \boxed{y^2 = \frac{1-x}{1+x}}, \quad \boxed{y = \frac{4x}{1+x^2}}$$





Question 7

Eliminate the parameter θ to obtain a Cartesian equation for each of the following parametric expressions.

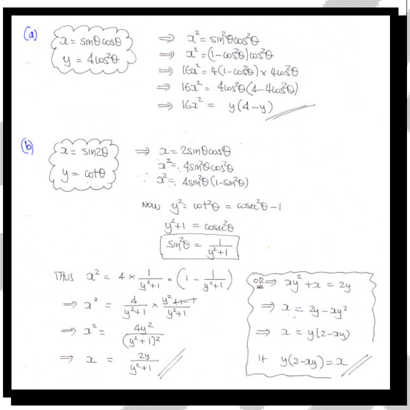
a) $x = \sin\theta\cos\theta, \quad y = 4\cos^2\theta, \qquad 0 \le \theta < 2\pi$

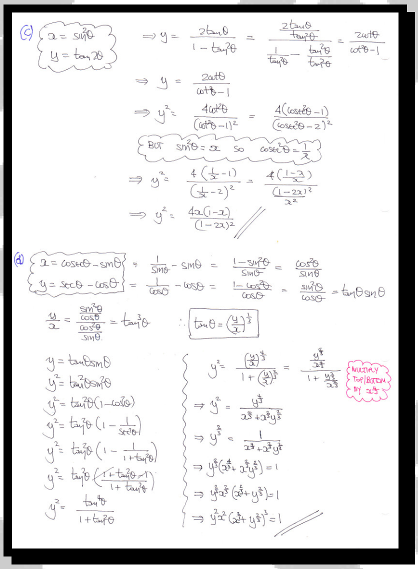
b) $x = \sin 2\theta, \quad y = \cot\theta, \qquad 0 \le \theta < 2\pi$

c) $x = \sin^2\theta, \quad y = \tan 2\theta, \qquad 0 \le \theta < 2\pi$

d) $x = \operatorname{cosec}\theta - \sin\theta, \quad y = \sec\theta - \cos\theta, \qquad 0 \le \theta < 2\pi$

$$\boxed{16x^2 = y(4-y)}, \quad \boxed{y(2-xy) = x}, \quad \boxed{y^2 = \frac{4x(1-x)}{(1-2x)^2}}, \quad \boxed{y^2x^2\left(x^{\frac{2}{3}} + y^{\frac{2}{3}}\right)^3 = 1}$$





PARAMETRIC ALGEBRA

Question 1

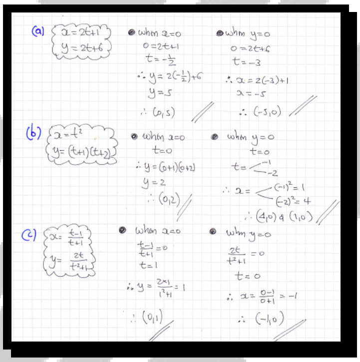
Find the x and y intercepts for each pair of parametric equations.

a) $x = 2t + 1, \quad y = 2t + 6, \quad t \in \mathbb{R}$

b) $x = t^2, \quad y = (t+1)(t+2), \quad t \in \mathbb{R}$

c) $x = \dfrac{t-1}{t+1}, \quad y = \dfrac{2t}{t^2+1}, \quad t \in \mathbb{R}, \quad t \neq -1$

$\boxed{(0,5) \,\&\, (-5,0)}$, $\boxed{(0,2) \,\&\, (4,0),(1,0)}$, $\boxed{(0,1) \,\&\, (-1,0)}$



Question 2

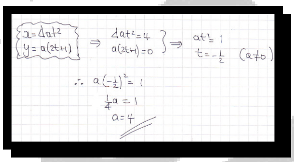
A curve is defined by the following parametric equations

$$x = 4at^2, \quad y = a(2t+1), \quad t \in \mathbb{R}.$$

where a is non zero constant.

Given the curves passes through the point $A(4,0)$, find the value of a.

$\boxed{a = 4}$



Question 3

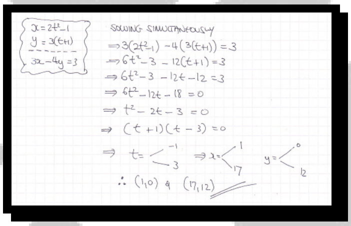
A curve is given by the parametric equations

$$x = 2t^2 - 1, \quad y = 3(t+1), \quad t \in \mathbb{R}.$$

Find the coordinates of the points of intersection of this curve and the line with equation

$$3x - 4y = 3.$$

$\boxed{(17,12) \ \& \ (1,0)}$



Question 4

The curve C_1 has Cartesian equation

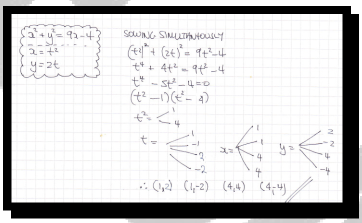
$$x^2 + y^2 = 9x - 4.$$

The curve C_2 has parametric equations

$$x = t^2, \quad y = 2t \quad , \quad t \in \mathbb{R}.$$

Find the coordinates of the points of intersection of C_1 and C_2.

$\boxed{(4,4), \ (4,-4), \ (1,2), \ (1,-2)}$



Question 5

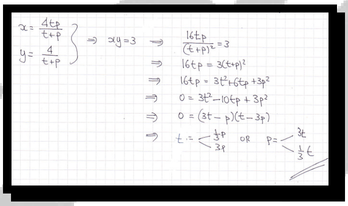
The curve with Cartesian equation $xy = 3$ is also traced by the following parametric equations

$$x = \frac{4tp}{t+p}, \quad y = \frac{4}{t+p}, \quad t, p \in \mathbb{R}, \; t \neq p$$

where t and p are parameters.

Find the relationship between the two parameters t and p in the form $p = f(t)$.

$$\boxed{p = 3t \text{ or } p = \frac{1}{3}t}$$



PARAMETRIC DIFFERENTIATION

Question 1

A curve is given parametrically by the equations

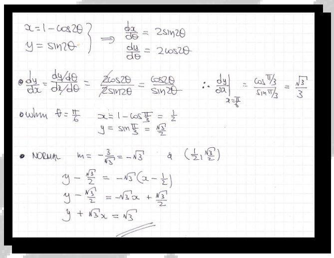
$$x = 1 - \cos 2\theta, \quad y = \sin 2\theta, \quad 0 \le \theta < 2\pi .$$

The point P lies on this curve, and the value of θ at that point is $\frac{\pi}{6}$.

Show that an equation of the normal at the point P is given by

$$y + \sqrt{3}x = \sqrt{3} .$$

proof



Question 2

A curve is given parametrically by the equations

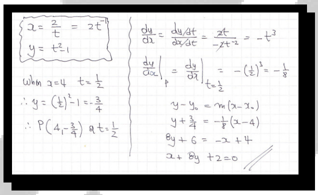
$$x = \frac{2}{t}, \;\; y = t^2 - 1, \;\; t \in \mathbb{R}, \; t \neq 0.$$

The point $P(4, y)$ lies on this curve.

Show that an equation of the tangent at the point P is given by

$$x + 8y + 2 = 0.$$

proof



Question 3

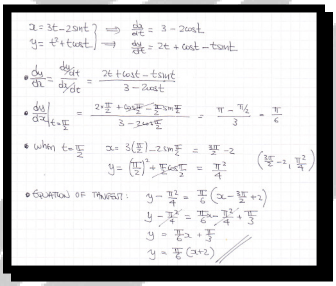
A curve is given parametrically by the equations

$$x = 3t - 2\sin t\,,\;\; y = t^2 + t\cos t\,,\;\; 0 \le t < 2\pi\,.$$

Show that an equation of the tangent at the point on the curve where $t = \frac{\pi}{2}$ is given by

$$y = \frac{\pi}{6}\left(x+2\right).$$

proof



Question 4

Find the turning points of the curve given parametrically by the equations

$$x = 1 - \cos 2t\,,\;\; y = \sin 2t\,,\;\; 0 \le t < 2\pi\,.$$

Determine the nature of these turning points.

$\max\left(1,1\right),\ \ \min\left(1,-1\right)$

Question 5

A curve is given parametrically by the equations

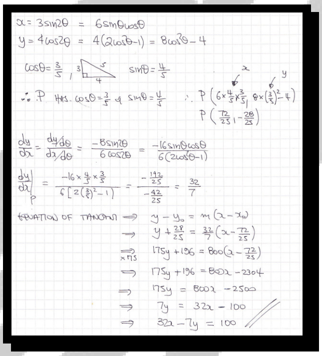
$$x = 3\sin 2\theta, \quad y = 4\cos 2\theta, \quad 0 \le \theta \le 2\pi.$$

The point P is such so that $\cos\theta = \frac{3}{5}$ with $0 \le \theta \le \frac{\pi}{2}$.

Show that an equation of the tangent at the point P is

$$32x - 7y = 100.$$

proof



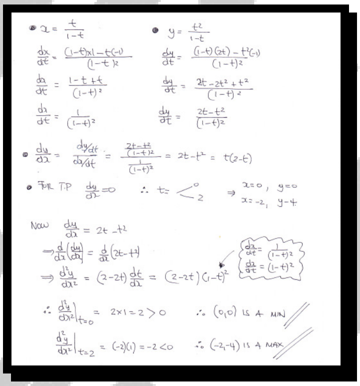
Question 6

For the curve given parametrically by

$$x = \frac{t}{1-t}, \quad y = \frac{t^2}{1-t} \quad , \quad t \in \mathbb{R},\ t \neq 1,$$

find the coordinates of the turning points and determine their nature.

$$\boxed{\max(-2,-4),\ \ \min(0,0)}$$



Question 7

A curve is given parametrically by the equations

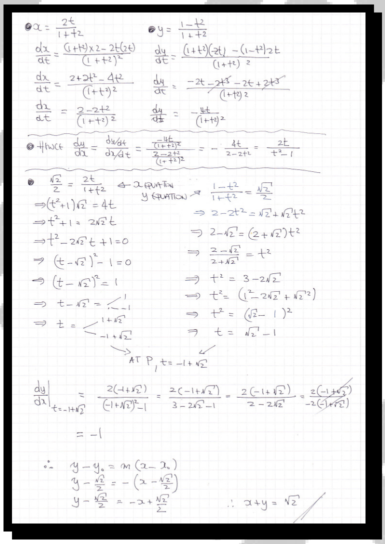
$$x = \frac{2t}{1+t^2}, \quad y = \frac{1-t^2}{1+t^2}, \quad t \in \mathbb{R}.$$

The point $P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lies on this curve.

Show that an equation of the tangent at the point P is given by

$$x + y = \sqrt{2}.$$

proof



$x = \frac{2t}{1+t^2}$ $\qquad$ $y = \frac{1-t^2}{1+t^2}$

$\frac{dx}{dt} = \frac{(1+t^2)\times 2 - 2t(2t)}{(1+t^2)^2}$ $\qquad$ $\frac{dy}{dt} = \frac{(1+t^2)(-2t) - (1-t^2)2t}{(1+t^2)^2}$

$\frac{dx}{dt} = \frac{2+2t^2-4t^2}{(1+t^2)^2}$ $\qquad$ $\frac{dy}{dt} = \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2}$

$\frac{dx}{dt} = \frac{2-2t^2}{(1+t^2)^2}$ $\qquad$ $\frac{dy}{dt} = -\frac{4t}{(1+t^2)^2}$

Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{-4t}{(1+t^2)^2}}{\frac{2-2t^2}{(1+t^2)^2}} = -\frac{4t}{2-2t^2} = \frac{2t}{t^2-1}$

$\frac{\sqrt{2}}{2} = \frac{2t}{1+t^2}$ ← x equation $\qquad$ y equation → $\frac{1-t^2}{1+t^2} = \frac{\sqrt{2}}{2}$

$\Rightarrow (t^2+1)\sqrt{2} = 4t$ $\qquad$ $\Rightarrow 2-2t^2 = \sqrt{2}+\sqrt{2}t^2$

$\Rightarrow t^2+1 = 2\sqrt{2}t$ $\qquad$ $\Rightarrow 2-\sqrt{2} = (2+\sqrt{2})t^2$

$\Rightarrow t^2 - 2\sqrt{2}t + 1 = 0$ $\qquad$ $\Rightarrow \frac{2-\sqrt{2}}{2+\sqrt{2}} = t^2$

$\Rightarrow (t-\sqrt{2})^2 - 1 = 0$ $\qquad$ $\Rightarrow t^2 = 3-2\sqrt{2}$

$\Rightarrow (t-\sqrt{2})^2 = 1$ $\qquad$ $\Rightarrow t^2 = (1^2 - 2\sqrt{2} + \sqrt{2}^2)$

$\Rightarrow t-\sqrt{2} = 1$ or -1 $\qquad$ $\Rightarrow t^2 = (\sqrt{2}-1)^2$

$\Rightarrow t = 1+\sqrt{2}$ or $-1+\sqrt{2}$ $\qquad$ $\Rightarrow t = \sqrt{2}-1$

AT P, $t = -1+\sqrt{2}$

$\left.\frac{dy}{dx}\right|_{t=-1+\sqrt{2}} = \frac{2(-1+\sqrt{2})}{(-1+\sqrt{2})^2-1} = \frac{2(-1+\sqrt{2})}{3-2\sqrt{2}-1} = \frac{2(-1+\sqrt{2})}{2-2\sqrt{2}} = \frac{2(-1+\sqrt{2})}{-2(-1+\sqrt{2})}$

$= -1$

$\therefore \; y - y_0 = m(x-x_0)$

$y - \frac{\sqrt{2}}{2} = -\left(x - \frac{\sqrt{2}}{2}\right)$

$y - \frac{\sqrt{2}}{2} = -x + \frac{\sqrt{2}}{2}$

$\therefore x + y = \sqrt{2}$