DISTRIBUTIONAL APPROXIMATIONS

BINOMIAL TO POISSON

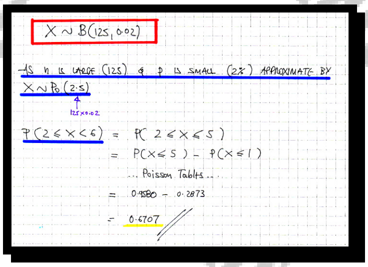
Question 1 (**)

The discrete random variable X has probability distribution

$$X \sim B(125, 0.02).$$

Use a distributional approximation, to find $P(2 \le X < 6)$.

$\boxed{}$, $\boxed{P(2 \le X < 6) = 0.6707}$

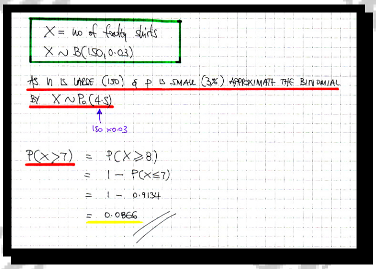


Question 2 (**)

The probability that a *Lake Island* shirt will have a fault is 3%.

Use a distributional approximation to find the probability that in a batch of 150 *Lake Island* shirts there will be more than 7 faulty shirts.

$\boxed{}$, $\boxed{0.0866}$



Question 3 (**)

8 people in every 10000 possess a rare gene.

There are 7500 patients registered in Dr Jarajah's surgery.

Using a distributional approximation, find the probability that there will be more than 5 but at most 10 patients registered in this surgery, that carry this gene.

$\boxed{}$, $\boxed{0.5117}$

X = NUMBER OF PATIENTS WITH RARE GENE

$X \sim B(7500, 0.0008)$

AS n IS LARGE, & p IS SMALL, WE APPROXIMATE BY $X \sim Po(\underset{np}{6})$

$$
\begin{aligned}
P(5 < X \le 10) &= P(6 \le X \le 10) \\
&= P(X \le 10) - P(X \le 5) \\
&= 0.957379\ldots - 0.4467966\ldots \\
&= 0.5117
\end{aligned}
$$

Question 4 (**)

On a certain day, postman Mat has 200 letters to deliver. In general, 2% of the letters that postman Mat delivers, are delivered to the wrong address.

a) Determine the probability that postman Mat delivers 6 letters to the wrong address that day.

b) Use a distributional approximation, to find the probability that postman Mat delivers more than 8 letters to the wrong address that day.

$\boxed{}$, $\boxed{0.1047}$, $\boxed{0.0214}$

a) X = NUMBER OF LETTERS DELIVERED INCORRECTLY

$X \sim B(200, 0.02)$

$P(X=6) = \binom{200}{6}(0.02)^6(0.98)^{194} \simeq 0.1047$

b) AS n IS LARGE & p IS SMALL APPROXIMATE BY $X \sim Po(\underset{np}{4})$

$$
\begin{aligned}
P(X>8) &= P(X \ge 9) \\
&= 1 - P(X \le 8) \\
&= 1 - 0.9786365\ldots \\
&= 0.0214
\end{aligned}
$$

Question 5 **(**)**

At a certain safari park, it is known that 42% of the cars come from London.

a) Show that in a random sample of 37 cars, the probability that more than 22 cars came from London is approximately 1%.

At the same safari park, it is known that 0.5% of the cars come from France.

b) Use a suitable approximation, to determine the probability that in a random sample of 80 cars, exactly 2 came from France.

$\boxed{}$, $\boxed{0.0536}$

a) X = NUMBER OF CARS FROM LONDON

$X \sim B(37, 0.42)$

$P(X > 22) = P(X \geqslant 23) = 1 - P(X \leqslant 22)$

$= 1 - 0.9893466...$

$= 0.01065...$

$\simeq 0.01$ i.e. 1%

b) Y = NUMBER OF CARS FROM FRANCE

$Y \sim B(80, 0.005)$

AS n IS LARGE & p IS SMALL APPROXIMATE BY POISSON $Po(0.4)$

$P(Y=2) = \dfrac{e^{-0.4} \times 0.4^2}{2!} = 0.0536$

Question 6 **(**)**

A store sold 400 furniture items last month.

It has been established over a long period of time that the probability of a customer returning a furniture items back to the store is 1.5%.

Use a distributional approximation, to find the probability that more than 6 but less than 13 furniture items will be returned to the store.

$\boxed{}$, $\boxed{0.3849}$

X = NUMBER OF RETURNED ITEMS

$X \sim B(400, 0.015)$

AS n IS LARGE & p IS SMALL APPROXIMATE BY $X \sim Po(np)$

i.e. $X \sim Po(6)$

$P(6 < X < 13) = P(7 \leqslant X \leqslant 12)$

$= P(X \leqslant 12) - P(X \leqslant 6)$

$= 0.991725... - 0.6063028...$

$= 0.3849$

Question 7 (**)

The probability that a certain brand of mobile phone, selected at random from the production line, will be faulty is 0.0228.

A random sample of 200 such phones is examined.

Use a distributional approximation to find the probability that the number of faulty phones in the sample will be at most 2.

$\boxed{}$, $\boxed{0.1669}$

X = NUMBER OF FAULTY MOBILE PHONES
$X \sim B(200, 0.0228)$

$np = 200 \times 0.0228$

AS n IS LARGE & p IS SMALL, APPROXIMATE BY $X \sim Po(4.56)$

$P(X \leq 2) = \dots$ calculator $\dots = 0.1669$

OR BY DIRECT CALCULATIONS

$$P(X \leq 2) = P(X=0,1,2) = e^{-4.56} + \frac{e^{-4.56} \times 4.56^1}{1!} + \frac{e^{-4.56} \times 4.56^2}{2!}$$
$$= e^{-4.56}\left[1 + 4.56 + 10.3968\right]$$
$$= e^{-4.56} \times 15.9568$$
$$= 0.1669$$

AS BEFORE

BINOMIAL TO NORMAL

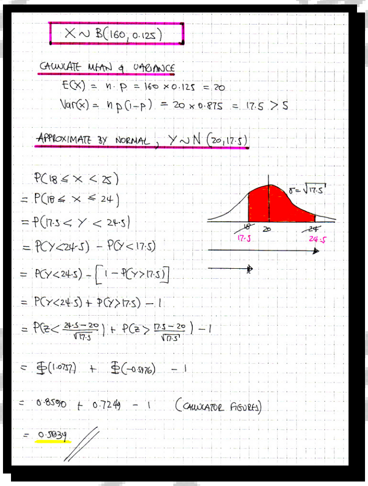
Question 1 **(**+)**

The discrete random variable X has probability distribution

$$X \sim \mathrm{B}(160, 0.125).$$

Use a distributional approximation, to find $\mathrm{P}(18 \le X < 25)$.

$\square$, $\boxed{\mathrm{P}(18 \le X < 25) = 0.584}$



$\boxed{X \sim B(160, 0.125)}$

CALCULATE MEAN & VARIANCE

$E(X) = n \cdot p = 160 \times 0.125 = 20$

$\mathrm{Var}(X) = np(1-p) = 20 \times 0.875 = 17.5 > 5$

APPROXIMATE BY NORMAL, $Y \sim N(20, 17.5)$

$P(18 \le X < 25)$

$= P(18 \le X \le 24)$

$= P(17.5 < Y < 24.5)$

$= P(Y < 24.5) - P(Y < 17.5)$

$= P(Y < 24.5) - [1 - P(Y > 17.5)]$

$= P(Y < 24.5) + P(Y > 17.5) - 1$

$= P\left(Z < \frac{24.5 - 20}{\sqrt{17.5}}\right) + P\left(Z > \frac{17.5 - 20}{\sqrt{17.5}}\right) - 1$

$= \Phi(1.0757) + \Phi(-0.5976) - 1$

$= 0.8590 + 0.7249 - 1$ (CALCULATOR FIGURES)

$= 0.5839$

Question 2 (**+)

It has been established over a long period of time, that in Enzo's Restaurant 30% of the orders are vegetarian.

Using a distributional approximation, find the probability that in a given day with 80 orders, there will be more than 30 vegetarian orders.

N.S.I , 0.056

X = NUMBER OF VEGETARIAN ORDERS

$X \sim B(80, 0.3)$

MEAN $= E(X) = np = 80 \times 0.3 = 24$

VARIANCE $= Var(X) = np(1-p) = 24 \times 0.7 = 16.8 > 5$

APPROXIMATE BY $Y \sim N(24, 16.8)$

$\Rightarrow P(X > 30)$

$= P(X \geqslant 31)$

$= P(Y > 30.5)$

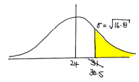
$= 1 - P(Y < 30.5)$

$= 1 - P\left(Z < \frac{30.5 - 24}{\sqrt{16.8}}\right)$

$= 1 - \Phi(1.585837...)$

$= 1 - 0.9436118...$

$= 0.0564$



Question 3 (**+)

The probability that a certain type of rose bush will exceed 2 metres in height is 0.25.

Sixty such rose bushes are planted.

Using a distributional approximation, find the probability that more than 13 but no more than 18 of these bushes, will exceed a height of 2 metres.

, 0.524

X = NUMBER OF BUSHES, EXCEEDING 2 METRES IN HEIGHT

$X \sim B(60, 0.25)$

- MEAN $= E(X) = np = 60 \times 0.25 = 15$
- Variance $= Var(X) = np(1-p) = 15 \times 0.75 = 11.25 > 5$

APPROXIMATE BY NORMAL $Y \sim N(15, 11.25)$

$P(13 < X \leqslant 18)$

$= P(14 \leqslant X \leqslant 18)$

$= P(13.5 < Y < 18.5)$

$= P(Y < 18.5) - P(Y < 13.5)$

$= P(Y < 18.5) - [1 - P(Y > 13.5)]$

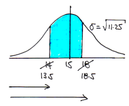
$= P(Y < 18.5) + P(Y > 13.5) - 1$

$= P\left(Z < \frac{18.5 - 15}{\sqrt{11.25}}\right) + P\left(Z > \frac{13.5 - 15}{\sqrt{11.25}}\right) - 1$

$= \Phi(1.0434983...) + \Phi(-0.4472135...) - 1$

$= 0.85164... + 0.67264... - 1$

$= 0.5243$



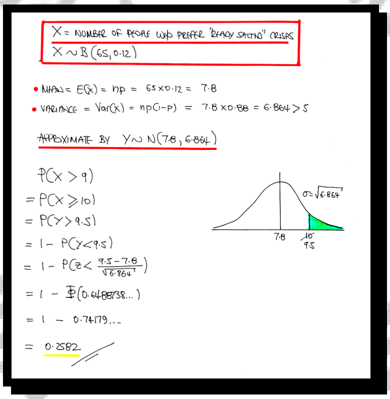
Question 4 (**+)

A shop owner has established over a long period of time, that 12% of the people who buy crisps, prefer the "ready salted" variety.

On a certain day 65 customers bought crisps.

Using a distributional approximation, find the probability that more than 9 of these 65 customers bought crisps of the "ready salted" variety.

$\boxed{}$, $\boxed{0.258}$



X = NUMBER OF PEOPLE WHO PREFER "READY SALTED" CRISPS

$X \sim B(65, 0.12)$

- MEAN $= E(X) = np = 65 \times 0.12 = 7.8$
- VARIANCE $= Var(X) = np(1-p) = 7.8 \times 0.88 = 6.864 > 5$

APPROXIMATE BY $Y \sim N(7.8, 6.864)$

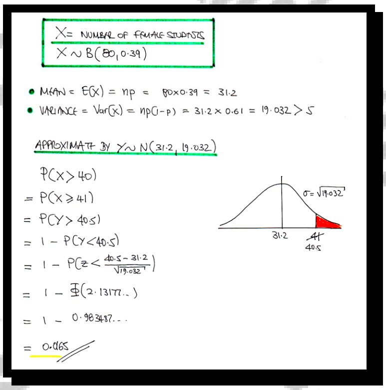
$P(X > 9)$
$= P(X \geqslant 10)$
$= P(Y > 9.5)$
$= 1 - P(Y < 9.5)$
$= 1 - P\left(Z < \frac{9.5 - 7.8}{\sqrt{6.864}}\right)$
$= 1 - \Phi(0.6488738\ldots)$
$= 1 - 0.74179\ldots$
$= 0.2582$

Question 5 (**+)

In a large university 39% of the students are female and the rest are male. A random sample of 80 students is selected from this university.

Use a distributional approximation, to find the probability that more than half the students in the sample are female.

$\boxed{}$, $\boxed{0.0165}$



X = NUMBER OF FEMALE STUDENTS

$X \sim B(80, 0.39)$

- MEAN $= E(X) = np = 80 \times 0.39 = 31.2$
- VARIANCE $= Var(X) = np(1-p) = 31.2 \times 0.61 = 19.032 > 5$

APPROXIMATE BY $Y \sim N(31.2, 19.032)$

$P(X > 40)$
$= P(X \geqslant 41)$
$= P(Y > 40.5)$
$= 1 - P(Y < 40.5)$
$= 1 - P\left(Z < \frac{40.5 - 31.2}{\sqrt{19.032}}\right)$
$= 1 - \Phi(2.13177\ldots)$
$= 1 - 0.983487\ldots$
$= 0.0165$

Question 6 (**+)

A popular bag of confectionary contains 20 sweets, of which $\frac{1}{5}$ are expected to be orange in flavour.

a) Find the probability that once such bag selected at random will contain at least 3 but no more than 7 orange flavoured sweets.

A family size bag of the same confectionary contains 90 sweets. The proportion of the orange flavoured sweets in these bags is also expected to be $\frac{1}{5}$.

b) Use a distributional approximation, to find the probability that a randomly selected family size bag, will contain less than 25 orange flavoured sweets.

$\boxed{}$, $\boxed{0.7818}$, $\boxed{0.9567}$

a) $\boxed{\begin{array}{l} X = \text{NUMBER OF ORANGE SWEETS} \\ X \sim B(20, 0.2) \end{array}}$

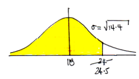
$P(3 \leqslant X \leqslant 7) = P(X \leqslant 7) - P(X \leqslant 2)$
$= 0.967857... - 0.2060847...$
$= 0.7618$

b) NOW $X \sim B(90, 0.2)$

- MEAN $= E(X) = np = 90 \times 0.2 = 18$
- VARIANCE $= \text{Var}(X) = np(1-p) = 18 \times 0.8 = 14.4$

APPROXIMATE BY $Y \sim N(18, 14.4)$

$P(X < 25)$
$= P(X \leqslant 24)$
$= P(Y < 24.5)$
$= P\left(z < \frac{24.5 - 18}{\sqrt{14.4}}\right)$
$= \Phi(1.712900...)$
$= 0.9566$



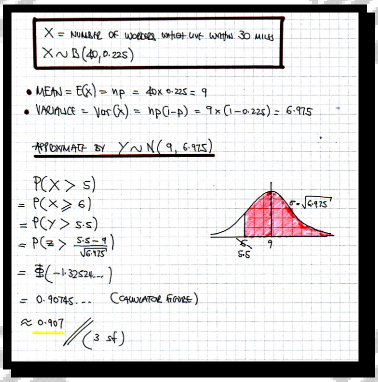
Question 7 (**+)

Of the workforce of a factory 22.5% live within 30 miles of the factory.

A random sample of 40 workers is selected.

Use a distributional approximation to show that the probability, of more than 5 workers in this sample live within 30 miles of the factory, is 0.907.

$\boxed{}$, $\boxed{\text{proof}}$



Question 8 (***+)

A garden centre sells bags which contain large number of seeds for a flowering plant. This plant only produces white or red flowers.

- Type A bags contain seeds which on average 80% will produce red flowers and 20% white flowers.
- Type B bags contain seeds which on average 60% will produce red flowers and 40% white flowers.

The manager finds an unlabelled bag. She plants 100 seeds picked at random from the bag and decides to label it as A if more than 68 red flowers are produced, otherwise she plans to label the bag as B.

Use a distributional approximation, to determine the probability that the manager …

a) …will label the bag A when in fact it should have been B.

b) … will label the bag B when in fact it should have been A.

$\boxed{0.0413}$, $\boxed{0.0020}$

a) SUPPOSE THE BAG WAS A "TYPE B" BAG

$\boxed{\begin{array}{l} X = \text{NUMBER OF RED FLOWERS} \\ X \sim B(100, 0.6) \end{array}}$

$\Rightarrow P(X > 68) = P(X \geqslant 69)$

APPROXIMATE BY NORMAL

$Y \sim N(\underbrace{60}_{np}, \underbrace{24}_{np(1-p)})$

$\ldots = P(Y > 68.5)$
$= 1 - P(Y < 68.5)$
$= 1 - P\left(Z < \frac{68.5 - 60}{\sqrt{24}}\right)$
$= 1 - \Phi(1.73505\ldots)$
$= 1 - 0.958634\ldots$
$= 0.041365\ldots$
$= 0.0414$

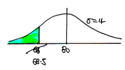
b) SUPPOSE THE BAG WAS A "TYPE A" BAG

$\boxed{\begin{array}{l} X = \text{NUMBER OF RED FLOWERS} \\ X \sim B(100, 0.8) \end{array}}$

$\Rightarrow P(X \leqslant 68) = \ldots$

APPROXIMATE BY NORMAL

$Y \sim N(\underbrace{80}_{np}, \underbrace{16}_{np(1-p)})$



$\ldots = P(Y < 68.5)$
$= 1 - P(Y > 68.5)$
$= 1 - P\left(Z > \frac{68.5 - 80}{4}\right)$
$= 1 - \Phi(-2.875)$
$= 1 - 0.9979798\ldots$
$= 0.00202$

Question 9 (****)

The discrete random variable $X \sim \mathrm{B}(n, p)$.

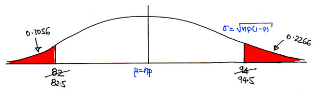
The value of n and the value of p are such so that X can be approximated by a Normal distribution.

- Using a Normal approximation, the probability that X is at most 82 is 0.1056.
- Using the same Normal approximation, the probability that X is less than 95 is 0.7734.

Determine the value of n and the value of p.

$\boxed{}$, $\boxed{n = 150}$, $\boxed{p = 0.6}$

$X \sim B(n,p)$ IS APPROXIMATED BY $Y \sim N(np, np(1-p))$



$\Rightarrow P(X \le 82) = 0.1056$
$\Rightarrow P(Y < 82.5) = 0.1056$
$\Rightarrow P(Y > 82.5) = 0.8944$
$\Rightarrow P\left(Z > \frac{82.5-\mu}{\sigma}\right) = 0.8944$

$\frac{82.5-\mu}{\sigma} = -\Phi^{-1}(0.8944)$

$\frac{82.5-\mu}{\sigma} = -1.25$

$82.5-\mu = -1.25\sigma$

$\mu = 82.5 + 1.25\sigma$

$\Rightarrow P(X < 95) = 0.7734$
$\Rightarrow P(X \le 94) = 0.7734$
$\Rightarrow P(Y < 94.5) = 0.7734$
$\Rightarrow P\left(Z < \frac{94.5-\mu}{\sigma}\right) = 0.7734$

$\frac{94.5-\mu}{\sigma} = +\Phi^{-1}(0.7734)$

$\frac{94.5-\mu}{\sigma} = +0.75$

$94.5-\mu = 0.75\sigma$

$\mu = 94.5 - 0.75\sigma$

SOLVING SIMULTANEOUSLY GIVES

$\Rightarrow 82.5 + 1.25\sigma = 94.5 - 0.75\sigma$

$2\sigma = 12$

$\sigma = 6$

$\Rightarrow \mu = 82.5 + 1.25 \times 6$

$\mu = 90$

FINALLY WE HAVE

$\left.\begin{array}{l}\sigma = 6\\ \mu = 90\end{array}\right\} \Rightarrow$

$\left.\begin{array}{l}\sqrt{np(1-p)} = 6\\ np = 90\end{array}\right\} \Rightarrow$

$\left.\begin{array}{l}np(1-p) = 36\\ np = 90\end{array}\right\} \Rightarrow \quad 90(1-p) = 36$

$1-p = 0.4$

$p = 0.6$

$n = 150$

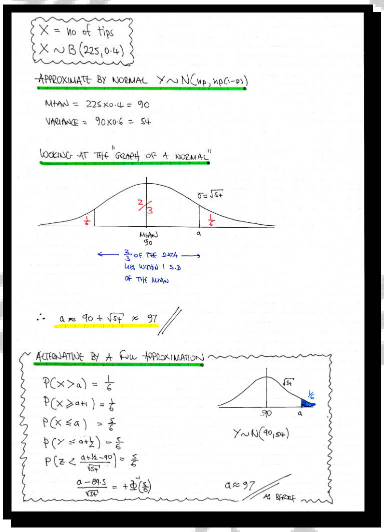
Question 110 (****)

The probability that a waiter gets a tip in a certain restaurant is thought to be constant at 0.4, and tipping is assumed to be independent from one customer to another.

The number of tips this waiter receives in a week with 225 orders is denoted by the discrete random variable X.

Estimate the value of a, given that $\mathrm{P}(X > a) > \frac{1}{6}$.

$\boxed{}$, $\boxed{a \approx 97}$



Question 11 (****+)

The sale records in "*Laptop World*", show that 35% of its customers buy insurance when they purchase a laptop.

A sample of 160 customers is considered.

The probability that less than x customers will buy insurance with their laptop purchase is 4.09%.

Determine the value of x.

$\boxed{}$, $\boxed{x = 46}$

• START BE DEFINING VARIABLES & DISTRIBUTIONS

$\boxed{\begin{array}{l} X = \text{NO OF CUSTOMERS WHO BUY INSURANCE} \\ X \sim B(160, 0.35) \end{array}}$

• APPROXIMATE BY NORMAL

- $E(X) = np = 160 \times 0.35 = 56$
- $\text{Var}(X) = np(1-p) = 56 \times 0.65 = 36.4$

$\boxed{Y \sim N(56, 36.4)}$

• WE NOW HAVE

$\Rightarrow P(X < x) = 4.09\%$

$\Rightarrow P(X \leqslant x-1) = 0.0409$

$\Rightarrow P(Y < x - \frac{1}{2}) = 0.0409$

$\Rightarrow P(Y > x - \frac{1}{2}) = 0.9591$

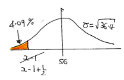
$\Rightarrow P\left(Z > \frac{x - \frac{1}{2} - 56}{\sqrt{36.4}}\right) = 0.9591$

$\downarrow$ INVERTING

$\Rightarrow \frac{x - 56.5}{\sqrt{36.4}} = -\Phi^{-1}(0.9591)$

$\Rightarrow \frac{x - 56.5}{\sqrt{36.4}} = -1.74$

$\Rightarrow x = 46.0021\ldots$



$\therefore \ x = 46$

Question 12 (*****)

A multiple choice paper has n questions, where $n > 20$.

Each question has 5 options of which only 1 is correct.

A pass is obtained if at least 20 questions are answered correctly.

It is required that the probability of obtaining a pass by randomly guessing the answers is less than 2.5%.

By using a distributional approximation, calculate the greatest value of n.

$\boxed{}$, $\boxed{n = 65}$

X = NUMBER OF CORRECT QUESTIONS (OUT OF n)

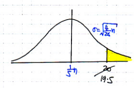
$X \sim B(n, 0.2)$

WE REQUIRE THAT THE PROBABILITY OF A PASS IS LESS THAN 2.5%

$\Rightarrow P(X \geqslant 20) < 0.025$

APPROXIMATE BY A NORMAL DISTRIBUTION

$$\left.\begin{array}{l} E(X) = np = n \times 0.2 = \frac{1}{5}n \\ \text{Var}(X) = np(1-p) = \frac{1}{5}n \times \frac{4}{5} = \frac{4}{25}n \end{array}\right\} \Rightarrow Y \sim N\left(\tfrac{1}{5}n, \tfrac{4}{25}n\right)$$



HENCE WE NOW HAVE

$\Rightarrow P(X \geqslant 20) < 0.025$

$\Rightarrow P(Y > 19.5) < 0.025$

$\Rightarrow 1 - P(Y > 19.5) < 0.025$

$\Rightarrow -P(Y < 19.5) < -0.975$

$\Rightarrow P(Y < 19.5) > 0.975$

$\Rightarrow P\left(Z < \frac{19.5 - \frac{1}{5}n}{\sqrt{\frac{4}{25}n}}\right) > 0.975$

$\Downarrow$ INVERTING

$\frac{19.5 - \frac{1}{5}n}{\frac{2}{5}\sqrt{n}} > +\Phi^{-1}(0.975)$

$\Rightarrow \frac{19.5 - \frac{1}{5}n}{\frac{2}{5}\sqrt{n}} > 1.96$

$\Rightarrow 19.5 - \frac{1}{5}n > 0.784\sqrt{n}$

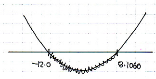
$\Rightarrow 0 > \frac{1}{5}n + 0.784\sqrt{n} - 19.5$

$\Rightarrow n + 3.92\sqrt{n} - 97.5 < 0$

BY QUADRATIC FORMULA OBTAIN CRITICAL VALUES

$\sqrt{n} = \frac{-3.92 \pm \sqrt{3.92^2 - 4 \times 1 \times (-97.5)}}{2 \times 1}$

$\sqrt{n} = \left< \begin{array}{l} 8.10685\ldots \\ -12.0268\ldots \end{array}\right.$



$\sqrt{n} < 8.1068\ldots$ (IGNORING NEGATIVE)

$n < 65.72\ldots$

$n = 65$

POISSON TO NORMAL

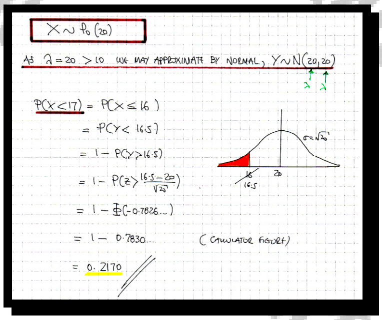
Question 1 (+)**

The discrete random variable X has probability distribution

$$X \sim \text{Po}(20).$$

Use a distributional approximation, to find $\text{P}(X < 17)$.

$$\boxed{\text{P}(X < 17) = 0.217}$$



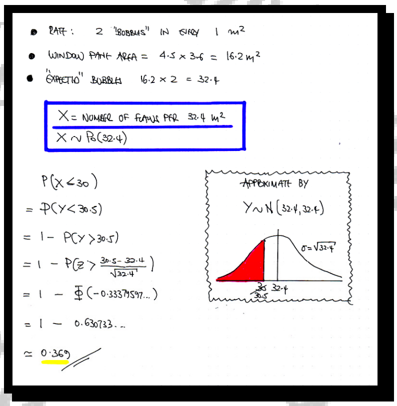
Question 2 (+)**

Minor flaws (air bubbles) in the glass manufacture of windows occur at the rate of two per square metre of glass.

A rectangular glass pane measures 4.5 metres by 3.6 metres.

Using a distributional approximation, find the probability that there will be at most 30 flaws in this window pane.

$\boxed{}$, $\boxed{0.369}$

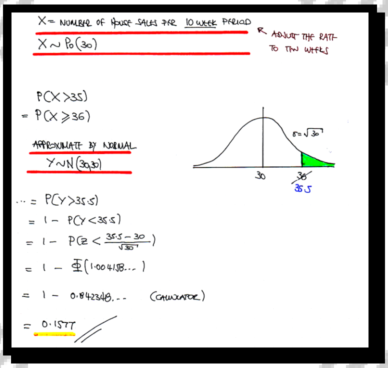


Question 3 (**+)

The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 3 houses per week. The estate agent will receive a bonus if he sells more than 35 houses in the next 10 weeks.

Use a suitable distributional approximation to estimate the probability that the estate agent receives a bonus.

$\boxed{}$, $\boxed{0.159}$



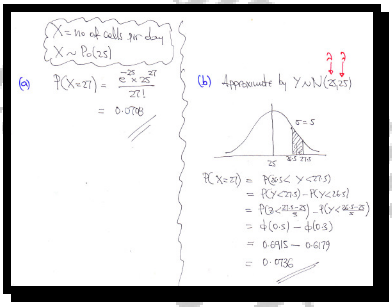
Question 4 (***)

A car breakdown company receives on average 25 calls per day.

a) Determine the probability that on a given day there will be exactly 27 calls.

b) Using a distributional approximation, find the probability that on a given day there will be exactly 27 calls.

$\boxed{0.0708}$, $\boxed{0.0736}$

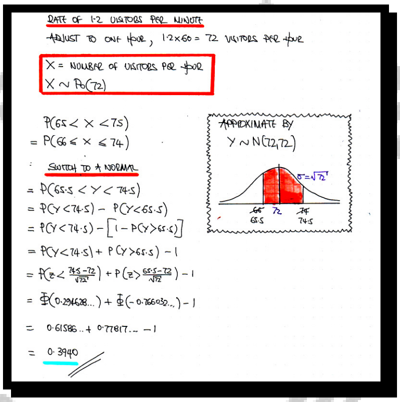


Question 5 (***)

A website receives visitors at the constant rate of 1.2 per minute.

Using a distributional approximation, find the probability that during a randomly selected hour the website will receive more than 65 but less than 75 visitors.

$\boxed{}$, $\boxed{0.3940}$



Question 6 (***+)

Minor defects occur at random in planks of wood with a constant rate of 0.5 per 10 cm length.

Noah buys a plank of length 100 cm.

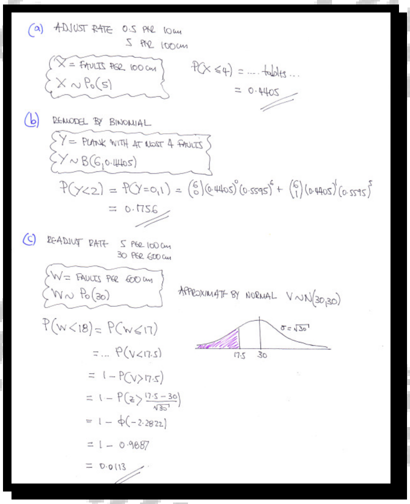
a) Find the probability that Noah's plank contains at most 4 minor defects.

Kallife buys 6 planks of wood, each of length 100 cm.

b) Find the probability that fewer than 2 of Kallife's planks of wood contain at most 4 minor defects.

c) Using a suitable distributional approximation, estimate the probability that the total number of defects on Kallife's 6 planks of wood is less than 18.

$\boxed{0.4405}$, $\boxed{0.1756}$, $\boxed{0.0113}$



(a) ADJUST RATE 0.5 PER 10 cm
5 PER 100 cm

X = FAULTS PER 100 cm
$X \sim Po(5)$

$P(X \leqslant 4) = \dots \text{tables} \dots = 0.4405$

(b) REMODEL BY BINOMIAL

Y = PLANK WITH AT MOST 4 FAULTS
$Y \sim B(6, 0.4405)$

$P(Y<2) = P(Y=0,1) = \binom{6}{0}(0.4405)^0(0.5595)^6 + \binom{6}{1}(0.4405)^1(0.5595)^5 = 0.1756$

(c) READJUST RATE 5 PER 100 cm
30 PER 600 cm

W = FAULTS PER 600 cm
$W \sim Po(30)$

APPROXIMATE BY NORMAL $V \sim N(30,30)$

$\sigma = \sqrt{30}$

17.5 30

$P(W<18) = P(W \leqslant 17)$
$= \dots P(V<17.5)$
$= 1 - P(V>17.5)$
$= 1 - P\left(z > \frac{17.5-30}{\sqrt{30}}\right)$
$= 1 - \Phi(-2.2822)$
$= 1 - 0.9887$
$= 0.0113$

Question 7 (***+)

The number of customer complaints received by a company is thought to follow a Poisson distribution, with a mean of 1.8 complaints per day.

In a randomly chosen 5 day week, the probability that there will be at least n customer complaints is 12.42%.

a) Determine the value of n.

b) Use a distributional approximation to find the probability that in a period of 20 working days there fewer than 30 customer complaints.

A week of 5 working days is called a "bad week" if at least n customer complaints are received, where n is the value found in part **(a)**.

c) Find the probability that in 40 randomly chosen weeks more than 2 are "bad".

$\boxed{n=13}$, $\boxed{\approx 0.140}$, $\boxed{\approx 0.889}$

(a) ADJUST RATE $5 \times 1.8 = 9$

X = COMPLAINTS PER 5 DAYS
$X \sim Po(9)$

$P(X \geq n) = 12.42\%$
$\Rightarrow 1 - P(X \leq n-1) = 0.1242$
$\Rightarrow -P(X \leq n-1) = -0.8758$
$\Rightarrow P(X \leq n-1) = 0.8758$

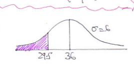
LOOKING AT $Po(9)$ TABLES
$\Rightarrow n-1 = 12$
$\Rightarrow n = 13$

(b) ADJUST RATE $20 \times 1.8 = 36$

Y = COMPLAINTS PER 20 DAYS
$Y \sim Po(36)$

APPROXIMATE BY NORMAL
$W \sim N(36, 36)$

$P(Y<30) = P(Y \leq 29) = P(W < 29.5)$
$= 1 - P(W > 29.5)$
$= 1 - P\left(Z > \frac{29.5-36}{6}\right)$
$= 1 - \Phi(-1.0833)$
$= 1 - 0.8599$
$= 0.140$



(c) REMODEL BY BINOMIAL

V = NO OF "BAD" WEEKS
$V \sim B(40, 0.1242)$

$P(V>2) = P(V \geq 3)$
$= 1 - P(V \leq 2)$
$= 1 - P(V=0,1,2)$
$= 1 - \left\{ \binom{40}{0}(0.1242)^0(0.8758)^{40} + \binom{40}{1}(0.1242)^1(0.8758)^{39} + \binom{40}{2}(0.1242)^2(0.8758)^{38} \right\}$
$= 1 - 0.1111$
$= 0.889$

Question 8 (***+)

The number of errors per page typed by Lena is assumed to follow a Poisson distribution with a mean of 0.45.

a) State two conditions, for a Poisson distribution to be a suitable model for the number of errors per page, typed by Lena.

A page typed by Lena is picked at random.

b) Calculate the probability of having exactly 2 errors on this page.

c) Calculate the probability of having at least 2 errors on this page.

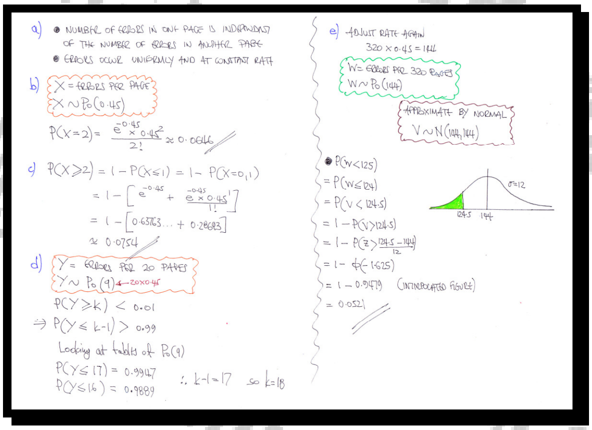
20 pages typed by Lena are next picked at random.

d) Determine the least integer k such that the probability of having k or more typing errors, in these 20 pages typed by Lena, is less than 1%.

Finally, 320 pages typed by Lena are picked at random.

e) Use a distributional approximation to find the probability of having less than 125 typing errors, in these 320 pages typed by Lena.

$\boxed{\approx 0.0646}$, $\boxed{\approx 0.0754}$, $\boxed{k = 18}$, $\boxed{\approx 0.052}$



a) • NUMBER OF ERRORS IN ONE PAGE IS INDEPENDENT OF THE NUMBER OF ERRORS IN ANOTHER PAGE
• ERRORS OCCUR UNIFORMLY AND AT CONSTANT RATE

b) X = ERRORS PER PAGE
$X \sim Po(0.45)$

$P(X=2) = \frac{e^{-0.45} \times 0.45^2}{2!} \approx 0.0646$

c) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - P(X = 0, 1)$
$= 1 - \left[e^{-0.45} + \frac{e^{-0.45} \times 0.45^1}{1!}\right]$
$= 1 - [0.63763... + 0.28693]$
≈ 0.0754

d) Y = ERRORS PER 20 PAGES
$Y \sim Po(9) \leftarrow 20 \times 0.45$

$P(Y \geq k) < 0.01$
$\Rightarrow P(Y \leq k-1) > 0.99$
Looking at tables of $Po(9)$
$P(Y \leq 17) = 0.9947$
$P(Y \leq 16) = 0.9889$
$\therefore k - 1 = 17$ so $k = 18$

e) ADJUST RATE AGAIN
$320 \times 0.45 = 144$

W = ERRORS PER 320 PAGES
$W \sim Po(144)$

APPROXIMATE BY NORMAL
$V \sim N(144, 144)$

• $P(W < 125)$
$= P(W \leq 124)$
$= P(V < 124.5)$
$= 1 - P(V > 124.5)$
$= 1 - P\left(Z > \frac{124.5 - 144}{12}\right)$
$= 1 - \Phi(-1.625)$
$= 1 - 0.9479$ (INTERPOLATED FIGURE)
$= 0.0521$

Question 9 (***+)

A radioactive substance during its decay emits radioactive particles. The number of particles emitted per second follows a Poisson distribution with mean 100. A warning alarm sounds if more than 6200 particles have been emitted in a continuous minute.

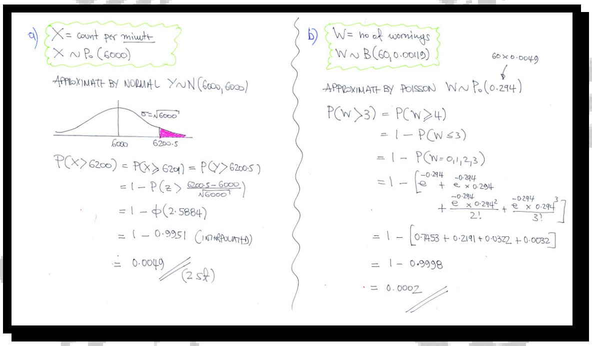
A random **one minute** interval is chosen.

a) Use a Normal distributional approximation to calculate the probability that the alarm will sound in that minute interval.

A random **one hour** interval is chosen.

b) Use a distributional approximation to calculate the probability that the alarm will sound on more than 3 occasions during this hour.

$\boxed{\approx 0.005}$, $\boxed{\approx 0.0002}$



a) X = count per minute

$X \sim Po(6000)$

APPROXIMATE BY NORMAL $Y \sim N(6000, 6000)$

$P(X > 6200) = P(X \geqslant 6201) = P(Y > 6200.5)$

$= 1 - P\left(z > \frac{6200.5 - 6000}{\sqrt{6000}}\right)$

$= 1 - \Phi(2.5884)$

$= 1 - 0.9951$ (INTERPOLATED)

$\doteq 0.0049$ (2 sf)

b) W = no of warnings

$W \sim B(60, 0.0049)$

APPROXIMATE BY POISSON $W \sim Po(\underbrace{0.294}_{60 \times 0.0049})$

$P(W > 3) = P(W \geqslant 4)$

$= 1 - P(W \leqslant 3)$

$= 1 - P(W = 0, 1, 2, 3)$

$= 1 - \left[e^{-0.294} + e^{-0.294} \times 0.294 + \frac{e^{-0.294} \times 0.294^2}{2!} + \frac{e^{-0.294} \times 0.294^3}{3!}\right]$

$= 1 - [0.7453 + 0.2191 + 0.0322 + 0.0032]$

$= 1 - 0.9998$

$= 0.0002$

Question 10 (****)

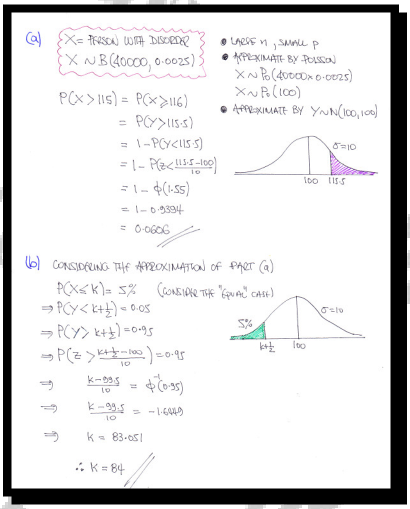
A gene for a rare blood disorder is known to occur in 0.0025 of the population.

A random sample of 40000 individuals is screened for this gene.

a) Calculate the probability that in this sample, more than 115 individuals will be carrying this gene.

b) Find the least value of k such that the probability that there are at most k individuals carrying this gene is greater than 5%.

$\boxed{\approx 0.0606}$, $\boxed{k = 84}$



(a) X = PERSON WITH DISORDER

$X \sim B(40000, 0.0025)$

- LARGE n, SMALL p
- APPROXIMATE BY POISSON

$X \sim Po(40000 \times 0.0025)$

$X \sim Po(100)$

- APPROXIMATE BY $Y \sim N(100, 100)$

$$P(X > 115) = P(X \geqslant 116) = P(Y > 115.5) = 1 - P(Y < 115.5) = 1 - P\left(Z < \frac{115.5 - 100}{10}\right) = 1 - \Phi(1.55) = 1 - 0.9394 = 0.0606$$

(b) CONSIDERING THE APPROXIMATION OF PART (a)

$P(X \leqslant k) = 5\%$ (CONSIDER THE "EQUAL" CASE)

$\Rightarrow P(Y < k + \frac{1}{2}) = 0.05$

$\Rightarrow P(Y > k + \frac{1}{2}) = 0.95$

$\Rightarrow P\left(Z > \frac{k + \frac{1}{2} - 100}{10}\right) = 0.95$

$\Rightarrow \frac{k - 99.5}{10} = \Phi^{-1}(0.95)$

$\Rightarrow \frac{k - 99.5}{10} = -1.6449$

$\Rightarrow k = 83.051$

$\therefore k = 84$

Question 11 (****)

The rate of failed connections, in madasmaths.com during the peak exam season, is λ per 10 minutes, where $1 < \lambda < 10$.

The probability that in a 2 hour period there will be less than 29 failed connection is approximated by a Normal distribution to be 0.1056.

Determine the value of λ.

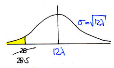
$\square$, $\boxed{\lambda = 3}$

RATE : λ DISCONNECTIONS EVERY 10 MINUTES
X = NUMBER OF DISCONNECTIONS PER TWO HOURS
$X \sim Po(12\lambda)$

NOTE AS $1 < \lambda < 10$
$12 < 12\lambda < 120$
AT LEAST 12 SO WE CAN APPROXIMATE BY A NORMAL DISTRIBUTION

LOOKING AT THE DIAGRAM BELOW

$P(X < 29) = 0.1056$
$P(X \leqslant 28) = 0.1056$



$Y \sim N(12\lambda, 12\lambda)$

SWITCHING INTO NORMAL

$P(Y < 28.5) = 0.1056$
$P(Y > 28.5) = 0.8944$
$P\left(Z > \frac{28.5 - 12\lambda}{\sqrt{12\lambda}}\right) = 0.8944$

INVERTING YIELDS

$\Rightarrow \frac{28.5 - 12\lambda}{\sqrt{12\lambda}} = -\Phi^{-1}(0.8944)$

$\Rightarrow \frac{28.5 - 12\lambda}{\sqrt{12\lambda}} = -1.25$

$\Rightarrow \frac{28.5 - t^2}{t} = -1.25$ $\quad t = \sqrt{12\lambda}$

$\Rightarrow 28.5 - t^2 = -1.25t$

$\Rightarrow 0 = t^2 - 1.25t - 28.5$

BY THE QUADRATIC FORMULA (OR FACTORIZATION)

$\Rightarrow 4t^2 - 5t - 114 = 0$

$\Rightarrow (t - 6)(4t + 19) = 0$

$\Rightarrow t = 6 \text{ or } -\frac{19}{4}$

$\Rightarrow \sqrt{12\lambda} = 6 \text{ or } \cancel{-\frac{19}{4}}$

$\Rightarrow 12\lambda = 36$

$\Rightarrow \lambda = 3$

Question 12 (****+)

Tiny faults, usually small blockages and cracks, occur in the pipeline of an oil refinery at the rate of 1 fault per 25 metres of pipe.

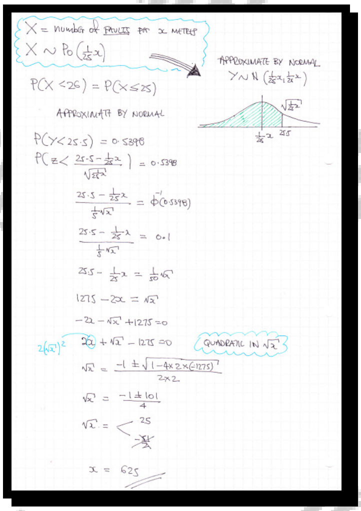
These faults are modelled by a Poisson variable.

A pipeline of length x metres is to be examined.

Using a normal approximation, the probability that this pipeline has fewer than 26 faults, is 0.5398.

Determine the value of x.

$\boxed{x = 625}$



X = number of FAULTS per x METRES

$X \sim Po\left(\frac{1}{25}x\right)$

APPROXIMATE BY NORMAL

$Y \sim N\left(\frac{1}{25}x, \frac{1}{25}x\right)$

$P(X < 26) = P(X \leq 25)$

APPROXIMATE BY NORMAL

$P(Y < 25.5) = 0.5398$

$P\left(z < \frac{25.5 - \frac{1}{25}x}{\sqrt{\frac{1}{25}x}}\right) = 0.5398$

$\frac{25.5 - \frac{1}{25}x}{\frac{1}{5}\sqrt{x}} = \Phi^{-1}(0.5398)$

$\frac{25.5 - \frac{1}{25}x}{\frac{1}{5}\sqrt{x}} = 0.1$

$25.5 - \frac{1}{25}x = \frac{1}{50}\sqrt{x}$

$1275 - 2x = \sqrt{x}$

$-2x - \sqrt{x} + 1275 = 0$

$2(\sqrt{x})^2$

$2x + \sqrt{x} - 1275 = 0$ QUADRATIC IN $\sqrt{x}$

$\sqrt{x} = \frac{-1 \pm \sqrt{1 - 4 \times 2 \times (-1275)}}{2 \times 2}$

$\sqrt{x} = \frac{-1 \pm 101}{4}$

$\sqrt{x} = 25$ or $\cancel{-\frac{51}{2}}$

$x = 625$