DATA ANALYSIS

DISCRETE DATA

Question 1

The following set of data is given

$$11,\ 45,\ 47,\ 49,\ 50,\ 52,\ 53,\ 59,\ 60,\ 94,\ 117.$$

For this set of data, ...

a) ... determine the value of the median and the quartiles.

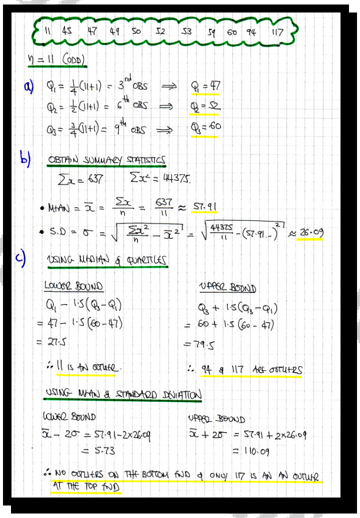
b) ... calculate the mean and the standard deviation.

c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

e) ... draw a suitably labelled box plot.

$\boxed{}$, $\boxed{Q_1 = 47}$, $\boxed{Q_2 = 52}$, $\boxed{Q_3 = 60}$, $\boxed{\bar{x} = 57.91}$, $\boxed{\sigma \approx 26.09}$, $\boxed{\text{(11, 94) and 117 are outliers depending on method}}$, $\boxed{\text{positive skew}}$



11 45 47 49 50 52 53 59 60 94 117

$n = 11$ (ODD)

a) $Q_1 = \frac{1}{4}(11+1) = 3^{rd}$ OBS $\Rightarrow$ $Q_1 = 47$

$Q_2 = \frac{1}{2}(11+1) = 6^{th}$ OBS $\Rightarrow$ $Q_2 = 52$

$Q_3 = \frac{3}{4}(11+1) = 9^{th}$ OBS $\Rightarrow$ $Q_3 = 60$

b) OBTAIN SUMMARY STATISTICS

$\sum x = 637 \qquad \sum x^2 = 44375.$

• MEAN $= \bar{x} = \frac{\sum x}{n} = \frac{637}{11} \approx 57.91$

• S.D $= \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{44375}{11} - (57.91..)^2} \approx 26.09$

c) USING MEDIAN & QUARTILES

LOWER BOUND

$Q_1 - 1.5(Q_3 - Q_1)$

$= 47 - 1.5(60 - 47)$

$= 27.5$

$\therefore$ 11 IS AN OUTLIER.

UPPER BOUND

$Q_3 + 1.5(Q_3 - Q_1)$

$= 60 + 1.5(60 - 47)$

$= 79.5$

$\therefore$ 94 & 117 ARE OUTLIERS

USING MEAN & STANDARD DEVIATION

LOWER BOUND

$\bar{x} - 2\sigma = 57.91 - 2 \times 26.09$

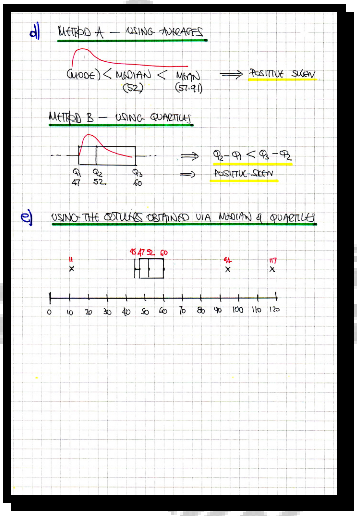
$= 5.73$

UPPER BOUND

$\bar{x} + 2\sigma = 57.91 + 2 \times 26.09$

$= 110.09$

$\therefore$ NO OUTLIERS ON THE BOTTOM END & ONLY 117 IS AN AN OUTLIER AT THE TOP END



d) METHOD A – USING AVERAGES

(MODE) < MEDIAN < MEAN $\Rightarrow$ POSITIVE SKEW

(52) (57.91)

METHOD B – USING QUARTILES

$\Rightarrow Q_2 - Q_1 < Q_3 - Q_2$

$\Rightarrow$ POSITIVE SKEW

e) USING THE OUTLIERS OBTAINED VIA MEDIAN & QUARTILES

Question 2

The following set of data is given

$$25,\ 47,\ 47,\ 49,\ 50,\ 52,\ 54,\ 55,\ 56,\ 56,\ 56,\ 59.$$

For this set of data, ...

a) ... determine the value of the median and the quartiles.

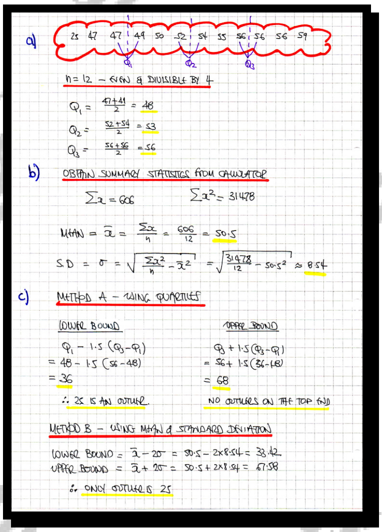
b) ... calculate the mean and the standard deviation.

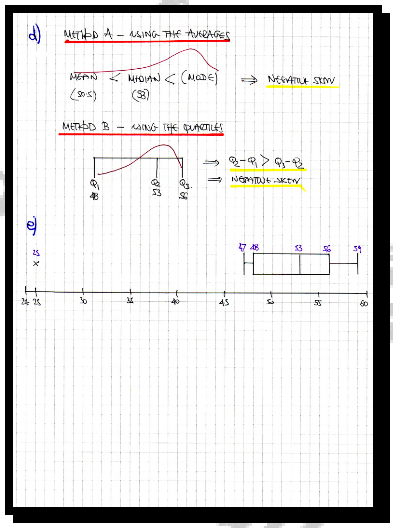
c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

e) ... draw a suitably labelled box plot.

$\boxed{}$, $\boxed{Q_1 = 48}$, $\boxed{Q_2 = 53}$, $\boxed{Q_3 = 56}$, $\boxed{\bar{x} = 50.5}$, $\boxed{\sigma \approx 8.54}$, $\boxed{\text{25 is an outlier}}$, $\boxed{\text{negative skew}}$





Question 3

The following set of data is given

$$78,\ 79,\ 79,\ 79,\ 80,\ 82,\ 82,\ 85,\ 86,\ 88,\ 89,\ 92,\ 97.$$

For this set of data, ...

a) ... determine the value of the median and the quartiles.

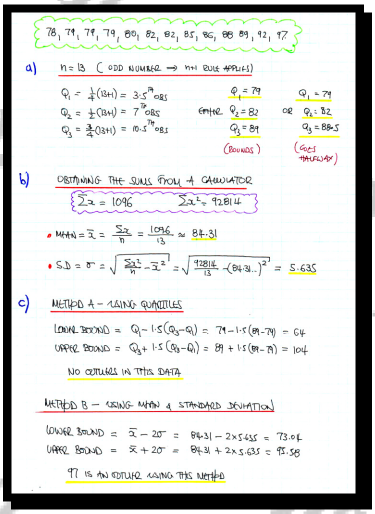
b) ... calculate the mean and the standard deviation.

c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

e) ... draw a suitably labelled box plot.

$\boxed{}$, $\boxed{Q_1 = 79}$, $\boxed{Q_2 = 82}$, $\boxed{Q_3 = 89}$, $\boxed{\bar{x} = 84.31}$, $\boxed{\sigma \approx 5.635}$,
$\boxed{\text{no outliers or 97 is an outlier depenending on method}}$, $\boxed{\text{positive skew}}$



78, 79, 79, 79, 80, 82, 82, 85, 86, 88 89, 92, 97

a) $n = 13$ (ODD NUMBER $\Rightarrow$ n+1 RULE APPLIES)

$Q_1 = \frac{1}{4}(13+1) = 3.5^{\text{TH}}$ OBS
$Q_2 = \frac{1}{2}(13+1) = 7^{\text{TH}}$ OBS
$Q_3 = \frac{3}{4}(13+1) = 10.5^{\text{TH}}$ OBS

EITHER $Q_1 = 79$, $Q_2 = 82$, $Q_3 = 89$ (ROUNDS) OR $Q_1 = 79$, $Q_2 = 82$, $Q_3 = 88.5$ (GOES HALFWAY)

b) OBTAINING THE SUMS FROM A CALCULATOR

$\sum x = 1096 \qquad \sum x^2 = 92814$

- MEAN $= \bar{x} = \frac{\sum x}{n} = \frac{1096}{13} \approx 84.31$
- S.D $= \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{92814}{13} - (84.31..)^2} = 5.635$

c) METHOD A – USING QUARTILES

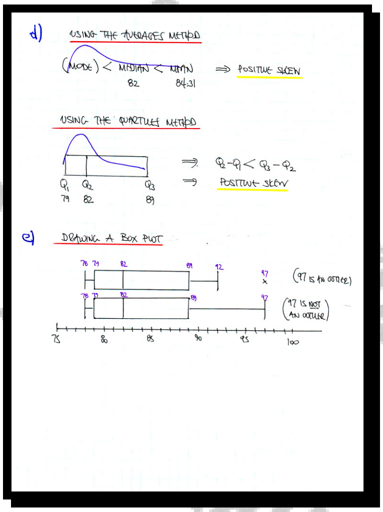
LOWER BOUND $= Q_1 - 1.5(Q_3 - Q_1) = 79 - 1.5(89-79) = 64$
UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 89 + 1.5(89-79) = 104$

NO OUTLIERS IN THIS DATA

METHOD B – USING MEAN & STANDARD DEVIATION

LOWER BOUND $= \bar{x} - 2\sigma = 84.31 - 2\times5.635 = 73.04$
UPPER BOUND $= \bar{x} + 2\sigma = 84.31 + 2\times5.635 = 95.58$

97 IS AN OUTLIER USING THIS METHOD



d) USING THE AVERAGES METHOD

(MODE) < MEDIAN < MEAN $\Rightarrow$ POSITIVE SKEW
82 84.31

USING THE QUARTILES METHOD

$\Rightarrow Q_2 - Q_1 < Q_3 - Q_2$
$\Rightarrow$ POSITIVE SKEW

e) DRAWING A BOX PLOT

Question 4

The following set of data is given

$$50,\ 52,\ 52,\ 52,\ 53,\ 54,\ 56,\ 58,\ 61,\ 64,\ 65,\ 69,\ 75,\ 77.$$

For this set of data, ...

a) ... determine the value of the median and the quartiles.

b) ... calculate the mean and the standard deviation.

c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

e) ... draw a suitably labelled box plot.

$\boxed{\ }$, $\boxed{Q_1 = 52}$, $\boxed{Q_2 = 57}$, $\boxed{Q_3 = 65}$, $\boxed{\bar{x} = 59.86}$, $\boxed{\sigma \approx 8.59}$, $\boxed{\text{no outliers}}$, $\boxed{\text{positive skew}}$

50 52 52 52 53 54 56 ¦ 58 61 64 65 69 75 77
Q_2

a) $n = 14$ (EVEN BUT NOT DIVISIBLE BY 4)

- $Q_1 = \frac{1}{4}(14+1) = 3.75 = 4^{\text{TH}}$ OBS $\Rightarrow Q_1 = 52$
- $Q_2 = \frac{56+58}{2} = 57 \Rightarrow Q_2 = 57$
- $Q_3 = \frac{3}{4}(14+1) = 11.25 = 11^{\text{TH}}$ OBS $\Rightarrow Q_3 = 65$

b) OBTAIN SUMMARY STATISTICS FROM CALCULATOR

$\Sigma x = 838 \qquad \Sigma x^2 = 51194$

- MEAN $= \bar{x} = \frac{\Sigma x}{n} = \frac{838}{14} = 59.86$
- S.D $= \sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{51194}{14} - (59.86...)^2} \simeq 8.59$

c) USING THE QUARTILES METHOD

LOWER BOUND $= Q_1 - 1.5(Q_3 - Q_1) = 52 - (65-52)\times 1.5 = 32.5$
UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 65 - (65-52)\times 1.5 = 84.5$

NO OUTLIERS

USING THE MEAN & STANDARD DEVIATION METHOD

LOWER BOUND $= \bar{x} - 2\sigma = 59.86 - 2\times 8.59 = 42.65$
UPPER BOUND $= \bar{x} + 2\sigma = 59.86 + 2\times 8.59 = 77.04$

NO OUTLIERS

d) USING THE AVERAGES METHOD

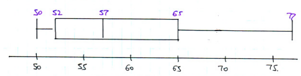
(MODE) < MEDIAN < MEAN $\Rightarrow$ POSITIVE SKEW
57 59.86

USING THE QUARTILES METHOD

Q_1 (52) Q_2 (57) Q_3 (65)

$\Rightarrow Q_2 - Q_1 < Q_3 - Q_2$
$\Rightarrow$ POSITIVE SKEW.

e) BOX PLOT



Question 5

The following set of data is given

$$7,\ 8,\ 10,\ 12,\ 16,\ 19,\ 19,\ 20,\ 23,\ 24,\ 30.$$

For this set of data, ...

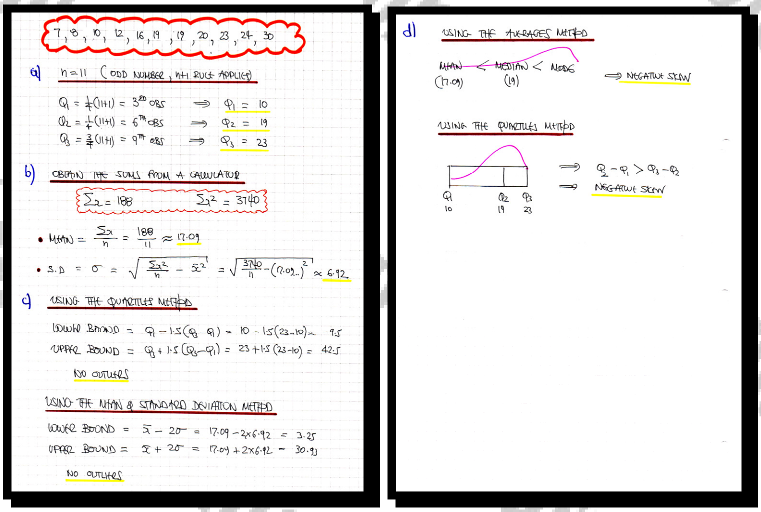
a) ... determine the value of the median and the quartiles.

b) ... calculate the mean and the standard deviation.

c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

$\boxed{\ }$, $\boxed{Q_1 = 10}$, $\boxed{Q_2 = 19}$, $\boxed{Q_3 = 23}$, $\boxed{\bar{x} \approx 17.09}$, $\boxed{\sigma \approx 6.92}$, $\boxed{\text{no outliers}}$, $\boxed{\text{negative skew}}$



7, 8, 10, 12, 16, 19, 19, 20, 23, 24, 30

a) $n = 11$ (ODD NUMBER, $n+1$ RULE APPLIES)

$Q_1 = \frac{1}{4}(11+1) = 3^{RD}$ OBS $\Longrightarrow Q_1 = 10$

$Q_2 = \frac{1}{4}(11+1) = 6^{TH}$ OBS $\Longrightarrow Q_2 = 19$

$Q_3 = \frac{3}{4}(11+1) = 9^{TH}$ OBS $\Longrightarrow Q_3 = 23$

b) OBTAIN THE SUMS FROM A CALCULATOR

$\sum x = 188 \qquad \sum x^2 = 3740$

• MEAN $= \frac{\sum x}{n} = \frac{188}{11} \approx 17.09$

• S.D $= \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{3740}{11} - (17.09\ldots)^2} \approx 6.92$

c) USING THE QUARTILES METHOD

LOWER BOUND $= Q_1 - 1.5(Q_3 - Q_1) = 10 - 1.5(23-10) = -9.5$

UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 23 + 1.5(23-10) = 42.5$

NO OUTLIERS

USING THE MEAN & STANDARD DEVIATION METHOD

LOWER BOUND $= \bar{x} - 2\sigma = 17.09 - 2\times 6.92 = 3.25$

UPPER BOUND $= \bar{x} + 2\sigma = 17.09 + 2\times 6.92 = 30.93$

NO OUTLIERS

d) USING THE AVERAGES METHOD

MEAN < MEDIAN < MODE
(17.09) (19)

$\Longrightarrow$ NEGATIVE SKEW

USING THE QUARTILES METHOD

Q_1 10, Q_2 19, Q_3 23

$\Longrightarrow Q_2 - Q_1 > Q_3 - Q_2$

$\Longrightarrow$ NEGATIVE SKEW

Question 6

The following set of data is given

$$5,\ 10,\ 11,\ 11,\ 14,\ 17,\ 19,\ 24,\ 30,\ 36,\ 40,\ 45.$$

For this set of data, ...

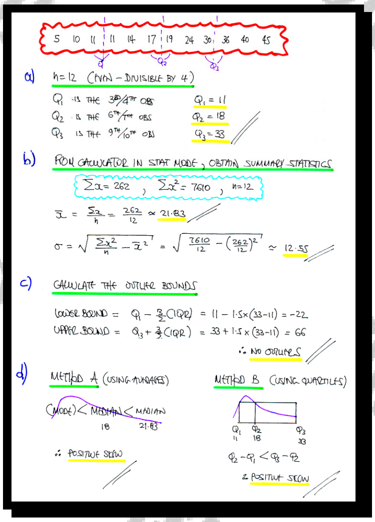
a) ... determine the value of the median and the quartiles.

b) ... calculate the mean and the standard deviation.

c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

$\boxed{}$, $\boxed{Q_1 = 11}$, $\boxed{Q_2 = 18}$, $\boxed{Q_3 = 33}$, $\boxed{\bar{x} \approx 21.83}$, $\boxed{\sigma \approx 12.55}$, $\boxed{\text{no outliers}}$, $\boxed{\text{positive skew}}$



5 10 11 | 11 14 17 | 19 24 30 | 36 40 45

Q_1 Q_2 Q_3

a) $n = 12$ (EVEN – DIVISIBLE BY 4)

Q_1 IS THE 3RD/4TH OBS $\quad Q_1 = 11$

Q_2 IS THE 6TH/7TH OBS $\quad Q_2 = 18$

Q_3 IS THE 9TH/10TH OBS $\quad Q_3 = 33$

b) FROM CALCULATOR IN STAT MODE, OBTAIN SUMMARY STATISTICS

$\sum x = 262$, $\sum x^2 = 7610$, $n = 12$

$\bar{x} = \frac{\sum x}{n} = \frac{262}{12} \approx 21.83$

$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{7610}{12} - \left(\frac{262}{12}\right)^2} \simeq 12.55$

c) CALCULATE THE OUTLIER BOUNDS

LOWER BOUND $= Q_1 - \frac{3}{2}(\text{IQR}) = 11 - 1.5 \times (33-11) = -22$

UPPER BOUND $= Q_3 + \frac{3}{2}(\text{IQR}) = 33 + 1.5 \times (33-11) = 66$

$\therefore$ NO OUTLIERS

d) METHOD A (USING AVERAGES)

(MODE) < MEDIAN < MEAN

18 $\quad$ 21.83

$\therefore$ POSITIVE SKEW

METHOD B (USING QUARTILES)

Q_1 Q_2 Q_3

11 18 33

$Q_2 - Q_1 < Q_3 - Q_2$

$\therefore$ POSITIVE SKEW

Question 7

The following set of data shows the number of posts made, in a given day, in a social media site by a group of individuals.

$$1,\ 12,\ 13,\ 14,\ 16,\ 17,\ 20,\ 21,\ 23,\ 24,\ 26,\ 39,\ 55.$$

For this set of data, ...

a) ... determine the value of the median and the quartiles.

b) ... calculate the mean and the standard deviation.

c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

$\boxed{}$, $\boxed{(Q_1, Q_2, Q_3) = (14, 20, 26) \quad \text{or} \quad (Q_1, Q_2, Q_3) = (13.5, 20, 25)}$, $\boxed{\bar{x} \approx 21.6}$, $\boxed{\sigma \approx 12.9}$, $\boxed{\text{55 is an outlier}}$, $\boxed{\text{no skew or positive skew depending on the method}}$

1, 12, 13, 14, 16, 17, 20, 21, 23, 24, 26, 39, 55

$n = 13$ (ODD)

a) MEDIAN & QUARTILES (n+1) RULE APPLIES

$Q_1 = \frac{1}{4}(13+1) = 3.5$ I.E $3^{RD}/4^{TH}$ OR 4^{TH}

$Q_2 = \frac{1}{2}(13+1) = 7^{th}$ OBS

$Q_3 = \frac{3}{4}(13+1) = 10.5$, I.E $10^{TH}/11^{TH}$ OR 11^{TH} OBS

$\therefore$ $Q_1 = 13.5$, $Q_2 = 20$, $Q_3 = 25$

OR

$Q_1 = 14$, $Q_2 = 20$, $Q_3 = 26$

b) USING CALCULATOR IN STAT MODE WE OBTAIN

$\sum x = 281$, $\sum x^2 = 8223$

- $\bar{x} = \frac{\sum x}{n} = \frac{281}{13} \simeq 21.6$
- $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{8223}{13} - \left(\frac{281}{13}\right)^2} \approx 12.9$

c) USING THE MOST COMMON CRITERION

- LOWER BOUND $= Q_1 - \frac{3}{2}(IQR) = 14 - \frac{3}{2}(26-14) = -4$
- UPPER BOUND $= Q_3 + \frac{3}{2}(IQR) = 26 + \frac{3}{2}(26-14) = 44$

$\therefore$ 55 IS A OUTLIER

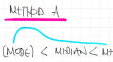
ALTERNATIVE METHOD FOR OUTLIERS

LOWER BOUND $= \bar{x} - 2\sigma = 21.6 - 2 \times 12.9 \simeq -4$

UPPER BOUND $= \bar{x} + 2\sigma = 21.6 + 2 \times 12.9 \simeq 47$

$\therefore$ 55 IS AN OUTLIER

d) METHOD A

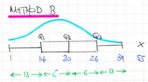


(MODE) < MEDIAN < MEAN

20 21.6

$\therefore$ POSITIVE SKEW

METHOD B



$Q_2 - Q_1 = Q_3 - Q_2$

$\therefore$ SYMMETRICAL (NO SKEW)

THE TWO METHODS HERE DISAGREE (SOMETIMES THIS HAPPENS)
THIS IS DUE TO THE OUTLIER AT 55.

IF THE OUTLIER IS CONSIDERED THERE IS SMALL POSITIVE SKEW; NOTE THAT MEAN & MEDIAN ARE STILL CLOSE.

IF THE OUTLIER IS NOT CONSIDERED THE DISTRIBUTION IS VERY SYMMETRICAL

Question 8

The following set of data is given

$$21,\ 50,\ 51,\ 53,\ 54,\ 57,\ 58,\ 60,\ 62,\ 63,\ 64,\ 68,\ 82,\ 97.$$

For this set of data, ...

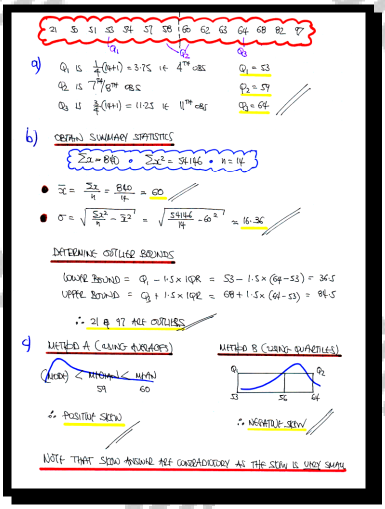
a) ... determine the value of the median and the quartiles.

b) ... calculate the mean and the standard deviation.

c) ... determine with justification whether there are any outliers.

d) ... state with justification if there is any type of skew.

$\boxed{}$, $\boxed{Q_1 = 53}$, $\boxed{Q_2 = 59}$, $\boxed{Q_3 = 64}$, $\boxed{\bar{x} = 60}$, $\boxed{\sigma \approx 16.36}$, $\boxed{\text{21 and 97 are outliers}}$, $\boxed{\text{positive/negative skew}}$



21 50 51 53 54 57 58 | 60 62 63 64 68 82 97

a) Q_1 is $\frac{1}{4}(14+1) = 3.75$ ie 4^{TH} OBS $Q_1 = 53$

Q_2 is $7^{TH}/8^{TH}$ OBS $Q_2 = 59$

Q_3 is $\frac{3}{4}(14+1) = 11.25$ ie 11^{TH} OBS $Q_3 = 64$

b) OBTAIN SUMMARY STATISTICS

$\sum x = 840 \quad \sum x^2 = 54146 \quad n = 14$

- $\bar{x} = \frac{\sum x}{n} = \frac{840}{14} = 60$
- $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{54146}{14} - 60^2} \approx 16.36$

DETERMINE OUTLIER BOUNDS

LOWER BOUND $= Q_1 - 1.5 \times IQR = 53 - 1.5 \times (64-53) = 36.5$

UPPER BOUND $= Q_3 + 1.5 \times IQR = 68 + 1.5 \times (64-53) = 84.5$

$\therefore$ 21 & 97 ARE OUTLIERS

c) METHOD A (USING AVERAGES)

(MODE) < MEDIAN < MEAN
59 60

$\therefore$ POSITIVE SKEW

METHOD B (USING QUARTILES)

Q_1 Q_2
53 56 64

$\therefore$ NEGATIVE SKEW

NOTE THAT SKEW ANSWER ARE CONTRADICTORY AS THE SKEW IS VERY SMALL

Question 9

The % marks, rounded to the nearest integer, of a recent Mathematics test taken by 16 students, were summarised in an ordered stem and leaf diagram.

$$\begin{array}{c||l}
4 & 7 \\
5 & 2,3,8 \\
6 & 0,3,4,\ a,\ b \\
7 & 3,6,c,d,8 \\
8 & 1,9
\end{array} \qquad \text{where } 5\|2 = 52\,.$$

a) Determine the lower quartile of the data.

b) Given the median is 68 and $a \neq b$, find the value of a and the value of b.

It is further given that $c \neq d$.

c) Find the possible values of the upper quartile.

$\boxed{Q_1 = 59}$, $\boxed{a = 7,\ \ b = 9}$, $\boxed{76.5,\ 77,\ 77.5}$

a)

$$\begin{array}{c|l l}
4 & 7 & (1) \\
5 & 2\,3\,8 & (3) \\
6 & 0\,3\,4\,a\,b & (5) \\
7 & 3\,6\,c\,d\,8 & (5) \\
8 & 1\,9 & (2)
\end{array}$$

TOTAL OF 16 OBSERVATIONS

Q_1 IS THE $4^{TH}/5^{TH}$ OBS

$\therefore Q_1 = \frac{58+60}{2} = 59$

b) MEDIAN IS $8^{TH}/9^{TH}$ OBSERVATION

IE $Q_2 = \frac{\text{"}6a\text{"} + \text{"}6b\text{"}}{2} = 68$

$\therefore a = 7$ & $b = 9$

$\cancel{68} - \cancel{68}$ $a \neq b$

$67 - 69$

$\cancel{66} - \cancel{70}$

ETC

c) Q_3 IS $12^{TH}/13^{TH}$ OBSERVATION

IE $Q_3 = \frac{\text{"}7c\text{"} + \text{"}7d\text{"}}{2}$

$\cancel{76} - \cancel{76}$ $c \neq d$

$76 - 77 \Rightarrow Q_3 = 76.5$

$76 - 78 \Rightarrow Q_3 = 77$

$\cancel{77} - \cancel{77}$ $c \neq d$

$77 - 78 \Rightarrow Q_3 = 77.5$

$\therefore$ POSSIBLE VALUES OF Q_3 ARE 76.5, 77 & 77.5

Question 10

The concentration of lactic acid, in appropriate units, after a period of intense exercise was measured in the blood of 12 marathon runners.

Athlete	A	B	C	D	E	F	G	H	I	J	K	L
Lactic Acid Concentration	180	172	110	175	256	140	241	450	205	375	402	195

a) Determine the value of the median and the quartiles.

b) Find the mean and the standard deviation of the data.

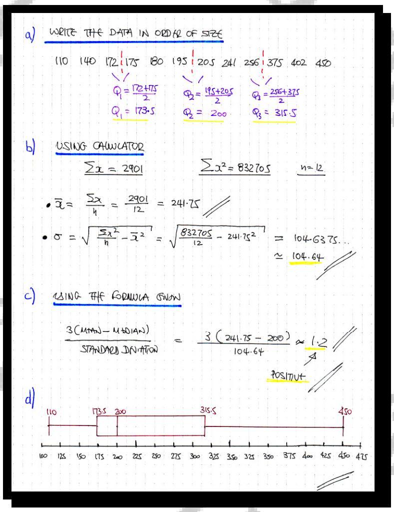
The skewness of data can be determined by the formula

$$\frac{3(\text{mean} - \text{median})}{\text{standard deviation}}.$$

c) Evaluate this expression for this data and hence state its skew.

d) Draw a suitably labelled box plot for this data.
You may assume that there are no outliers in this data.

$\boxed{\bar{x} = 241.75}$, $\boxed{\sigma \approx 104.64}$, $\boxed{Q_1 = 173.5}$, $\boxed{Q_2 = 200}$, $\boxed{Q_3 = 315.5}$, $\boxed{1.20}$, $\boxed{\text{positive skew}}$


a) WRITE THE DATA IN ORDER OF SIZE

110 140 172 175 180 195 205 241 256 375 402 450

$Q_1 = \frac{172+175}{2}$, $Q_1 = 173.5$ $Q_2 = \frac{195+205}{2}$, $Q_2 = 200$ $Q_3 = \frac{256+375}{2}$, $Q_3 = 315.5$

b) USING CALCULATOR

$\sum x = 2901$ $\sum x^2 = 832705$ $n = 12$

• $\bar{x} = \frac{\sum x}{n} = \frac{2901}{12} = 241.75$

• $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{832705}{12} - 241.75^2} = 104.6375\ldots \simeq 104.64$

c) USING THE FORMULA GIVEN

$\frac{3(\text{MEAN} - \text{MEDIAN})}{\text{STANDARD DEVIATION}} = \frac{3(241.75 - 200)}{104.64} \approx 1.2$

POSITIVE

d)

Question 11

The number of phone text messages send by 11 different students is given below.

14, 25, 31, 36, 37, 41, 51, 52, 55, 79, 112.

a) Find the lower quartile, the median and the upper quartile of the data.

b) Show clearly that there is only one outlier in the data.

c) Draw a suitably labelled box plot for this data, clearly indicating any outliers.

d) Determine with justification the skewness of the data.

$\boxed{Q_1 = 3}$, $\boxed{Q_2 = 41}$, $\boxed{Q_3 = 55}$, $\boxed{112 \text{ is the only outlier}}$, $\boxed{\text{positive skew}}$

a) 14 25 31 36 37 41 51 52 55 79 112

N=11, N+1 RULE APPLY

$Q_1 = \frac{1}{4}(11+1) = 3^{RD}$ OBS $\Rightarrow Q_1 = 31$

$Q_2 = \frac{1}{2}(11+1) = 6^{TH}$ OBS $\Rightarrow Q_2 = 41$

$Q_3 = \frac{3}{4}(11+1) = 9^{TH}$ OBS $\Rightarrow Q_3 = 55$

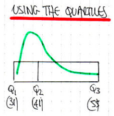
b) LOWER BOUND $= Q_1 - 1.5(Q_3 - Q_1) = 31 - 1.5(55-31) = -5$

UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 55 + 1.5(55-31) = 91$

$\therefore$ ONLY 112 IS AN OUTLIER

c)

d) USING THE QUARTILES



POSITIVE SKEW SINCE $Q_2 - Q_1 < Q_3 - Q_2$

Question 12

The number of bottles of red wine sold by a local supermarket over a two week period is shown below.

$$22,\ 14,\ 11,\ 33,\ 32,\ 45,\ 4,\ 12,\ 13,\ 20,\ 27,\ 44,\ 30,\ 15.$$

a) Display the above data in an ordered stem and leaf diagram.

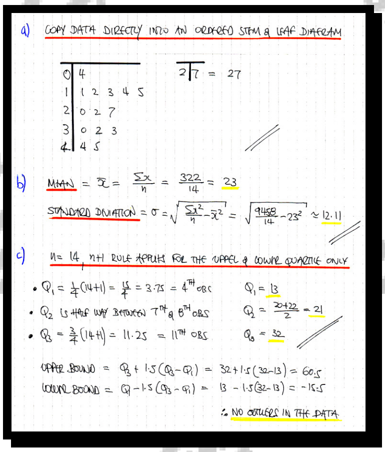
b) Calculate the mean and the standard deviation of the data.

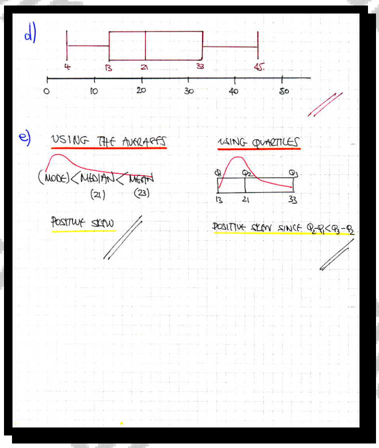
c) Find the median and the quartiles of the data and use them to determine if there are any outliers.

d) Draw a suitably labelled box plot for this data.

e) Determine with justification the skewness of the data.

$\boxed{\bar{x}=23}$, $\boxed{\sigma=12.11}$, $\boxed{Q_1=13}$, $\boxed{Q_2=21}$, $\boxed{Q_3=33}$, $\boxed{\text{no outliers}}$, $\boxed{\text{positive skew}}$





Question 13

A company decides to give their 23 employees a skills test in order to decide if any of these employees need to be retrained.

The maximum possible score in this test is 50 and the results are summarised in an ordered stem and leaf diagram.

0	5
1	9,9
2	1,6,8
3	3,4,5,7
4	2,3,4,4,8,9,9
5	0,0,0,0,0,0

where $2\|9 = 29$.

a) Find the median score of the test.

b) Determine the interquartile range of the scores.

The company decides to retrain any employee whose score is less than the **lower quartile minus the interquartile range**.

c) Show clearly that only one employee will undergo retraining.

d) Draw a suitably labelled box plot for this data, clearly indicating any outliers, as found in part (**c**).

e) Determine with justification the skewness of the scores.

$\boxed{Q_2 = 43}$, $\boxed{IQR = 22}$, $\boxed{\text{05 is the only outlier}}$, $\boxed{\text{negative skew}}$

a) LOOKING AT THE STEM & LEAF DIAGRAM

0	5	(1)
1	9 9	(2)
2	1 6 8	(3)
3	3 4 5 7	(4)
4	2 3 4 4 8 9 9	(7)
5	0 0 0 0 0	(5)

TOTAL OF 23 OBSERVATION

$Q_2 = \frac{1}{2}(23+1) = 12^{TH}$ OBS

$\therefore Q_2 = 43$

b) FIND THE QUARTILES

$Q_1 = \frac{1}{4}(23+1) = 6^{TH}$ OBS $\quad \therefore Q_1 = 28$

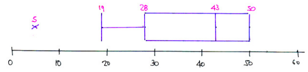
$Q_3 = \frac{3}{4}(23+1) = 18^{TH}$ OBS $\quad \therefore Q_3 = 50$

$\therefore IQR = Q_3 - Q_1 = 50 - 28 = 22$

c) LOWER BOUND $= Q_1 - IQR = 28 - 22 = 6$

$\therefore$ ONLY ONE EMPLOYEE WITH A SCORE OF 5 WILL UNDERGO RETRAINING

d)



e) LOOKING AT THE BOX PLOT

$Q_3 - Q_2 < Q_2 - Q_1 \implies$ NEGATIVE SKEW

Question 14

The ages of the residents of Arnold Street are denoted by x the ages of the residents of Benedict Street are denoted by y.

These are summarized in the following back to back stem and leaf diagram.

x		y
5	0	
5, 5, 3, 3	1	
9, 9, 1	2	5
9, 8, 6, 5, 5, 4, 3, 2, 2, 2, 1	3	6, 7, 8
6, 4, 1, 0, 0, 0, 0	4	1, 2, 2, 3, 4, 8
9	5	1, 4, 4, 4, 4, 5, 8, 8
	6	1, 3, 4, 4, 5, 9, 9
	7	2, 6, 9

where 2|3|9 = 32 in Arnold Street and 39 in Benedict Street.

a) Find separately for the residents of Arnold Street and Benedict Street, ...

i. ... the mode.

ii. ... the lower quartile, the median and the upper quartile.

iii. ... the mean and the standard deviation.

You may assume $\sum x = 866$, $\sum x^2 = 31514$, $\sum y = 1516$, $\sum y^2 = 86880$.

[continues overleaf]

[continued from overleaf]

A coefficient of skewness is defined as

$$\frac{\text{mean} - \text{mode}}{\text{standard deviation}}.$$

b) Evaluate this coefficient for the ages in each street.

c) Compare the distribution of the ages between the two streets.

mode $= 40$ $Q_1 = 29$ $Q_2 = 34$ $Q_3 = 40$ $\bar{x} \approx 32.07$ $\sigma_x \approx 11.77$ skew ≈ -0.67	,	mode $= 54$ $Q_1 = 42.5$ $Q_2 = 54$ $Q_3 = 64$ $\bar{y} \approx 54.14$ $\sigma_y \approx 13.09$ skew ≈ 0.01

	x ARNOLD ST		BENEDICT ST y	
(1)	5	0		
(4)	5 5 3 3	1		
(3)	9 9 1	2	5	(1)
(11)	9 8 6 5 5 4 3 2 2 2 1	3	6 7 8	(3)
(7)	6 4 1 0 0 0 0	4	1 2 2 3 4 8	(6)
(1)	9	5	1 4 4 4 4 5 8 8	(8)
		6	1 3 4 4 5 9 9	(7)
		7	2 6 9	(3)
27				28

$1|3 = 1|3|6 = 36$

a) ARNOLD STREET | BENEDICT STREET

ARNOLD STREET	BENEDICT STREET
MODE = 40	MODE = 54
$Q_1 = \frac{1}{4}(27+1) = 7^{\text{TH}}$ OBS	$Q_1 = 7^{\text{TH}}/8^{\text{TH}}$ OBS
$Q_1 = 29$	$Q_1 = 42.5$
$Q_2 = \frac{1}{2}(27+1) = 14^{\text{TH}}$ OBS	$Q_2 = 14^{\text{TH}}/15^{\text{TH}}$ OBS
$Q_2 = 34$	$Q_2 = 54$
$Q_3 = \frac{3}{4}(27+1) = 21^{\text{ST}}$ OBS	$Q_3 = 21^{\text{ST}}/22^{\text{ND}}$ OBS
$Q_3 = 40$	$Q_3 = 64$
$\bar{x} = \frac{\Sigma x}{n} = \frac{866}{27} = 32.07$	$\bar{y} = \frac{\Sigma y}{n} = \frac{1516}{28} = 54.14$
$\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = 11.77$	$\sigma_y = \sqrt{\frac{\Sigma y^2}{n} - \bar{y}^2} = 13.09$

b) USING THE FORMULA GIVEN

FOR ARNOLD STREET $= \frac{32.07 - 40}{11.77} = -0.67$

FOR BENEDICT STREET $= \frac{54.14 - 54}{13.09} = 0.01$

c)
- MEDIAN & MEAN IS HIGHER IN BENEDICT STREET, INDICATING OLDER PEOPLE LIVING THERE
- BENEDICT STREET AGES ARE SLIGHTLY MORE VARIED, AS INDICATED BY THE STANDARD DEVIATION
- DATA IN ARNOLD STREET IS NEGATIVELY SKEWED AS INDICATED BY PART (b), WHILE DATA IN BENEDICT STREET IS PRACTICALLY SYMMETRICAL (SLIGHT POSITIVE SKEW) AS INDICATED BY (b)

Question 15

The mean and standard deviation of 20 observations $x_1, x_2, x_3, \ldots, x_{20}$ are

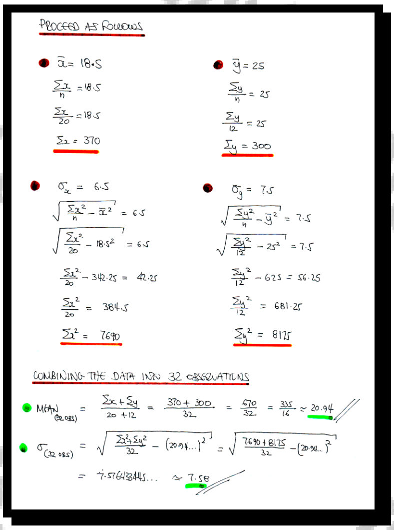
$$\bar{x} = 18.5 \quad \text{and} \quad \sigma_x = 6.5 .$$

The mean and standard deviation of 12 observations $y_1, y_2, y_3, \ldots, y_{12}$ are

$$\bar{y} = 25 \quad \text{and} \quad \sigma_y = 7.5 .$$

Determine the mean and the standard deviation of all 32 observations.

$\boxed{\text{mean} \approx 20.94}$, $\boxed{\text{standard deviation} \approx 7.58}$



PROCEED AS FOLLOWS

- $\bar{x} = 18.5$
  $\frac{\Sigma x}{n} = 18.5$
  $\frac{\Sigma x}{20} = 18.5$
  $\Sigma x = 370$
- $\bar{y} = 25$
  $\frac{\Sigma y}{n} = 25$
  $\frac{\Sigma y}{12} = 25$
  $\Sigma y = 300$
- $\sigma_x = 6.5$
  $\sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = 6.5$
  $\sqrt{\frac{\Sigma x^2}{20} - 18.5^2} = 6.5$
  $\frac{\Sigma x^2}{20} - 342.25 = 42.25$
  $\frac{\Sigma x^2}{20} = 384.5$
  $\Sigma x^2 = 7690$
- $\sigma_y = 7.5$
  $\sqrt{\frac{\Sigma y^2}{n} - \bar{y}^2} = 7.5$
  $\sqrt{\frac{\Sigma y^2}{12} - 25^2} = 7.5$
  $\frac{\Sigma y^2}{12} - 625 = 56.25$
  $\frac{\Sigma y^2}{12} = 681.25$
  $\Sigma y^2 = 8175$

COMBINING THE DATA INTO 32 OBSERVATIONS

- $\text{MEAN}_{(32 \text{ OBS})} = \frac{\Sigma x + \Sigma y}{20 + 12} = \frac{370 + 300}{32} = \frac{670}{32} = \frac{335}{16} \approx 20.94$
- $\sigma_{(32 \text{ OBS})} = \sqrt{\frac{\Sigma x^2 + \Sigma y^2}{32} - (20.94\ldots)^2} = \sqrt{\frac{7690 + 8175}{32} - (20.94\ldots)^2}$
  $= 7.57643445\ldots \approx 7.58$

Question 16

The mean and standard deviation of the test marks of 40 pupils in a Mathematics class are 65 and 18, respectively.

The mean and standard deviation of the test marks of the 24 boys of the class are 72 and 20, respectively.

Find the mean and standard deviation of the test marks of the 16 girls of the class.

$\boxed{\text{mean} = 54.5}$, $\boxed{\text{standard deviation} \approx 5.12}$

LOOKING AT THE INFORMATION GIVEN FOR THE WHOLE CLASS

- $\bar{x} = 65$

$$\frac{\Sigma x}{40} = 65$$

$$\Sigma x = 2600$$

- $\sigma = 18$

$$\sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = 18$$

$$\sqrt{\frac{\Sigma x^2}{40} - 65^2} = 18$$

$$\frac{\Sigma x^2}{40} - 4225 = 18^2$$

$$\Sigma x^2 = 181960$$

NOW REPEAT FOR THE 24 BOYS

- $\bar{x}_B = 72$

$$\frac{\Sigma x_B}{24} = 72$$

$$\Sigma x_B = 1728$$

- $\sigma_B = 20$

$$\sqrt{\frac{\Sigma x_B^2}{n} - \bar{x}_B^2} = 20$$

$$\sqrt{\frac{\Sigma x_B^2}{24} - 72^2} = 20$$

$$\frac{\Sigma x_B^2}{24} - 5184 = 20^2$$

$$\Sigma x_B^2 = 134016$$

SUBTRACTING THE SUMS FOUND & CALCULATE THE MEAN & STANDARD DEVIATION OF THE 16 GIRLS

$$\Sigma x_G = 2600 - 1728 = 872$$

$$\Sigma x_G^2 = 181960 - 134016 = 47944$$

$$\Rightarrow \bar{x}_G = \frac{\Sigma x_G}{n} = \frac{872}{16} = 54.5$$

$$\Rightarrow \sigma_G = \sqrt{\frac{\Sigma x_G^2}{n} - \bar{x}_G^2} = \sqrt{\frac{47944}{16} - 54.5^2} \approx 5.12$$

Question 17

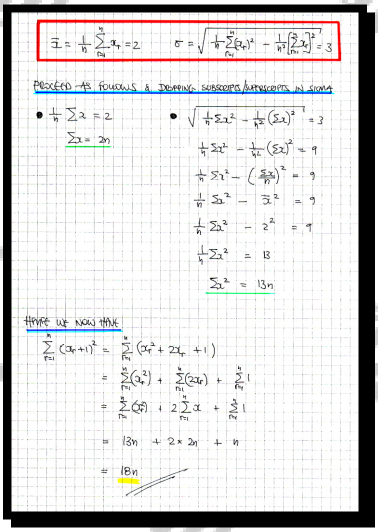
It is given that for a sample of data $x_1,\ x_2,\ x_3, x_4, x_5, \dots x_n$ the mean $\bar{x}$ and standard deviation σ are

$$\bar{x} = \frac{1}{n}\sum_{r=1}^{n} x_r = 2 \qquad \text{and} \qquad \sigma = \sqrt{\frac{1}{n}\sum_{r=1}^{n}(x_r)^2 - \frac{1}{n^2}\left(\sum_{r=1}^{n} x_r\right)^2} = 3.$$

Determine, in terms of n, the value of

$$\sum_{r=1}^{n}(x_r+1)^2.$$

$$\boxed{\sum_{r=1}^{n}(x_r+1)^2 = 18n}$$


$\bar{x} = \frac{1}{n}\sum_{r=1}^{n} x_r = 2 \qquad \sigma = \sqrt{\frac{1}{n}\sum_{r=1}^{n}(x_r)^2 - \frac{1}{n^2}\left[\sum_{r=1}^{n} x_r\right]^2} = 3$

PROCEED AS FOLLOWS & DROPPING SUBSCRIPTS/SUPERSCRIPTS IN SIGMA

- $\frac{1}{n}\sum x = 2$

  $\sum x = 2n$

- $\sqrt{\frac{1}{n}\sum x^2 - \frac{1}{n^2}(\sum x)^2} = 3$

  $\frac{1}{n}\sum x^2 - \frac{1}{n^2}(\sum x)^2 = 9$

  $\frac{1}{n}\sum x^2 - \left(\frac{\sum x}{n}\right)^2 = 9$

  $\frac{1}{n}\sum x^2 - \bar{x}^2 = 9$

  $\frac{1}{n}\sum x^2 - 2^2 = 9$

  $\frac{1}{n}\sum x^2 = 13$

  $\sum x^2 = 13n$

HENCE WE NOW HAVE

$$\sum_{r=1}^{n}(x_r+1)^2 = \sum_{r=1}^{n}(x_r^2 + 2x_r + 1)$$
$$= \sum_{r=1}^{n}(x_r^2) + \sum_{r=1}^{n}(2x_r) + \sum_{r=1}^{n}1$$
$$= \sum_{r=1}^{n}(x_r^2) + 2\sum_{r=1}^{n}x + \sum_{r=1}^{n}1$$
$$= 13n + 2 \times 2n + n$$
$$= 18n$$

CONTINUOUS DATA

Question 1

The distances achieved by a group of javelin throwers, rounded to the nearest metre, is summarized in the table below.

Distance (nearest metre)	Frequency
30 – 35	12
36 – 38	18
39 – 40	21
41	37
42 – 49	10
50 – 60	2

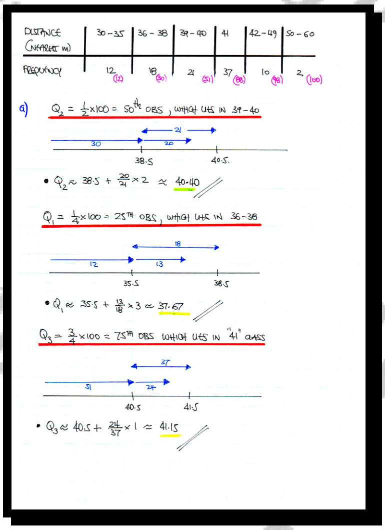
a) Estimate by linear interpolation, the value of the median and the quartiles.

b) Estimate the mean and the standard deviation of these data.

c) Determine with justification the skewness of the data.

d) Investigate the possibility of any outliers

$\boxed{Q_1 \approx 37.67}$, $\boxed{Q_2 \approx 40.40}$, $\boxed{Q_3 \approx 41.15}$, $\boxed{\bar{x} = 39.675}$, $\boxed{\sigma \approx 4.026}$, $\boxed{\text{negative skew}}$



DISTANCE (NEAREST m)	30 – 35	36 – 38	39 – 40	41	42 – 49	50 – 60
FREQUENCY	12 (12)	18 (30)	21 (51)	37 (88)	10 (98)	2 (100)

a) $Q_2 = \frac{1}{2} \times 100 = 50^{th}$ OBS, WHICH LIES IN 39 – 40

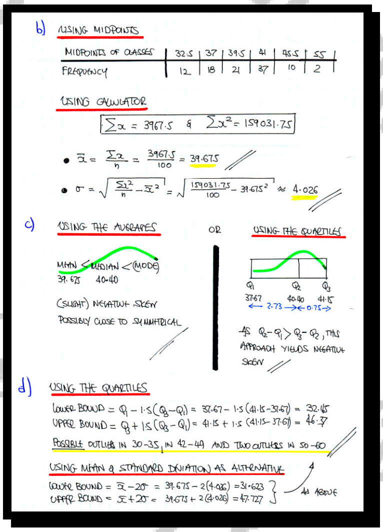
• $Q_2 \approx 38.5 + \frac{20}{21} \times 2 \approx 40.40$

$Q_1 = \frac{1}{4} \times 100 = 25^{TH}$ OBS, WHICH LIES IN 36 – 38

• $Q_1 \approx 35.5 + \frac{13}{18} \times 3 \approx 37.67$

$Q_3 = \frac{3}{4} \times 100 = 75^{TH}$ OBS WHICH LIES IN "41" CLASS

• $Q_3 \approx 40.5 + \frac{24}{37} \times 1 \approx 41.15$



b) USING MIDPOINTS

MIDPOINTS OF CLASSES	32.5	37	39.5	41	45.5	55
FREQUENCY	12	18	21	37	10	2

USING CALCULATOR

$\boxed{\sum x = 3967.5 \ \& \ \sum x^2 = 159031.75}$

• $\bar{x} = \frac{\sum x}{n} = \frac{3967.5}{100} = 39.675$

• $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{159031.75}{100} - 39.675^2} \approx 4.026$

c) USING THE AVERAGES

MEAN < MEDIAN < (MODE)
39.675 40.40

(SLIGHT) NEGATIVE SKEW
POSSIBLY CLOSE TO SYMMETRICAL

OR

USING THE QUARTILES

Q_1 Q_2 Q_3
37.67 40.40 41.15
2.73 0.75

AS $Q_2 - Q_1 > Q_3 - Q_2$, THIS APPROACH YIELDS NEGATIVE SKEW

d) USING THE QUARTILES

LOWER BOUND $= Q_1 - 1.5(Q_3 - Q_1) = 37.67 - 1.5(41.15 - 37.67) = 32.45$
UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 41.15 + 1.5(41.15 - 37.67) = 46.37$

POSSIBLE OUTLIERS IN 30 – 35, IN 42 – 49 AND TWO OUTLIERS IN 50 – 60

USING MEAN & STANDARD DEVIATION AS ALTERNATIVE

LOWER BOUND $= \bar{x} - 2\sigma = 39.675 - 2(4.026) = 31.623$
UPPER BOUND $= \bar{x} + 2\sigma = 39.675 + 2(4.026) = 47.727$ } AS ABOVE

Question 2

The number of hours worked in a given week by a group of 64 individuals is summarized in the table below.

Hours (nearest hour)	Frequency
1 – 10	5
11 – 20	16
21 – 25	14
26 – 30	17
31 – 40	10
41 – 59	2

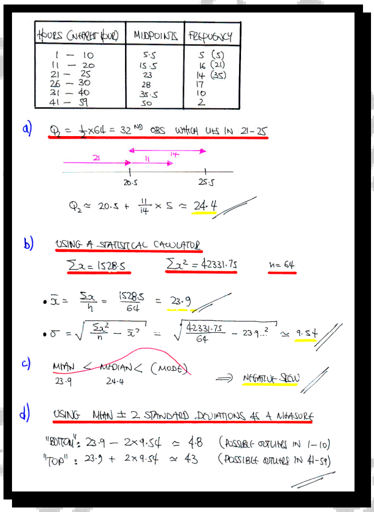
a) Estimate, by linear interpolation, the value of the median.

b) Estimate the mean and the standard deviation of these data.

c) Establish, with justification, the skewness of the data.

d) Determine the possibility whether the data contain any outliers.

$\boxed{Q_2 \approx 24.4}$, $\boxed{\bar{x} \approx 23.88}$, $\boxed{\sigma \approx 9.54}$, $\boxed{\text{negative skew}}$



HOURS (NEAREST HOUR)	MIDPOINTS	FREQUENCY
1 – 10	5.5	5 (5)
11 – 20	15.5	16 (21)
21 – 25	23	14 (35)
26 – 30	28	17
31 – 40	35.5	10
41 – 59	50	2

a) $Q_2 = \frac{1}{2} \times 64 = 32^{ND}$ OBS WHICH LIES IN 21 – 25

21, 11, 14, 20.5, 25.5

$Q_2 \approx 20.5 + \frac{11}{14} \times 5 \approx 24.4$

b) USING A STATISTICAL CALCULATOR

$\sum x = 1528.5$ $\quad \sum x^2 = 42331.75$ $\quad n = 64$

• $\bar{x} = \frac{\sum x}{n} = \frac{1528.5}{64} = 23.9$

• $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{42331.75}{64} - 23.9...^2} \approx 9.54$

c) MEAN < MEDIAN < (MODE) $\Rightarrow$ NEGATIVE SKEW

23.9 $\quad$ 24.4

d) USING MEAN $\pm$ 2 STANDARD DEVIATIONS AS A MEASURE

"BOTTOM": $23.9 - 2 \times 9.54 \approx 4.8$ (POSSIBLE OUTLIERS IN 1 – 10)

"TOP": $23.9 + 2 \times 9.54 \approx 43$ (POSSIBLE OUTLIERS IN 41 – 59)

Question 3

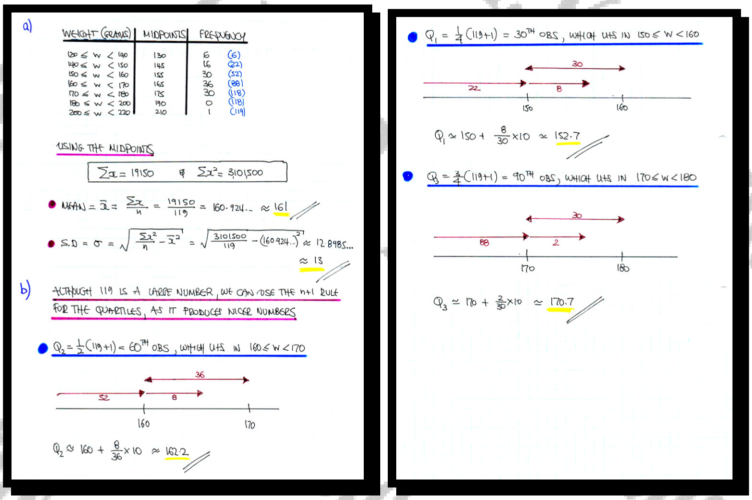
The weights of a random sample of a variety of apples, in grams, is summarised in the table below.

Weight (grams)	Frequency
$120 \le w < 140$	6
$140 \le w < 150$	16
$150 \le w < 160$	30
$160 \le w < 170$	36
$170 \le w < 180$	30
$180 \le w < 200$	0
$200 \le w < 220$	1

a) Estimate the mean and the standard deviation of these weights.

b) Estimate the median and the quartiles of these weights.

$\boxed{}$, $\boxed{\bar{x} \approx 161}$, $\boxed{\sigma \approx 13}$, $\boxed{Q_1 \approx 152.6 - 152.7}$, $\boxed{Q_2 \approx 162.1 - 162.2}$, $\boxed{Q_3 \approx 170.4 - 170.7}$



a)

WEIGHT (GRAMS)	MIDPOINTS	FREQUENCY	
$120 \le w < 140$	130	6	(6)
$140 \le w < 150$	145	16	(22)
$150 \le w < 160$	155	30	(52)
$160 \le w < 170$	165	36	(88)
$170 \le w < 180$	175	30	(118)
$180 \le w < 200$	190	0	(118)
$200 \le w < 220$	210	1	(119)

USING THE MIDPOINTS

$\boxed{\sum x = 19150 \quad \& \quad \sum x^2 = 3{,}101{,}500}$

- MEAN $= \bar{x} = \frac{\sum x}{n} = \frac{19150}{119} = 160.924... \approx 161$
- S.D $= \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{3101500}{119} - (160.924...)^2} \approx 12.8985... \approx 13$

b) ALTHOUGH 119 IS A LARGE NUMBER, WE CAN USE THE n+1 RULE FOR THE QUARTILES, AS IT PRODUCES NICER NUMBERS

- $Q_2 = \frac{1}{2}(119+1) = 60^{TH}$ OBS, WHICH LIES IN $160 \le w < 170$

36, 52, 8, 160, 170

$Q_2 \approx 160 + \frac{8}{36} \times 10 \approx 162.2$

- $Q_1 = \frac{1}{4}(119+1) = 30^{TH}$ OBS, WHICH LIES IN $150 \le w < 160$

30, 22, 8, 150, 160

$Q_1 \simeq 150 + \frac{8}{30} \times 10 \simeq 152.7$

- $Q_3 = \frac{3}{4}(119+1) = 90^{TH}$ OBS, WHICH LIES IN $170 \le w < 180$

30, 88, 2, 170, 180

$Q_3 \simeq 170 + \frac{2}{30} \times 10 \simeq 170.7$

Question 4

The mileages of 120 journeys covered by a minicab driver over a monthly period are summarized in the table below.

Mileages	Frequency
$10 \le m < 12$	2
$12 \le m < 17$	54
$17 \le m < 19$	28
$19 \le m < 21$	16
$21 \le m < 23$	13
$23 \le m < 25$	7

a) Estimate by linear interpolation, the value of the median.

b) Estimate the mean and the standard deviation of these data.

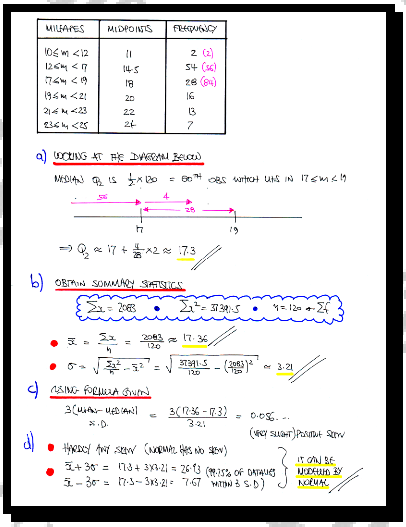
A skewness coefficient can be determined by

$$\frac{3(\text{mean} - \text{median})}{\text{standard deviation}}.$$

c) Evaluate this coefficient for this data and hence state its skew.

d) Determine whether the data could be modelled by a Normal distribution.

$\boxed{}$, $\boxed{Q_2 \approx 17.3}$, $\boxed{\bar{x} \approx 17.36}$, $\boxed{\sigma \approx 3.21}$, $\boxed{\text{slight positive skew/no skew}}$



MILEAGES	MIDPOINTS	FREQUENCY
$10 \le m < 12$	11	2 (2)
$12 \le m < 17$	14.5	54 (56)
$17 \le m < 19$	18	28 (84)
$19 \le m < 21$	20	16
$21 \le m < 23$	22	13
$23 \le m < 25$	24	7

a) LOOKING AT THE DIAGRAM BELOW

MEDIAN Q_2 IS $\frac{1}{2} \times 120 = 60^{\text{TH}}$ OBS WHICH LIES IN $17 \le m < 19$

$\Rightarrow Q_2 \approx 17 + \frac{4}{28} \times 2 \approx 17.3$

b) OBTAIN SUMMARY STATISTICS

$\sum x = 2083$ • $\sum x^2 = 37391.5$ • $n = 120 \leftarrow \sum f$

• $\bar{x} = \frac{\sum x}{n} = \frac{2083}{120} \approx 17.36$

• $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{37391.5}{120} - \left(\frac{2083}{120}\right)^2} \approx 3.21$

c) USING FORMULA GIVEN

$\frac{3(\text{MEAN} - \text{MEDIAN})}{\text{S.D.}} = \frac{3(17.36 - 17.3)}{3.21} = 0.056\ldots$

(VERY SLIGHT) POSITIVE SKEW

d) • HARDLY ANY SKEW (NORMAL HAS NO SKEW)

• $\bar{x} + 3\sigma = 17.3 + 3 \times 3.21 = 26.93$ (99.75% OF DATA LIES

$\bar{x} - 3\sigma = 17.3 - 3 \times 3.21 = 7.67$ WITHIN 3 S.D)

IT CAN BE MODELLED BY NORMAL

Question 5

The daily commuting distances of 125 individuals, rounded to the nearest mile, is summarised in the table below.

Distance (nearest mile)	Frequency
0 – 9	12
10 – 19	22
20 – 29	48
30 – 39	26
40 – 49	8
50 – 59	5
60 – 69	3
70 – 79	1

a) Estimate the mean and the standard deviation of these commuting distances.

b) Use linear interpolation to estimate the value of the median.

c) Determine with justification the skewness of the data.

d) Explain which out of the mean and standard deviation or the median and the interquartile range are more appropriate measures to summarize this data.

$\boxed{\ }$, $\boxed{\bar{x} \approx 26.74}$, $\boxed{\sigma \approx 13.85}$, $\boxed{Q_2 = 25.3 - 25.5}$, $\boxed{\text{positive skew}}$, $\boxed{\text{median \& IQR}}$

DISTANCE (NEAREST MILE)	MIDPOINT	FREQUENCY
0 – 9	4.5	12 (12)
10 – 19	14.5	22 (34)
20 – 29	24.5	48 (82)
30 – 39	34.5	26 (108)
40 – 49	4.5	8 (116)
50 – 59	54.5	5 (121)
60 – 69	64.5	3 (124)
70 – 79	74.5	1 (125)

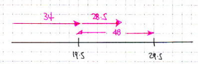
a) OBTAIN SUMMARY STATISTICS FROM CALCULATOR

• $\sum fx = 3342.5$ • $\sum fx^2 = 113351.25$ • $\sum f = 125$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{3342.5}{125} = 26.74$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{113351.25}{125} - 26.74^2} \simeq 13.85$$

b) BY LINEAR INTERPOLATION – Q_2 IS $\frac{1}{2} \times 125 = 62.5$ OBS IN 20–29



$$\therefore Q_2 = 19.5 + \frac{28.5}{48} \times 10 \simeq 25.44$$

c) USING THE AVERAGES

(MODE) < MEDIAN < MEAN

25.44 26.74

∴ POSITIVE SKEW

d) WE TEND TO USE MEAN & STANDARD DEVIATION, IF THERE ARE NO EXTREME VALUES & NEGLIGIBLE SKEW, OTHERWISE WE TEND TO USE MEDIAN & IQR

HERE WE HAVE LITTLE SKEW, AS MEDIAN IS NUMERICALLY CLOSE TO THE MEAN, BUT THERE ARE EXTREME VALUES AT THE "TOP END"

∴ MEDIAN & IQR ARE PREFERABLE

HISTOGRAMS

Question 1

A group of patients with a certain respiratory condition were asked to hold their breath for as long as they could.

The results are summarized in the table below.

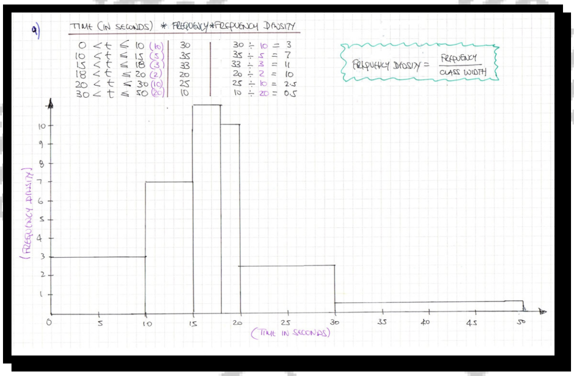
Time t (in seconds)	Frequency
$0 < t \le 10$	30
$10 < t \le 15$	35
$15 < t \le 18$	33
$18 < t \le 20$	20
$20 < t \le 30$	25
$30 < t \le 50$	10

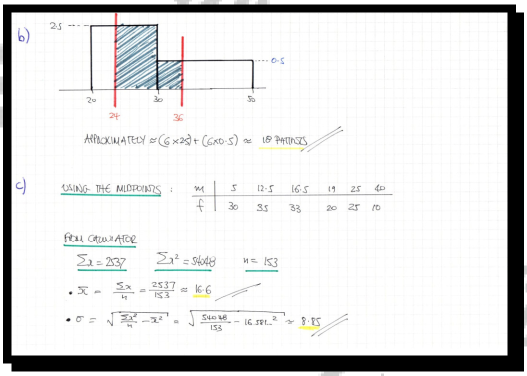
a) Draw an accurate histogram to represent this data.

b) Use the histogram to estimate the number of patients that managed to hold their breath between 24 and 36 seconds.

c) Calculate estimates for the mean and standard deviation of this data.

$\boxed{\approx 18}$, $\boxed{\bar{x} \approx 16.6}$, $\boxed{\sigma \approx 8.85}$





Question 2

The number of hours worked in a given week by a group of 64 freelance electricians is summarized in the table below.

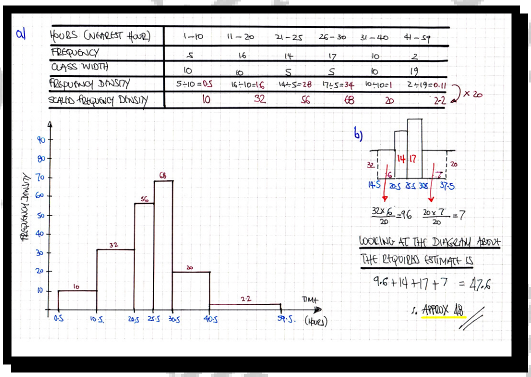
Hours (nearest hour)	Frequency
1 – 10	5
11 – 20	16
21 – 25	14
26 – 30	17
31 – 40	10
41 – 59	2

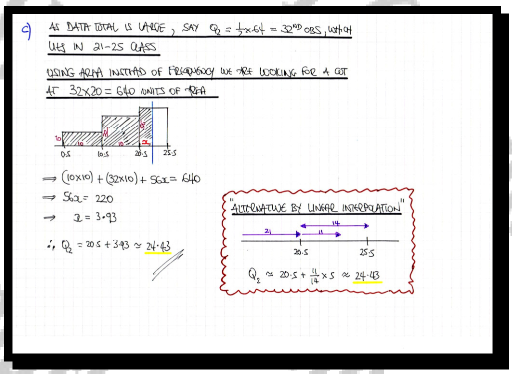
a) Draw an accurate histogram to represent this data.

b) Use the histogram to estimate the number of freelance electricians that worked between 15 and 37 hours during that week.

c) Estimate the median of the data.

$\boxed{\approx 48}$, $\boxed{Q_2 \approx 24.4}$





Question 3

The times taken to complete a 3 mile run, in minutes, by the members of a jogging club are summarized in the table below.

Times (nearest hour)	Frequency
11 – 14	24
15 – 17	24
18 – 19	19
20	11
21 – 23	21
24 – 28	15

a) Estimate the mean and standard deviation of this data.

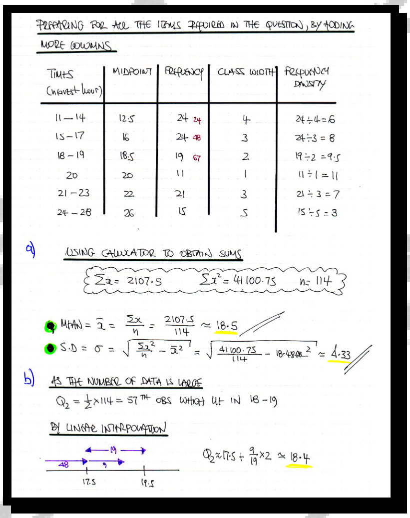
b) Estimate, by linear interpolation, the median of this data.

c) Draw an accurate histogram to represent this data.

d) Find the proportion of data which lies within 3 standard deviations of the mean.

e) Discuss briefly whether this data could be modelled by a Normal distribution.

$\boxed{\bar{x} \approx 18.5}$, $\boxed{\sigma \approx 4.33}$, $\boxed{Q_2 \approx 18.4}$, $\boxed{100\%}$



PREPARING FOR ALL THE ITEMS REQUIRED IN THE QUESTION, BY ADDING MORE COLUMNS

TIMES (nearest hour)	MIDPOINT	FREQUENCY	CLASS WIDTH	FREQUENCY DENSITY
11 – 14	12.5	24 24	4	$24 \div 4 = 6$
15 – 17	16	24 48	3	$24 \div 3 = 8$
18 – 19	18.5	19 67	2	$19 \div 2 = 9.5$
20	20	11	1	$11 \div 1 = 11$
21 – 23	22	21	3	$21 \div 3 = 7$
24 – 28	26	15	5	$15 \div 5 = 3$

a) USING CALCULATOR TO OBTAIN SUMS

$\sum x = 2107.5 \qquad \sum x^2 = 41100.75 \qquad n = 114$

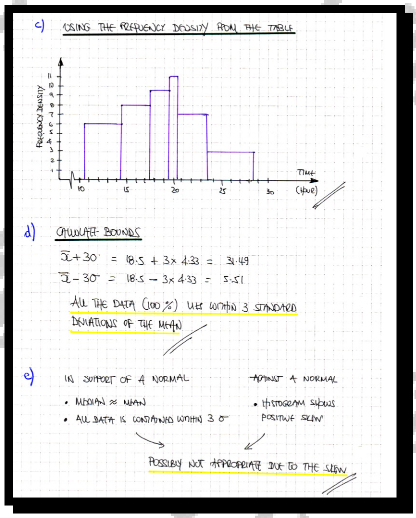
- MEAN $= \bar{x} = \frac{\sum x}{n} = \frac{2107.5}{114} \approx 18.5$
- S.D $= \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{41100.75}{114} - 18.4868...^2} \approx 4.33$

b) AS THE NUMBER OF DATA IS LARGE

$Q_2 = \frac{1}{2} \times 114 = 57^{\text{TH}}$ OBS WHICH LIE IN 18 – 19

BY LINEAR INTERPOLATION

$Q_2 \approx 17.5 + \frac{9}{19} \times 2 \approx 18.4$



c) USING THE FREQUENCY DENSITY FROM THE TABLE

d) CALCULATE BOUNDS

$\bar{x} + 3\sigma = 18.5 + 3 \times 4.33 = 31.49$

$\bar{x} - 3\sigma = 18.5 - 3 \times 4.33 = 5.51$

ALL THE DATA (100%) LIES WITHIN 3 STANDARD DEVIATIONS OF THE MEAN

e) IN SUPPORT OF A NORMAL

- MEDIAN ≈ MEAN
- ALL DATA IS CONTAINED WITHIN 3σ

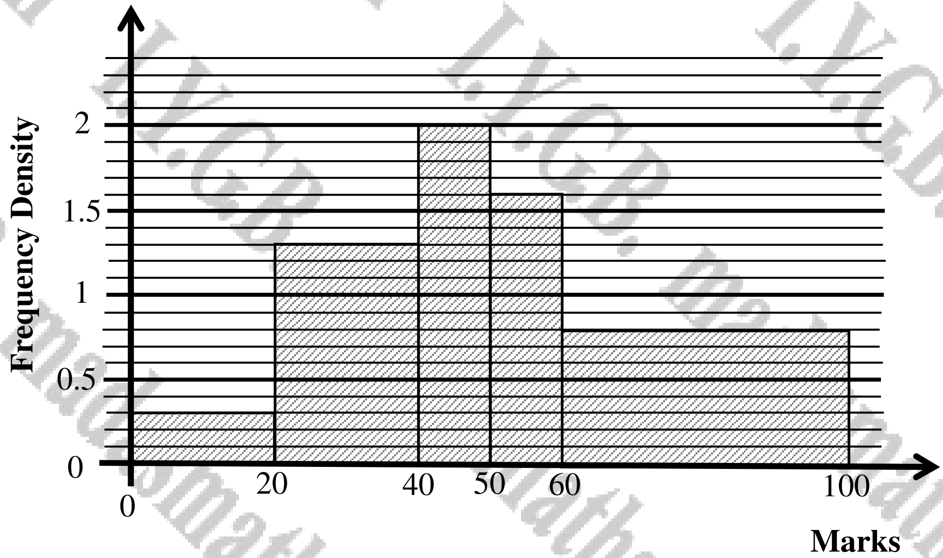
AGAINST A NORMAL

- HISTOGRAM SHOWS POSITIVE SKEW

POSSIBLY NOT APPROPRIATE DUE TO THE SKEW

Question 4

The histogram below shows the distribution of the marks of 250 students.

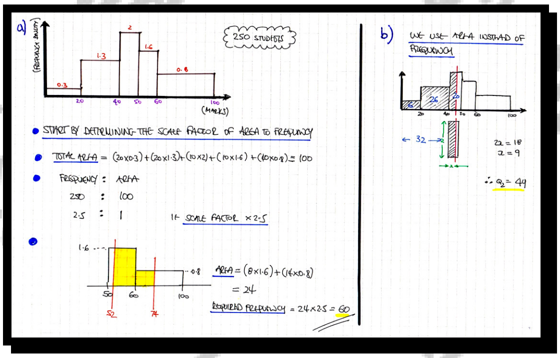


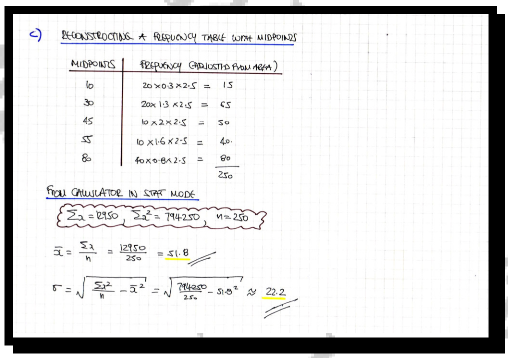
a) Estimate how many students scored between 52 and 74 marks.

b) Use the histogram estimate the median.

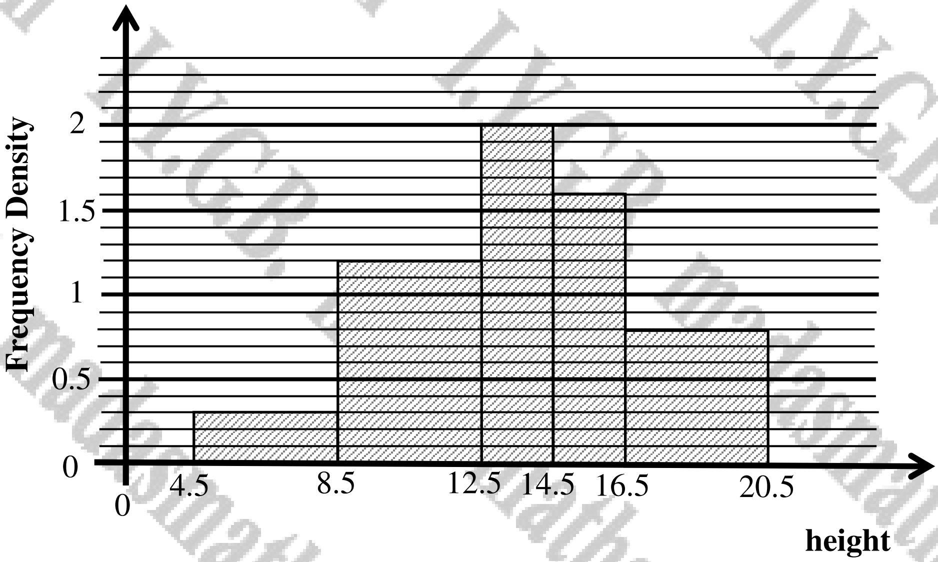
c) Calculate estimates for the mean and standard deviation of the marks of these students.

$\boxed{60}$, $\boxed{49}$, $\boxed{\bar{x} \approx 51.8}$, $\boxed{\sigma \approx 22.22}$





Question 5



The histogram above shows the distribution of the heights, to the nearest cm, of some plants in a garden centre. It is further given that there were 18 plants with a height between 5 cm and 8 cm, rounded to the nearest cm.

a) Use the histogram to estimate the median.

b) Estimate, by calculation, the mean and the standard deviation of the heights of these plants.

$\boxed{\text{median} \approx 13.6}$, $\boxed{\bar{x} \approx 13.48}$, $\boxed{\sigma \approx 3.45}$

"HEIGHT BETWEEN 5cm AND 8 cm" $\Rightarrow 4.5 \leqslant h < 8.5$

AREA OF 4×0.3 CORRESPONDS 18 PLANTS

AREA : PLANTS
1.2 : 18
1 : 15

(THIS IS NEEDED FOR THE MEAN & STANDARD DEVIATION)

a) FIND THE MEDIAN, WITHOUT INVOLVING FREQUENCIES.

TOTAL AREA
$0.3 \times 4 = 1.2$
$1.2 \times 4 = 4.8$
$2 \times 2 = 4$
$1.6 \times 2 = 3.2$
$0.8 \times 4 = 3.2$
16.4

$\frac{1}{2}$ THE AREA IS 8.2

$2x = 2.2$
$x = 1.1$

$\therefore Q_2 = 12.5 + 1.1$

$Q_2 = 13.6$

b) RECONSTRUCT THE "TABLE OF DATA"

PLANT HEIGHT (NEAREST cm)	MIDPOINTS	AREA	FREQUENCY (AREA×15)
5 – 8	6.5	1.2	18
9 – 12	10.5	4.8	72
13 – 14	13.5	4	60
15 – 16	15.5	3.2	48
17 – 20	18.5	3.2	48
			246

OBTAINING THE SUMS BY CALCULATOR

$\sum fx = 3315$
$\sum fx^2 = 47593.5$

- $\bar{x} = \frac{\sum fx}{\sum f} = \frac{3315}{246} = 13.48$
- $S.D = \sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}} = \sqrt{\frac{47593.5}{246} - (13.48...)^2} \approx 3.45$

Question 6

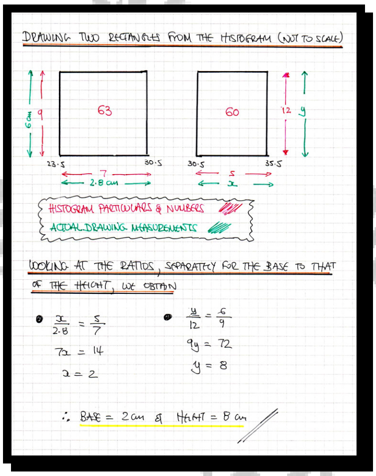
In a histogram the weights of baby hamsters, correct to the nearest gram, are plotted on the x axis.

In this histogram the class $24-30$ has a frequency of 63 and is represented by a rectangle of base 2.8 cm and height 6 cm.

In the same histogram the class $31-35$ has a frequency of 60.

Determine the measurements, in cm, of the rectangle that represents the class $31-35$.

$\boxed{\text{base} = 2 \text{ cm}}$, $\boxed{\text{height} = 8 \text{ cm}}$



Question 7

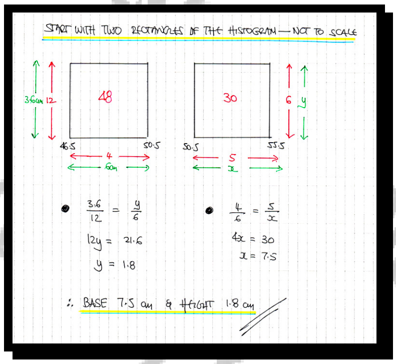
In a histogram the commuting times of a group of individuals, correct to the nearest minute, are plotted on the x axis.

In this histogram the class $47-50$ has a frequency of 48 and is represented by a rectangle of base 6 cm and height 3.6 cm.

In the same histogram the class $51-55$ has a frequency of 30.

Determine the measurements, in cm, of the rectangle that represents the class $51-55$.

$\boxed{\text{base} = 7.5 \text{ cm}}$, $\boxed{\text{height} = 1.8 \text{ cm}}$



Question 8

In a histogram the weights of apples, W grams, are plotted on the x axis.

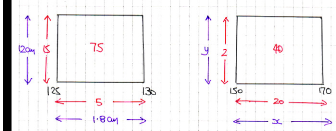
In this histogram the class $125 \le W < 130$ has a frequency of 75 and is represented by a rectangle of base 1.8 cm and height 12 cm.

In the same histogram the class $150 \le W < 170$ has a frequency of 40.

Find the measurements, in cm, of the rectangle that represents the class $150 \le W < 170$.

$\boxed{\text{base} = 7.2 \text{ cm}}$, $\boxed{\text{height} = 1.6 \text{ cm}}$

DRAWING TWO RECTANGLES TO INPUT INFORMATION (NOT TO SCALE)



USING RATIOS/PROPORTION

- $\frac{5}{1.8} = \frac{20}{x}$

$\Rightarrow 5x = 36$

$\Rightarrow x = 7.2$

- $\frac{12}{15} = \frac{y}{2}$

$15y = 24$

$y = 1.6$

$\therefore$ BASE 7.2 cm AND HEIGHT 1.6 cm.

Question 9

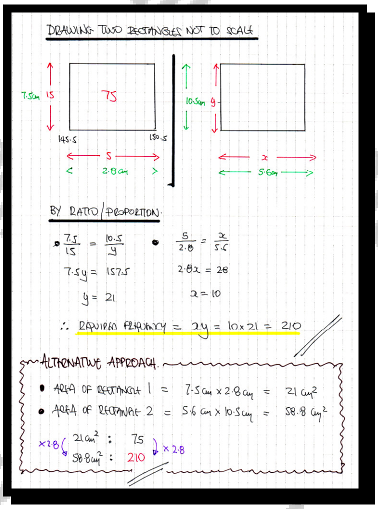
In a histogram the weights of peaches, correct to the nearest gram, are plotted on the x axis.

In this histogram the class $146-150$ has a frequency of 75 and is represented by a rectangle of base 2.8 cm and height 7.5 cm.

In the same histogram a different class is represented by a rectangle of base 5.6 cm and height 10.5 cm.

Determine the frequency of this class.

$$\boxed{f = 210}$$



DRAWING TWO RECTANGLES NOT TO SCALE

7.5cm 15 | 75 | 145.5 | 150.5 | 5 | 2.8cm

10.5cm y | x | 5.6cm

BY RATIO/PROPORTION.

- $\frac{7.5}{15} = \frac{10.5}{y}$
  $7.5y = 157.5$
  $y = 21$
- $\frac{5}{2.8} = \frac{x}{5.6}$
  $2.8x = 28$
  $x = 10$

$\therefore$ REQUIRED FREQUENCY $= xy = 10 \times 21 = 210$

ALTERNATIVE APPROACH.

- AREA OF RECTANGLE 1 $= 7.5\text{ cm} \times 2.8\text{ cm} = 21\text{ cm}^2$
- AREA OF RECTANGLE 2 $= 5.6\text{ cm} \times 10.5\text{ cm} = 58.8\text{ cm}^2$

$\times 2.8$ $21\text{ cm}^2 : 75$ $\times 2.8$
$58.8\text{ cm}^2 : 210$

Question 10

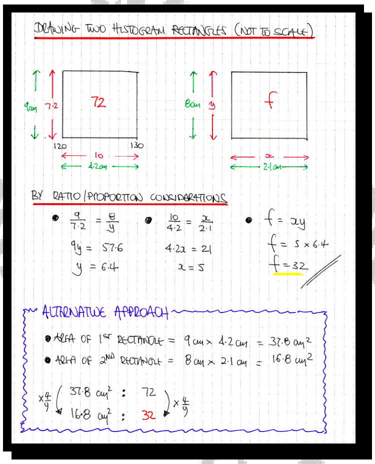
In a histogram the heights, h cm, of primary school pupils are plotted on the x axis.

In this histogram the class $120 \le h < 130$ has a frequency of 72 and is represented by a rectangle of base 4.2 cm and height 9 cm.

In the same histogram a different class is represented by a rectangle of base 2.1 cm and height 8 cm.

Determine the frequency of this class.

$\boxed{f = 32}$



DATA CODING

Question 1

The monthly mileages of a sales rep are summarised in the table below.

Mileages (m)	Frequency
$3250 \le m < 3300$	19
$3300 \le m < 3350$	45
$3350 \le m < 3400$	16
$3400 \le m < 3450$	5
$3450 \le m < 3500$	2

By using the coding

$$y = \frac{x - 3325}{50},$$

where x represents the midpoint of each class, estimate the mean and the standard deviation of this data.

$\boxed{\bar{x} \approx 3332}$, $\boxed{\sigma \approx 45.2}$

RECONSTRUCT THE TABLE

MILEAGES	MIDPOINTS (x)	$y = \frac{x-3325}{50}$	FREQUENCY (f)
$3250 \le m < 3300$	3275	-1	19
$3300 \le m < 3350$	3325	0	45
$3350 \le m < 3400$	3375	1	16
$3400 \le m < 3450$	3425	2	5
$3450 \le m < 3500$	3475	3	2

CALCULATE SUMMARY STATISTICS IN y

$\sum fy = 13$ $\quad$ $\sum fy^2 = 73$ $\quad$ $\sum f = 87$

CALCULATE MEAN & STANDARD DEVIATION IN y

- $\bar{y} = \frac{\sum fy}{\sum f} = \frac{13}{87} \approx 0.1494\ldots$
- $\sigma_y = \sqrt{\frac{\sum fy^2}{\sum f} - \bar{y}^2} = \sqrt{\frac{73}{87} - \left(\frac{13}{87}\right)^2} \approx 0.90374\ldots$

UNCODING BACK INTO x

- $\bar{x} = \bar{y} \times 50 + 3325 \approx 3332$
- $\sigma_x = \sigma_y \times 50 \approx 45.187\ldots \approx 45.2$

Question 2

The masses of 68 cows, in kg, are summarised in the table below.

Mass (m)	Frequency
$600 < m \le 625$	11
$625 < m \le 650$	14
$650 < m \le 675$	28
$675 < m \le 700$	7
$700 < m \le 725$	5
$725 < m \le 750$	2
$750 < m \le 775$	1

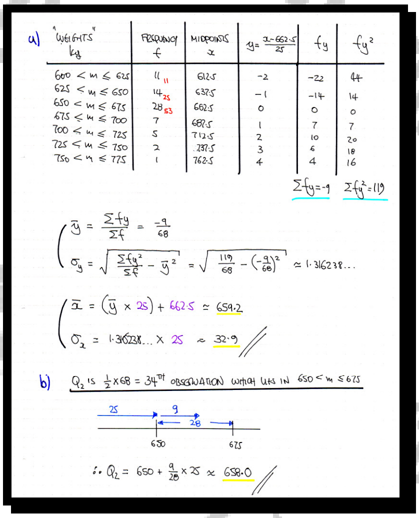
a) By using the coding

$$y = \frac{x - 662.5}{25},$$

where x represents the midpoint of each class, estimate the mean and standard deviation of this data.

b) Estimate, by the method of linear interpolation, the median mass of these cows.

$\boxed{\bar{x} \approx 659.19}$, $\boxed{\sigma \approx 32.91}$, $\boxed{Q_2 = 658.0}$



a)

"WEIGHTS" kg	FREQUENCY f	MIDPOINTS x	$y = \frac{x-662.5}{25}$	fy	fy^2
$600 < m \le 625$	11 (11)	612.5	−2	−22	44
$625 < m \le 650$	14 (25)	637.5	−1	−14	14
$650 < m \le 675$	28 (53)	662.5	0	0	0
$675 < m \le 700$	7	687.5	1	7	7
$700 < m \le 725$	5	712.5	2	10	20
$725 < m \le 750$	2	737.5	3	6	18
$750 < m \le 775$	1	762.5	4	4	16

$\Sigma fy = -9$ $\Sigma fy^2 = 119$

$$\bar{y} = \frac{\Sigma fy}{\Sigma f} = \frac{-9}{68}$$

$$\sigma_y = \sqrt{\frac{\Sigma fy^2}{\Sigma f} - \bar{y}^2} = \sqrt{\frac{119}{68} - \left(-\frac{9}{68}\right)^2} \simeq 1.316238\ldots$$

$$\bar{x} = (\bar{y} \times 25) + 662.5 \simeq 659.2$$

$$\sigma_x = 1.316238\ldots \times 25 \approx 32.9$$

b) Q_2 IS $\frac{1}{2} \times 68 = 34^{\text{TH}}$ OBSERVATION WHICH LIES IN $650 < m \le 675$

25, 9, 28, 650, 675

$$\therefore Q_2 = 650 + \frac{9}{28} \times 25 \approx 658.0$$

Question 3

The diameters of fine sand particles, in mm, are summarised in the table below.

Diameters (d)	Frequency
$0.02 < d \le 0.04$	25
$0.04 < d \le 0.06$	76
$0.06 < d \le 0.08$	111
$0.08 < d \le 0.10$	255
$0.10 < d \le 0.12$	33

a) By using the coding

$$y = 50(x - 0.09),$$

where x represents the midpoint of each class, estimate the mean and the standard deviation of this data.

b) Estimate, by linear interpolation, the median diameter of these sand particles.

c) Describe, with justification, the skewness of the data.

$\boxed{\bar{x} \approx 0.0778}$, $\boxed{\sigma \approx 0.0197}$, $\boxed{Q_2 = 0.08298}$

a) RECONSTRUCT THE TABLE

DIAMETERS (mm)	MIDPOINTS (x)	$y = 50(x-0.09)$	FREQUENCY (f)
$0.02 < d \le 0.04$	0.03	−3	25 (25)
$0.04 < d \le 0.06$	0.05	−2	76 (101)
$0.06 < d \le 0.08$	0.07	−1	111 (212)
$0.08 < d \le 0.10$	0.09	0	255 (467)
$0.10 < d \le 0.12$	0.11	1	33 (500)

CALCULATE SUMMARY STATISTICS IN y

$\Sigma fy = -305 \qquad \Sigma fy^2 = 673 \qquad \Sigma f = 500$

CALCULATE THE MEAN & STANDARD DEVIATION IN y

- $\bar{y} = \dfrac{\Sigma fy}{\Sigma f} = \dfrac{-305}{500} = -0.61$
- $\sigma_y = \sqrt{\dfrac{\Sigma fy^2}{\Sigma f} - \bar{y}^2} = \sqrt{\dfrac{673}{500} - (-0.61)^2} \simeq 0.9868637191\ldots$

UNCODE BACK INTO x

- $\bar{x} = \bar{y} \div 50 + 0.09 = -0.61 \div 50 + 0.09 = 0.0778$
- $\sigma_x = \sigma_y \div 50 = 0.986863\ldots \div 50 \simeq 0.0197$

b) Q_2 IS $\frac{1}{2} \times 500 = 250$ OBS, WHICH LIES IN $0.08 < d \le 0.10$

$\xleftrightarrow{\ 255\ }$

$\xrightarrow{\ 212\ } \ \xrightarrow{\ 38\ }$

0.08 0.10

$\Longrightarrow Q_2 = 0.08 + \dfrac{38}{255} \times 0.02 \approx 0.0830$

c) USING THE AVERAGES

MEAN < MEDIAN < (MODE) $\Longrightarrow$ POSITIVE SKEW

0.0778 0.0830

Question 4

The masses, x kg, of 40 students were measured and the results were summarized using the notation below.

$$\sum_{n=1}^{40}(x_n - 50) = 140 \quad \text{and} \quad \sum_{n=1}^{40}(x_n - 50)^2 = 4490 .$$

Calculate the mean and standard deviation of the masses of these 40 students.

$\boxed{\bar{x} = 53.5}$, $\boxed{\sigma = 10}$

LOOKING AT THE CODED-SUMMARY STATISTICS

$$\sum_{n=1}^{40}(x_n - 50) = 140 \qquad \sum_{n=1}^{40}(x_n - 50)^2 = 4490$$

LET $y = x - 50$

$$\sum y = 140 \qquad \sum y^2 = 4490 \qquad n = 40$$

CALCULATE THE MEAN & STANDARD DEVIATION IN y

$$\bar{y} = \frac{\sum y}{n} = \frac{140}{40} = 3.5$$

$$\sigma_y = \sqrt{\frac{\sum y^2}{n} - \bar{y}^2} = \sqrt{\frac{4490}{40} - 3.5^2} = 10$$

UNCODE BACK INTO x

- $\bar{x} = \bar{y} + 50$

  $\bar{x} = 3.5 + 50$

  $\bar{x} = 53.5$

- $\sigma_x = \sigma_y$

  $\sigma_x = 10$

  (STANDARD DEVIATION DOES NOT GET AFFECTED BY ADDITION/SUBTRACTION)

Question 5

The test marks, x, of 20 students were coded and their results were summarized as

$$\sum(x-10) = 220 \quad \text{and} \quad \sum(x-10)^2 = 2720.$$

a) Use a detailed method to show that

$$\sum x^2 = 9120.$$

b) Calculate the mean and standard deviation of the test marks of these students.

$\boxed{\bar{x} = 21}$, $\boxed{\sigma = \sqrt{15} \approx 3.87}$

a) $\sum_{r=1}^{20}(x_r-10) = 220 \qquad \sum_{r=1}^{20}(x_r-10)^2 = 2720 \qquad n=20$

$$\sum_{r=1}^{20}(x_r-10)^2 = \sum_{r=1}^{20}\left[x_r^2 - 20x_r + 100\right]$$

$$2720 = \sum_{r=1}^{20}x_r^2 - 20\sum_{r=1}^{20}x_r + 100\sum_{r=1}^{20}1$$

$$2720 = \sum_{r=1}^{20}x_r^2 - 20\sum_{r=1}^{20}x_r + 100\times 20$$

BY INSPECTION $\sum_{r=1}^{20}x = 220 + 20\times 10 = 420$ OR BY USING A DETAILED METHOD

$$\sum_{r=1}^{20}(x_r-10) = 220$$

$$\sum_{r=1}^{20}x_r - 10\sum_{r=1}^{20}1 = 220$$

$$\sum_{r=1}^{20}x_r - 10\times 20 = 220$$

$$\sum_{r=1}^{20}x_r = 420$$

RETURNING TO THE MAIN LINE

$$\Rightarrow 2720 = \sum_{r=1}^{20}x_r^2 - 20\times 420 + 2000$$

$$\Rightarrow \sum_{r=1}^{20}x_r^2 = 9120$$

b)
- $\bar{x} = \frac{\sum x}{n} = \frac{420}{20} = 21$
- $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{9120}{20} - 21^2} = \sqrt{15}$

≈ 3.87

ALTERNATIVE USING THE CODED VALUES

$y = x - 10$ so $\sum y = 220$ & $\sum y^2 = 2720$

$$\bar{y} = \frac{\sum y}{n} = \frac{220}{20} = 11$$

$$\sigma_y = \sqrt{\frac{\sum y^2}{n} - \bar{y}^2} = \sqrt{\frac{2720}{20} - 11^2} = \sqrt{15}$$

UNCODING

$\bar{x} = \bar{y} + 10 = 21$

$\sigma_x = \sigma_y = \sqrt{15}$ (UNAFFECTED BY SUBTRACTION)

Question 6

The following information about 5 observations of x is shown below.

$$\sum_{i=1}^{5}\left(\frac{x_i - 255}{2}\right) = 50 \qquad \text{and} \qquad \sum_{i=1}^{5}\left(\frac{x_i - 255}{2}\right)^2 = 1650 .$$

Calculate the mean and standard deviation of x.

$\boxed{\bar{x} = 275}$, $\boxed{\sigma = 2\sqrt{230} \approx 30.3}$

LOOKING AT THE SUMMARY STATISTICS

$$\sum_{i=1}^{5}\left(\frac{x_i - 255}{2}\right) = 50 \qquad \sum_{i=1}^{5}\left(\frac{x_i - 255}{2}\right)^2 = 1650$$

LET $y = \frac{x - 255}{2}$

$$\sum y = 50 \qquad \sum y^2 = 1650 \qquad n = 5$$

CALCULATE THE MEAN & STANDARD DEVIATION IN y

$$\bar{y} = \frac{\sum y}{n} = \frac{50}{5} = 10$$

$$\sigma_y = \sqrt{\frac{\sum y^2}{n} - \bar{y}^2} = \sqrt{\frac{1650}{5} - 10^2} = \sqrt{230}$$

UNCODE THE MEAN & STANDARD DEVIATION BACK INTO x.

- $\bar{x} = \bar{y} \times 2 + 255$
  $\bar{x} = 10 \times 2 + 255$
  $\bar{x} = 275$
- $\sigma_x = \sigma_y \times 2$
  $\sigma_x = 2\sqrt{230}$
  $\sigma_x \approx 30.3$