NORMAL VARIABLES

Question 1 (**)

The random variables X and Y are Normally distributed, such that

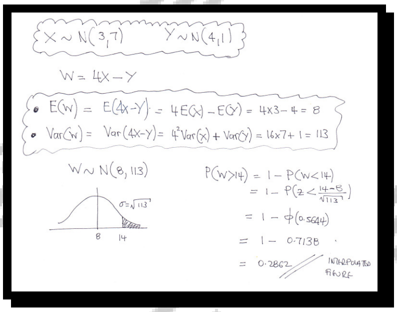
$$X \sim \mathrm{N}(3,7) \text{ and } Y \sim \mathrm{N}(4,1).$$

The random variable W is defined as

$$W \equiv 4X - Y.$$

Given that X and Y are independent of one another, determine $\mathrm{P}(W > 14)$.

$\boxed{0.2862}$



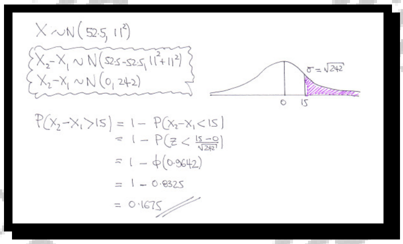
Question 2 (**+)

The continuous random variable X is Normally distributed with mean 52.5 and standard deviation 11.

Two randomly chosen observations of X are denoted by X_1 and X_2.

Determine the probability that X_2 will exceed X_1 by at least 15.

$\boxed{0.1675}$



Question 3 (**+)

The continuous random variables X and Y are Normally distributed, such that

$$X \sim \mathrm{N}(40,9) \quad \text{and} \quad Y \sim \mathrm{N}(50,16).$$

Determine, correct to 4 decimal places,

a) $\mathrm{P}(X+Y>100)$.

b) $\mathrm{P}(X-Y>0)$.

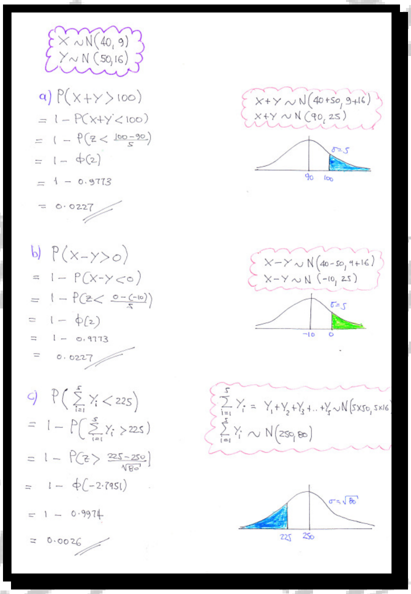
c) $\mathrm{P}\left[\left(\sum_{i=1}^{5} Y_i\right)<225\right]$.

d) $\mathrm{P}(5X-4Y<50)$.

e) $\mathrm{P}(2X>Y_1+Y_2)$.

You may assume that X and Y are independent of one another.

$\boxed{0.0227}$, $\boxed{0.0227}$, $\boxed{0.0026}$, $\boxed{0.9887}$, $\boxed{0.0076}$


$X \sim N(40, 9)$
$Y \sim N(50, 16)$

a) $P(X+Y>100)$
$= 1 - P(X+Y<100)$
$= 1 - P\left(Z < \frac{100-90}{5}\right)$
$= 1 - \Phi(2)$
$= 1 - 0.9773$
$= 0.0227$

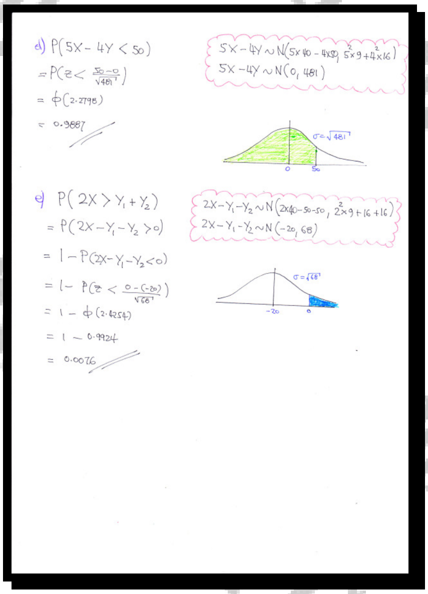
$X+Y \sim N(40+50, 9+16)$
$X+Y \sim N(90, 25)$

b) $P(X-Y>0)$
$= 1 - P(X-Y<0)$
$= 1 - P\left(Z < \frac{0-(-10)}{5}\right)$
$= 1 - \Phi(2)$
$= 1 - 0.9773$
$= 0.0227$

$X-Y \sim N(40-50, 9+16)$
$X-Y \sim N(-10, 25)$

c) $P\left(\sum_{i=1}^{5} Y_i < 225\right)$
$= 1 - P\left(\sum_{i=1}^{5} Y_i > 225\right)$
$= 1 - P\left(Z > \frac{225-250}{\sqrt{80}}\right)$
$= 1 - \Phi(-2.7951)$
$= 1 - 0.9974$
$= 0.0026$

$\sum_{i=1}^{5} Y_i = Y_1+Y_2+Y_3+\dots+Y_5 \sim N(5\times50, 5\times16)$
$\sum_{i=1}^{5} Y_i \sim N(250, 80)$


d) $P(5X-4Y<50)$
$= P\left(Z < \frac{50-0}{\sqrt{481}}\right)$
$= \Phi(2.2798)$
$= 0.9887$

$5X-4Y \sim N(5\times40 - 4\times50, 5^2\times9 + 4^2\times16)$
$5X-4Y \sim N(0, 481)$

e) $P(2X > Y_1+Y_2)$
$= P(2X-Y_1-Y_2>0)$
$= 1 - P(2X-Y_1-Y_2<0)$
$= 1 - P\left(Z < \frac{0-(-20)}{\sqrt{68}}\right)$
$= 1 - \Phi(2.4254)$
$= 1 - 0.9924$
$= 0.0076$

$2X-Y_1-Y_2 \sim N(2\times40-50-50, 2^2\times9+16+16)$
$2X-Y_1-Y_2 \sim N(-20, 68)$

Question 4 (***)

The continuous random variables X and Y are Normally distributed, such that

$$X \sim N\left(250, 5^2\right) \text{ and } Y \sim N\left(20, 2^2\right).$$

Determine, correct to 4 decimal places,

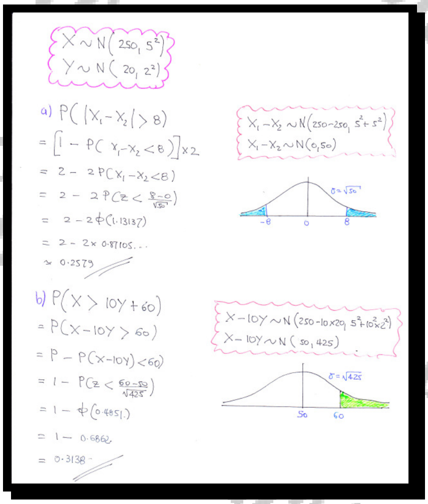
a) $P\left(|X_1 - X_2| > 8\right)$.

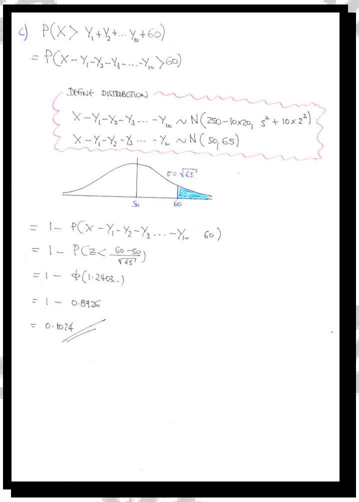
b) $P\left(X > 10Y + 60\right)$.

c) $P\left(X > Y_1 + Y_2 + Y_3 + \ldots + Y_{10} + 60\right)$.

You may assume that X and Y are independent of one another.

$\boxed{0.2579}$, $\boxed{0.3138}$, $\boxed{0.1074}$





Question 5 (***)

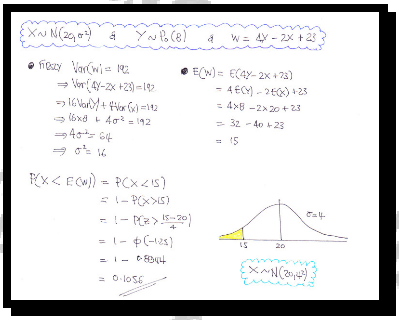
The random variables X and Y are independent and have respective distributions $\mathrm{N}\left(20, \sigma^2\right)$ and $\mathrm{Po}(8)$.

The random variable W is defined as

$$W = 4Y - 2X + 23.$$

Given that $\mathrm{Var}(W) = 192$, determine $\mathrm{P}\left[X < \mathrm{E}(W)\right]$.

$\boxed{0.1056}$



Question 6 (***)

The continuous random variables X and Y are independent of one another and have Normal distributions

$$X \sim \mathrm{N}\left(40, 3.3^2\right) \quad \text{and} \quad Y \sim \mathrm{N}\left(\mu, 5.6^2\right).$$

Given further that $P(Y - X < 5) = 5\%$, determine the value of μ.

$\boxed{}$, $\boxed{\mu \approx 55.7}$

$\boxed{X \sim N(40, 3.3^2) \quad \text{AND} \quad Y \sim N(\mu, 5.6^2)}$

DEFINE A NEW VARIABLE $W = Y - X$

$E(W) = E(Y-X) = \mu - 40$

$\text{Var}(W) = \text{Var}(Y-X) = 5.6^2 + 3.3^2 = 42.25 = 6.5^2$

NOW WE HAVE

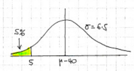
$P(Y-X<5) = 5\%$

$P(W<5) = 5\%$

$P(W>5) = 95\%$

$P\left(Z > \frac{5-\mu+40}{6.5}\right) = 0.95$

$P\left(Z > \frac{45-\mu}{6.5}\right) = 0.95$



INVERTING NOTING THE "NEGATIVE"

$\frac{45-\mu}{6.5} = -\Phi^{-1}(0.95)$

$\frac{45-\mu}{6.5} = -1.6449$

$45 - \mu = -10.69185$

$\mu = 55.69185$

$\therefore \mu \approx 55.7$

Question 7 (***)

The random variables A and B are Normally distributed, such that

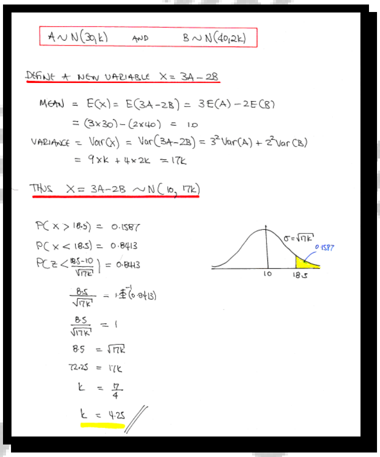
$$A \sim \mathrm{N}(30, k) \quad \text{and} \quad B \sim \mathrm{N}(40, 2k),$$

where k is a positive constant.

Determine the value of k if $\mathrm{P}(3A - 2B > 18.5) = 0.1587$.

You may assume that A and B are independent of one another.

$\boxed{\text{FS2-Q}}$, $\boxed{k = \dfrac{17}{4} = 4.25}$



$A \sim N(30, k)$ AND $B \sim N(40, 2k)$

DEFINE A NEW VARIABLE $X = 3A - 2B$

MEAN $= E(X) = E(3A - 2B) = 3E(A) - 2E(B)$
$= (3 \times 30) - (2 \times 40) = 10$

VARIANCE $= \mathrm{Var}(X) = \mathrm{Var}(3A - 2B) = 3^2\mathrm{Var}(A) + 2^2\mathrm{Var}(B)$
$= 9 \times k + 4 \times 2k = 17k$

THUS $X = 3A - 2B \sim N(10, 17k)$

$P(X > 18.5) = 0.1587$

$P(X < 18.5) = 0.8413$

$P\left(Z < \frac{18.5 - 10}{\sqrt{17k}}\right) = 0.8413$

$\frac{8.5}{\sqrt{17k}} = \Phi^{-1}(0.8413)$

$\frac{8.5}{\sqrt{17k}} = 1$

$8.5 = \sqrt{17k}$

$72.25 = 17k$

$k = \frac{17}{4}$

$k = 4.25$

Question 8 (***)

The random variables S and T are Normally distributed, such that

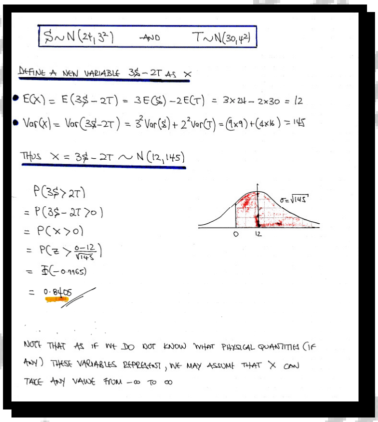
$$S \sim \mathrm{N}\left(24, 3^2\right) \quad \text{and} \quad T \sim \mathrm{N}\left(30, 4^2\right),$$

where k is a positive constant.

Determine $\mathrm{P}(3S > 2T)$.

You may assume that S and T are independent of one another.

$\boxed{}$, $\boxed{\mathrm{P}(3S > 2T) = 0.8405}$



$\boxed{S \sim N(24, 3^2) \quad \text{AND} \quad T \sim N(30, 4^2)}$

DEFINE A NEW VARIABLE $3S - 2T$ AS X

- $E(X) = E(3S - 2T) = 3E(S) - 2E(T) = 3 \times 24 - 2 \times 30 = 12$
- $\mathrm{Var}(X) = \mathrm{Var}(3S - 2T) = 3^2 \mathrm{Var}(S) + 2^2 \mathrm{Var}(T) = (9 \times 9) + (4 \times 16) = 145$

THUS $X = 3S - 2T \sim N(12, 145)$

$P(3S > 2T)$
$= P(3S - 2T > 0)$
$= P(X > 0)$
$= P\left(Z > \frac{0 - 12}{\sqrt{145}}\right)$
$= \Phi(-0.9965)$
$= 0.8405$

NOTE THAT AS IF WE DO NOT KNOW WHAT PHYSICAL QUANTITIES (IF ANY) THESE VARIABLES REPRESENT, WE MAY ASSUME THAT X CAN TAKE ANY VALUE FROM $-\infty$ TO ∞

Question 9 (***+)

The continuous random variable X is defined as

$$X = 5Y - 3W,$$

where Y and W are independent of one another, and $Y \sim \mathrm{N}(45,10)$, $W \sim \mathrm{N}(60,15)$.

a) Find $\mathrm{P}(X > 80)$.

X_1, X_2 and X_3 are three independent observations of X.

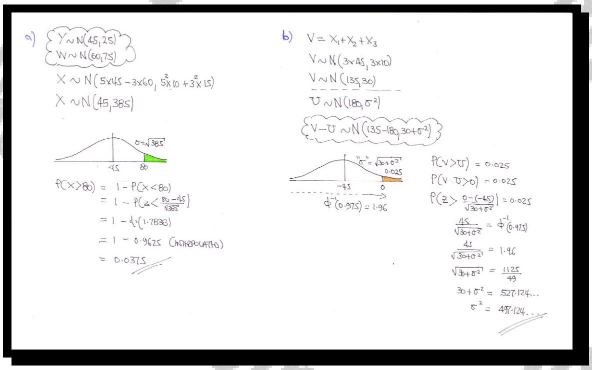
The random variable V is defined as

$$V = \sum_{i=1}^{3} X_i$$

The random variable U is such that $U \sim N\left(180, \sigma^2\right)$

b) Given that U and V are independent of one another, and $\mathrm{P}(V > U) = 0.025$ determine the variance U.

$\boxed{0.0375}$, $\boxed{\sigma^2 = 497.124...}$



Question 10 (***+)

It is given that the continuous random variables A, B and C are independent of one another and have distribution

$$A \sim \mathrm{N}\left(10, 4^2\right), \quad B \sim \mathrm{N}\left(15, 5^2\right) \quad \text{and} \quad C \sim \mathrm{N}\left(20, 6^2\right).$$

The random variable X is defined as

$$X \equiv A + B + 3C.$$

a) Determine $\mathrm{P}(X < 76)$.

Three independent observations of C are made, denoted by C_1, C_2 and C_3.

The random variable Y is defined as

$$Y \equiv A + B + \sum_{i=1}^{3} C_i.$$

b) Determine $\mathrm{P}(Y < 76)$.

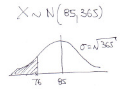
$\boxed{0.3188}$, $\boxed{0.2304}$

(a) $A \sim N(10, 4^2)$ $B \sim N(15, 5^2)$ $C \sim N(20, 6^2)$

$X = A + B + 3C$

- $E(X) = 10 + 15 + 3 \times 20 = 85$
- $\mathrm{Var}(X) = 16 + 25 + 3^2 \times 36 = 365$

$X \sim N(85, 365)$

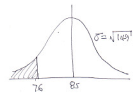


$$\begin{aligned} P(X < 76) &= 1 - P(X > 76) \\ &= 1 - P\left(Z > \frac{76 - 85}{\sqrt{365}}\right) \\ &= 1 - \Phi(-0.4711) \\ &= 1 - 0.6812 \\ &= 0.3188 \end{aligned}$$

INTERPOLATED FIGURE

(b) $Y = A + B + \sum_{i=1}^{3} C_i$

- $E(Y) = 10 + 15 + 3 \times 20 = 85$
- $\mathrm{Var}(Y) = 16 + 25 + 3 \times 36 = 149$



$$\begin{aligned} P(Y < 76) &= 1 - P(Y > 76) \\ &= 1 - P\left(Z > \frac{76 - 85}{\sqrt{149}}\right) \\ &= 1 - \Phi(-0.7373) \\ &= 1 - 0.7696 \\ &= 0.2304 \end{aligned}$$

INTERPOLATED FIGURE

Question 11 (***+)

Bottles of mineral water are delivered in crates containing 12 bottles each.

The weights of bottles are Normally distributed with mean weight 2 kg and standard deviation 0.05 kg. The weights of empty crates are also Normally distributed with mean weight 2 kg and standard deviation 0.5 kg.

a) Assuming that the weights of the bottles and the empty crates are independent of one another, determine the probability that a full crate will weigh between 26 kg and 27 kg.

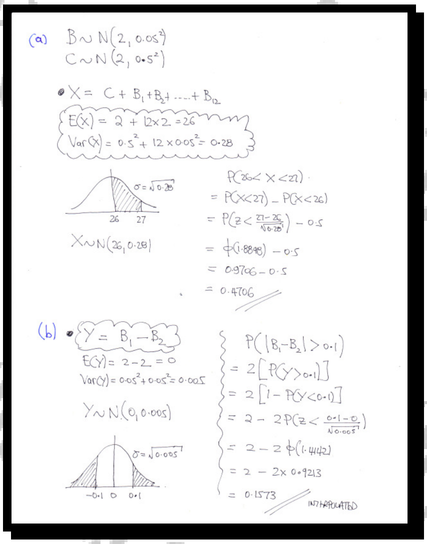
Two bottles are selected at random from one crate.

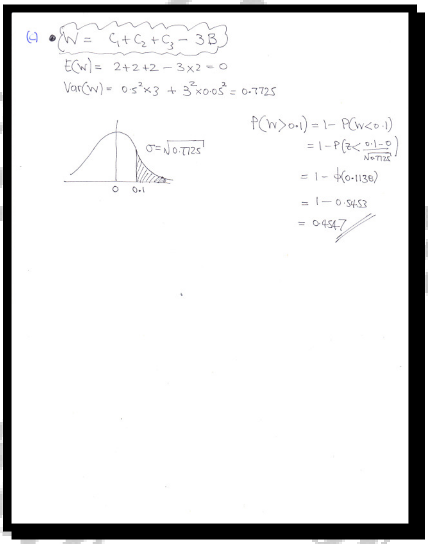
b) Find the probability that their weight will differ by at least 0.1 kg.

Another three bottles are selected at random together with one empty crate.

c) Find the probability that the weight of the three bottles will be at least 0.1 kg heavier than three times the weight of the empty crate.

$\boxed{0.4706}$, $\boxed{0.1573}$, $\boxed{0.4547}$





Question 12 (***+)

The weight of a bag of sugar, of a particular supermarket brand, is modelled by a Normal distribution with mean 1008 grams and standard deviation of 4 grams.

a) Determine the probability that the weight of 2 randomly chosen bags of sugar will differ by more than 12 grams.

Bags of sugar are randomly selected and packed shrink-wrapped onto wooden pallets.

The weight of a pallet and the wrapping is also modelled by a Normal distribution with mean 9.6 kg and standard deviation of 0.5 kg

Each full pallet is loaded with 160 bags of sugar.

b) Find the probability that the weight of a randomly selected, fully loaded pallet, will be less than 170 kg .

$\boxed{}$, $\boxed{0.0340}$, $\boxed{0.0400}$

a) • DEFINING VARIABLES & DISTRIBUTIONS

$\boxed{\begin{array}{l} X = \text{WEIGHT OF A BAG OF SUGAR} \\ X \sim N(1008, 4^2) \end{array}}$

- $X_1 - X_2 \sim N(1008 - 1008, 4^2 + 4^2)$
- $X_1 - X_2 \sim N(0, 32)$

• WE REQUIRE $P(X_1 - X_2 > 12)$ OR $P(X_2 - X_1 > 12)$

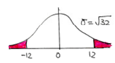
$P(X_1 - X_2 > 12) = 1 - P(X_1 - X_2 < 12)$

$= 1 - P\left(Z < \frac{12-0}{\sqrt{32}}\right)$

$= 1 - \Phi(2.1213)$

$= 1 - 0.9830$

$= 0.017$



• HENCE, THE REQUIRED PROBABILITY (BY SYMMETRY) IS $2 \times 0.017 = 0.034$

b) • LET $T = X_1 + X_2 + X_3 + \dots + X_{160} + Y$

WHERE Y = weight of the pallet/wrapping
$Y \sim N(9600, 500^2)$
(WORKING IN GRAMS)

- $E(T) = (160 \times 1008) + 9600 = 170880$
- $Var(T) = 160 \times 4^2 + 500^2 = 252560$

• HENCE WE NOW HAVE

$\boxed{T \sim N(170880, 252560)}$

$P(T < 170000)$

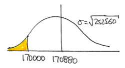
$= 1 - P(T > 170000)$

$= 1 - P\left(Z > \frac{170000 - 170880}{\sqrt{252560}}\right)$

$= 1 - \Phi(-1.7511)$

$= 1 - 0.9600$

$= 0.0400$



Question 13 (***+)

A car hire company washes their cars after being returned by clients.

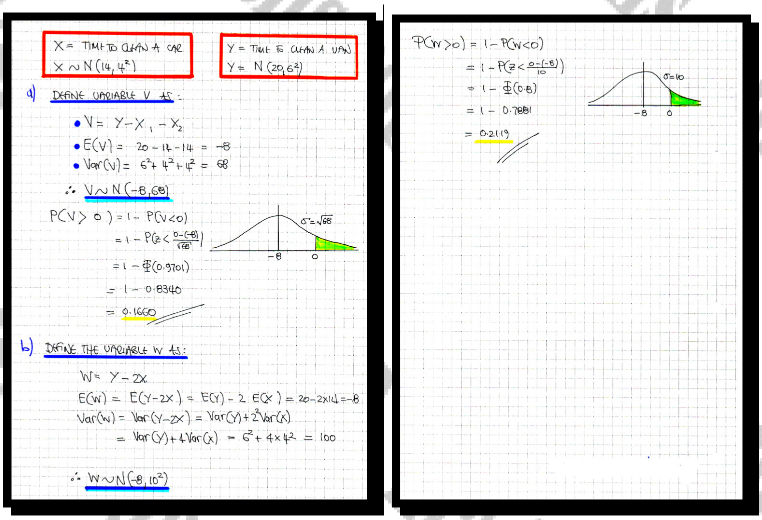
The time it takes to wash a car, in minutes, is assumed to be a Normal variable with mean 14 and standard deviation of 4.

The time it takes to wash a van, in minutes, is also assumed to be a Normal variable with mean 20 and standard deviation of 6.

a) Determine the probability that the time it takes to wash the next van will be greater than the total time taken to wash the next 2 cars.

b) Find the probability that the time it takes to wash the next van will more than twice the time taken to wash the next car.

$\boxed{}$, $\boxed{0.1660}$, $\boxed{0.2119}$



Question 14 (***+)

It is thought that the weight of males of a certain type of dogs is a Normal variable with mean 25 kg and standard deviation 2 kg.

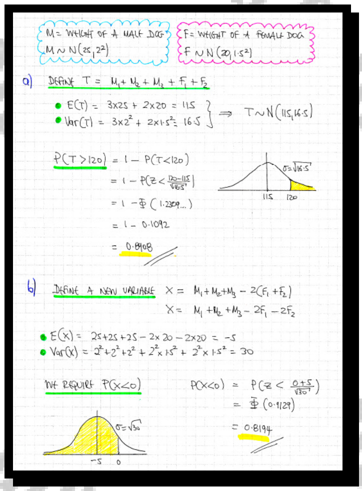
It is also thought that the weight of females of the same type of dogs is a Normal variable with mean 20 kg and standard deviation 1.5 kg.

Three male dogs and two female dogs of the above type are sampled.

a) Determine the probability that the combined weight of all five dogs in the sample will exceed 120 kg.

b) Find the probability that the combined weight of the three male dogs will be less than twice the combined weight of the two female dogs.

$\boxed{}$, $\boxed{\approx 0.8908}$, $\boxed{\approx 0.8194}$



M = WEIGHT OF A MALE DOG
$M \sim N(25, 2^2)$

F = WEIGHT OF A FEMALE DOG
$F \sim N(20, 1.5^2)$

a) DEFINE $T = M_1 + M_2 + M_3 + F_1 + F_2$

- $E(T) = 3 \times 25 + 2 \times 20 = 115$
- $Var(T) = 3 \times 2^2 + 2 \times 1.5^2 = 16.5$

$\Rightarrow T \sim N(115, 16.5)$

$$P(T > 120) = 1 - P(T < 120) = 1 - P\left(Z < \frac{120-115}{\sqrt{16.5}}\right) = 1 - \Phi(1.2309\ldots) = 1 - 0.1092 = 0.8908$$

b) DEFINE A NEW VARIABLE $X = M_1 + M_2 + M_3 - 2(F_1 + F_2)$
$X = M_1 + M_2 + M_3 - 2F_1 - 2F_2$

- $E(X) = 25 + 25 + 25 - 2 \times 20 - 2 \times 20 = -5$
- $Var(X) = 2^2 + 2^2 + 2^2 + 2^2 \times 1.5^2 + 2^2 \times 1.5^2 = 30$

WE REQUIRE $P(X < 0)$

$$P(X < 0) = P\left(Z < \frac{0+5}{\sqrt{30}}\right) = \Phi(0.9129) = 0.8194$$

Question 15 (****)

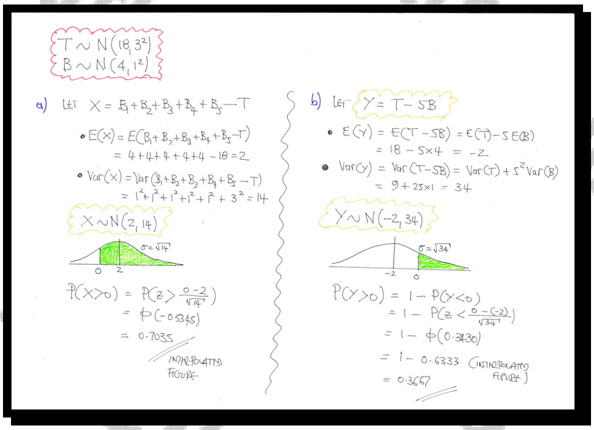
The lengths of the leaves of the Tirch trees are Normally distributed with mean 18 cm and standard deviation 3 cm. The lengths of the leaves of the Belm trees are also Normally distributed with mean 4 cm and standard deviation 1 cm.

a) Determine the probability that the **total** length of five randomly picked leaves from a Belm tree is greater than the length of a randomly picked leave from a Tirch tree.

A new leaf is selected from a Tirch tree and a new leaf is selected from a Belm tree.

b) Find the probability that the length of the leaf from the Tirch tree is more than five times the length of the leaf from the Belm tree.

$\boxed{0.7035}$, $\boxed{0.3667}$



Question 16 (****)

The continuous random variables X and Y have the following distributions.

$$X \sim \mathrm{N}\left(100, 5^2\right) \qquad \text{and} \qquad Y \sim \mathrm{N}\left(90, 3^2\right).$$

Determine the value of ...

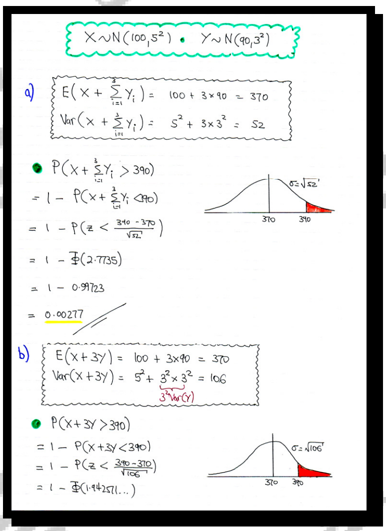
a) ... $\mathrm{P}\left(X + \sum_{i=1}^{3} Y_i \; > \; 390 \right)$.

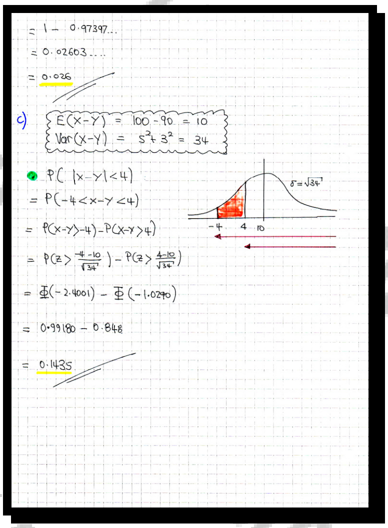
b) ... $\mathrm{P}\left(X + 3Y > 390 \right)$.

c) ... $\mathrm{P}\left(|X - Y| < 4 \right)$.

You may assume that X and Y are independent of one another.

$\boxed{}$, $\boxed{0.00277}$, $\boxed{0.026}$, $\boxed{0.1435}$





Question 17 (****)

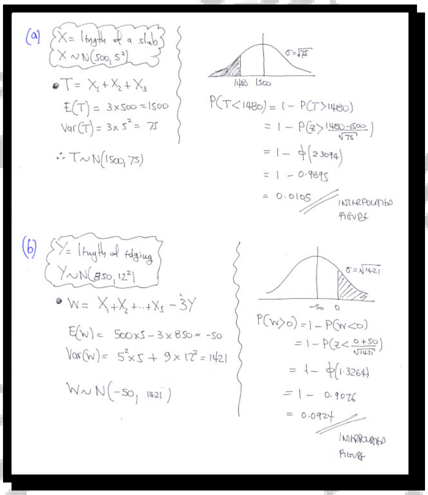
The lengths of paving slabs are Normally distributed with mean 500 mm and standard deviation 5 mm.

a) Find the probability that the total length of three randomly picked paving slabs will be less than 1.48 m.

The length of edging pieces which are placed between paving slabs are also Normally distributed with mean 850 mm and standard deviation 12 mm.

b) Find the probability that the total length of five randomly picked paving slabs will be greater than three times the length of randomly picked piece of edging.

$\boxed{0.0105}$, $\boxed{0.0924}$


(a) X = length of a slab
$X \sim N(500, 5^2)$
• $T = X_1 + X_2 + X_3$
$E(T) = 3 \times 500 = 1500$
$Var(T) = 3 \times 5^2 = 75$
$\therefore T \sim N(1500, 75)$

$P(T < 1480) = 1 - P(T > 1480)$
$= 1 - P\left(Z > \frac{1480-1500}{\sqrt{75}}\right)$
$= 1 - \Phi(2.3094)$
$= 1 - 0.9895$
$= 0.0105$
INTERPOLATED FIGURE

(b) Y = length of edging
$Y \sim N(850, 12^2)$
• $W = X_1 + X_2 + \dots + X_5 - 3Y$
$E(W) = 500 \times 5 - 3 \times 850 = -50$
$Var(W) = 5^2 \times 5 + 9 \times 12^2 = 1421$
$W \sim N(-50, 1421)$

$P(W > 0) = 1 - P(W < 0)$
$= 1 - P\left(Z < \frac{0+50}{\sqrt{1421}}\right)$
$= 1 - \Phi(1.3264)$
$= 1 - 0.9076$
$= 0.0924$
INTERPOLATED FIGURE

Question 18 (****)

Some biology students are measuring the length of plant leaves.

The lengths of leaves from a certain bush are Normally distributed with mean 2.2 cm and standard deviation 0.2 cm.

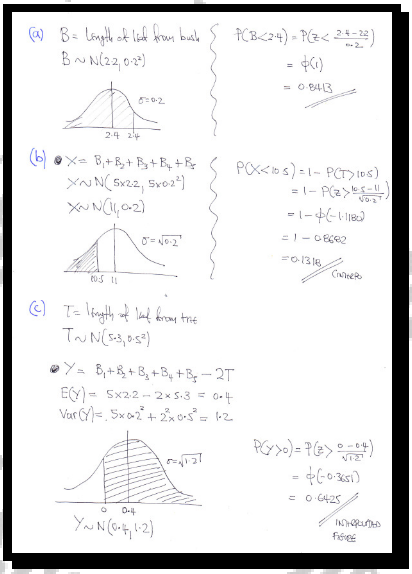
a) Find the probability that the length of one such leaf picked at random, will be at most 2.4 cm .

b) Find the probability that the **total length** of 5 such leaves picked at random, will be less than 10.5 cm .

The lengths of leaves from a certain tree are Normally distributed with mean 5.3 cm and standard deviation 0.5 cm.

c) Determine the probability that the **total length** of 5 leaves picked at random from the bush is more than twice the length of a **single** randomly picked leaf from the tree.

$\boxed{0.8413}$, $\boxed{0.1318}$, $\boxed{0.6425}$



Question 19 (****)

The weights of female bodybuilders are Normally distributed with mean 60 kg and standard deviation 4 kg. The weights of male bodybuilders are Normally distributed with mean 100 kg and standard deviation 6 kg.

Determine the probability that the total weight of 6 randomly selected female bodybuilders is more than 10 kg lighter than 4 times the weight of a randomly selected male bodybuilder.

$\boxed{}$, $\boxed{0.8764}$

X = weight of female bodybuilder
$X \sim N(60, 4^2)$

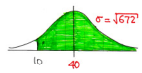
Y = weight of male bodybuilder
$Y \sim N(100, 6^2)$

DEFINE A NEW VARIABLE W

- $W = 4Y - (X_1 + X_2 + X_3 + X_4 + X_5 + X_6)$
- $E(W) = (4 \times 100) - (6 \times 60) = 40$
- $\text{Var}(W) = 4^2 \times \text{Var}(Y) + 6 \times \text{Var}(X)$
  $= (16 \times 36) + (6 \times 16)$
  $= 672$

THE WAY W IS DEFINED, WE MODEL AS
"4 TIMES MALE WEIGHT MORE THAN 10kg THAT OF 6 FEMALES"

$P(W > 10)$
$= P\left(z > \frac{10 - 40}{\sqrt{672}}\right)$
$= \Phi(-1.1573)$
$= 0.8764$



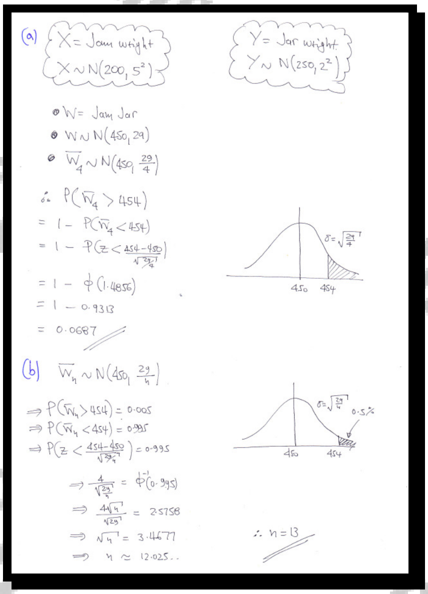
Question 20 (****)

The weight of an empty jar is Normally distributed with mean 250 grams and standard deviation 2 grams. The weight of jam delivered in a jar is Normally distributed with mean 200 grams and standard deviation 5 grams.

a) Find the probability that the mean weight of 4 jam filled jars is greater than 454 grams.

b) Determine the least number of jam filled jars that have to be sampled so that there is at most a 0.5% chance that their mean is greater than 454 grams.

$\boxed{0.0687}$, $\boxed{n = 13}$



Question 21 (****)

The thickness of paperback books in a certain section of a library is Normally distributed with mean 1.9 cm and variance 0.90 cm^2.

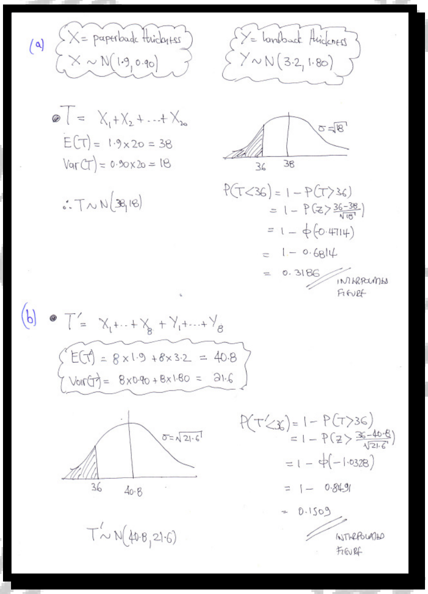
The thickness of hardback books in the same section of the library is Normally distributed with mean 3.2 cm and variance 1.80 cm^2.

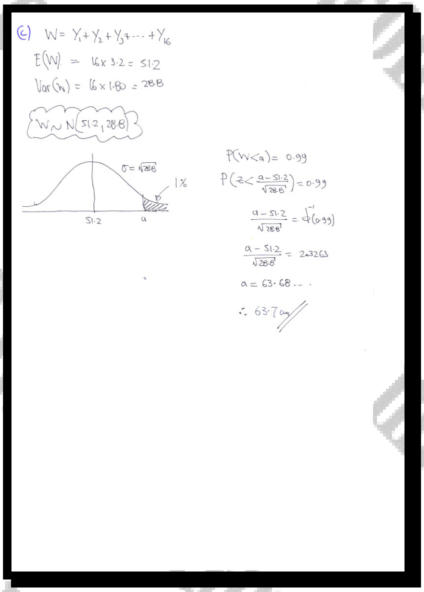
a) Find the probability that 20 paperback books will fit on a 36 cm shelf.

b) Determine the probability that 8 paperback books and 8 hardback books will fit on a 36 cm shelf.

c) Determine the smallest length of shelving so that 16 hardback books have at least a 99% chance to fit on.

$\boxed{0.3186}$, $\boxed{0.1509}$, $\boxed{\approx 63.7 \text{ cm}}$





Question 22 (****+)

The discrete random variables X and Y are independent of one another and have the following distributions

$$X \sim \mathrm{B}\left(40, \tfrac{3}{4}\right) \text{ and } Y \sim \mathrm{Po}(17.5).$$

a) Determine, with full justification, an approximate distribution for $X - Y$.

b) Calculate $\mathrm{P}\left(|X - Y| \geq 4\right)$.

$\boxed{X - Y \sim \mathrm{N}\left(12.5, 5^2\right)}$, $\boxed{\mathrm{P}\left(|X - Y| \geq 4\right) \approx 0.9648}$

(a) $X \sim B(40, \frac{3}{4})$ $E(X) = np = 40 \times \frac{3}{4} = 30 > 5$

$Var(X) = np(1-p) = 30 \times \frac{1}{4} = 7.5 > 5$

$Y \sim Po(17.5)$ $E(X) = \lambda = 17.5 > 15$

$Var(Y) = \lambda = 17.5 > 15$

THUS APPROXIMATELY $X \sim N(30, 7.5)$

$Y \sim N(17.5, 17.5)$

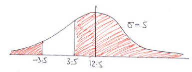
THUS $X - Y \sim N(30 - 17.5, 7.5 + 17.5)$

$X - Y \sim N(12.5, 25)$

(b) $P(|X-Y| \geq 4) = P(X-Y \geq 4) + P(Y-X \geq 4)$

$= P(X-Y \geq 4) + P(X-Y \leq -4)$

APPLY CONTINUITY CORRECTION



$= P(X-Y > 3.5) + P(X-Y < -3.5)$

$= P(X-Y > 3.5) + [1 - P(X-Y > -3.5)]$

$= P(X-Y > 3.5) + 1 - P(X-Y > -3.5)$

$= P(Z > \frac{3.5-12.5}{5}) - P(Z > \frac{-3.5-12.5}{5}) + 1$

$= \Phi(-1.8) - \Phi(-3.2) + 1$

$= 0.9641 - 0.9993 + 1$

$= 0.9648$

Question 23 (****+)

A simple random sample X_1, X_2, X_3 is taken from the population of a Normal variable X, whose mean is μ and variance is σ^2.

The sample mean of X_1, X_2 and X_3 is denoted by $\bar{X}$.

Given further that $P\left(X_1 < \bar{X} - \beta\sigma\right) = 20\%$, calculate an approximate value of β.

$\boxed{}$, $\boxed{\beta \approx 0.687}$

$X \sim N(\mu, \sigma^2)$

DEFINE VARIABLE $Y = X_1 - \bar{X} + \beta\sigma$

$Y = X_1 - \frac{X_1 + X_2 + X_3}{3} + \beta\sigma$

$Y = \frac{2}{3}X_1 - \frac{1}{3}X_2 - \frac{1}{3}X_3 + \beta\sigma$ — constant

DETERMINE THE MEAN & VARIANCE OF Y

$E(Y) = \cancel{\frac{2}{3}\mu - \frac{1}{3}\mu - \frac{1}{3}\mu} + \beta\sigma$

$\text{Var}(Y) = \frac{4}{9}\sigma^2 + \frac{1}{9}\sigma^2 + \frac{1}{9}\sigma^2 = \frac{2}{3}\sigma^2$

FINALLY WE HAVE

$\Rightarrow P(X_1 < \bar{X} - \beta\sigma) = 0.2$

$\Rightarrow P(X_1 - \bar{X} + \beta\sigma < 0) = 0.2$

$\Rightarrow P(Y < 0) = 0.2$

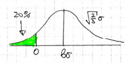
$\Rightarrow P(Y > 0) = 0.8$

$\Rightarrow P\left(Z > \frac{0 - \beta\sigma}{\sqrt{\frac{2}{3}}\sigma}\right) = 0.8$

$\Rightarrow \frac{-\beta\cancel{\sigma}}{\sqrt{\frac{2}{3}}\cancel{\sigma}} = -\Phi^{-1}(0.8)$ NEGATIVE INVERSION

$\Rightarrow \frac{-\beta}{\sqrt{\frac{2}{3}}} = -0.8416$

$\Rightarrow \beta = 0.68716\ldots$



$\therefore \beta \approx 0.687$