PROBABILITY

VENN DIAGRAMS INTRODUCTION

Question 1 (**)

The events A and B are such so that

$$P(A) = 0.3, \quad P(B) = 0.5 \quad \text{and} \quad P(A \cup B) = 0.6.$$

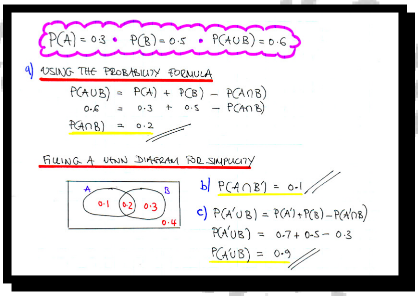
Determine …

a) … $P(A \cap B)$.

b) … $P(A \cap B')$.

c) … $P(A' \cup B)$.

$\boxed{P(A \cap B) = 0.2}$, $\boxed{P(A \cap B') = 0.1}$, $\boxed{P(A' \cup B) = 0.9}$



Question 2 (**)

The events A and B are such so that

$$\mathrm{P}(A) = 0.7\,, \quad \mathrm{P}(B) = 0.4 \quad \text{and} \quad \mathrm{P}(A \cup B) = 0.8\,.$$

a) Illustrate this information in a fully completed Venn diagram.

b) Determine the probability …

i. … of either event A or event B occurring, but not both.

ii. … of neither event A nor event B occurring.

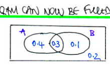
$\boxed{0.5}$, $\boxed{0.2}$

$\mathrm{P}(A) = 0.7$ • $\mathrm{P}(B) = 0.4$ • $\mathrm{P}(A \cup B) = 0.8$

a) USING THE STANDARD PROBABILITY FORMULA

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$
$$0.8 = 0.7 + 0.4 - \mathrm{P}(A \cap B)$$
$$\mathrm{P}(A \cap B) = 0.3$$

A DIAGRAM CAN NOW BE FILLED IN



b) I) $0.4 + 0.1 = \underline{0.5}$

II) $\underline{0.2}$

(BOTH FROM VENN DIAGRAM)

Question 3 (**)

The events A and B are such so that

$$\mathrm{P}(A)=0.48,\ \ \mathrm{P}(B)=0.38 \ \text{ and } \ \mathrm{P}(A\cap B)=0.28.$$

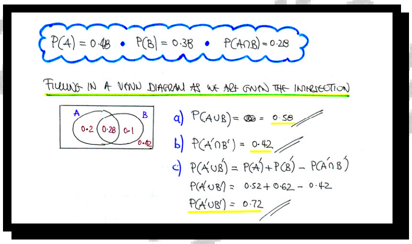
Determine the value of …

a) … $\mathrm{P}(A\cup B)$.

b) … $\mathrm{P}(A'\cap B')$.

c) … $\mathrm{P}(A'\cup B')$.

$\boxed{\mathrm{P}(A\cup B)=0.58}$, $\boxed{\mathrm{P}(A'\cap B')=0.42}$, $\boxed{\mathrm{P}(A'\cup B')=0.72}$



Question 4 (**)

The events A and B are such so that

$$\mathrm{P}(A' \cap B) = 0.35, \quad \mathrm{P}(A \cap B') = 0.25 \quad \text{and} \quad \mathrm{P}(A' \cap B') = 0.1.$$

a) Illustrate the above information in a fully completed Venn diagram.

b) Determine …

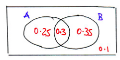
i. … $\mathrm{P}(A)$.

ii. … $\mathrm{P}(A' \cup B)$.

$\boxed{\mathrm{P}(A) = 0.55}$, $\boxed{\mathrm{P}(A' \cup B) = 0.75}$

$P(A' \cap B) = 0.35$ • $P(A \cap B') = 0.25$ • $P(A' \cap B') = 0.1$

a) FILLING IN THE VENN DIAGRAM AS WE HAVE ENOUGH DIRECT INFO



b) $P(A) = 0.25 + 0.3 = 0.55$

c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$
$P(A' \cup B) = 0.45 + 0.65 - 0.35$
$P(A' \cup B) = 0.75$

Question 5 (**)

The events A and B are such so that

$$\mathrm{P}(A)=0.45, \quad \mathrm{P}(A \cap B')=0.3 \quad \text{and} \quad \mathrm{P}(A \cup B)=0.8.$$

a) Illustrate the above information in a fully completed Venn diagram.

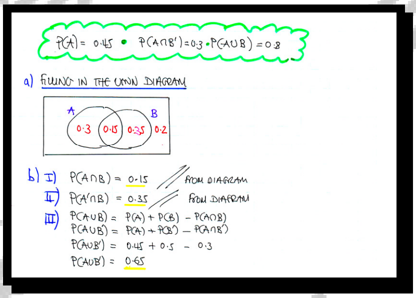
b) Determine …

i. … $\mathrm{P}(A \cap B)$.

ii. … $\mathrm{P}(A' \cap B)$.

iii. … $\mathrm{P}(A \cup B')$.

$\boxed{\mathrm{P}(A \cap B)=0.15}$, $\boxed{\mathrm{P}(A' \cap B)=0.35}$, $\boxed{\mathrm{P}(A \cup B')=0.65}$



Question 6 (**)

Of the families in a village, 70% have dogs and 45% have dogs and cats.

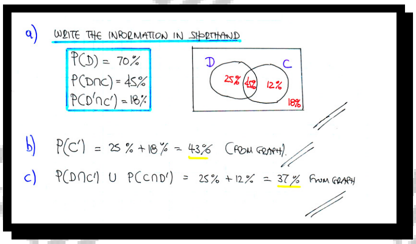
Of the families in this village, 18% have neither a dog, nor a cat.

a) Illustrate this information in a fully completed Venn diagram.

b) Find the percentage of families that do not have a cat.

c) Determine the probability that a family picked at random will own dogs or cats, but not both.

$\boxed{43\%}$, $\boxed{37\%}$



a) WRITE THE INFORMATION IN SHORTHAND

$P(D) = 70\%$
$P(D \cap C) = 45\%$
$P(D' \cap C') = 18\%$

b) $P(C') = 25\% + 18\% = 43\%$ (FROM GRAPH)

c) $P(D \cap C') \cup P(C \cap D') = 25\% + 12\% = 37\%$ FROM GRAPH

Question 7 (**+)

The events A and B are such so that

$$\mathrm{P}(A)=0.7, \quad \mathrm{P}(B')=0.8 \quad \text{and} \quad \mathrm{P}(A\cup B)=0.8.$$

a) Find $\mathrm{P}(A\cap B)$.

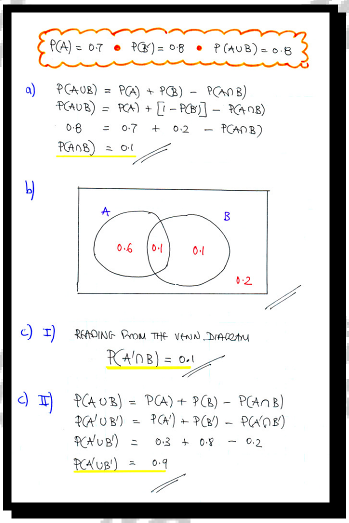
b) Illustrate the above information in a fully completed Venn diagram.

c) Determine …

i. … $\mathrm{P}(A'\cap B)$.

ii. … $\mathrm{P}(A'\cup B')$.

$\boxed{\mathrm{P}(A\cap B)=0.1}$, $\boxed{\mathrm{P}(A'\cap B)=0.1}$, $\boxed{\mathrm{P}(A'\cup B')=0.9}$



$\mathrm{P}(A) = 0.7$ • $\mathrm{P}(B') = 0.8$ • $\mathrm{P}(A\cup B) = 0.8$

a) $\mathrm{P}(A\cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A\cap B)$

$\mathrm{P}(A\cup B) = \mathrm{P}(A) + [1-\mathrm{P}(B')] - \mathrm{P}(A\cap B)$

$0.8 = 0.7 + 0.2 - \mathrm{P}(A\cap B)$

$\mathrm{P}(A\cap B) = 0.1$

b)

c) I) READING FROM THE VENN DIAGRAM

$\mathrm{P}(A'\cap B) = 0.1$

c) II) $\mathrm{P}(A\cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A\cap B)$

$\mathrm{P}(A'\cup B') = \mathrm{P}(A') + \mathrm{P}(B') - \mathrm{P}(A'\cap B')$

$\mathrm{P}(A'\cup B') = 0.3 + 0.8 - 0.2$

$\mathrm{P}(A'\cup B') = 0.9$

Question 8 (**+)

The events C and D are such so that

$$\mathrm{P}(C) = 0.4, \quad \mathrm{P}(D) = 0.5 \quad \text{and} \quad \mathrm{P}(C' \cap D') = 0.25.$$

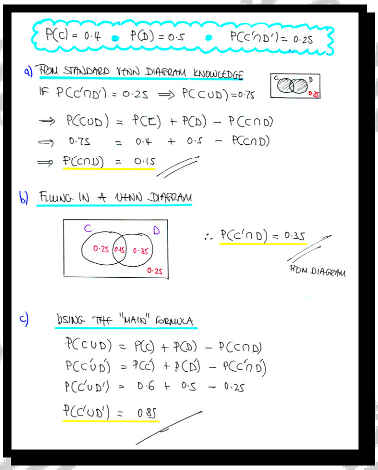
Determine the value of …

a) … $\mathrm{P}(C \cap D)$.

b) … $\mathrm{P}(C' \cap D)$.

c) … $\mathrm{P}(C' \cup D')$.

$$\boxed{\mathrm{P}(C \cap D) = 0.15}, \ \boxed{\mathrm{P}(C' \cap D) = 0.35}, \ \boxed{\mathrm{P}(C' \cup D') = 0.85}$$



$P(C) = 0.4$ • $P(D) = 0.5$ • $P(C' \cap D') = 0.25$

a) FROM STANDARD VENN DIAGRAM KNOWLEDGE

IF $P(C' \cap D') = 0.25 \Rightarrow P(C \cup D) = 0.75$

$\Rightarrow P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$\Rightarrow 0.75 = 0.4 + 0.5 - P(C \cap D)$

$\Rightarrow P(C \cap D) = 0.15$

b) FILLING IN A VENN DIAGRAM

$\therefore P(C' \cap D) = 0.35$

FROM DIAGRAM

c) USING THE "MAIN" FORMULA

$P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$P(C' \cup D') = P(C') + P(D') - P(C' \cap D')$

$P(C' \cup D') = 0.6 + 0.5 - 0.25$

$P(C' \cup D') = 0.85$

Question 9 (**+)

The events E and F satisfy

$$\mathrm{P}(E)=0.35,\ \mathrm{P}(F)=0.45 \text{ and } \mathrm{P}(E\cap F')=0.15.$$

a) By completing a suitably labelled Venn diagram, or otherwise, find $\mathrm{P}(E\cap F)$.

b) Determine …

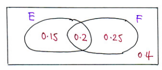
i. … $\mathrm{P}(E|F)$.

ii. … $\mathrm{P}(F|E')$.

$\boxed{\mathrm{P}(E\cap F)=0.2=\frac{1}{5}}$, $\boxed{\mathrm{P}(E|F)=\frac{4}{9}}$, $\boxed{\mathrm{P}(F|E')=\frac{5}{13}}$

$\mathrm{P}(E)=0.35$ • $\mathrm{P}(F)=0.45$ • $\mathrm{P}(E\cap F')=0.15$

a) THERE IS ENOUGH INFORMATION TO FILL IN A VENN DIAGRAM



$\therefore\ \mathrm{P}(E\cap F)=0.2$ // FROM DIAGRAM

b) I) $\mathrm{P}(E|F)=\frac{\mathrm{P}(E\cap F)}{\mathrm{P}(F)}=\frac{0.2}{0.45}=\frac{4}{9}$ //

II) $\mathrm{P}(F|E')=\frac{\mathrm{P}(F\cap E')}{\mathrm{P}(E')}=\frac{0.25}{0.65}=\frac{5}{13}$ //

Question 10 (**+)

The events C and D satisfy

$$\mathrm{P}(C \cap D') = 0.1, \quad \mathrm{P}(C' \cap D) = 0.15 \quad \text{and} \quad \mathrm{P}(C' \cap D') = 0.2.$$

a) Illustrate the above information in a fully completed Venn diagram.

b) Determine …

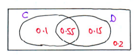
i. … $\mathrm{P}(C|D')$.

ii. … $\mathrm{P}(D'|C)$.

$\boxed{\mathrm{P}(C|D') = \frac{1}{3}}$, $\boxed{\mathrm{P}(D'|C) = \frac{2}{13}}$

$\mathrm{P}(C \cap D') = 0.1$ • $\mathrm{P}(C' \cap D) = 0.15$ • $\mathrm{P}(C' \cap D') = 0.2$

a) THERE IS ENOUGH INFORMATION TO FILL IN A VENN DIAGRAM



b) I) $\mathrm{P}(C|D') = \frac{\mathrm{P}(C \cap D')}{\mathrm{P}(D')} = \frac{0.1}{0.3} = \frac{1}{3}$

II) $\mathrm{P}(D'|C) = \frac{\mathrm{P}(D' \cap C)}{\mathrm{P}(C)} = \frac{0.1}{0.65} = \frac{2}{13}$

Question 11 (+)**

The events A and B are independent with

$$\mathrm{P}(A) = 0.3 \text{ and } \mathrm{P}(B) = 0.5 .$$

Determine …

a) … $\mathrm{P}(A \cup B)$.

b) … $\mathrm{P}(A' \cap B)$.

c) … $\mathrm{P}(B|A')$.

$\boxed{\mathrm{P}(A \cup B) = 0.65}$, $\boxed{\mathrm{P}(A' \cap B) = 0.35}$, $\boxed{\mathrm{P}(B|A') = \mathrm{P}(B) = 0.5}$

$\boxed{P(A) = 0.3 \;\bullet\; P(B) = 0.5 \;\bullet\; A \,\&\, B \text{ INDEPENDENT}}$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ INDEPENDENT EVENTS

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$P(A \cup B) = 0.3 + 0.5 - 0.3 \times 0.5$

$P(A \cup B) = 0.65$

b) AS THE EVENTS ARE INDEPENDENT

$P(A' \cap B) = P(A') \times P(B) = 0.7 \times 0.5 = 0.35$

c) AS THE EVENTS ARE INDEPENDENT

$P(B|\cancel{A'}) = P(B) = 0.5$

Question 12 (**+)

The quality controller in a factory examined 250 components for three types of minor faults, known as fault A, fault B and fault C.

His results are summarized as follows.

- 4 components with type C fault only.
- 8 components with type A and C but no B fault.
- 9 components with type A fault only.
- 7 components with type B and C but no A fault.
- 1 component with type B fault only.
- 5 components with type A and B but no C fault.
- 2 components with all three types of fault.

a) Draw a fully completed Venn diagram to represent this information.

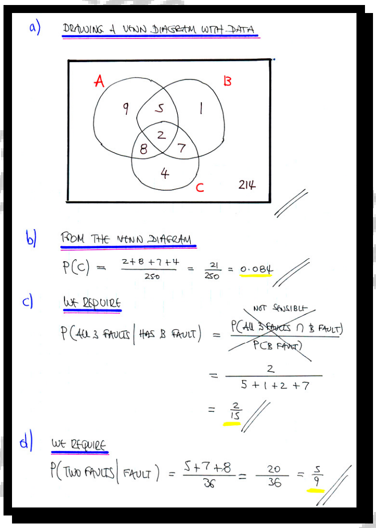
A component is selected at random from the quality controller's sample.

b) Find the probability that the selected component has a type C fault.

c) Given that the selected component has a type B fault, find the probability that the component has all three types of fault.

d) Given instead that the selected component has a fault, find the probability it has two faults.

$\boxed{0.084}$, $\boxed{\frac{2}{15}}$, $\boxed{\frac{5}{9}}$



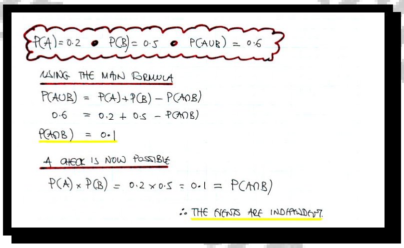
Question 13 (**+)

The events A and B are such so that

$$P(A) = 0.2, \quad P(B) = 0.5 \quad \text{and} \quad P(A \cup B) = 0.6.$$

Determine whether A and B are independent events.

$$\boxed{P(A \cap B) = 0.1 = P(A)P(B) \Leftrightarrow \text{independent}}$$



Question 14 (**+)

The following information is known about the families in a certain village.

The probability of a family owning cats only is 0.1, while the probability of owning dogs only is 0.15.

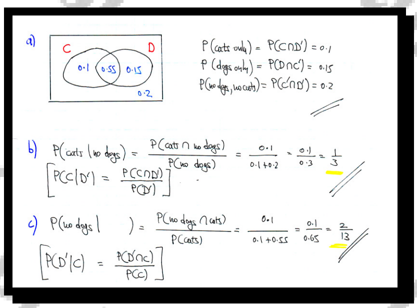
The probability of family owning no dogs or cats is 0.2.

a) Illustrate the above information in a fully completed Venn diagram.

b) Given a family owns no dogs determine the probability they own cats.

c) Given a family owns cats determine the probability they own no dogs.

$$\boxed{P(C|D') = \frac{1}{3}}, \quad \boxed{P(D'|C) = \frac{2}{13}}$$



Question 15 (**+)

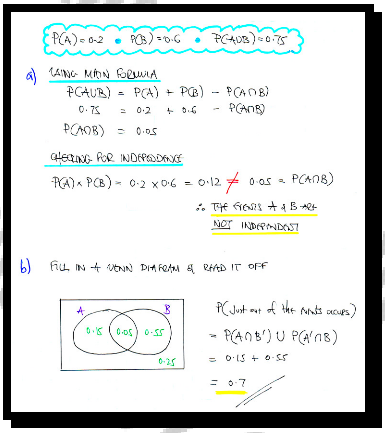
The events A and B are such so that

$$\mathrm{P}(A) = 0.2\,, \quad \mathrm{P}(B) = 0.6 \quad \text{and} \quad \mathrm{P}(A \cup B) = 0.75\,.$$

a) Determine by showing clear workings whether the events A and B are statistically independent.

b) Find the probability that just one of the two events occurs.

$\boxed{\text{not independent}}$, $\boxed{0.7}$



Question 16 (**+)

The events A and B are statistically independent and further satisfy

$$\mathrm{P}(A) = 0.4 \quad \text{and} \quad \mathrm{P}(A \cap B) = 0.12 .$$

Determine …

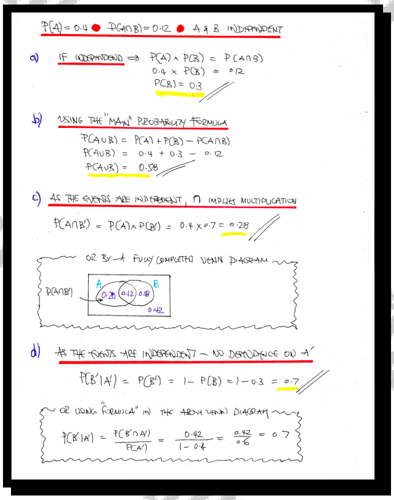
a) … $\mathrm{P}(B)$.

b) … $\mathrm{P}(A \cup B)$.

c) … $\mathrm{P}(A \cap B')$.

d) … $\mathrm{P}(B' | A')$.

$\boxed{}$, $\boxed{\mathrm{P}(B) = 0.3}$, $\boxed{\mathrm{P}(A \cup B) = 0.58}$, $\boxed{\mathrm{P}(A \cap B') = 0.28}$, $\boxed{\mathrm{P}(B' | A') = 0.7}$



P(A) = 0.4 • P(A∩B) = 0.12 • A & B INDEPENDENT

a) IF INDEPENDENT ⇒ $P(A) \times P(B) = P(A \cap B)$
$0.4 \times P(B) = 0.12$
$P(B) = 0.3$

b) USING THE "MAIN" PROBABILITY FORMULA
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cup B) = 0.4 + 0.3 - 0.12$
$P(A \cup B) = 0.58$

c) AS THE EVENTS ARE INDEPENDENT, ∩ IMPLIES MULTIPLICATION
$P(A \cap B') = P(A) \times P(B') = 0.4 \times 0.7 = 0.28$

OR BY A FULLY COMPLETED VENN DIAGRAM

d) AS THE EVENTS ARE INDEPENDENT – NO DEPENDENCE ON A'
$P(B' | A') = P(B') = 1 - P(B) = 1 - 0.3 = 0.7$

OR USING "FORMULA" IN THE ABOVE VENN DIAGRAM
$P(B' | A') = \frac{P(B' \cap A')}{P(A')} = \frac{0.42}{1 - 0.4} = \frac{0.42}{0.6} = 0.7$

Question 17 (**+)

The results of 100 people taking part in a wine tasting survey are shown below.

- 90 people liked wine A.
- 90 people liked wine B.
- 92 people liked wine C.
- 88 people liked wine A and B.
- 86 people liked wine B and C.
- 87 people liked wine A and C.
- 85 people liked wine A, B and C.

a) Draw a fully completed Venn diagram to represent this data.

Find the probability that a randomly chosen person …

b) … likes only two out of the three wines.

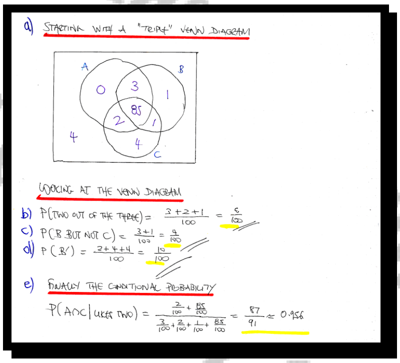
c) … likes wine B but not wine C.

d) … does not like wine B.

A person who likes at least two types of wine is selected.

e) Determine the probability that this person likes wines A and C.

$\boxed{}$, $\boxed{\frac{6}{100}}$, $\boxed{\frac{4}{100}}$, $\boxed{\frac{10}{100}}$, $\boxed{\frac{87}{91}}$



Question 18 (**+)

The events A and B satisfy

$$\text{P}(A) = 0.7 \quad \text{and} \quad \text{P}(B) = 0.3 .$$

Given that A and B are statistically independent, determine …

a) … $\text{P}(A \cup B)$.

b) … $\text{P}(A \cup B')$.

$\boxed{\text{P}(A \cup B) = 0.79}$, $\boxed{\text{P}(A \cup B') = 0.91}$

P(A) = 0.7 • P(B) = 0.3 • A & B ARE INDEPENDENT

a) USING MAIN PROBABILITY FORMULA

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$ ← INDEPENDENT EVENTS

$P(A \cup B) = 0.7 + 0.3 - 0.7 \times 0.3$

$P(A \cup B) = 0.79$

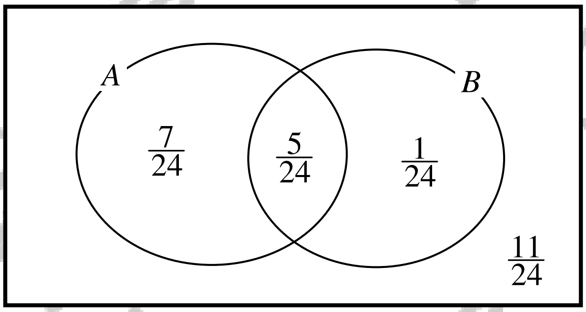
b) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ ← INDEPENDENT EVENTS

$P(A \cup B') = P(A) + P(B') - P(A)P(B')$

$P(A \cup B') = 0.7 + 0.7 - 0.7 \times 0.7$

$P(A \cup B') = 0.91$

Question 19 (**+)



The figure above shows the probability sample space for two events A and B, summarized by a Venn diagram.

Determine the following probabilities.

a) At least one of the events A, B occurs.

b) At most one of the events A, B occurs.

c) Only event B occurs.

d) Exactly one of the events A, B occurs.

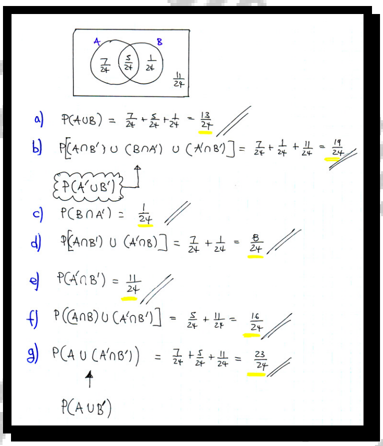
e) None of the events A, B occurs.

f) Either both events occur or neither event occurs.

g) Either event A occurs or neither event occurs.

The word "or" in this question only implies one or the other but not both

$\boxed{}$, $\boxed{\frac{13}{24}}$, $\boxed{\frac{19}{24}}$, $\boxed{\frac{1}{24}}$, $\boxed{\frac{8}{24}=\frac{1}{3}}$, $\boxed{\frac{11}{24}}$, $\boxed{\frac{16}{24}=\frac{2}{3}}$, $\boxed{\frac{23}{24}}$



a) $P(A \cup B) = \frac{7}{24} + \frac{5}{24} + \frac{1}{24} = \frac{13}{24}$

b) $P[(A \cap B') \cup (B \cap A') \cup (A' \cap B')] = \frac{7}{24} + \frac{1}{24} + \frac{11}{24} = \frac{19}{24}$

$P(A' \cup B')$

c) $P(B \cap A') = \frac{1}{24}$

d) $P[(A \cap B') \cup (A' \cap B)] = \frac{7}{24} + \frac{1}{24} = \frac{8}{24}$

e) $P(A' \cap B') = \frac{11}{24}$

f) $P[(A \cap B) \cup (A' \cap B')] = \frac{5}{24} + \frac{11}{24} = \frac{16}{24}$

g) $P(A \cup (A' \cap B')) = \frac{7}{24} + \frac{5}{24} + \frac{11}{24} = \frac{23}{24}$

$P(A \cup B')$

Question 20 **(***)**

The events A and B satisfy

$$\mathrm{P}(A) = 0.5, \ \mathrm{P}(B) = 0.2 \ \text{ and } \ \mathrm{P}(A|B) = 0.3.$$

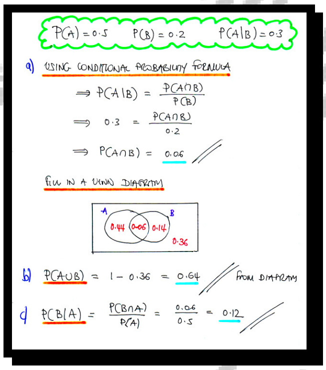
Determine …

a) … $\mathrm{P}(A \cap B)$.

b) … $\mathrm{P}(A \cup B)$.

c) … $\mathrm{P}(B|A)$.

$\boxed{}$, $\boxed{\mathrm{P}(A \cap B) = 0.06}$, $\boxed{\mathrm{P}(A \cup B) = 0.64}$, $\boxed{\mathrm{P}(B|A) = 0.12}$



Question 21 (***)

The events A and B satisfy

$$\mathrm{P}(A)=0.6,\ \mathrm{P}(B)=0.52 \ \text{ and } \ \mathrm{P}(A\cup B)=0.88.$$

a) Find the value of $\mathrm{P}(A\cap B)$ and hence illustrate this probability information in a fully completed Venn diagram.

b) Determine …

i. … $\mathrm{P}(B|A)$.

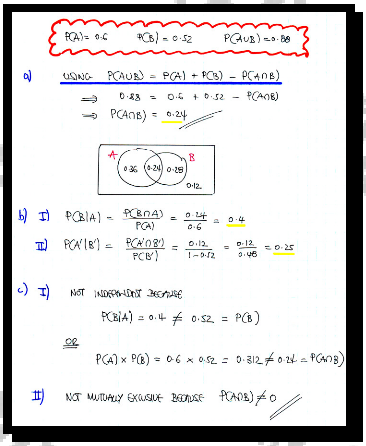
ii. … $\mathrm{P}(A'|B')$.

c) State, giving a reason, whether A and B are …

i. … statistically independent.

ii. … mutually exclusive.

$\boxed{}$, $\boxed{\mathrm{P}(A\cap B)=0.24}$, $\boxed{\mathrm{P}(B|A)=0.4}$, $\boxed{\mathrm{P}(A'|B')=0.25}$



P(A) = 0.6   P(B) = 0.52   P(A∪B) = 0.88

a) USING $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow 0.88 = 0.6 + 0.52 - P(A\cap B)$

$\Rightarrow P(A\cap B) = 0.24$

b) I) $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{0.24}{0.6} = 0.4$

II) $P(A'|B') = \frac{P(A'\cap B')}{P(B')} = \frac{0.12}{1-0.52} = \frac{0.12}{0.48} = 0.25$

c) I) NOT INDEPENDENT BECAUSE

$P(B|A) = 0.4 \neq 0.52 = P(B)$

OR

$P(A)\times P(B) = 0.6\times 0.52 = 0.312 \neq 0.24 = P(A\cap B)$

II) NOT MUTUALLY EXCLUSIVE BECAUSE $P(A\cap B)\neq 0$

Question 22 **(***)**

The events A and B are such so that

$$\mathrm{P}(A)=0.2\,,\quad \mathrm{P}(B)=0.6 \quad \text{and} \quad \mathrm{P}(A'\cap B')=0.25\,.$$

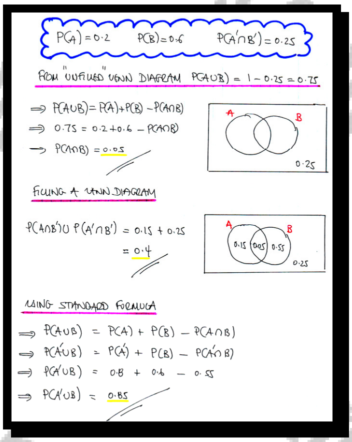
Determine …

a) … $\mathrm{P}(A\cap B)$.

b) … $\mathrm{P}(A\cap B')\cup \mathrm{P}(A'\cap B')$.

c) … $\mathrm{P}(A'\cup B)$

$\boxed{}$, $\boxed{\mathrm{P}(A\cap B)=0.05}$, $\boxed{\mathrm{P}(A\cap B')\cup \mathrm{P}(A'\cap B')=0.4}$, $\boxed{\mathrm{P}(A'\cup B)=0.85}$



$\mathrm{P}(A)=0.2 \qquad \mathrm{P}(B)=0.6 \qquad \mathrm{P}(A'\cap B')=0.25$

FROM "UNFILLED" VENN DIAGRAM $\mathrm{P}(A\cup B) = 1-0.25 = 0.75$

$\Rightarrow \mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$

$\Rightarrow 0.75 = 0.2+0.6-\mathrm{P}(A\cap B)$

$\rightarrow \mathrm{P}(A\cap B) = 0.05$

FILLING A VENN DIAGRAM

$\mathrm{P}(A\cap B')\cup \mathrm{P}(A'\cap B') = 0.15+0.25$

$= 0.4$

USING STANDARD FORMULA

$\Rightarrow \mathrm{P}(A\cup B) = \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$

$\Rightarrow \mathrm{P}(A'\cup B) = \mathrm{P}(A')+\mathrm{P}(B)-\mathrm{P}(A'\cap B)$

$\Rightarrow \mathrm{P}(A'\cup B) = 0.8+0.6-0.55$

$\Rightarrow \mathrm{P}(A'\cup B) = 0.85$

Question 23 (***)

The events A and B satisfy

$$\mathrm{P}(A)=0.4 \quad \text{and} \quad \mathrm{P}(A \cup B)=0.79 .$$

Determine $\mathrm{P}(B)$ in each of the two following cases.

a) If A and B are mutually exclusive.

b) If A and B are independent.

$\boxed{\text{MMS}}$, $\boxed{0.39}$, $\boxed{0.65}$

a) IF THE EVENTS ARE MUTUALLY EXCLUSIVE $P(A \cap B)=0$

$\Rightarrow P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}$

$\Rightarrow 0.79 = 0.4 + P(B)$

$\Rightarrow P(B) = 0.39$

b) IF THE EVENTS ARE INDEPENDENT $P(A \cap B) = P(A) \times P(B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\Rightarrow 0.79 = 0.4 + P(B) - 0.4P(B)$

$\Rightarrow 0.39 = 0.6P(B)$

$\Rightarrow P(B) = 0.65$

Question 24 (***)

The events E and F are such so that

$$\mathrm{P}(E)=0.5, \quad \mathrm{P}(F|E)=0.6, \quad \mathrm{P}(E' \cap F')=0.4.$$

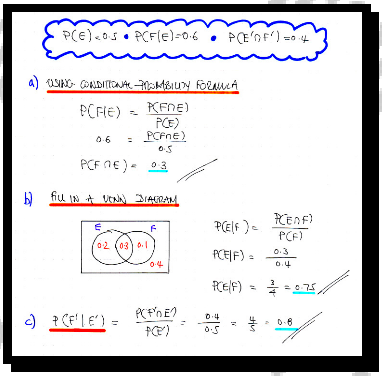
Determine the value of …

a) … $\mathrm{P}(E \cap F)$.

b) … $\mathrm{P}(E|F)$.

c) … $\mathrm{P}(F'|E')$.

$\square$, $\boxed{\mathrm{P}(E \cap F)=0.3}$, $\boxed{\mathrm{P}(E|F)=0.75}$, $\boxed{\mathrm{P}(F'|E')=0.8}$



$P(E)=0.5$ • $P(F|E)=0.6$ • $P(E' \cap F')=0.4$

a) USING CONDITIONAL PROBABILITY FORMULA

$$P(F|E) = \frac{P(F \cap E)}{P(E)}$$
$$0.6 = \frac{P(F \cap E)}{0.5}$$
$$P(F \cap E) = 0.3$$

b) FILL IN A VENN DIAGRAM

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$
$$P(E|F) = \frac{0.3}{0.4}$$
$$P(E|F) = \frac{3}{4} = 0.75$$

c) $P(F'|E') = \dfrac{P(F' \cap E')}{P(E')} = \dfrac{0.4}{0.5} = \dfrac{4}{5} = 0.8$

Question 25 (***)

The events A and B are such so that

$$\mathrm{P}(B) = 0.35, \qquad \mathrm{P}(A|B) = 0.1, \qquad \mathrm{P}(A \cap B') = 0.15.$$

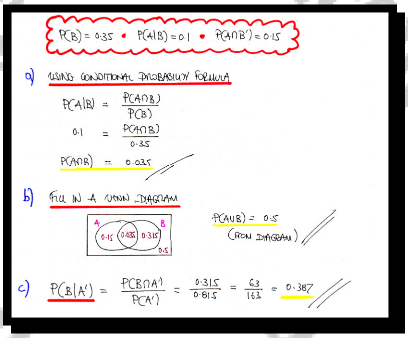
Determine …

a) … $\mathrm{P}(A \cap B)$.

b) … $\mathrm{P}(A \cup B)$.

c) … $\mathrm{P}(B|A')$.

$\boxed{\mathrm{P}(A \cap B) = 0.035}$, $\boxed{\mathrm{P}(A \cup B) = 0.5}$, $\boxed{\mathrm{P}(B|A') = \frac{63}{163} \approx 0.387}$



$\mathrm{P}(B) = 0.35$ • $\mathrm{P}(A|B) = 0.1$ • $\mathrm{P}(A \cap B') = 0.15$

a) USING CONDITIONAL PROBABILITY FORMULA

$$\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$
$$0.1 = \frac{\mathrm{P}(A \cap B)}{0.35}$$
$$\mathrm{P}(A \cap B) = 0.035$$

b) FILL IN A VENN DIAGRAM

$\mathrm{P}(A \cup B) = 0.5$

(FROM DIAGRAM)

c) $\mathrm{P}(B|A') = \frac{\mathrm{P}(B \cap A')}{\mathrm{P}(A')} = \frac{0.315}{0.815} = \frac{63}{163} = 0.387$

Question 26 (***)

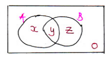
The events A and B are such so that

$$\mathrm{P}(A) = 0.72\,, \qquad \mathrm{P}(B) = 0.56\,.$$

Given that the events are exhaustive, determine $\mathrm{P}(A|B)$.

$\square$, $\boxed{\mathrm{P}(A|B) = \frac{1}{2}}$

EXHAUSTIVE $\Rightarrow$ $P(A' \cap B') = 0$



① $x + y = 0.72$
② $y + z = 0.56$
③ $x + y + z = 1$

SUB ① INTO ③ $\Rightarrow 0.72 + z = 1$
$\Rightarrow z = 0.28$
$\Rightarrow y = 0.28$
$\Rightarrow (x = 0.44)$
$\Rightarrow P(A \cap B) = 0.28$

USING CONDITIONAL PROBABILITY FORMULA

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.28}{0.56} = 0.5 \;//$$

ALTERNATIVE TO FIND THE VALUE OF $P(A \cap B)$

- EXHAUSTIVE $\Rightarrow P(A' \cap B') = 0$
  $\Rightarrow P(A \cup B) = 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  $1 = 0.72 + 0.56 - P(A \cap B)$
  $P(A \cap B) = 0.72 + 0.56 - 1$
  $P(A \cap B) = 0.28$

Question 27 (***)

The events A and B satisfy

$$\mathrm{P}(A)=0.3, \quad \mathrm{P}(B)=0.2, \quad \mathrm{P}(A \cup B)=0.4.$$

a) Determine $\mathrm{P}(A \cap B)$.

b) Illustrate the events A and B in a fully completed Venn diagram.

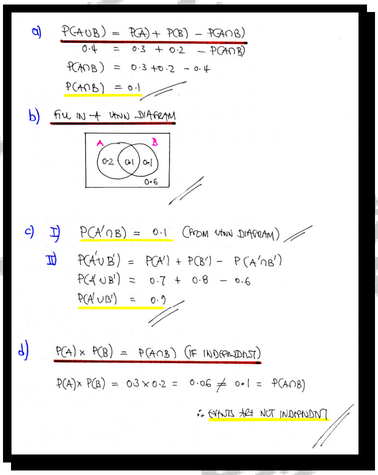
c) Find …

i. … $\mathrm{P}(A' \cap B)$.

ii. … $\mathrm{P}(A' \cup B')$

d) Determine, with a reason, whether or not the events A and B are statistically independent.

$\boxed{}$, $\boxed{\mathrm{P}(A \cap B)=0.1}$, $\boxed{\mathrm{P}(A' \cap B)=0.1}$, $\boxed{\mathrm{P}(A' \cup B')=0.9}$, $\boxed{\text{not independent}}$


a) P(A∪B) = P(A) + P(B) − P(A∩B)
0.4 = 0.3 + 0.2 − P(A∩B)
P(A∩B) = 0.3 + 0.2 − 0.4
P(A∩B) = 0.1

b) FILL IN A VENN DIAGRAM

c) I) P(A′∩B) = 0.1 (FROM VENN DIAGRAM)
II) P(A′∪B′) = P(A′) + P(B′) − P(A′∩B′)
P(A′∪B′) = 0.7 + 0.8 − 0.6
P(A′∪B′) = 0.9

d) P(A) × P(B) = P(A∩B) (IF INDEPENDENT)
P(A) × P(B) = 0.3 × 0.2 = 0.06 ≠ 0.1 = P(A∩B)
∴ EVENTS ARE NOT INDEPENDENT

Question 28 (***)

A total of 60 students went on long coach trip on which the coach stopped at two separate motorway service stations.

Of those students 28 bought refreshments at the first service station, 42 bought refreshments at the second service station and 24 bought refreshments at both service stations.

A student that went on this trip is selected at random.

a) Determine the probability that the student …

i. … bought refreshments at least once.

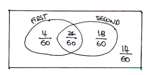
ii. … bought refreshments once.

b) If the selected student did **not** buy refreshments twice, find the probability (s)he did not buy refreshments at all.

$\boxed{}$, $\boxed{\dfrac{23}{30}}$, $\boxed{\dfrac{11}{30}}$, $\boxed{\dfrac{7}{18}}$

USING A VENN DIAGRAM

$P(\text{FIRST}) = \frac{28}{60}$, $P(\text{SECOND}) = \frac{42}{60}$ $P(\text{FIRST} \cap \text{SECOND}) = \frac{24}{60}$



a) I) $P(\text{AT LEAST ONCE}) = P(\text{FIRST} \cup \text{SECOND}) = \frac{4}{60} + \frac{24}{60} + \frac{18}{60} = \frac{46}{60} = \frac{23}{30}$

II) $P(\text{ONCE}) = \frac{4}{60} + \frac{18}{60} = \frac{22}{60} = \frac{11}{30}$

b) $P(\text{NO REFRESHMENTS} \mid \text{NOT BUY TWICE}) = \dfrac{\frac{14}{60}}{\frac{4}{60} + \frac{18}{60} + \frac{14}{60}} = \dfrac{14}{4+18+14}$

$= \frac{14}{36} = \frac{7}{18}$

Question 29 (***)

- The probability that one of the members of the W.M.T. Club reads the Economist is 0.65.
- The probability that one of the members of the W.M.T. Club reads the Financial Times is 0.45.
- The probability that one of the members of the W.M.T Club only reads the Economist is 0.4.

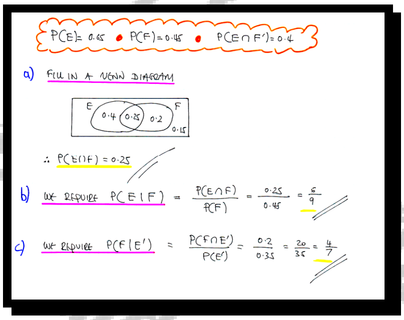
A member of the W.M.T. Club is selected at random.

a) Determine the probability that this member reads the Economist and the Financial Times.

b) If this member reads the Financial Times determine the probability that he also reads the Economist.

c) Given instead that this member does not read the Economist, determine the probability he reads the Financial Times.

$\square$, $\boxed{\mathrm{P}(E \cap F) = 0.25 = \frac{1}{4}}$, $\boxed{\mathrm{P}(E|F) = \frac{5}{9}}$, $\boxed{\mathrm{P}(F|E') = \frac{4}{7}}$



Question 30 (***+)

The events A and B are such so that

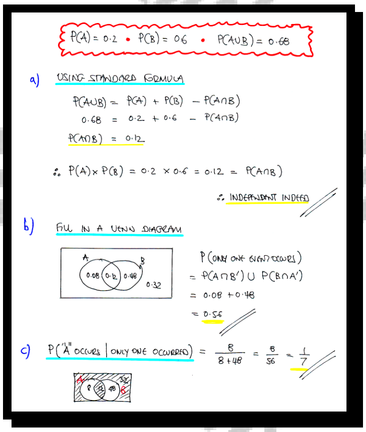
$$\mathrm{P}(A) = 0.2, \qquad \mathrm{P}(B) = 0.6, \qquad \mathrm{P}(A \cup B) = 0.68.$$

a) Determine whether the events A and B are statistically independent.

b) Find the probability that exactly one of the two events A and B occurs.

c) Find the probability that the event A occurs, given that exactly one of the two events A and B has already occurred.

$\boxed{}$, $\boxed{\text{independent}}$, $\boxed{0.56}$, $\boxed{\frac{1}{7}}$



Question 31 (***+)

The probability that James wears a tie at work is 0.3. The probability he wears a jacket at work is 0.5. If James wears a jacket at work the probability he wears a tie is 0.4.

a) Find the probability that, on a randomly selected day, James wears at work …

i. … a jacket and a tie.

ii. … no jacket or no tie (or neither).

On a given day James did not wear a tie at work.

b) Find the probability that he was wearing a jacket on that day.

$\boxed{}$, $\boxed{0.2}$, $\boxed{0.8}$, $\boxed{\frac{3}{7}}$

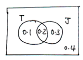
$P(T) = 0.3 \quad \bullet \quad P(J) = 0.5 \quad \bullet \quad P(T|J) = 0.4$

a) I) USING CONDITIONAL FORMULA

$$P(T|J) = \frac{P(T \cap J)}{P(J)}$$

$$0.4 = \frac{P(T \cap J)}{0.5}$$

$$P(T \cap J) = 0.2$$



II) USING THE MAIN PROBABILITY FORMULA

$$\Rightarrow P(T \cup J) = P(T) + P(J) - P(T \cap J)$$

$$\Rightarrow P(T' \cup J') = P(T') + P(J') - P(T' \cap J')$$

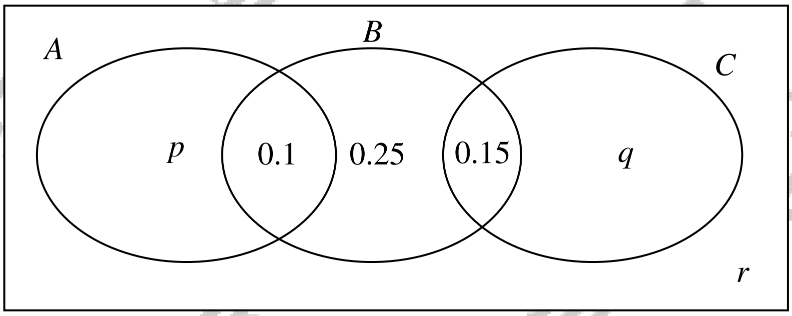
$$\Rightarrow P(T' \cup J') = 0.7 + 0.5 - 0.4$$

$$\Rightarrow P(T' \cup J') = 0.8$$

b) WE REQUIRE $P(J|T')$

$$\Rightarrow P(J|T') = \frac{P(J \cap T')}{P(T')} = \frac{0.3}{0.7} = \frac{3}{7}$$

Question 32 (***+)



The Venn diagram in the figure above shows the probabilities associated with three events A, B and C.

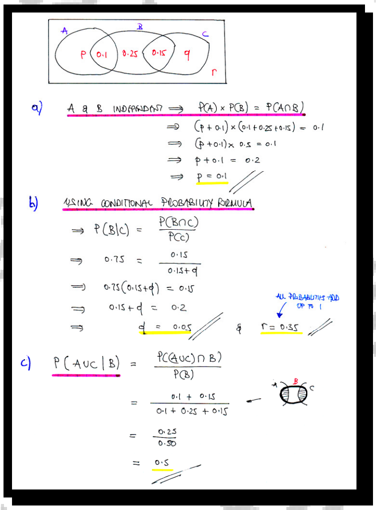
Some of the probabilities in the Venn diagram are given in terms of the constants p, q and r.

a) Given that the events A and B are independent, calculate the value of p.

b) Given further that $\mathrm{P}(B|C) = 0.75$, find the value of q and the value of r.

c) Determine $\mathrm{P}(A \cup C|B)$.

$\boxed{}$, $\boxed{p = 0.1}$, $\boxed{q = 0.05}$, $\boxed{r = 0.35}$, $\boxed{\mathrm{P}(A \cup C|B) = 0.5}$



a) A & B INDEPENDENT $\Rightarrow$ $P(A) \times P(B) = P(A \cap B)$

$\Rightarrow (p + 0.1) \times (0.1 + 0.25 + 0.15) = 0.1$

$\Rightarrow (p + 0.1) \times 0.5 = 0.1$

$\Rightarrow p + 0.1 = 0.2$

$\Rightarrow p = 0.1$

b) USING CONDITIONAL PROBABILITY FORMULA

$\Rightarrow P(B|C) = \dfrac{P(B \cap C)}{P(C)}$

$\Rightarrow 0.75 = \dfrac{0.15}{0.15 + q}$

$\Rightarrow 0.75(0.15 + q) = 0.15$

$\Rightarrow 0.15 + q = 0.2$

$\Rightarrow q = 0.05$ & $r = 0.35$

ALL PROBABILITIES ADD UP TO 1

c) $P(A \cup C | B) = \dfrac{P((A \cup C) \cap B)}{P(B)}$

$= \dfrac{0.1 + 0.15}{0.1 + 0.25 + 0.15}$

$= \dfrac{0.25}{0.50}$

$= 0.5$

Question 33 (***+)

The events A and B are independent and further satisfy

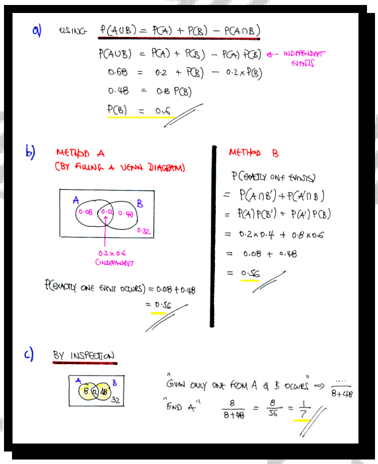
$$\mathrm{P}(A) = 0.2 \qquad \text{and} \qquad \mathrm{P}(A \cup B) = 0.68 .$$

a) Determine $\mathrm{P}(B)$.

b) Find the probability that exactly one of the two events occur.

c) Given that exactly one of the two events occur, find the probability that event A occurs.

$\boxed{}$, $\boxed{\mathrm{P}(B) = 0.6}$, $\boxed{0.56}$, $\boxed{\frac{1}{7}}$


a) USING $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\cup B) = P(A) + P(B) - P(A)P(B)$ ← INDEPENDENT EVENTS

$0.68 = 0.2 + P(B) - 0.2\times P(B)$

$0.48 = 0.8\,P(B)$

$P(B) = 0.6$

b) METHOD A
(BY FILLING A VENN DIAGRAM)

P(EXACTLY ONE EVENT OCCURS) $= 0.08 + 0.48$

$= 0.56$

METHOD B

P(EXACTLY ONE EVENTS)

$= P(A\cap B') + P(A'\cap B)$

$= P(A)P(B') + P(A')P(B)$

$= 0.2\times 0.4 + 0.8\times 0.6$

$= 0.08 + 0.48$

$= 0.56$

c) BY INSPECTION

"GIVEN ONLY ONE FROM A & B OCCURS" $\Rightarrow \dfrac{\cdots}{8+48}$

"FIND A" $\dfrac{8}{8+48} = \dfrac{8}{56} = \dfrac{1}{7}$

Question 34 (***+)

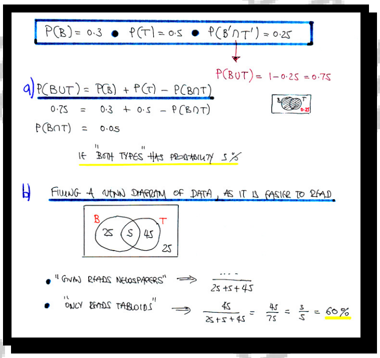
A survey on the reading habits of train commuters revealed that on a regular basis

- 30% read broadsheet newspapers.
- 50% read tabloid newspapers.
- 25% do not read newspapers.

a) Find the probability that a randomly selected commuter reads both broadsheet and tabloid newspapers.

b) Given that a commuter reads newspapers on a regular basis, determine the probability that he only reads tabloid newspapers.

$\boxed{}$, $\boxed{5\%}$, $\boxed{60\%}$



Question 35 (***+)

The events A and B satisfy

$$P(A) = 0.2, \qquad P(B) = 0.7, \qquad P(A' \cap B') = 0.25.$$

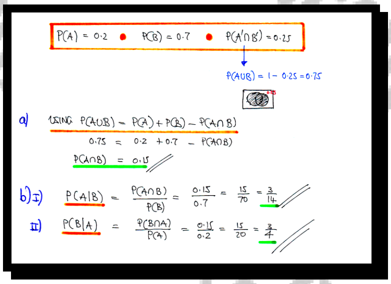
a) Determine $P(A \cap B)$.

b) Find the value of …

i. … $P(A|B)$.

ii. … $P(B|A)$

$\boxed{}$, $\boxed{P(A \cap B) = 0.15}$, $\boxed{P(A|B) = \frac{3}{14}}$, $\boxed{P(B|A) = \frac{3}{4}}$



$P(A) = 0.2$ • $P(B) = 0.7$ • $P(A' \cap B') = 0.25$

$P(A \cup B) = 1 - 0.25 = 0.75$

a) USING $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.75 = 0.2 + 0.7 - P(A \cap B)$

$P(A \cap B) = 0.15$

b) I) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70} = \frac{3}{14}$

II) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.15}{0.2} = \frac{15}{20} = \frac{3}{4}$

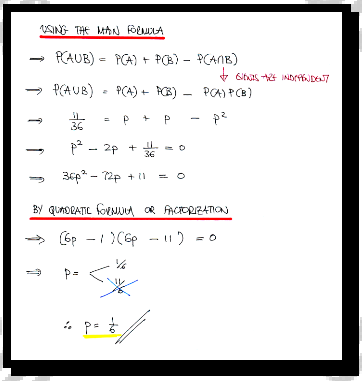
Question 36 (***+)

The events A and B satisfy

$$\mathrm{P}(A) = \mathrm{P}(B) = p \quad \text{and} \quad \mathrm{P}(A \cup B) = \frac{11}{36}.$$

Given that A and B are statistically independent, determine the value of p.

$\boxed{X}$, $\boxed{p = \frac{1}{6}}$



USING THE MAIN FORMULA

$\Rightarrow \mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

EVENTS ARE INDEPENDENT

$\Rightarrow \mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$

$\Rightarrow \frac{11}{36} = p + p - p^2$

$\Rightarrow p^2 - 2p + \frac{11}{36} = 0$

$\Rightarrow 36p^2 - 72p + 11 = 0$

BY QUADRATIC FORMULA OR FACTORIZATION

$\Rightarrow (6p - 1)(6p - 11) = 0$

$\Rightarrow p = \frac{1}{6}, \ \cancel{\frac{11}{6}}$

$\therefore p = \frac{1}{6}$

Question 37 (***+)

- The probability that one of the sixth-formers of the Oakwood Boys School studies Art is 0.12.
- The probability that one of the sixth-formers of the Oakwood Boys School studies Biology is 0.2.
- If one of the sixth-formers of the Oakwood Boys School studies Biology, the probability he studies Art is 0.2.

A sixth-former of the Oakwood Boys School is selected at random.

a) Find the probability that this student studies …

  i. … Art and Biology.

  ii. … Art or Biology.

b) Given that the selected student studies Art, determine the probability he also studies Biology.

$\boxed{}$, $\boxed{\mathrm{P}(A \cap B) = 0.04}$, $\boxed{\mathrm{P}(A \cup B) = 0.28}$, $\boxed{\mathrm{P}(B|A) = \frac{1}{3}}$

$P(A) = 0.12$ • $P(B) = 0.2$ • $P(A|B) = 0.2$

a) STARTING WITH THE CONDITIONAL PROBABILITY FORMULA

$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow 0.2 = \frac{P(A \cap B)}{0.2}$

$\Rightarrow P(A \cap B) = 0.04$

USING THE MAIN PROBABILITY FORMULA

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = 0.12 + 0.2 - 0.04$

$\Rightarrow P(A \cup B) = 0.28$

b) USING THE CONDITIONAL PROBABILITY FORMULA

$\Rightarrow P(B|A) = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow P(B|A) = \frac{0.04}{0.12}$

$\Rightarrow P(B|A) = \frac{1}{3}$

Question 38 (***+)

The probability that one of the members of the E.C. Club reads the Economist is 0.5.

The probability that one of the members of the E.C. Club does not read the Economist or the Financial Times is 0.2.

If a member of the E.C. Club reads the Economist, the probability he reads the Financial Times is 0.9.

A member of the E.C. Club is selected at random.

a) Determine the probability that this member reads the Economist and the Financial Times.

b) If the member selected reads the Financial Times determine the probability that he also reads the Economist.

c) Given instead that the member selected does not read the Economist, determine the probability he does not read the Financial Times either.

$\boxed{}$, $\boxed{P(E \cap F) = 0.45}$, $\boxed{P(E|F) = 0.6}$, $\boxed{P(F'|E') = 0.4}$

$\boxed{P(E) = 0.5 \quad \bullet \quad P(E' \cap F') = 0.2 \quad \bullet \quad P(F|E) = 0.9}$

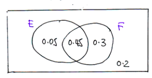
a) USING THE CONDITIONAL PROBABILITY FORMULA

$$P(F|E) = \frac{P(F \cap E)}{P(E)}$$

$$0.9 = \frac{P(F \cap E)}{0.5}$$

$$P(F \cap E) = 0.45$$

b) USING A VENN DIAGRAM TO FIND P(F)



$P(E) = 0.5$
$P(F) = 0.75$

USING THE CONDITIONAL PROBABILITY

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.45}{0.75} = \frac{45}{75} = 0.6$$

c) FINALLY

$$P(F'|E') = \frac{P(F' \cap E')}{P(E')} = \frac{0.2}{0.5} = \frac{2}{5} = 0.4$$

Question 39 (***+)

The events A and B are such so that

$$\mathrm{P}(A) = 0.6, \qquad \mathrm{P}(B) = 0.3, \qquad \mathrm{P}(A \cup B) = 0.72.$$

a) Illustrate this information in a fully completed Venn diagram.

b) Find the value of …

i. … $\mathrm{P}(A' \cap B')$.

ii. … $\mathrm{P}(A \cup B')$.

iii. … $\mathrm{P}(A | A \cup B)$.

c) Determine whether A and B are statistically independent.

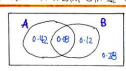
$\boxed{}$, $\boxed{\mathrm{P}(A' \cap B') = 0.28}$, $\boxed{\mathrm{P}(A \cup B') = 0.88}$, $\boxed{\mathrm{P}(A | A \cup B) = \frac{5}{6}}$, $\boxed{\text{independent}}$

a) USING $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.72 = 0.6 + 0.3 - P(A \cap B)$

$P(A \cap B) = 0.18$

HENCE A VENN DIAGRAM CAN BE COMPLETED



b) I) $P(A' \cap B') = 0.28$ (FROM DIAGRAM)

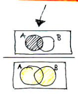
II) $P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$= 0.6 + 0.7 - 0.42$

$= 0.88$

III) $P(A | A \cup B) = \ldots$ WITHOUT A "FORMULA"

$= \dfrac{P(A)}{P(A \cup B)} = \dfrac{0.6}{0.72} = \dfrac{5}{6}$



c) USING THE STANDARD METHOD

$P(A) \times P(B) = 0.6 \times 0.3 = 0.18 = P(A \cap B)$

EVENTS ARE INDEPENDENT

Question 40 (***+)

In a particular day, the following information is given about the 125 customers of a ladies leather goods store.

- 25 customers did not buy a pair of shoes, nor a handbag, nor a belt.
- No customer bought a pair of shoes and a belt.
- 60 customers bought pairs of shoes.
- 20 customers bought belts.
- 24 customers bought a pair of shoes and a handbag.
- 6 customers bought a belt and a handbag.

One of these 125 customers is selected at random.

a) Draw a Venn diagram to represent the above information and hence determine the probability that this customer bought ...

  i. ... a pair of shoes.

  ii. ... a belt.

  iii. ... two different leather items.

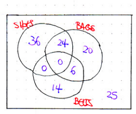
b) If a customer bought a handbag, find the probability she also bought shoes.

c) If a customer bought a handbag, find the probability she also bought a belt.

d) State, with justification, two types of leather purchases which are statistically independent.

$\boxed{}$, $\boxed{0.48}$, $\boxed{0.16}$, $\boxed{0.24}$, $\boxed{0.48}$, $\boxed{0.12}$

a) FILL IN A VENN DIAGRAM



- $P(\text{SHOES}) = \frac{60}{125} = 0.48$
- $P(\text{BELT}) = \frac{20}{125} = 0.16$

- $P(\text{EXACTLY TWO ITEMS}) = \frac{24+6}{125} = \frac{30}{125} = \frac{6}{25} = 0.24$

b) $P(\text{SHOES} \mid \text{BAG}) = \frac{24}{24+20+6} = \frac{24}{50} = \frac{12}{25} = 0.48$

c) $P(\text{BELT} \mid \text{BAG}) = \frac{6}{24+20+6} = \frac{6}{50} = \frac{3}{25} = 0.12$

d) AS $P(\text{SHOES} \mid \text{BAG}) = P(\text{SHOES}) = 0.48$, THE EVENTS "BUY SHOES" & "BUY BAG" ARE INDEPENDENT

ALTERNATIVE

$P(\text{SHOES} \cap \text{BAG}) = \frac{24}{125} = 0.192$

$P(\text{SHOES}) = \frac{60}{125} = 0.48$

$P(\text{BAG}) = \frac{50}{125} = 0.4$

$P(\text{SHOES}) \times P(\text{BAG}) = 0.48 \times 0.4 = 0.192 = P(\text{SHOES} \cap \text{BAG})$

$\therefore$ INDEPENDENT.

Question 41 (***+)

The events A and B are such so that

$$\mathrm{P}(A) = 0.15 \qquad \text{and} \qquad \mathrm{P}(B) = 0.4 .$$

a) Find $\mathrm{P}(A \cup B)$ if A and B are …

i. … mutually exclusive.

ii. … independent.

b) Given instead that the events A and B are neither mutually exclusive nor independent, determine …

i. … $\mathrm{P}(A|B)$ with $\mathrm{P}(A \cup B) = 0.45$.

ii. … the minimum value of $\mathrm{P}(A \cup B)$, fully justifying your answer.

$\square$, $\boxed{\mathrm{P}(A \cup B) = 0.55}$, $\boxed{\mathrm{P}(A \cup B) = 0.49}$, $\boxed{\mathrm{P}(A|B) = 0.25}$, $\boxed{0.4}$

a) I) MUTUALLY EXCLUSIVE $\Rightarrow$ $P(A \cap B) = 0$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = 0.15 + 0.4 - 0$

$\Rightarrow P(A \cup B) = 0.55$

II) INDEPENDENT $\Rightarrow$ $P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$\Rightarrow P(A \cup B) = 0.15 + 0.4 - 0.15 \times 0.4$

$\Rightarrow P(A \cup B) = 0.49$

b) I) CALCULATE $P(A \cap B)$ FIRST

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.45 = 0.15 + 0.4 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 0.1$

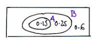
USING THE CONDITIONAL PROBABILITY FORMULA

$\Rightarrow P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.4} = 0.25$

II) LOOKING AT THE VENN DIAGRAM BELOW TOGETHER WITH THE STANDARD FORMULA

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

NEED TO MINIMIZE THIS; FIXED; FIXED; WE NEED TO MAKE THIS AS LARGE AS POSSIBLE



$\Rightarrow$ MIN VALUE OF $P(A \cup B) = 0.4$

Question 42 (****)

The events A and B are such so that

$$\mathrm{P}(A) = 0.3, \qquad \mathrm{P}(A \cap B') = 0.1, \qquad \mathrm{P}(A \cup B') = 0.55.$$

a) Find $\mathrm{P}(B)$.

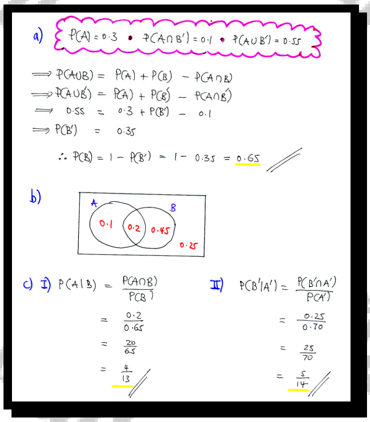
b) Illustrate the above information in a fully completed Venn diagram.

c) Determine …

i. … $\mathrm{P}(A|B)$.

ii. … $\mathrm{P}(B'|A')$.

$\boxed{}$, $\boxed{\mathrm{P}(B) = 0.65}$, $\boxed{\mathrm{P}(A|B) = \frac{4}{13}}$, $\boxed{\mathrm{P}(B'|A') = \frac{5}{14}}$



a) $P(A) = 0.3 \bullet P(A \cap B') = 0.1 \bullet P(A \cup B') = 0.55$

$\Longrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Longrightarrow P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$\Longrightarrow 0.55 = 0.3 + P(B') - 0.1$

$\Longrightarrow P(B') = 0.35$

$\therefore P(B) = 1 - P(B') = 1 - 0.35 = 0.65$

b)

c) I) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.65} = \frac{20}{65} = \frac{4}{13}$

II) $P(B'|A') = \frac{P(B' \cap A')}{P(A')} = \frac{0.25}{0.70} = \frac{25}{70} = \frac{5}{14}$

Question 43 (****)

The events A and B satisfy

$$\mathrm{P}(A)=\frac{3}{5}, \qquad \mathrm{P}(B)=\frac{5}{8}, \qquad \mathrm{P}(A\cup B)=\frac{7}{10}.$$

a) Illustrate this information in a fully completed Venn diagram.

b) Determine, showing all the relevant workings, whether A and B are statistically independent.

The events A and C satisfy

$$\mathrm{P}(A\cup C)=\frac{7}{10} \quad \text{and} \quad \mathrm{P}(C|A)=\frac{1}{3}.$$

c) Determine …

i. … $\mathrm{P}(A|C)$.

ii. … $\mathrm{P}\left[(A\cap C)'\right]$.

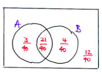
$\boxed{}$, $\boxed{\text{dependent}}$, $\boxed{\mathrm{P}(A|C)=\frac{2}{3}}$, $\boxed{\mathrm{P}\left[(A\cap C)'\right]=\frac{4}{5}}$

$\mathrm{P}(A)=\frac{3}{5}$ • $\mathrm{P}(B)=\frac{5}{8}$ • $\mathrm{P}(A\cup B)=\frac{7}{10}$

a) USING $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$

$\Rightarrow \frac{7}{10} = \frac{3}{5}+\frac{5}{8}-\mathrm{P}(A\cap B)$

$\Rightarrow \mathrm{P}(A\cap B) = \frac{21}{40}$

$\Rightarrow$ 

b) TO CHECK FOR INDEPENDENCE

$\mathrm{P}(A)\times\mathrm{P}(B) = \frac{3}{5}\times\frac{5}{8} = \frac{15}{40} \neq \frac{21}{40} = \mathrm{P}(A\cap B)$

$\therefore$ EVENTS ARE NOT INDEPENDENT

c) COLLECTING ALL INFORMATION FOR A & C

$\mathrm{P}(A)=\frac{3}{5}$ $\quad \mathrm{P}(A\cup C)=\frac{7}{10}$ $\quad \mathrm{P}(C|A)=\frac{1}{3}$

$\Rightarrow \mathrm{P}(C|A) = \frac{\mathrm{P}(C\cap A)}{\mathrm{P}(A)}$

$\Rightarrow \frac{1}{3} = \frac{\mathrm{P}(C\cap A)}{\frac{3}{5}}$

$\Rightarrow \mathrm{P}(C\cap A) = \frac{1}{5}$

USING $\mathrm{P}(A\cup C)=\mathrm{P}(A)+\mathrm{P}(C)-\mathrm{P}(A\cap C)$

$\Rightarrow \frac{7}{10} = \frac{3}{5}+\mathrm{P}(C)-\frac{1}{5}$

$\Rightarrow \mathrm{P}(C) = 0.3$

THUS $\mathrm{P}(A|C) = \frac{\mathrm{P}(A\cap C)}{\mathrm{P}(C)}$

$\Rightarrow \mathrm{P}(A|C) = \frac{1/5}{0.3}$

$\Rightarrow \mathrm{P}(A|C) = \frac{2}{3}$

II) AS $\mathrm{P}(A\cap C)=\frac{1}{5}$

$\mathrm{P}\left[(A\cap C)'\right] = 1-\frac{1}{5} = \frac{4}{5}$

Question 44 (****)

The events A and B are such so that

$$\mathrm{P}(A) = 0.7 \qquad \text{and} \qquad \mathrm{P}(B) = 0.5 .$$

a) Show mathematically that A and B cannot be mutually exclusive.

It is further given that $\mathrm{P}(A'|B') = 0.4$.

b) Determine $\mathrm{P}(A \cap B)$.

$\boxed{}$, $\boxed{\mathrm{P}(A \cap B) = 0.4}$

a) IF MUTUALLY EXCLUSIVE $P(A \cap B) = 0$

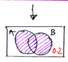
$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow P(A \cup B) = 0.7 + 0.5 - 0$$
$$\Rightarrow P(A \cup B) = 1.2 > 1$$

WHICH IS NOT POSSIBLE, SO A & B CANNOT BE MUTUALLY EXCLUSIVE

b) USING THE CONDITIONAL PROBABILITY FORMULA

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$\Rightarrow P(A'|B') = \frac{P(A' \cap B')}{P(B')}$$
$$\Rightarrow 0.4 = \frac{P(A' \cap B')}{1 - 0.5}$$
$$\Rightarrow P(A' \cap B') = 0.2$$

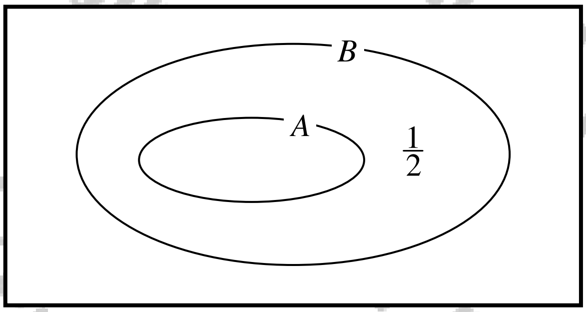
HENCE $P(A \cup B) = 1 - P(A' \cap B') = 1 - 0.2 = 0.8$



FINALLY WE HAVE

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.8 = 0.7 + 0.5 - P(A \cap B)$$
$$P(A \cap B) = 0.4$$

Question 45 (****)



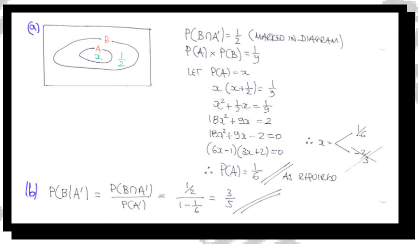
The Venn diagram above, shows the probability sample space for two events A and B, where

$$A \subseteq B \ , \quad \mathrm{P}(A) \times \mathrm{P}(B) = \frac{1}{9} \quad \text{and} \quad \mathrm{P}(B \cap A') = \frac{1}{2} .$$

a) Show clearly that $\mathrm{P}(A) = \frac{1}{6}$.

b) Determine $\mathrm{P}(B|A')$.

$$\boxed{\mathrm{P}(B|A') = \frac{3}{5}}$$



(a)

$P(B \cap A') = \frac{1}{2}$ (MARKED IN DIAGRAM)

$P(A) \times P(B) = \frac{1}{9}$

LET $P(A) = x$

$x\left(x + \frac{1}{2}\right) = \frac{1}{9}$

$x^2 + \frac{1}{2}x = \frac{1}{9}$

$18x^2 + 9x = 2$

$18x^2 + 9x - 2 = 0$

$(6x-1)(3x+2) = 0$

$\therefore x = \frac{1}{6}, \ \cancel{-\frac{2}{3}}$

$\therefore P(A) = \frac{1}{6}$ AS REQUIRED

(b) $P(B|A') = \frac{P(B \cap A')}{P(A')} = \frac{\frac{1}{2}}{1 - \frac{1}{6}} = \frac{3}{5}$

Question 46 (****)

The events A and B are such so that

$$P(A) = P(B), \quad P(A' \cap B') = \frac{17}{24}, \quad P(A' \cup B') = \frac{19}{24}.$$

Find the value of $P(A \cap B)$.

$\boxed{}$, $\boxed{P(A \cap B) = \frac{5}{24}}$

$P(A) = P(B)$ • $P(A' \cap B') = \frac{17}{24}$ • $P(A' \cup B') = \frac{19}{24}$

- USING $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$$
$$\Rightarrow \frac{19}{24} = P(A') + P(A') - \frac{17}{24}$$
$$\Rightarrow 2P(A') = \frac{36}{24}$$
$$\Rightarrow P(A') = \frac{18}{24} = \frac{3}{4}$$
$$\Rightarrow P(A) = \frac{1}{4} \quad \& \quad P(B) = \frac{1}{4}$$

- AS $P(A' \cap B') = \frac{17}{24} \Rightarrow P(A \cup B) = \frac{7}{24}$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\Rightarrow \frac{7}{24} = \frac{1}{4} + \frac{1}{4} - P(A \cap B)$$
$$\Rightarrow P(A \cap B) = \frac{5}{24}$$

Question 47 (****)

In the Southgate Academy Sixth Form the students are either left handed or right handed. The following information is also known.

- 0.6 of the students are female.
- 0.11 of the students are left handed.
- 0.10 of the female students are left handed.

a) Draw a fully completed Venn Diagram to display the above information.

A single student is selected at random from the Southgate Academy Sixth Form.

b) Determine the probability that the student is female and right handed.

c) Given that the student is left handed, determine the probability that the student is female.

d) Given that the student is male, determine the probability that the student is left handed.

$\boxed{}$, $\boxed{0.54}$, $\boxed{\frac{6}{11} \approx 0.545}$, $\boxed{\frac{1}{8} = 0.125}$

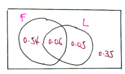
$P(\text{Female}) = 0.6$, $P(\text{Left handed}) = 0.11$, $P(\text{Left handed} \mid \text{Female}) = 0.1$

a) USING $P(L|F) = \frac{P(L \cap F)}{P(F)}$

$0.1 = \frac{P(L \cap F)}{0.6}$

$P(L \cap F) = 0.06$

THE VENN DIAGRAM CAN NOW BE COMPLETED



b) FROM PART (a) OR THE VENN DIAGRAM

$P(F \cap L') = 0.54$

c) $P(F|L) = \frac{P(F \cap L)}{P(L)} = \frac{0.06}{0.11} = \frac{6}{11} \simeq 0.545$

d) $P(L|F') = \frac{P(L \cap F')}{P(F')} = \frac{0.05}{0.40} = \frac{1}{8} = 0.125$

Question 48 (****)

The events A and B satisfy

$$\mathrm{P}(A) = 0.45, \qquad \mathrm{P}(A \cap B') = 0.25, \qquad \mathrm{P}(A \cup B) = 0.8.$$

a) Illustrate the above information in a fully completed Venn diagram.

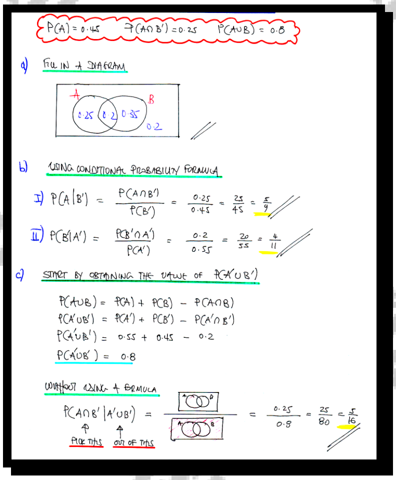
b) Determine …

i. … $\mathrm{P}(A|B')$.

ii. … $\mathrm{P}(B'|A')$

c) Find $\mathrm{P}(A \cap B' | A' \cup B')$.

$\boxed{\text{MS}}$, $\boxed{\mathrm{P}(A|B') = \frac{5}{9}}$, $\boxed{\mathrm{P}(B'|A') = \frac{4}{11}}$, $\boxed{\mathrm{P}(A \cap B'|A' \cup B') = \frac{5}{16}}$


$P(A) = 0.45 \quad P(A \cap B') = 0.25 \quad P(A \cup B) = 0.8$

a) FILL IN A DIAGRAM

b) USING CONDITIONAL PROBABILITY FORMULA

I) $P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{0.25}{0.45} = \frac{25}{45} = \frac{5}{9}$

II) $P(B'|A') = \frac{P(B' \cap A')}{P(A')} = \frac{0.2}{0.55} = \frac{20}{55} = \frac{4}{11}$

c) START BY OBTAINING THE VALUE OF $P(A' \cup B')$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$
$P(A' \cup B') = 0.55 + 0.45 - 0.2$
$P(A' \cup B') = 0.8$

WITHOUT USING A FORMULA

$P(A \cap B' | A' \cup B') = \frac{\square}{\square} = \frac{0.25}{0.8} = \frac{25}{80} = \frac{5}{16}$

PICK THIS OUT OF THIS

Question 49 (****)

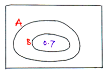
The events A and B satisfy

$$\mathrm{P}(A|B)=1, \qquad \mathrm{P}(A|B')=\frac{1}{4}, \qquad \mathrm{P}(B)=\frac{7}{10}.$$

Find the value of $\mathrm{P}(B'|A)$.

$\boxed{}$, $\boxed{\mathrm{P}(B'|A)=\frac{3}{31}}$

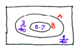
$\mathrm{P}(A|B)=1 \quad \bullet \quad \mathrm{P}(A|B')=\frac{1}{4} \quad \bullet \quad \mathrm{P}(B)=\frac{7}{10}$



$\mathrm{P}(A|B')=\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}$

$\Rightarrow \dfrac{1}{4}=\dfrac{\mathrm{P}(A\cap B')}{0.3}$

$\Rightarrow \mathrm{P}(A\cap B')=\dfrac{3}{40}$



FINALLY $\mathrm{P}(B'|A)=\dfrac{\mathrm{P}(B'\cap A)}{\mathrm{P}(A)}$

$=\dfrac{\frac{3}{40}}{\frac{3}{40}+0.7}$

$=\dfrac{\frac{3}{40}}{\frac{31}{40}}$

$=\dfrac{3}{31}$

Question 50 (****)

The events A and B are such so that

$$\mathrm{P}(A \cap B') = 0.25, \quad \mathrm{P}(A) = 2\mathrm{P}(B) \quad \text{and} \quad \mathrm{P}(A \cup B) = 0.45.$$

Determine …

a) … $\mathrm{P}(A \cap B)$.

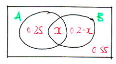
b) … $\mathrm{P}(A \cup B')$.

$\boxed{\mathrm{P}(A \cap B) = 0.15}$, $\boxed{\mathrm{P}(A \cup B') = 0.95}$

$\mathrm{P}(A \cap B') = 0.25$ $\quad$ $\mathrm{P}(A) = 2\mathrm{P}(B)$ $\quad$ $\mathrm{P}(A \cup B) = 0.45$

$\mathrm{P}(A' \cap B') = 1 - 0.45$
$\mathrm{P}(A' \cap B') = 0.55$

FILL IN A VENN DIAGRAM (PARTLY)



LET $\mathrm{P}(A \cap B) = x$

$\Rightarrow \mathrm{P}(B \cap A') = 1 - 0.25 - 0.55 - x$
$\Rightarrow \mathrm{P}(B \cap A') = 0.2 - x$

USING $\mathrm{P}(A) = 2\mathrm{P}(B)$

$\Rightarrow 0.25 + x = 2[x + 0.2 - x]$
$\Rightarrow 0.25 + x = 0.4$
$\Rightarrow x = 0.15$

$\therefore \mathrm{P}(A \cap B) = 0.15$ //

USING THE "STANDARD FORMULA

$\Rightarrow \mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
$\Rightarrow \mathrm{P}(A \cup B') = \mathrm{P}(A) + \mathrm{P}(B') - \mathrm{P}(A \cap B')$
$\Rightarrow \mathrm{P}(A \cup B') = (0.25 + x) + (\cancel{0.25} + 0.55) - \cancel{0.25}$
$\Rightarrow \mathrm{P}(A \cup B') = 0.25 + 0.55 + x$
$\Rightarrow \mathrm{P}(A \cup B') = 0.95$ //

Question 51 (****)

The events A and B are such so that

$$\mathrm{P}(A \cup B) = \tfrac{3}{5} \qquad \text{and} \qquad \mathrm{P}(A) = \tfrac{2}{5}.$$

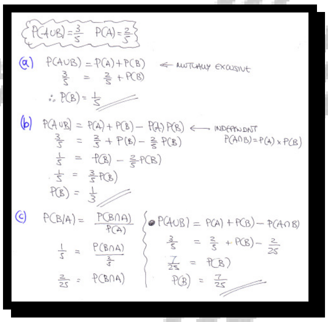
Determine $\mathrm{P}(B)$ in each of the following three cases.

a) If A and B are mutually exclusive.

b) If A and B are independent.

c) If $\mathrm{P}(B|A) = \tfrac{1}{5}$.

$$\boxed{\mathrm{P}(B) = \tfrac{1}{5}},\ \boxed{\mathrm{P}(B) = \tfrac{1}{3}},\ \boxed{\mathrm{P}(B) = \tfrac{7}{25}}$$



$\mathrm{P}(A \cup B) = \frac{3}{5}$ $\quad$ $\mathrm{P}(A) = \frac{2}{5}$

(a) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ ← MUTUALLY EXCLUSIVE

$\frac{3}{5} = \frac{2}{5} + \mathrm{P}(B)$

$\therefore \mathrm{P}(B) = \frac{1}{5}$

(b) $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\mathrm{P}(B)$ ⟵ INDEPENDENT $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$

$\frac{3}{5} = \frac{2}{5} + \mathrm{P}(B) - \frac{2}{5}\mathrm{P}(B)$

$\frac{1}{5} = \mathrm{P}(B) - \frac{2}{5}\mathrm{P}(B)$

$\frac{1}{5} = \frac{3}{5}\mathrm{P}(B)$

$\mathrm{P}(B) = \frac{1}{3}$

(c) $\mathrm{P}(B|A) = \frac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)}$

$\frac{1}{5} = \frac{\mathrm{P}(B \cap A)}{\frac{2}{5}}$

$\frac{2}{25} = \mathrm{P}(B \cap A)$

• $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\frac{3}{5} = \frac{2}{5} + \mathrm{P}(B) - \frac{2}{25}$

$\frac{7}{25} = \mathrm{P}(B)$

$\mathrm{P}(B) = \frac{7}{25}$

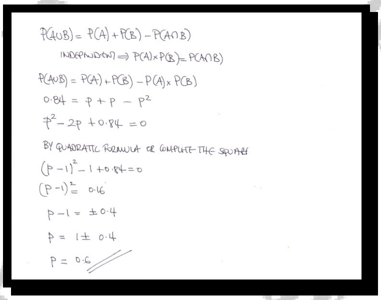
Question 52 (****)

The events A and B are such so that

$$P(A) = P(B) = p \quad \text{and} \quad P(A \cup B) = 0.84 .$$

Given that A and B are independent events determine the value of p.

$$\boxed{p = 0.6}$$



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

INDEPENDENT $\Rightarrow P(A) \times P(B) = P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$0.84 = p + p - p^2$

$p^2 - 2p + 0.84 = 0$

BY QUADRATIC FORMULA OR COMPLETE THE SQUARE

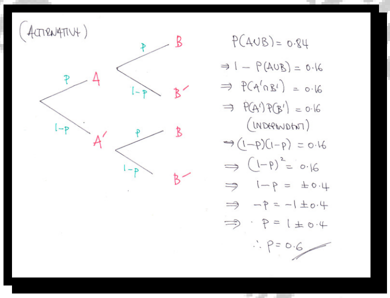
$(p-1)^2 - 1 + 0.84 = 0$

$(p-1)^2 = 0.16$

$p - 1 = \pm 0.4$

$p = 1 \pm 0.4$

$p = 0.6$



$P(A \cup B) = 0.84$

$\Rightarrow 1 - P(A \cup B) = 0.16$

$\Rightarrow P(A' \cap B') = 0.16$

$\Rightarrow P(A')P(B') = 0.16$

(INDEPENDENT)

$\Rightarrow (1-p)(1-p) = 0.16$

$\Rightarrow (1-p)^2 = 0.16$

$\Rightarrow 1 - p = \pm 0.4$

$\Rightarrow -p = -1 \pm 0.4$

$\Rightarrow p = 1 \pm 0.4$

$\therefore p = 0.6$

Question 53 **(****)**

The events A and B are such so that

$$\mathrm{P}(A)=\frac{3}{5}, \quad \mathrm{P}(B|A)=\frac{1}{6} \quad \text{and} \quad \mathrm{P}(A|B)=\frac{3}{13}.$$

Determine the value of $\mathrm{P}(A' \cap B')$.

$\boxed{}$, $\boxed{\mathrm{P}(A' \cap B')=\frac{1}{15}}$

- USING THE CONDITIONAL PROBABILITY FORMULA

$$\Rightarrow P(B|A) = \frac{P(A\cap B)}{P(A)}$$
$$\Rightarrow \frac{1}{6} = \frac{P(A\cap B)}{\frac{3}{5}}$$
$$\Rightarrow P(A\cap B) = \frac{1}{10}$$

- USING THE PROBABILITY FOUND AND THE CONDITIONAL FORMULA AGAIN

$$\Rightarrow P(A|B) = \frac{P(A\cap B)}{P(B)}$$
$$\Rightarrow \frac{3}{13} = \frac{\frac{1}{10}}{P(B)}$$
$$\Rightarrow P(B) = \frac{\frac{1}{10}}{\frac{3}{13}}$$
$$\Rightarrow P(B) = \frac{13}{30}$$

- FINALLY WE HAVE

$$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$\Rightarrow P(A\cup B) = \frac{3}{5} + \frac{13}{30} - \frac{1}{10}$$
$$\Rightarrow P(A\cup B) = \frac{14}{15}$$

- HENCE $P(A'\cap B') = \frac{1}{15}$

Question 54 (****)

The events A and B are such so that

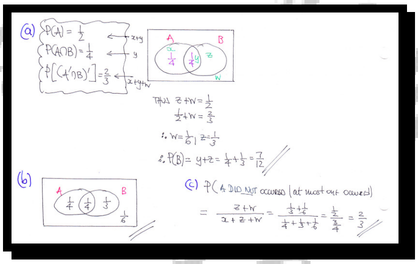
$$\mathrm{P}(A)=\frac{1}{2}, \qquad \mathrm{P}(A\cap B)=\frac{1}{4}, \qquad \mathrm{P}\left[(A'\cap B)'\right]=\frac{2}{3}.$$

a) Find the value of $\mathrm{P}(B)$.

b) Illustrate the probability space of A and B in a suitably labelled, fully completed Venn diagram.

c) Given that at most one of the two events occurred, determine the probability that event A did not occur.

$\boxed{\mathrm{P}(B)=\frac{7}{12}}$, $\boxed{\frac{2}{3}}$



Question 55 (****+)

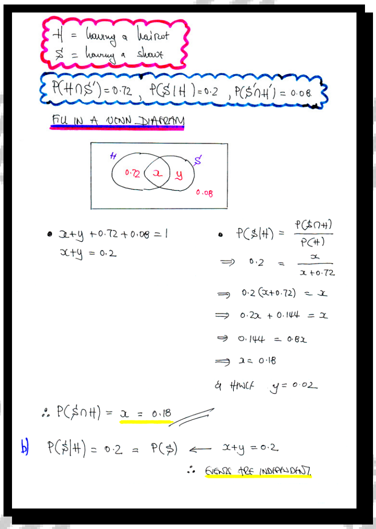
Abel usually visits his hairdresser at most once a week. On any given week, there is a probability of 0.08 that Abel will not go to his hairdresser.

The probability that, on his weekly visit to his hairdresser, Abel has a haircut but not a shave is 0.72. On his weekly visit, if Abel has a haircut the probability he will also have a shave is 0.2.

a) Find the probability that on a week picked at random Abel has a haircut and a shave at his hairdresser.

b) Determine whether the events "having a haircut at his hairdresser" and "having a shave at his hairdresser" are statistically independent.

$\boxed{}$, $\boxed{0.18}$, $\boxed{\text{independent}}$



Question 56 (****+)

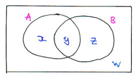
The events A and B are such so that

$$\mathrm{P}(A)=\frac{1}{2}, \qquad \mathrm{P}(B)=\frac{1}{4}, \qquad \mathrm{P}(A \cap B')+\mathrm{P}(A' \cap B)=\frac{1}{3}.$$

Illustrate the probability space of these two events in a fully completed Venn diagram.

$\boxed{}$, $\boxed{\text{diagram}}$

STARTING WITH A VENN DIAGRAM



① $x+y=\frac{1}{2}$
② $y+z=\frac{1}{4}$
③ $x+z=\frac{1}{3}$
④ $x+y+z+w=1$

$\Rightarrow x=\frac{1}{2}-y$

SUB $x=\frac{1}{2}-y$ INTO THE OTHER

② $y+z=\frac{1}{4}$
③ $\frac{1}{2}-y+z=\frac{1}{3}$
④ $\frac{1}{2}-\cancel{y}+\cancel{y}+z+w=1$

TIDY

① $y+z=\frac{1}{4}$
② $-y+z=-\frac{1}{6}$
③ $z+w=\frac{1}{2}$

ADDING $\Rightarrow$ $2z=\frac{1}{12}$, $z=\frac{1}{24}$

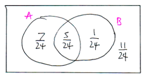
HENCE WE NOW HAVE

$y=\frac{1}{4}-z=\frac{1}{4}-\frac{1}{24}=\frac{5}{24}$

$w=\frac{1}{2}-z=\frac{1}{2}-\frac{1}{24}=\frac{11}{24}$

$x=\frac{1}{2}-y=\frac{1}{2}-\frac{5}{24}=\frac{7}{24}$

FINALLY WE CAN COMPLETE THE DIAGRAM



Question 57 (****+)

The events A, B and C are defined in the same probability space so that

$$\mathrm{P}(A) = \mathrm{P}(C) = \mathrm{P}(B' \cap C') = 0.4 \qquad \text{and} \qquad \mathrm{P}(A \cup B) = 0.58.$$

It is further given that A and B are independent, and A and C are mutually exclusive.

Find the value of $\mathrm{P}\left[(B \cap C') \cup (B' \cap C' \cap A')\right]$.

$\boxed{}$, $\boxed{\mathrm{P}\left[(B \cap C') \cup (B' \cap C' \cap A')\right] = 0.32}$

$P(A) = 0.4 \quad P(A \cup B) = 0.58 \quad P(C) = 0.4 \quad P(B' \cap C') = 0.4$

A & B ARE INDEPENDENT   A & C ARE MUTUALLY EXCLUSIVE

STARTING FROM "A & B ARE INDEPENDENT"

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

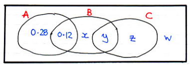
$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)P(B) \leftarrow$ INDEPENDENCE

$\Rightarrow 0.58 = 0.4 + P(B) - 0.4P(B)$

$\Rightarrow 0.18 = 0.6P(B)$

$\Rightarrow P(B) = 0.3$

PUTTING THE KNOWN INFORMATION INTO A VENN DIAGRAM, NOTING THAT A & C ARE MUTUALLY EXCLUSIVE & $P(A \cap B) = P(A)P(B) = 0.12$



- $P(C) = 0.4 \Rightarrow y + z = 0.4$ — I
- $P(B' \cap C') = 0.4 \Rightarrow 0.28 + w = 0.4$ — II
- $P(A \cup B) = 0.58 \Rightarrow z + w = 0.42$ — III
  $\Rightarrow x + y + 0.28 + 0.12 = 0.58$ — IV

FROM II : $w = 0.12$
FROM III : $z = 0.3$
FROM I : $y = 0.1$
FROM IV : $x = 0.08$

$\therefore P(B \cap C' \cup B' \cap C' \cap A')$
$= (x + 0.12) + (w)$
$= 0.32$

Question 58 (****+)

The events A and B are such so that

$$\mathrm{P}(B) = 0.76\,, \qquad \mathrm{P}(B|A) = 0.6\,, \qquad \mathrm{P}(A' \cap B') = 0\,.$$

Find the value of $\mathrm{P}(A)$

$\boxed{}$, $\boxed{\mathrm{P}(A) = 0.6}$

$\boxed{P(B) = 0.76 \qquad P(B|A) = 0.6 \qquad P(A' \cap B') = 0}$

FORMING TWO EQUATIONS FROM THE EQUATIONS GIVEN

- IF $P(A' \cap B') = 0 \Rightarrow P(A \cup B) = 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$1 = P(A) + 0.76 - P(A \cap B)$$

$$\boxed{0.24 = P(A) - P(A \cap B)}$$

ALSO FROM THE CONDITIONAL PROBABILITY

- $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$

$$0.6 = \frac{P(B \cap A)}{P(A)}$$

$$P(B \cap A) = 0.6\,P(A)$$

$$\boxed{P(A \cap B) = 0.6\,P(A)}$$

COMBINING RESULTS

$\Rightarrow 0.24 = P(A) - P(A \cap B)$

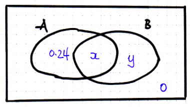
$\Rightarrow 0.24 = P(A) - 0.6\,P(A)$

$\Rightarrow 0.24 = 0.4\,P(A)$

$\Rightarrow P(A) = \dfrac{0.24}{0.4}$

$\Rightarrow P(A) = 0.6$

ALTERNATIVE METHOD BY SETTING EQUATIONS DIRECTLY FROM A VENN DIAGRAM



$P(A' \cap B') = 0$
$P(B) = 0.76$
$P(B|A) = 0.6$

$x + y + 0.24 = 1$ & $\dfrac{x}{x + 0.24} = 0.6$ ← CONDITIONAL

$x + y = 0.76$

$$x = 0.6x + 0.144$$

$$0.4x = 0.144$$

$$x = 0.36$$

$\therefore P(A) = x + 0.36$

$P(A) = 0.6$

AS BEFORE

Question 59 (**+)**

The events A and B satisfy

$$\mathrm{P}(A) = x, \qquad \mathrm{P}(B) = y, \qquad \mathrm{P}(A \cup B) = 0.6, \qquad \mathrm{P}(B|A) = 0.2.$$

a) Show clearly that

$$4x + 5y = 3.$$

The events B and C are mutually exclusive such that

$$\mathrm{P}(B \cup C) = 0.9, \qquad \mathrm{P}(C) = x + y.$$

b) Find the value of x and the value of y.

c) Determine, showing all the relevant workings, whether A and B are statistically independent.

$\boxed{}$, $\boxed{x = 0.5}$, $\boxed{y = 0.2}$, $\boxed{\text{independent}}$

$\mathrm{P}(A) = x \quad \bullet \quad \mathrm{P}(B) = y \quad \bullet \quad \mathrm{P}(A \cup B) = 0.6 \quad \bullet \quad \mathrm{P}(B|A) = 0.2$

a) PROCEED AS FOLLOWS

- $\mathrm{P}(B|A) = \dfrac{\mathrm{P}(B \cap A)}{\mathrm{P}(A)}$

$0.2 = \dfrac{\mathrm{P}(B \cap A)}{x}$

$\mathrm{P}(B \cap A) = 0.2x$

- $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

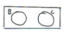
$0.6 = x + y - 0.2x$

$0.6 = 0.8x + y \quad \times 5$

$3 = 4x + 5y$

$4x + 5y = 3$ // AS REQUIRED

b) MUTUALLY EXCLUSIVE $\Rightarrow$ $\mathrm{P}(B) + \mathrm{P}(C) = \mathrm{P}(B \cup C)$



$\Rightarrow y + (x + y) = 0.9$

$\Rightarrow x + 2y = 0.9$

$\Rightarrow x = 0.9 - 2y$

SUB INTO THE EQUATION OF PART (a)

$\Rightarrow 4(0.9 - 2y) + 5y = 3$

$\Rightarrow 3.6 - 8y + 5y = 3$

$\Rightarrow 0.6 = 3y$

$\Rightarrow y = 0.2$ //

&

$\Rightarrow x = 0.5$ //

c) USING $\mathrm{P}(B|A) = 0.2$

$\mathrm{P}(B|A) = \mathrm{P}(B) = y = 0.2$

EVENTS ARE INDEPENDENT

ALTERNATIVE

$\left.\begin{array}{l} \mathrm{P}(A) = x = 0.5 \\ \mathrm{P}(B) = y = 0.2 \\ \mathrm{P}(A \cap B) = 0.2x = 0.1 \end{array}\right\} \quad \mathrm{P}(A) \times \mathrm{P}(B) = 0.5 \times 0.2 = 0.1 = \mathrm{P}(A \cap B)$

EVENTS ARE INDEPENDENT //

Question 60 (****+)

The events A and B satisfy

$$\mathrm{P}(A) = 0.4, \qquad \mathrm{P}(A|B) = 0.6, \qquad \mathrm{P}(A \cup B) = 2\mathrm{P}(A \cap B).$$

Find the value of $\mathrm{P}(B|A')$.

$\boxed{}$, $\boxed{\mathrm{P}(B|A') = \frac{1}{3}}$

TRYING TO FORM SOME EQUATIONS FROM THE INFORMATION GIVEN

$P(A) = 0.4 \quad P(A|B) = 0.6 \quad P(A\cup B) = 2P(A\cap B)$

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

$\Rightarrow 2P(A\cap B) = 0.4 + P(B) - P(A\cap B)$

$\Rightarrow 3P(A\cap B) = 0.4 + P(B) \quad - \mathrm{I}$

AND ALSO WE HAVE

$$\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$$

$\Rightarrow 0.6 = \frac{P(A\cap B)}{P(B)}$

$\Rightarrow P(A\cap B) = 0.6P(B) \quad - \mathrm{II}$

COMBINING EQUATIONS (I) & (II) WE OBTAIN

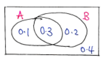
$\Rightarrow 3[0.6P(B)] = 0.4 + P(B)$

$\Rightarrow 1.8P(B) = 0.4 + P(B)$

$\Rightarrow 0.8P(B) = 0.4$

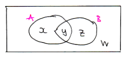
$\Rightarrow P(B) = 0.5 \quad \& \quad P(A\cap B) = 0.3$

FINALLY COMPLETING A VENN DIAGRAM



$P(B|A') = \frac{P(B\cap A')}{P(A')} = \frac{0.2}{0.6} = \frac{1}{3}$

ALTERNATIVE / VARIATION



$P(A) = 0.4 \Rightarrow x + y = 0.4$

$P(A|B) = 0.6 \Rightarrow \frac{y}{y+z} = 0.6$

$P(A\cup B) = 2P(A\cap B) \Rightarrow x + y + z = 2y$

$x + y + z + w = 1$

TIDY EQUATIONS

$$\left.\begin{array}{ll} ① & x+y = 0.4 \\ ② & y = 0.6(y+z) \\ ③ & x+y+z = 2y \\ ④ & x+y+z+w = 1 \end{array}\right\} \Rightarrow \begin{array}{ll} ① & x+y = 0.4 \\ ② & 0.4y = 0.6z \\ ③ & x-y+z = 0 \\ ④ & x+y+z+w = 1 \end{array} \rightarrow \boxed{y = \tfrac{3}{2}z}$$

SUB $y = \frac{3}{2}z$ INTO THE OTHER 3 EQUATIONS

$$\left.\begin{array}{ll} ① & x + \frac{3}{2}z = 0.4 \\ ③ & x - \frac{3}{2}z + z = 0 \\ ④ & x + \frac{3}{2}z + z + w = 1 \end{array}\right\} \Rightarrow \begin{array}{ll} ① & x + \frac{3}{2}z = 0.4 \\ ② & x - \frac{1}{2}z = 0 \\ ③ & x + \frac{5}{2}z + w = 1 \end{array}\left.\vphantom{\begin{array}{l}1\\1\end{array}}\right\} \text{SUBTRACT } \begin{array}{l} 2z = 0.4 \\ z = 0.2 \end{array}$$

HENCE WE OBTAIN x, y & w

$x = \frac{1}{2}z = \frac{1}{2}\times 0.2 = 0.1$

$y = \frac{3}{2}z = \frac{3}{2}\times 0.2 = 0.3$

$w = 1 - x - \frac{5}{2}z = 1 - 0.1 - \frac{5}{2}\times 0.2 = 0.4$

FINALLY

$P(B|A') = \frac{P(B\cap A')}{P(A')} = \ldots$ looking at venn diagram

$= \frac{z}{z+w} = \frac{0.2}{0.6} = \frac{1}{3}$

Question 61 (****+)

The events A and B are such so that

$$\mathrm{P}(B|A)=\frac{3}{8}, \qquad \mathrm{P}(A|B)=\frac{4}{9}, \qquad \mathrm{P}(B|A')=\frac{15}{28}.$$

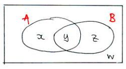
Determine the value of $\mathrm{P}(A)$.

$\boxed{}$, $\boxed{\mathrm{P}(A)=\frac{8}{15}}$

$\boxed{P(B|A)=\frac{3}{8} \qquad P(A|B)=\frac{4}{9} \qquad P(B|A')=\frac{15}{28}}$

① $\frac{P(B\cap A)}{P(A)}=\frac{3}{8}$ ② $\frac{P(A\cap B)}{P(B)}=\frac{4}{9}$ ③ $\frac{P(B\cap A')}{P(A')}=\frac{15}{28}$

FILL IN A VENN DIAGRAM



① $\frac{y}{x+y}=\frac{3}{8}$ ② $\frac{y}{y+z}=\frac{4}{9}$ ③ $\frac{z}{z+w}=\frac{15}{28}$

$8y=3x+3y$ $\quad 9y=4y+4z$ $\quad 28z=15z+15w$

$5y=3x$ $\quad 5y=4z$ $\quad 13z=15w$

REWRITE EQUATIONS & TIDY

① $5y=3x$
② $5y=4z$ $\Longrightarrow$ $\boxed{y=\frac{3}{5}x}$ & SUB INTO THE OTHER 3
③ $13z=15w$
④ $x+y+z+w=1$

② $5\times\frac{3}{5}x=4z$
③ $13z=15w$ $\Big\}$ TIDY
④ $x+\frac{3}{5}x+z+w=1$

② $3x=4z$
③ $13z=15w$
④ $\frac{8}{5}x+z+w=1$

$\Longrightarrow x=\frac{4}{3}z$ & SUBSTITUTE INTO THE OTHER 2 EQUATIONS

③ $13z=15w$
④ $\frac{8}{5}\times\frac{4}{3}z+z+w=1$ $\Big\}\Longrightarrow$ TIDY $\quad 13z=15w$, $\quad \frac{47}{15}z+w=1$

③ $13z=15w$
④ $\frac{47}{15}z=1-w$ $\Big\}\Longrightarrow$ $13z=15w$, $\quad 47z=15-15w$ $\Big\}\Longrightarrow$ ADDING

$60z=15$

$z=0.25$

HENCE

$13z=15w$ $\quad x=\frac{4}{3}z$ $\quad y=\frac{3}{5}x$

$\frac{13}{4}=15w$ $\quad x=\frac{4}{3}\times\frac{1}{4}$ $\quad y=\frac{3}{5}\times\frac{1}{3}$

$w=\frac{13}{60}$ $\quad x=\frac{1}{3}$ $\quad y=\frac{1}{5}$

(NOT ACTUALLY NEEDED)

FINALLY $P(A)=x+y=\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$

Question 62 (*****)

In a summer camp sport games weekend, some kids played football, some played rugby, some played football **and** rugby and some played other games.

The probability, that a kid selected at random, played football but no rugby is $\frac{1}{4}$.

The probability, that a kid selected at random, played rugby but no football is $\frac{1}{6}$.

It is thought that the events "a kid playing rugby" or "a kid playing football" are independent.

Given further that 324 kids played football and rugby, determine two possible estimates for the number of kids that played no rugby and no football.

$\boxed{}$, $\boxed{54 \text{ or } 1944}$

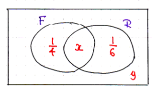
$P(\text{football but no rugby}) = \frac{1}{4}$

$P(\text{rugby but no football}) = \frac{1}{6}$

! INDEPENDENT EVENTS !

324 PLAY FOOTBALL & RUGBY — ESTIMATE NO FOOTBALL & NO RUGBY

PUTTING THE INFORMATION INTO A VENN DIAGRAM



- $x + y + \frac{1}{4} + \frac{1}{6} = 1$

  $x + y = \frac{7}{12}$

- INDEPENDENCE

  $P(F) \times P(R) = P(F \cap R)$

USING THE INDEPENDENCE EQUATION.

$$\Rightarrow P(F \cap R) = P(F) \times P(R)$$
$$\Rightarrow x = \left(x + \frac{1}{4}\right)\left(x + \frac{1}{6}\right)$$
$$\Rightarrow x = x^2 + \frac{1}{6}x + \frac{1}{4}x + \frac{1}{24} \quad \Big) \times 24$$
$$\Rightarrow 24x = 24x^2 + 4x + 6x + 1$$
$$\Rightarrow 0 = 24x^2 - 14x + 1$$
$$\Rightarrow 0 = (2x - 1)(12x - 1)$$
$$\Rightarrow x = \begin{cases} \frac{1}{2} \\ \frac{1}{12} \end{cases} \qquad y = \begin{cases} \frac{1}{12} \\ \frac{1}{2} \end{cases}$$

NOW IF WE USE RATIO

IF $x : y$ & BY SYMMETRY $x : y$

$\frac{1}{2} : \frac{1}{12}$ $\qquad$ $\frac{1}{12} : \frac{1}{2}$

$6 : 1$ $\qquad$ $1 : 6$

$324 : 54$ $\qquad$ $324 : 1944$

Question 63 (*****)

In Tawlfy Towers hotel, residents may choose to have breakfast in the hotel and similarly residents may choose to have dinner in the hotel.

Non residents to the hotel can also book to have dinner in the hotel.

On a certain day, a person who resides in the hotel or had dinner in the hotel is selected at random.

The probability that this person had breakfast in the hotel is 0.5.

The probability that this person had breakfast and dinner in the hotel is 0.2.

The probability that this person resides in the hotel and had dinner in the hotel is 0.32.

It is thought that the events "a person has breakfast in the hotel" or "a person has dinner at the hotel" are independent.

Determine the probability that this person ...

a) ... resides in the hotel and had no breakfast in the hotel.

b) ... had dinner at the hotel given he had breakfast in the hotel.

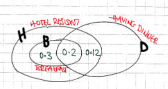
c) ... resides in the hotel and had no breakfast in the hotel given he had dinner in the hotel.

$\boxed{}$, $\boxed{0.42}$, $\boxed{0.4}$, $\boxed{0.3}$

- $P(\text{BREAKFAST}) = 0.5$
- $P(\text{BREAKFAST} \cap \text{DINNER}) = 0.2$
- $P(\text{HOTEL} \cap \text{DINNER}) = 0.32$

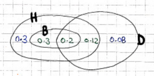
- ONLY HOTEL RESIDENTS MAY HAVE BREAKFAST IN THE HOTEL "B" $\subset$ "H"
- B & D ARE INDEPENDENT

a) START BY FILLING A "VENN DIAGRAM" OF EXHAUSTIVE EVENTS



USING THE INDEPENDENCE OF B & D

$$P(D) \times P(B) = P(D \cap B)$$
$$P(D) \times 0.5 = 0.2$$
$$P(D) = 0.4$$



HENCE $P(H \cap B') = 0.42$

b) $P(D|B) = \dfrac{P(D \cap B)}{P(B)} = \dfrac{0.2}{0.5} = 0.4$

(OR SIMPLY $P(D)$ AS B & D ARE INDEPENDENT

c) $P(H \cap B'|D) = \dfrac{P(H \cap B' \cap D)}{P(D)} = \dfrac{0.12}{0.4} = 0.3$

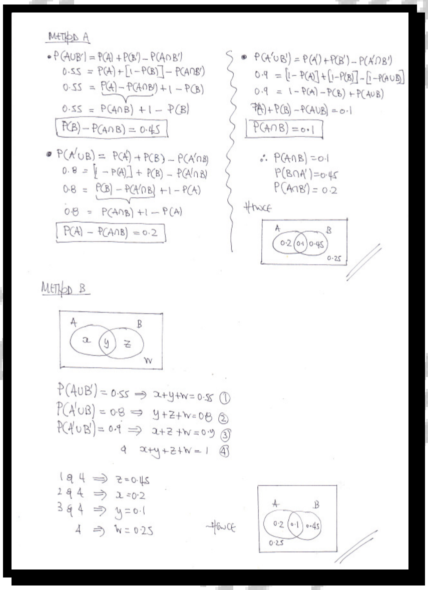
Question 64 (*****)

The events A and B are such so that

$$\mathrm{P}\left(A \cup B'\right) = 0.55 , \quad \mathrm{P}\left(A' \cup B\right) = 0.8 \quad \text{and} \quad \mathrm{P}\left(A' \cup B'\right) = 0.9 .$$

Illustrate the above information in a fully completed Venn diagram.

diagram



METHOD A

• $P(A \cup B') = P(A) + P(B') - P(A \cap B')$
$0.55 = P(A) + [1 - P(B)] - P(A \cap B')$
$0.55 = \underbrace{P(A) - P(A \cap B')} + 1 - P(B)$
$0.55 = P(A \cap B) + 1 - P(B)$
$\boxed{P(B) - P(A \cap B) = 0.45}$

• $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$
$0.8 = [1 - P(A)] + P(B) - P(A' \cap B)$
$0.8 = \underbrace{P(B) - P(A' \cap B)} + 1 - P(A)$
$0.8 = P(A \cap B) + 1 - P(A)$
$\boxed{P(A) - P(A \cap B) = 0.2}$

• $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$
$0.9 = [1 - P(A)] + [1 - P(B)] - [1 - P(A \cup B)]$
$0.9 = 1 - P(A) - P(B) + P(A \cup B)$
$P(A) + P(B) - P(A \cup B) = 0.1$
$\boxed{P(A \cap B) = 0.1}$

$\therefore P(A \cap B) = 0.1$
$P(B \cap A') = 0.45$
$P(A \cap B') = 0.2$

HENCE

A B
0.2 0.1 0.45
0.25

METHOD B

A B
x y z
w

$P(A \cup B') = 0.55 \Rightarrow x + y + w = 0.55$ ①
$P(A' \cup B) = 0.8 \Rightarrow y + z + w = 0.8$ ②
$P(A' \cup B') = 0.9 \Rightarrow x + z + w = 0.9$ ③
& $x + y + z + w = 1$ ④

1 & 4 $\Rightarrow z = 0.45$
2 & 4 $\Rightarrow x = 0.2$
3 & 4 $\Rightarrow y = 0.1$
4 $\Rightarrow w = 0.25$

HENCE

A B
0.2 0.1 0.45
0.25

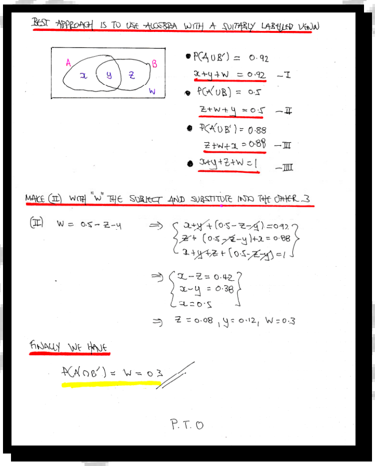
Question 65 (*****)

The events A and B are such so that

$$\mathrm{P}(A \cup B') = 0.92, \quad \mathrm{P}(A' \cup B) = 0.5 \quad \text{and} \quad \mathrm{P}(A' \cup B') = 0.88.$$

Determine the value of $\mathrm{P}(A' \cap B')$.

$\boxed{}$, $\boxed{\mathrm{P}(A' \cap B') = 0.3}$



BEST APPROACH IS TO USE ALGEBRA WITH A SUITABLY LABELLED VENN

- $\mathrm{P}(A \cup B') = 0.92$
  $x + y + w = 0.92$ — I
- $\mathrm{P}(A' \cup B) = 0.5$
  $z + w + y = 0.5$ — II
- $\mathrm{P}(A' \cup B') = 0.88$
  $z + w + x = 0.88$ — III
- $x + y + z + w = 1$ — IIII

MAKE (II) WITH "w" THE SUBJECT AND SUBSTITUTE INTO THE OTHER 3

(II) $w = 0.5 - z - y \implies \begin{cases} x + \cancel{y} + (0.5 - z - \cancel{y}) = 0.92 \\ \cancel{z} + (0.5 - \cancel{z} - y) + x = 0.88 \\ x + \cancel{y} + \cancel{z} + (0.5 - \cancel{z} - \cancel{y}) = 1 \end{cases}$

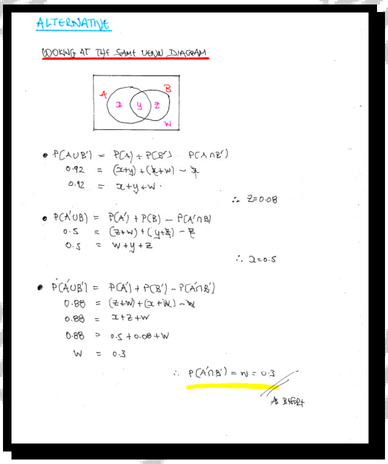
$\implies \begin{cases} x - z = 0.42 \\ x - y = 0.38 \\ x = 0.5 \end{cases}$

$\implies z = 0.08, \; y = 0.12, \; w = 0.3$

FINALLY WE HAVE

$\mathrm{P}(A' \cap B') = w = 0.3$

P.T.O



ALTERNATIVE

LOOKING AT THE SAME VENN DIAGRAM

- $\mathrm{P}(A \cup B') = \mathrm{P}(A) + \mathrm{P}(B') - \mathrm{P}(A \cap B')$
  $0.92 = (x + y) + (x + w) - x$
  $0.92 = x + y + w$
  $\therefore z = 0.08$
- $\mathrm{P}(A' \cup B) = \mathrm{P}(A') + \mathrm{P}(B) - \mathrm{P}(A' \cap B)$
  $0.5 = (z + w) + (y + z) - z$
  $0.5 = w + y + z$
  $\therefore x = 0.5$
- $\mathrm{P}(A' \cup B') = \mathrm{P}(A') + \mathrm{P}(B') - \mathrm{P}(A' \cap B')$
  $0.88 = (z + w) + (x + w) - w$
  $0.88 = x + z + w$
  $0.88 = 0.5 + 0.08 + w$
  $w = 0.3$

$\therefore \mathrm{P}(A' \cap B') = w = 0.3$

AS BEFORE

Question 66 (*****)

The events C and D are such so that

$$\mathrm{P}(C)=\frac{1}{3}, \qquad \mathrm{P}(D)=\frac{7}{36}, \qquad \mathrm{P}\big[(C\cap D')\cup(C'\cap D)\big]=\frac{13}{36}.$$

a) Find, showing a full clear method, the value of $\mathrm{P}(C\cap D)$.

If **instead** the events C and D satisfy

$$\mathrm{P}(C)=\frac{k}{k+2}, \qquad \mathrm{P}(D)=\frac{2}{k},$$

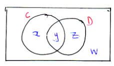
where k is a positive constant such that $\mathrm{P}(C)<1$, $\mathrm{P}(D)<1$.

b) Show that C and D cannot be mutually exclusive.

$\boxed{}$, $\boxed{\mathrm{P}(C\cap D)=\frac{1}{12}}$

$P(C)=\frac{1}{3}$ • $P(D)=\frac{7}{36}$ • $P[(C\cap D')\cup(C'\cap D)]=\frac{13}{36}$

a) FILL IN A VENN DIAGRAM



$P(C)=\frac{1}{3} \Rightarrow x+y=\frac{1}{3}$

$P(D)=\frac{7}{36} \Rightarrow y+z=\frac{7}{36}$

$P[(C\cap D')\cup(C'\cap B)]=\frac{13}{36} \Rightarrow x+z=\frac{13}{36}$

& $x+y+z+w=1$

SOLVING THE 4 EQUATIONS

$$\left.\begin{aligned} x+y&=\tfrac{1}{3}\\ y+z&=\tfrac{7}{36}\\ x+z&=\tfrac{13}{36}\end{aligned}\right\} \text{ ADD THESE} \qquad 2x+2y+2z=\frac{8}{9}$$

$x+y+z+w=1$ ← DOUBLE THIS $\qquad \underbrace{2x+2y+2z}_{\frac{8}{9}}+2w=2$

$2w=\frac{10}{9}$

$w=\frac{5}{9}$

USING

$x+y+z+w=1$

$x+y+z+\frac{5}{9}=1$

$x+y+z=\frac{4}{9}$ BUT $x+z=\frac{13}{36}$

$\frac{13}{36}+y=\frac{4}{9}$

$y=\frac{1}{12}$

$\therefore P(C\cap D)=\frac{1}{12}$

b) PROCEED AS FOLLOWS

$P(C)=\frac{k}{k+2}$ $\quad P(D)=\frac{2}{k}$

$\Rightarrow P(C\cup D)=P(C)+P(D)-\cancel{P(C\cap D)}$ IF MUTUALLY EXCLUSIVE

$\Rightarrow P(C\cup D)=P(C)+P(D)$

$\Rightarrow P(C\cup D)=\frac{k}{k+2}+\frac{2}{k}$

$\Rightarrow P(C\cup D)=\frac{k^2+2k+2}{k(k+2)}$

$\Rightarrow P(C\cup D)=\frac{k^2+2k+4}{k^2+2k}$

$\Rightarrow P(C\cup D)=1+\frac{4}{k^2+2k}$

$\Rightarrow P(C\cup D)>1$

$\therefore$ C & D CANNOT BE MUTUALLY EXCLUSIVE

Question 67 (*****)

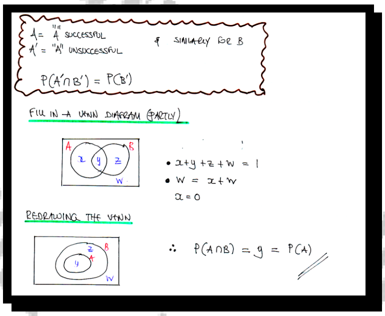
Two candidates, A and B, are about to be interviewed by a company.

The probability that **both** candidates will be **unsuccessful** is the same as the probability that candidate B will be **unsuccessful**.

Show that the probability that **both** candidates will be **successful** is the same as the probability that candidate A will be **successful**.

You may not assume independence in this question.

[ ], [proof]



TREE DIAGRAMS

Question 1 (**)

Two boxes A and B contain chocolates.

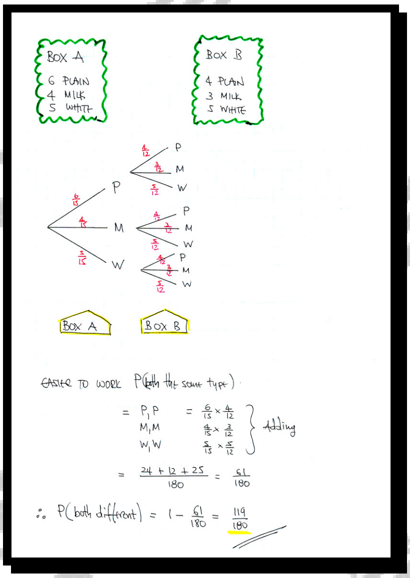
Box A contains 6 plain chocolates, 4 milk chocolates and 5 white chocolates.

Box B contains 4 plain chocolates, 3 milk chocolates and 5 white chocolates.

One chocolate is selected from each box at random.

Determine the probability that the two chocolates will be of different type.

$\boxed{\frac{119}{180}}$



Question 2 (+)**

Bag A contains 3 red balls and 4 green balls and bag B contains 2 red balls and 3 green balls.

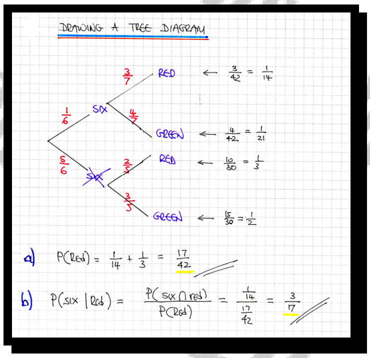
A fair dice numbered 1 to 6 is rolled.

- If the dice shows 6, a ball is drawn at random from bag A.
- If the dice does not show 6, a ball is drawn at random from bag B.

a) Find the probability that a red ball will be drawn.

b) Given that a red ball was drawn, determine the probability that the dice had previously shown a six.

$\boxed{}$, $\boxed{\frac{17}{42}}$, $\boxed{\frac{3}{17}}$



Question 3 (***)

During the winter, Ned attends weekly business meetings in Newcastle and always travels to these meetings by car.

The probability of being dry, raining or snowing during his travel to these meetings is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$, respectively. The respective probabilities of Ned arriving on time when is dry, raining or snowing are $\frac{4}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$.

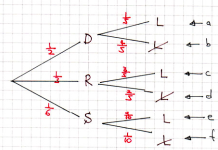
a) Determine the probability that Ned will arrive late for his next winter business meeting.

Ned arrived late for his meeting last week.

b) Find the probability that it was raining on that day.

$\boxed{}$, $\boxed{\frac{9}{20}}$, $\boxed{\frac{4}{9}}$

DRAWING A TREE DIAGRAM



a) $P(L) = a + c + e$

$= \left(\frac{1}{2}\times\frac{1}{5}\right) + \left(\frac{1}{3}\times\frac{3}{5}\right) + \left(\frac{1}{6}\times\frac{9}{10}\right)$

$= \frac{1}{10} + \frac{1}{5} + \frac{3}{20}$

$= \frac{9}{20} = 0.45$

b) $P(R|L) = \dfrac{P(R\cap L)}{P(L)} = \dfrac{\frac{1}{3}\times\frac{3}{5}}{\frac{9}{20}} = \dfrac{\frac{1}{5}}{\frac{9}{20}} = \frac{4}{9}$

OR

$\dfrac{c}{a+c+e}$

Question 4 (***)

A bag contains blue, yellow and red discs and these discs show on their face a single digit whole number.

The probability of drawing a blue disc is $\frac{1}{2}$, the probability of drawing a yellow disc is $\frac{1}{3}$ and the probability of drawing a red disc is $\frac{1}{6}$.

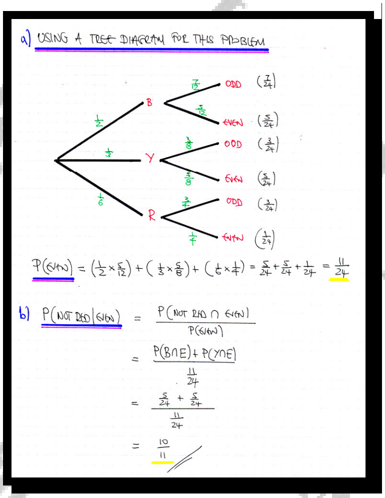
$\frac{5}{12}$ of the blue discs show an even number, $\frac{5}{8}$ of the yellow discs show an even number and $\frac{1}{4}$ of the red discs show an even number .

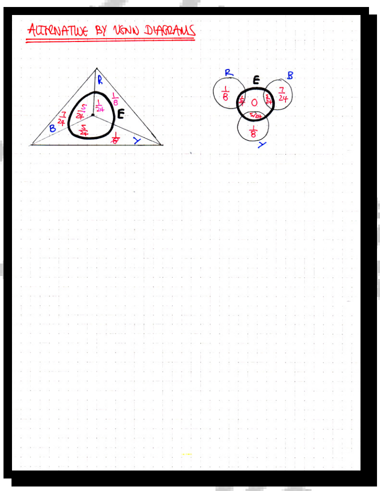
A disc is drawn at random from the bag.

a) Determine the probability that the disc will show an even number.

b) Given that the disc that was drawn was showing an even number, find the probability that the disc was not red.

$\boxed{}$, $\boxed{\frac{11}{24}}$, $\boxed{\frac{10}{11}}$





Question 5 (***)

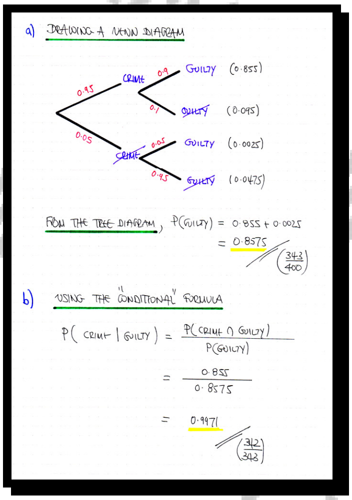
In a certain Crown Court 95% of the defendants being tried have actually committed the crime they are being tried for.

For those who committed the crime the probability of being found guilty is 90% and for those who did not commit the crime the probability of being found guilty is 5%.

a) Find the probability that a randomly chosen defendant will be found guilty.

b) Given that a randomly chosen defendant was found guilty, find the probability that the defendant committed the crime.

$\boxed{\dfrac{343}{400}}$, $\boxed{\dfrac{342}{343}}$



Question 6 (***)

A test is developed to determine whether someone has or has not got a disease, which is known to be present in 3% of the population.

Given a person has the disease the test is positive with probability of 98%.

Given a person does not have the disease the test is positive with probability of 5%.

a) Draw a tree diagram to represent this information.

A person is selected at random from the population and tested for the disease.

b) Find the probability that this person's test is positive.

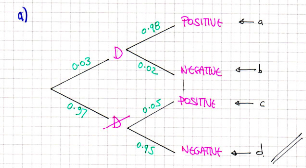
A person who tested positive is selected.

c) Find the probability that the person does not have the disease.

d) Comment on the effectiveness of this test with reference to the answer given in part (**c**).

$\boxed{}$, $\boxed{0.0779}$, $\boxed{0.6226}$

a)



b) $P(\text{POSITIVE}) = (0.03 \times 0.98) + (0.97 \times 0.05) = 0.0779$

c) $P(\cancel{D} \mid \text{POSITIVE}) = \dfrac{P(\cancel{D} \cap \text{POSITIVE})}{P(\text{POSITIVE})} = \dfrac{0.97 \times 0.05}{0.0779} = 0.6226$

OR $\boxed{\dfrac{c}{a+c}}$

d) TEST IS NOT EFFECTIVE AS IT PREDICTS A "HEALTHY PERSON" BEING ILL WITH PROBABILITY 0.6226 WHICH IS VERY HIGH

Question 7 **(***+)**

It is estimated that 20% of the cars sold in a certain auction have been involved in serious accidents. The auction has a test which produces a positive result to indicate that a car has been involved in serious accident.

If a car **has** been involved in a serious accident the test will produce a positive result in 80% of the cases. If a car **has not** been involved in a serious accident the test will produce a positive result in 10% of the cases.

a) Draw a tree diagram to represent the above information.

A car is selected at random.

b) Find the probability that this car will…

i. … test positive.

ii. … be classed correctly by the test.

c) Given that a car tests positive determine the probability it has been involved in a serious accident.

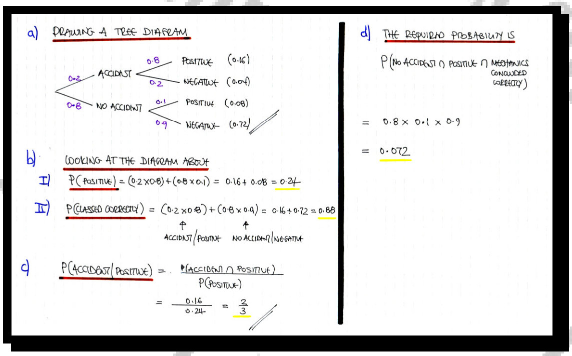
The cars that test positive are further examined by a team of mechanics.

These teams of mechanics have a 90% probability of **correctly** identifying whether a car **has** been involved in an accident or **not**.

Another car is selected in the auction.

d) Determine the probability that this car has not been involved in a serious accident, it tested positive but the team of mechanics concluded it had not been involved in a serious accident.

$\boxed{\text{MMS}}$, $\boxed{0.24}$, $\boxed{0.88}$, $\boxed{\frac{2}{3}}$, $\boxed{0.072}$



Question 8 (***+)

Bag X contains 3 one pound coins and 2 two pound coins.

Bag Y contains 1 one pound coin and 3 two pound coins.

A statistical experiment consists of

- a coin being picked **at random** from bag X and placed into bag Y.
- then a coin being picked **at random** from bag Y and placed back into bag X.

Find the probability …

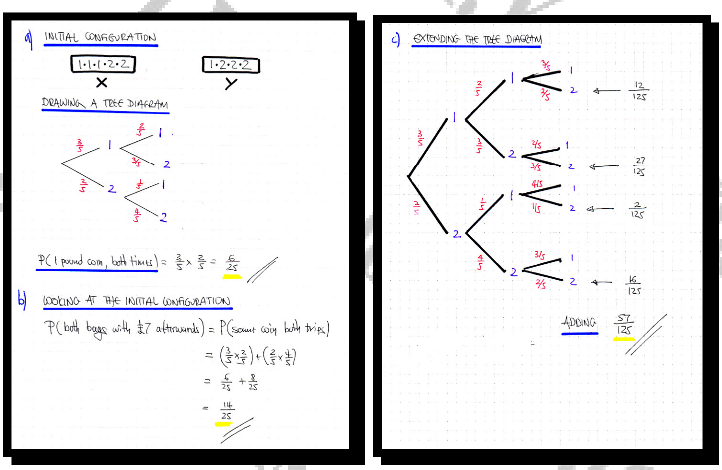
a) … that a one pound coin will be picked on both occasions in this experiment.

b) … that at the end of the experiment both bags contain £7 .

A third coin is picked **at random** from bag X at the end of the experiment.

c) Determine the probability that it will be a two pound coin.

$\boxed{}$, $\boxed{\frac{6}{25}}$, $\boxed{\frac{14}{25}}$, $\boxed{\frac{57}{125}}$



Question 9 (***+)

Markus is a health fanatic.

On a given day, the probabilities that he goes for a run, he uses the gym or he cycles are 0.5, 0.4 and 0.1, respectively.

Markus sometimes uses the sauna after these activities.

The probability he uses the sauna after he goes for a run is 0.1. The respective probabilities for using the sauna after using the gym or cycling are 0.6 and 0.3.

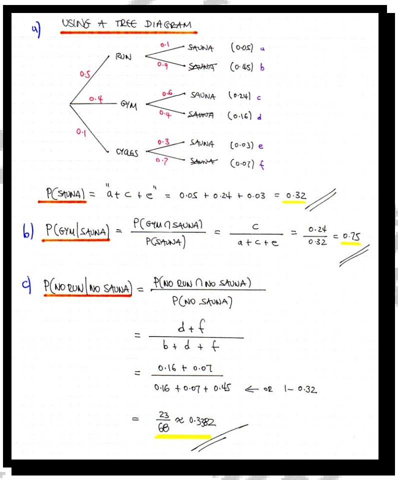
Find the probability that on a given day Markus …

a) … will use the sauna.

b) … used the gym, given he used the sauna.

c) … did not go for a run, given he did not use the sauna.

$\boxed{}$, $\boxed{0.32}$, $\boxed{0.75}$, $\boxed{\frac{23}{68} \approx 0.3382}$



Question 10 (*+)**

Taxis in Pajan have to pass an additional safety test consisting of three parts, one for the brakes, one for the tyres and one for the lights. A taxi must pass all three parts.

The individual probabilities that a taxi will fail the "brake part", the "tyre part" and the "light part" are $\frac{1}{6}$, $\frac{1}{4}$ and $\frac{1}{5}$, respectively.

These probabilities are independent of one another.

A taxi from Pajan is tested at random.

a) Find the probability that it will fail **exactly one** of the three parts of the test.

Safety regulations change so that the test has to be performed in the order "brake part" first, "tyre part" next and "lights part" last.

If the taxi fails one of the three parts the test results in failure, **without** any of remaining parts of the test having to be carried out.

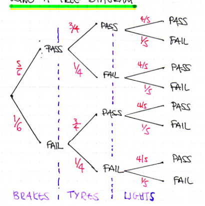
A taxi from Pajan is tested at random under these new regulations.

b) Find the probability that it will fail the test.

c) Given a taxi failed the test, determine the probability it failed the "lights part".

$\boxed{}$, $\boxed{\frac{47}{120}}$, $\boxed{\frac{1}{2}}$, $\boxed{\frac{1}{4}}$

a) USING A TREE DIAGRAM



P(FAIL EXACTLY ONE) = "FPP" $= \frac{1}{6}\times\frac{3}{4}\times\frac{4}{5} = \frac{12}{120}$

PFP $= \frac{5}{6}\times\frac{1}{4}\times\frac{4}{5} = \frac{20}{120}$

PPF $= \frac{5}{6}\times\frac{3}{4}\times\frac{1}{5} = \frac{15}{120}$

$\frac{47}{120}$

b) USING THE TREE DIAGRAM, NOTING THAT BRANCHES TERMINATE IF THEY END UP IN A FAIL

$$P(\text{FAIL}) = \underbrace{\frac{1}{6}}_{\text{FAILED BRAKES}} + \underbrace{\frac{5}{6}\times\frac{1}{4}}_{\text{FAILED TYRES}} + \underbrace{\frac{5}{6}\times\frac{3}{4}\times\frac{1}{5}}_{\text{FAILED LIGHTS}} = \frac{1}{2}$$

ALTERNATIVE APPROACH

$P(\text{FAIL}) = 1 - P(\text{PASSES}) = 1 - \frac{5}{6}\times\frac{3}{4}\times\frac{4}{5} = \frac{1}{2}$

c) $P(\text{FAILED LIGHTS} \mid \text{FAILED}) = \dfrac{P(\text{FAILED LIGHTS} \cap \text{FAILED})}{P(\text{FAILED})}$

$= \dfrac{\frac{5}{6}\times\frac{3}{4}\times\frac{1}{5}}{\frac{1}{2}}$ ← FOUND IN (b)

$= \dfrac{\frac{1}{8}}{\frac{1}{2}}$

$= \frac{1}{4}$

Question 11 (****)

Arnie and Ned play each other in a darts match, which consists of **up to three** games.

The winner is the first person to win two games.

The probability that Arnie wins the first game is 0.7.

Whenever Arnie wins a game the probability he wins the next is 0.6.

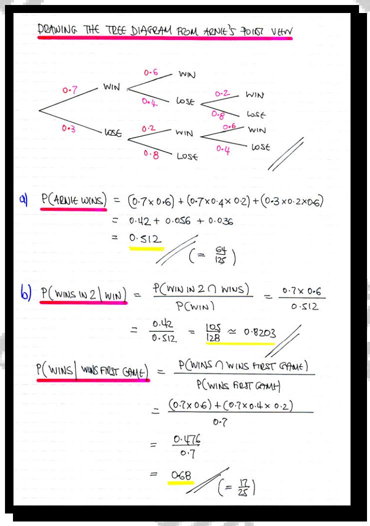
Whenever Ned wins a game the probability he wins the next is 0.8.

a) Find the probability that Arnie wins the match.

b) Given that Arnie won the match find the probability he won in two games.

c) Given Arnie won the first game find the probability he won the match.

$\boxed{}$, $\boxed{\frac{64}{125}}$, $\boxed{\frac{105}{128}}$, $\boxed{\frac{17}{25}}$


DRAWING THE TREE DIAGRAM FROM ARNIE'S POINT VIEW

a) $P(\text{ARNIE WINS}) = (0.7\times0.6)+(0.7\times0.4\times0.2)+(0.3\times0.2\times0.6)$
$= 0.42 + 0.056 + 0.036$
$= 0.512 \quad \left(=\frac{64}{125}\right)$

b) $P(\text{WINS IN 2}\,|\,\text{WIN}) = \dfrac{P(\text{WIN IN 2} \cap \text{WINS})}{P(\text{WIN})} = \dfrac{0.7\times0.6}{0.512}$
$= \dfrac{0.42}{0.512} = \dfrac{105}{128} \simeq 0.8203$

$P(\text{WINS}\,|\,\text{WINS FIRST GAME}) = \dfrac{P(\text{WINS} \cap \text{WINS FIRST GAME})}{P(\text{WINS FIRST GAME})}$
$= \dfrac{(0.7\times0.6)+(0.7\times0.4\times0.2)}{0.7}$
$= \dfrac{0.476}{0.7}$
$= 0.68 \quad \left(=\frac{17}{25}\right)$

Question 12 (****)

Trains are arriving at the Villerpool train station **every hour** and these trains are either late or on time.

- If a train is on time the probability the next train is on time is 0.9.
- If a train is late the probability the next train is late is 0.6.

On a particular day the 07.00 train arrives on time.

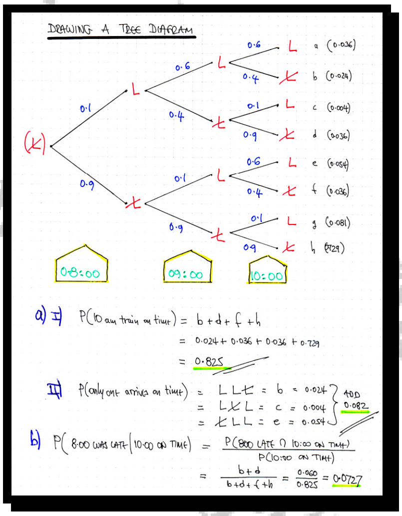
a) Determine the probability that…

  i. … the 10.00 train will arrive on time.

  ii. … **only one** out of the 08.00, 09.00 and 10.00 trains will arrive on time.

b) Given that the 10.00 train arrived on time, find the probability that the 08.00 train was late.

$\boxed{}$, $\boxed{0.825}$, $\boxed{0.082}$, $\boxed{0.0727}$



Question 13 (****)

The scheduled flight DM104, of a certain airline, can be delayed due to the following three reasons.

- aircraft technical problems, denoted by the event T
- weather conditions, denoted by the event W
- air traffic congestion, denoted by the event C.

These events are assumed to be independent of one another and the flight will be delayed if one or more of these events occur.

It is given that $\mathrm{P}(T) = 0.1$, $\mathrm{P}(W) = 0.05$ and $\mathrm{P}(C) = 0.2$.

a) Find the probability that the next DM104 flight …

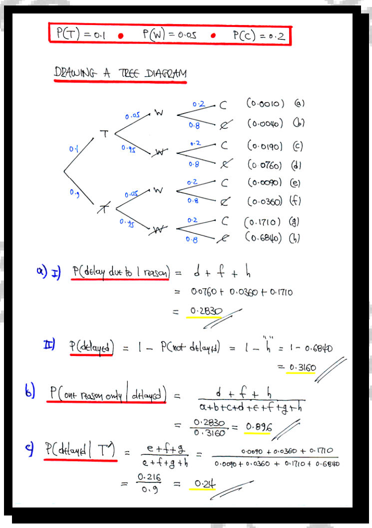
i. … will be delayed due to exactly one reason.

ii. … will be delayed.

b) Given that the next DM104 flight was delayed, find the probability that it was delayed due to **one** reason only.

c) Given that the next DM104 flight had no technical problems, find the probability that it was delayed.

$\boxed{}$, $\boxed{0.283}$, $\boxed{0.316}$, $\boxed{\approx 0.896}$, $\boxed{0.24}$



$\boxed{P(T) = 0.1 \quad \bullet \quad P(W) = 0.05 \quad \bullet \quad P(C) = 0.2}$

DRAWING A TREE DIAGRAM

a) I) $P(\text{delay due to 1 reason}) = d + f + h = 0.0760 + 0.0360 + 0.1710 = 0.2830$

II) $P(\text{delayed}) = 1 - P(\text{not delayed}) = 1 - \text{"}h\text{"} = 1 - 0.6840 = 0.3160$

b) $P(\text{one reason only} \mid \text{delayed}) = \dfrac{d+f+h}{a+b+c+d+e+f+g+h} = \dfrac{0.2830}{0.3160} = 0.896$

c) $P(\text{delayed} \mid T') = \dfrac{e+f+g}{e+f+g+h} = \dfrac{0.0090 + 0.0360 + 0.1710}{0.0090 + 0.0360 + 0.1710 + 0.6840} = \dfrac{0.216}{0.9} = 0.24$

Question 14 (****)

A candidate is attempting a test at which a **maximum of two attempts** is allowed.

The outcome of this test is deemed to be a pass or a fail.

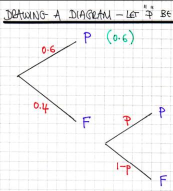
The probability that a candidate passes the test in the first attempt is 0.6

The probability that a candidate passes the test is 0.76.

If a candidate passed the test, determine the probability that he failed the first attempt.

$\boxed{}$, $\boxed{\frac{4}{19}}$

DRAWING A DIAGRAM – LET "p" BE THE PROBABILITY OF PASSING 2ND TIME



$\Rightarrow P(\text{passing}) = 0.76$

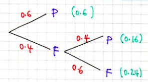
$\Rightarrow P(\text{passing 1st time}) + P(\text{passing 2nd time}) = 0.76$

$\Rightarrow 0.6 + 0.4p = 0.76$

$\Rightarrow 0.4p = 0.16$

$\Rightarrow p = 0.4$

HENCE THE DIAGRAM CAN BE FULLY DRAWN



$P(\text{FAILED 1ST ATTEMPT} \mid \text{PASSED TEST}) = \dfrac{P(\text{FAILED FIRST} \cap \text{PASSED TEST})}{P(\text{PASSED TEST})}$

$= \dfrac{0.16}{0.76} = \dfrac{16}{76} = \dfrac{4}{19}$

Question 15 (****)

A medical test for a certain disease will produce a positive result for 2% of the population and a negative result for 95% of the population. The test is classed inconclusive for 3% of the population.

95% of the people who test positive have the disease, 1% of the people who test negative have the disease, while 20% of the people whose test is classed as inconclusive have the disease.

a) Draw a tree diagram to represent the above information.

A person is selected at random.

b) Find the probability that this person will …

i. … test negative and will have the disease.

ii. … will have the disease.

c) Given a person has the disease, determine the probability he tested negative.

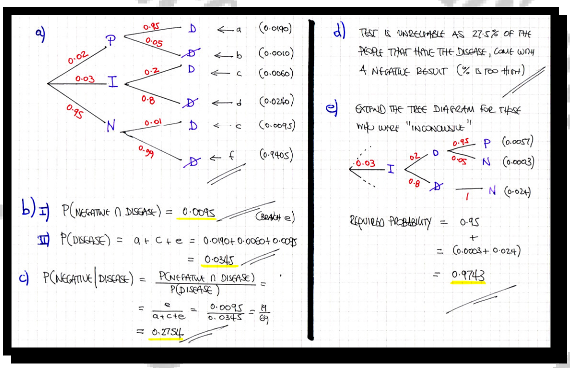
d) Comment on the effectiveness of the test with reference to part **(c)**.

For the people whose test is classed as inconclusive a second more expensive test is carried out. This test will always identify if a person does not have the disease. If a person has the disease the test will correctly identify this in 95% of the cases.

Another person is selected from the population.

e) Determine the probability that this person either tested negative originally or his test was inconclusive but eventually was told that he does not have the disease when the second test was carried out.

$\boxed{}$, $\boxed{0.0095}$, $\boxed{0.0345}$, $\boxed{\approx 0.2754}$, $\boxed{0.9743}$



Question 16 (****)

In the Southgate Academy Sixth Form the students are either left handed or right handed. The following information is also known.

- 0.6 of the students are female.
- 0.11 of the students are left handed.
- 0.10 of the female students are left handed.

a) Draw a fully completed tree diagram to display the above information.

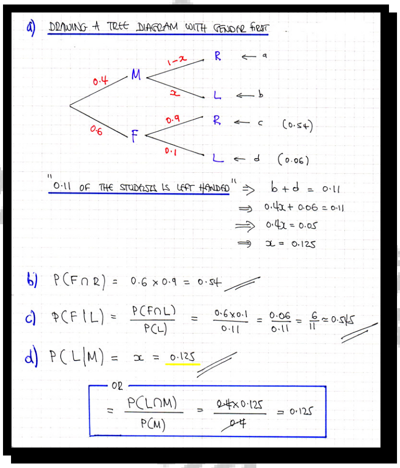
A student is selected at random from the Southgate Academy Sixth Form.

b) Determine the probability that the student is female and right handed.

c) If the student is left handed, find the probability that the student is female.

d) If the student is male, find the probability that the student is left handed.

$\boxed{}$, $\boxed{0.54}$, $\boxed{\frac{6}{11} \approx 0.545}$, $\boxed{\frac{1}{8} = 0.125}$



a) DRAWING A TREE DIAGRAM WITH GENDER FIRST

"0.11 OF THE STUDENTS IS LEFT HANDED" $\Rightarrow b + d = 0.11$

$\Rightarrow 0.4x + 0.06 = 0.11$

$\Rightarrow 0.4x = 0.05$

$\Rightarrow x = 0.125$

b) $P(F \cap R) = 0.6 \times 0.9 = 0.54$

c) $P(F \mid L) = \frac{P(F \cap L)}{P(L)} = \frac{0.6 \times 0.1}{0.11} = \frac{0.06}{0.11} = \frac{6}{11} \approx 0.545$

d) $P(L \mid M) = x = 0.125$

OR

$= \frac{P(L \cap M)}{P(M)} = \frac{\cancel{0.4} \times 0.125}{\cancel{0.4}} = 0.125$

Question 17 (****)

Dora, Tina and Flora are three girls which train daily in their local gym.

Dora's and Tina's daily attendances are independent of one another, with respective probabilities of 0.9 and 0.8.

Flora is Tina's friend, which also trains in the same gym as Tina.

The probability that Flora trains on any given day is 0.7 if Tina also trains on that day, but it is 0.4 if Tina does not train on that day.

a) Find the probability that on a given day ...

i. ... all three girls train.

ii. ... only two of the three girls train.

b) Determine the probability that on a given day...

i. ... if Flora trained, then Dora and Tina also trained.

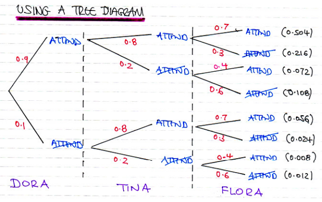
ii. ... if Dora and Tina trained, then Flora also trained.

iii. ... if Tina trained, then Dora and Flora also trained.

iv. ... if Tina and Flora trained, then Dora also trained.

$\boxed{0.504}$, $\boxed{0.344}$, $\boxed{0.7875}$, $\boxed{0.7}$, $\boxed{0.63}$, $\boxed{0.9}$

USING A TREE DIAGRAM



a) LOOKING AT THE DIAGRAM & THE PROBABILITIES CALCULATED AT THE END OF THE 8 BRANCHES

I) $P(\text{All } 3) = 0.504$

II) $P(\text{EXACTLY } 2) = 0.216 + 0.072 + 0.056 = 0.344$

b) I) $P(\text{DORA} \cap \text{TINA} \mid \text{FLORA}) = \dfrac{P(\text{DORA} \cap \text{TINA} \cap \text{FLORA})}{P(\text{FLORA})}$

$= \dfrac{0.504}{0.504 + 0.072 + 0.056 + 0.008}$

$= \dfrac{0.504}{0.640}$

$= \dfrac{63}{80} = 0.7878$

II) $P(\text{FLORA} \mid \text{DORA} \cap \text{TINA}) = \dfrac{P(\text{FLORA} \cap \text{DORA} \cap \text{TINA})}{P(\text{DORA} \cap \text{TINA})}$

$= \dfrac{0.504}{0.9 \times 0.8}$

$= \dfrac{0.504}{0.720}$

$= \dfrac{7}{10} = 0.7$

III) $P(\text{DORA} \cap \text{FLORA} \mid \text{TINA}) = \dfrac{P(\text{DORA} \cap \text{FLORA} \cap \text{TINA})}{P(\text{TINA})}$

$= \dfrac{0.504}{(0.9 \times 0.8) + (0.1 \times 0.8)}$

$= \dfrac{0.504}{0.800}$

$= \dfrac{63}{100} = 0.63$

IV) $P(\text{DORA} \mid \text{TINA} \cap \text{FLORA}) = \dfrac{P(\text{DORA} \cap \text{TINA} \cap \text{FLORA})}{P(\text{TINA} \cap \text{FLORA})}$

$= \dfrac{0.504}{0.504 + 0.056}$

$= \dfrac{0.504}{0.560}$

$= \dfrac{9}{10} = 0.9$

Question 18 (****)

A certain genetic disease is present in 6% of the population.

A certain test has been developed for detecting this disease.

> If a person has the disease the test returns a positive result in 92% of the cases.
>
> If a person does **not** have the disease, the test returns a positive result in 5% of the cases.

A randomly chosen person from the general population is tested.

a) Determine the probability that his test returns a positive result.

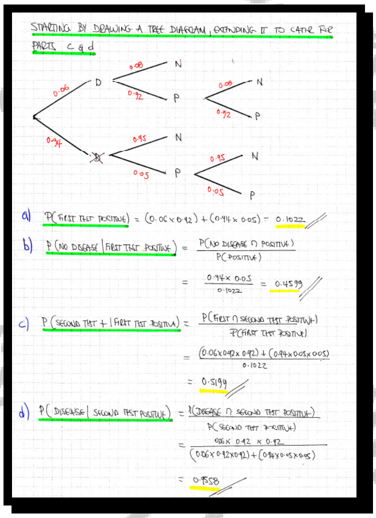
b) If this person's test returns a positive result, calculate the probability that he does not have the disease.

A randomly chosen person that tested positive is tested again for a second time.

c) Find the probability that this person's test will again return a positive result.

d) If the second test returns a positive result, calculate the probability that this person does have the disease.

$\boxed{}$, $\boxed{\approx 0.1022}$, $\boxed{\approx 0.4599}$, $\boxed{\approx 0.5199}$, $\boxed{\approx 0.9558}$



a) P(FIRST TEST POSITIVE) = $(0.06 \times 0.92) + (0.94 \times 0.05) = 0.1022$

b) P(NO DISEASE | FIRST TEST POSITIVE) = $\frac{\text{P(NO DISEASE} \cap \text{POSITIVE)}}{\text{P(POSITIVE)}}$

$= \frac{0.94 \times 0.05}{0.1022} = 0.4599$

c) P(SECOND TEST + | FIRST TEST POSITIVE) = $\frac{\text{P(FIRST} \cap \text{SECOND TEST POSITIVE)}}{\text{P(FIRST TEST POSITIVE)}}$

$= \frac{(0.06 \times 0.92 \times 0.92) + (0.94 \times 0.05 \times 0.05)}{0.1022}$

$= 0.5199$

d) P(DISEASE | SECOND TEST POSITIVE) = $\frac{\text{P(DISEASE} \cap \text{SECOND TEST POSITIVE)}}{\text{P(SECOND TEST POSITIVE)}}$

$= \frac{0.06 \times 0.92 \times 0.92}{(0.06 \times 0.92 \times 0.92) + (0.94 \times 0.05 \times 0.05)}$

$= 0.9558$

Question 19 (****+)

A box contains 3 black balls and 7 white balls, all identical in size.

An experiment consists of drawing a ball out of the box and recording its colour.

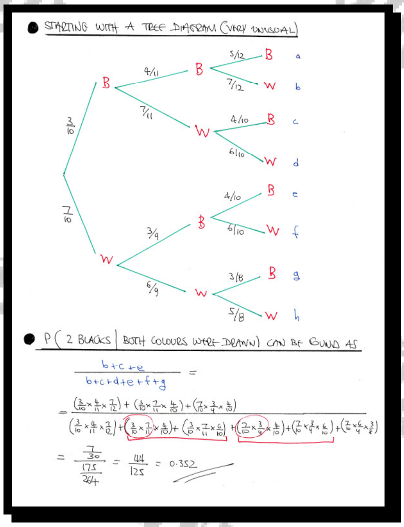
- If the ball drawn is black, after its colour is recorded, the ball is replaced back into the box and an extra black ball is also placed in the box.

- If the ball drawn is white, after its colour is recorded, the ball is not replaced back into the box.

The experiment is attempted 3 consecutive times.

Given that in these 3 attempts both colours were recorded, determine the probability that a black colour was recorded twice.

$\boxed{}$, $\boxed{\frac{44}{125} = 0.352}$



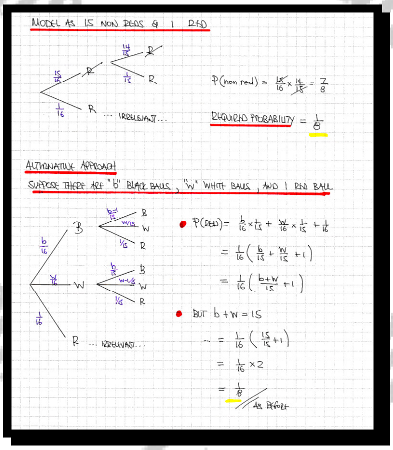
Question 20 (****+)

A box contains 16 balls, identical in size, of which w of them are white, b of them are black and 1 of them is red.

Two balls are drawn from the box one after the other, without being replaced.

Determine the probability that the red ball will be picked.

$\square$, $\boxed{\frac{1}{8}}$



Question 21 (****+)

The youth academy of Hottenham Football Club has introduced trials for new players. The performances of prospective players are judged at the end of every game as pass or fail by a group of football experts.

Each player is invited to play up to three games.

- The probability of a player passing the first game is 0.8.
- Players who pass any game have a probability 0.8 of passing the next game.
- Players who fail any game have a probability 0.6 of failing the next game.

Players who pass all three games will join the first team squad, players who pass any two games will join the second team squad and those who fail in two games will be asked to leave. A player can therefore be asked to leave after two games.

A player that is attempting the trials is selected at random.

a) Determine the probability that …

  i. … he will be joining the first team.

  ii. … he will be joining the second team.

  iii. … he will be asked to leave.

b) Given a player is asked to leave, find the probability that he is asked to leave after two games.

Alan, Ben and Charlie are three players having these trials.

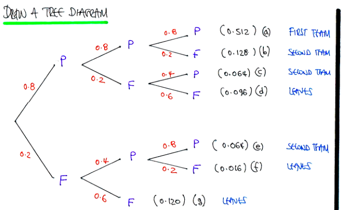
c) Assuming that their performances are independent find the probability, that …

  i. … at least one of them will be asked to leave.

  ii. … they will pass seven games between them.

$\boxed{}$, $\boxed{0.512}$, $\boxed{0.256}$, $\boxed{0.232}$, $\boxed{0.517}$, $\boxed{0.547}$, $\boxed{0.189}$

DRAW A TREE DIAGRAM



a) LOOKING AT THE DIAGRAM

I) P(FIRST TEAM) = 0.512

II) P(SECOND TEAM) = b + c + e = 0.128 + 0.064 + 0.064 = 0.256

III) P(LEAVES) = d + f + g = 0.096 + 0.016 + 0.120 = 0.232

b) P("LEAVES IN 2" | "LEAVES")

$$= \frac{g}{d+f+g} = \frac{0.120}{0.232} = \frac{15}{29} = 0.517$$

c) I) P("LEAVES") = 0.232

P("STAYS") = 0.768

REQUIRED PROBABILITY $= 1 - 0.768^3$

$= 0.547$

c) II) "PASS 7 GAMES BETWEEN THEM"

CAN ONLY HAPPEN AS

- "FIRST" | "SECOND" | "SECOND" [F S S], [S F S], [S S F]

  3 2 2

  $0.512 \times 0.256 \times 0.256 \times 3 \text{ WAYS} = 0.10066\ldots$

- "FIRST" "FIRST" "LEAVES AFTER 3 GAMES"

  3 3 1

  $0.512 \times 0.512 \times \underset{(0.096+0.016)}{0.112} \times 3 \text{ WAYS} = 0.08808\ldots$

$\therefore$ REQUIRED PROBABILITY $= 0.18874\ldots \approx 0.189$

Question 22 (*****)

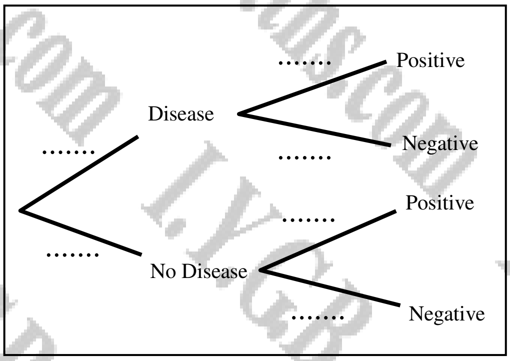
For people that spent a long time in the tropics, on their return a test is used to determine whether they have or have not contracted tropical diseases.

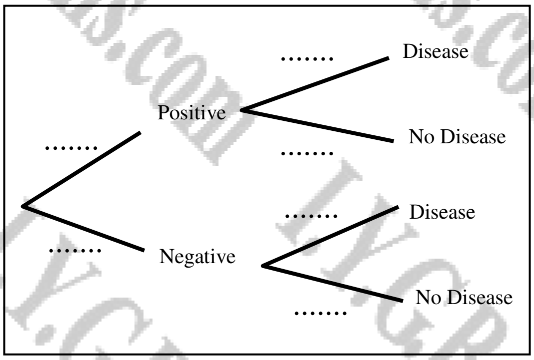
It has been established over time that $\frac{1}{10}$ of the people spending a long time in the tropics have contracted tropical diseases.

Given a person **has** contracted tropical diseases, the probability that the test will provide a positive result is $\frac{19}{20}$.

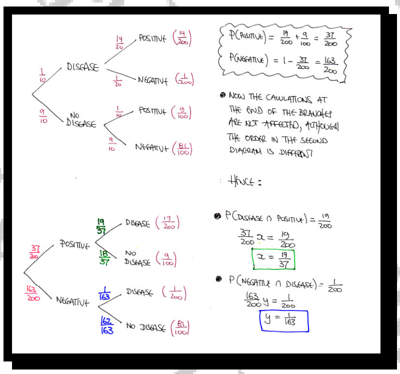
Given a person **has not** contracted tropical diseases, the probability that the test will provide a positive result is $\frac{1}{10}$.

Complete each of the following tree diagrams to illustrate the above information.





diagram

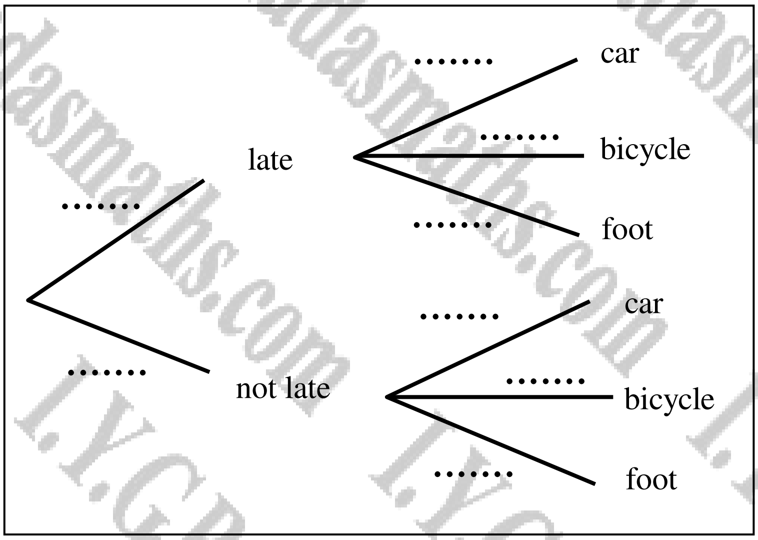


Question 23 (*****)

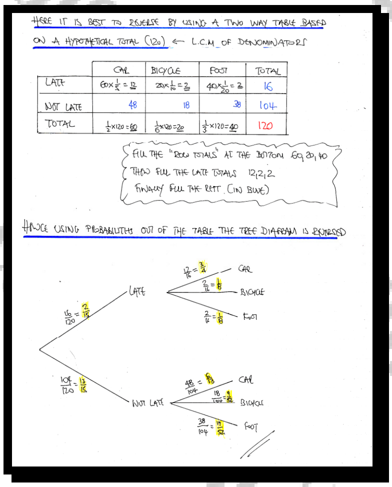
The probability that Phil goes to work by car, by bicycle or on foot are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively.

The respective probabilities of Phil being **late** when using these 3 forms of transport are $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$.

Complete the following tree diagram to illustrate the above information.



[   ], diagram



HERE IT IS BEST TO REVERSE BY USING A TWO WAY TABLE BASED ON A HYPOTHETICAL TOTAL (120) ← L.C.M OF DENOMINATORS

	CAR	BICYCLE	FOOT	TOTAL
LATE	$60 \times \frac{1}{5} = 12$	$20 \times \frac{1}{10} = 2$	$40 \times \frac{1}{20} = 2$	16
NOT LATE	48	18	38	104
TOTAL	$\frac{1}{2} \times 120 = 60$	$\frac{1}{6} \times 120 = 20$	$\frac{1}{3} \times 120 = 40$	120

FILL THE "ROW TOTALS" AT THE BOTTOM 60, 20, 40
THEN FILL THE LATE TOTALS 12, 2, 2
FINALLY FILL THE REST (IN BLUE)

HENCE USING PROBABILITIES OUT OF THE TABLE THE TREE DIAGRAM IS REVERSED

VARIOUS PROBLEMS

Question 1 (***)

A market research company is conducting a survey on pets.

There were 144 pet owners in the survey of whom 66 were dog owners, and of these dog owners 18 were cat owners too.

There were 42 pet owners that owned no dog or cat.

As an incentive for pet owners to participate in the survey, a year's free supply of pet food will be given to one of the entrants as a prize.

An entrant is drawn at random to determine the winner of the prize.

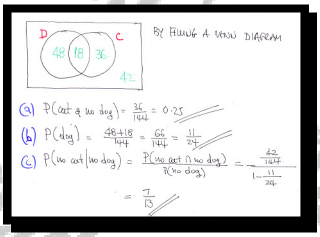
Find the probability that the prize-winner …

a) … owns a cat but does not own a dog.

b) … owns a dog.

c) … does not own a cat given he does not own a dog.

$\boxed{\frac{1}{4}}, \boxed{\frac{11}{24}}, \boxed{\frac{7}{13}}$



Question 2 (***)

People are watching a film in a cinema.

The counterfoil of their tickets was retained as they went in.

The management is going to draw two ticket counterfoils at random and the recipients of these tickets will receive free cinema tickets.

Of the people watching the film, there are 10 men, 20 women, 25 boys and 45 girls.

Calculate the probability that the two free tickets will be won by two children of the same gender.

$$\boxed{\frac{43}{165}}$$

$$\text{P(BOY, BOY)} + \text{P(GIRL, GIRL)} = \left(\frac{25}{100} \times \frac{24}{99}\right) + \left(\frac{45}{100} \times \frac{44}{99}\right) = \frac{2}{33} + \frac{1}{5} = \frac{43}{165}$$

Question 3 (***)

Two bags contain numbered cards.

Bag A contains 5 cards, numbered 1, 2, 3, 4 and 5.

Bag B contains 5 cards, numbered 2, 3, 4, 5 and 6.

A card is selected at random from bag A and placed into bag B. A card is then selected at random from bag B.

Determine the probability that …

a) … **both** the cards selected will show the number 3.

b) … **both** the cards selected will show the same number.

$$\boxed{\frac{1}{15}}, \boxed{\frac{3}{10}}$$

$$P(3\text{-}3) = \frac{1}{5} \times \frac{2}{6} = \frac{1}{15}$$

$$P(1\text{-}1, 2\text{-}2, 3\text{-}3, 4\text{-}4, 5\text{-}5) =$$

$$\left(\frac{1}{5} \times \frac{1}{6}\right) + \left(\frac{1}{5} \times \frac{2}{6}\right) + \left(\frac{1}{5} \times \frac{2}{6}\right) + \left(\frac{1}{5} \times \frac{2}{6}\right) + \left(\frac{1}{5} \times \frac{2}{6}\right)$$

$$\frac{1}{30} + \frac{2}{30} + \frac{2}{30} + \frac{2}{30} + \frac{2}{30} = \frac{9}{30} = \frac{3}{10}$$

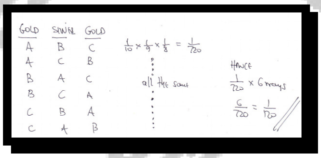
Question 4 (***)

In a school race ten boys are assumed of having **equal** chance of winning the race or finishing second or third. The boys that will finish in the first three positions will receive the customary gold, silver and bronze medal.

Adam, Ben and Carl are taking place in this race.

Calculate the probability that Adam, Ben and Carl will win the three medals.

$\boxed{\dfrac{1}{120}}$



Question 5 (***)

A school committee consists of 2 teachers, 3 boys and 4 girls.

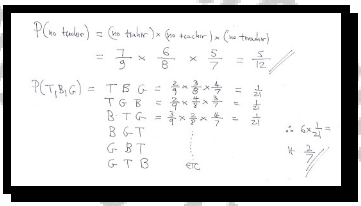
A subcommittee is to be formed by picking at random 3 members out of the original members of the school committee.

Calculate the probability that the subcommittee will contain …

a) … no teachers.

b) … one teacher, one boy and one girl.

$\boxed{\dfrac{5}{12}}$, $\boxed{\dfrac{2}{7}}$



Question 6 (***)

A box contains 6 jam doughnuts, 5 cream doughnuts and 4 plain doughnuts. The doughnuts all look the same on the outside. Three children pick a doughnut each.

a) Find the probability that the three doughnuts the children picked are all …

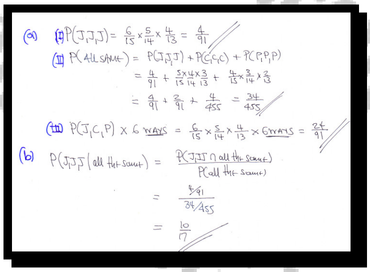
i. … jam doughnuts.

ii. … of the same type.

iii. … of different type.

b) Given that the doughnuts the children picked are all of the same type, find the probability they contained jam.

$\boxed{\frac{4}{91}}$, $\boxed{\frac{34}{455}}$, $\boxed{\frac{24}{91}}$, $\boxed{\frac{10}{17}}$



(a) (I) $P(J,J,J) = \frac{6}{15}\times\frac{5}{14}\times\frac{4}{13} = \frac{4}{91}$

(II) $P(\text{ALL SAME}) = P(J,J,J) + P(C,C,C) + P(P,P,P)$

$= \frac{4}{91} + \frac{5}{15}\times\frac{4}{14}\times\frac{3}{13} + \frac{4}{15}\times\frac{3}{14}\times\frac{2}{13}$

$= \frac{4}{91} + \frac{2}{91} + \frac{4}{455} = \frac{34}{455}$

(III) $P(J,C,P)\times 6 \text{ WAYS} = \frac{6}{15}\times\frac{5}{14}\times\frac{4}{13}\times 6 \text{ WAYS} = \frac{24}{91}$

(b) $P(J,J,J \mid \text{all the same}) = \frac{P(J,J,J \cap \text{all the same})}{P(\text{all the same})}$

$= \frac{4/91}{34/455}$

$= \frac{10}{17}$

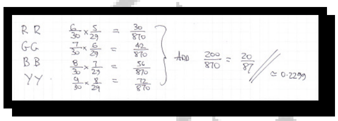
Question 7 (***)

A bag contains 6 red pegs, 7 green pegs, 8 blue pegs and 9 yellow pegs.

Andrea picks two pegs at random from the bag to hang a dress on a washing line.

Determine the probability that the pegs will be of the same colour.

$\boxed{\frac{20}{87} \approx 0.23}$



RR $\frac{6}{30}\times\frac{5}{29} = \frac{30}{870}$

GG $\frac{7}{30}\times\frac{6}{29} = \frac{42}{870}$

BB $\frac{8}{30}\times\frac{7}{29} = \frac{56}{870}$

YY $\frac{9}{30}\times\frac{8}{29} = \frac{72}{870}$

ADD $\frac{200}{870} = \frac{20}{87} \simeq 0.2299$

Question 8 (***)

A school's staff consists of 90 teachers and 10 administrators.

There are 7 female administrators and 53 female teachers.

Two members of staff are selected at random to attend a meeting.

Find the probability of the two members selected, one will be a female teacher and the other a male.

$$\boxed{\frac{212}{495} \approx 0.428}$$

FILL A TWO WAY TABLE OR SIMILAR

	ADMIN	TEACHER	TOTAL
MALE	3	37	40
FEMALE	7	53	60
TOTAL	10	90	100

P(FEMALE TEACHER – MALE) OR THE OTHER WAY ROUND

$= \frac{53}{100} \times \frac{40}{99} \times 2 \text{ WAYS} = \frac{212}{495} \approx 0.428$

Question 9 (***)

Michael opens a bag of sweets which contains 8 orange flavoured sweets and 12 lemon flavoured sweets.

Firstly, two of his friends pick a sweet at random from the bag and then Michael takes at random a sweet for himself.

Find the probability that Michael chose a lemon flavoured sweet.

$$\boxed{\frac{3}{5}}$$

BY A TREE DIAGRAM OR LISTING & NOTING THAT THERE IS NO REPLACEMENT

OOO
OOL $\leftarrow \frac{8}{20} \times \frac{7}{19} \times \frac{12}{18} = \frac{672}{6840}$
OLO
LOO
OLL $\leftarrow \frac{8}{20} \times \frac{12}{19} \times \frac{11}{18} = \frac{1056}{6840}$
LOL $\leftarrow \frac{12}{20} \times \frac{8}{19} \times \frac{11}{18} = \frac{1056}{6840}$
LLO
LLL $\leftarrow \frac{12}{20} \times \frac{11}{19} \times \frac{10}{18} = \frac{1320}{6840}$

add $= \frac{4104}{6840} = \frac{3}{5}$

Question 10 (***)

A bag contains 10 balls of which 6 are red, 3 are blue and 1 is yellow.

Three balls are selected at random without replacement.

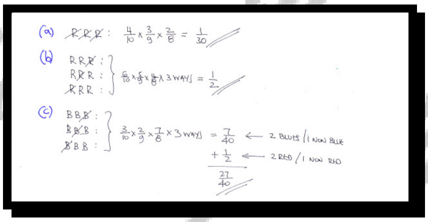
Determine the probability that …

a) … no red balls are selected.

b) … exactly 2 red balls are selected.

c) … the selection contains exactly 2 balls of the same colour.

$\boxed{\frac{1}{30}}, \boxed{\frac{1}{2}}, \boxed{\frac{27}{40}}$



(a) $\cancel{R}\cancel{R}\cancel{R}$: $\frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} = \frac{1}{30}$

(b) $RR\cancel{R}$: , $R\cancel{R}R$: , $\cancel{R}RR$: $\left.\right\}$ $\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \times 3 \text{ WAYS} = \frac{1}{2}$

(c) $BB\cancel{B}$: , $B\cancel{B}B$: , $\cancel{B}BB$: $\left.\right\}$ $\frac{3}{10} \times \frac{2}{9} \times \frac{7}{8} \times 3 \text{ WAYS} = \frac{7}{40}$ ← 2 BLUES / 1 NON BLUE

$+ \frac{1}{2}$ ← 2 RED / 1 NON RED

$\frac{27}{40}$

Question 11 (***)

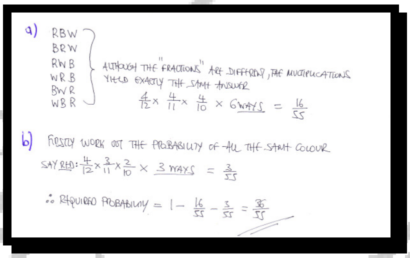
A box contains 4 black balls, 4 white balls and 4 red balls.

Margot takes three balls at random out of the box and places them on a table.

a) Determine the probability that a ball of each colour was picked.

b) Hence find the probability that two of the balls have the same colour but the third ball has different colour.

$\boxed{\frac{16}{55}}, \boxed{\frac{36}{55}}$



a) RBW, BRW, RWB, WRB, BWR, WBR

ALTHOUGH THE "FRACTIONS" ARE DIFFERENT, THE MULTIPLICATIONS YIELD EXACTLY THE SAME ANSWER

$\frac{4}{12} \times \frac{4}{11} \times \frac{4}{10} \times 6 \text{ WAYS} = \frac{16}{55}$

b) FIRSTLY WORK OUT THE PROBABILITY OF ALL THE SAME COLOUR

SAY RED: $\frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} \times 3 \text{ WAYS} = \frac{3}{55}$

∴ REQUIRED PROBABILITY $= 1 - \frac{16}{55} - \frac{3}{55} = \frac{36}{55}$

Question 12 (***+)

Marcia picks at random three bags of crisps, one for herself and two for her two friends.

She picks the three bags from a multi-pack which contains 6 ready salted flavour, 4 cheese and onion flavour and 2 salt and vinegar flavour.

Determine the probability that the three packs she picks are…

a) … all ready salted flavour.

b) … all of the same type.

c) … one of each type.

$\boxed{\frac{1}{11}}, \boxed{\frac{6}{55}}, \boxed{\frac{12}{55}}$

(a) $P[\text{READY SALTED} \times \text{READY SALTED} \times \text{READY SALTED}] = \frac{6}{12} \times \frac{5}{11} \times \frac{4}{10} = \frac{1}{11}$

(b) $P[C\&O \times C\&O \times C\&O] = \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} = \frac{1}{55}$

IT IS IMPOSIBLE TO PICK 3 SALT & VINEGAR

HENCE $P(\text{ALL OF THE SAME TYPE}) = \frac{1}{11} + \frac{1}{55} = \frac{6}{55}$

(c) $P(\text{ONE OF EACH TYPE}) = \text{READY SALTED} \times \text{CHEESE \& ONION} \times \text{SALT \& VINEGAR} \times \underline{\text{6 WAYS}}$

$= \left(\frac{6}{12} \times \frac{4}{11} \times \frac{2}{10}\right) \times 6$

$= \frac{12}{55}$

Question 13 (*+)**

A box contains 5 red and 4 black balls.

Three balls are selected from the box and are not replaced.

Determine the probability that …

a) … all three balls selected are red.

b) … more black balls are selected that red balls.

$\boxed{\frac{5}{42}}, \boxed{\frac{17}{42}}$

(a) $P(\text{red},\text{red},\text{red}) = \frac{5}{9}\times\frac{4}{8}\times\frac{3}{7} = \frac{60}{504} = \frac{5}{42}$

(b) $P(\text{Black},\text{Black},\text{Black}) = \frac{4}{9}\times\frac{3}{8}\times\frac{2}{7} = \frac{24}{504}$

$P(\text{Black},\text{Black},\text{Red}) = \frac{4}{9}\times\frac{3}{8}\times\frac{5}{7} = \frac{60}{504}$

$P(\text{Black},\text{Red},\text{Black}) = \frac{4}{9}\times\frac{5}{8}\times\frac{3}{7} = \frac{60}{504}$

$P(\text{Red},\text{Black},\text{Black}) = \frac{5}{9}\times\frac{4}{8}\times\frac{3}{7} = \frac{60}{504}$

add: $\frac{204}{504} = \frac{17}{42}$

Question 14 (*+)**

Katerina and Mathew are taking part in a quiz game where they will be asked two questions each.

The probability that Katerina answers any of her questions correctly is $\frac{2}{3}$.

The probability that Mathew answers any of his questions correctly is $\frac{3}{4}$.

Determine the probability that Katerina and Mathew will answer correctly exactly three out of the four questions.

$\boxed{\frac{5}{12}}$

$\cancel{K}\ K\ M\ M \quad \frac{1}{3}\times\frac{2}{3}\times\frac{3}{4}\times\frac{3}{4} = \frac{18}{144}$

$K\ \cancel{K}\ M\ M \quad \frac{2}{3}\times\frac{1}{3}\times\frac{3}{4}\times\frac{3}{4} = \frac{18}{144}$

$K\ K\ \cancel{M}\ M \quad \frac{2}{3}\times\frac{2}{3}\times\frac{1}{4}\times\frac{3}{4} = \frac{12}{144}$

$K\ K\ M\ \cancel{M} \quad \frac{2}{3}\times\frac{2}{3}\times\frac{3}{4}\times\frac{1}{4} = \frac{12}{144}$

ADD $\frac{60}{144} = \frac{5}{12}$

Question 15 (***+)

A bag contains 4 orange flavoured sweets, 3 berry flavoured sweets and 2 strawberry flavoured sweets.

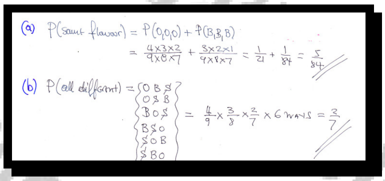
Three sweets are selected from the bag and are not replaced.

Determine the probability that …

a) … all three sweets are of the same flavour.

b) … the three sweets have different flavours.

$\boxed{\frac{5}{84}}$, $\boxed{\frac{2}{7}}$



Question 16 (***+)

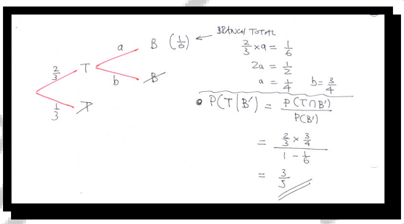
Ron is retired and lives in a country cottage. On some Saturdays he goes to town and when in town he sometimes goes to a betting shop.

The probability he goes to town on a given Saturday is $\frac{2}{3}$.

The probability he goes to town and goes to the betting shop on a given Saturday is $\frac{1}{6}$.

If Ron did not go to the betting shop one Saturday, determine the probability he did go to town that Saturday.

$\boxed{\frac{3}{5}}$



Question 17 (***+)

A box contains 4 fair coins and one coin which is double-headed.

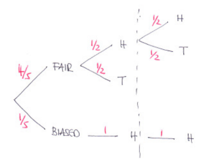
A coin is selected at random from the box and flipped once.

a) Determine the probability that a head is obtained.

b) Given that a head was obtained, find the probability that the double-headed coin was picked out of the box.

c) Given that a head was obtained, find the probability that when the selected coin is flipped again, a head will be obtained for a second time.

$\boxed{\frac{3}{5}}, \boxed{\frac{1}{3}}, \boxed{\frac{2}{3}}$



$P(H) = \left(\frac{4}{5} \times \frac{1}{2}\right) + \left(\frac{1}{5} \times 1\right) = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

$P(\text{BIASED} | \text{HEAD}) = \frac{P(\text{BIASED} \cap \text{HEAD})}{P(\text{HEAD})} = \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$

$P(\text{2ND HEAD} | \text{FIRST HEAD}) = \frac{P(\text{HEAD} - \text{HEAD})}{P(\text{FIRST HEAD})} = \frac{\left(\frac{4}{5} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{5} \times 1 \times 1\right)}{\frac{3}{5}}$

$= \frac{\frac{1}{5} + \frac{1}{5}}{\frac{3}{5}} = \frac{2}{3}$

Question 18 (****)

A bag contains four coins, of which three are fair while the fourth coin is double headed so that when this coin is tossed a head is always obtained.

One of these four coins is selected at random and tossed three times.

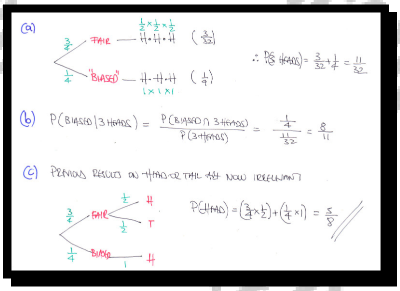
a) Find the probability that a head will be obtained on all three tosses.

b) Given three heads were obtained, determine the probability that the double headed coin was picked from the bag.

The selected coin is tossed a fourth time.

c) Find the probability that a head is obtained.

$\boxed{\frac{11}{32}}, \boxed{\frac{8}{11}}, \boxed{\frac{5}{8}}$



Question 19 (****)

Andrew and Bradley are tennis players playing each other regularly. Their matches can last up to 5 sets, the winner being the first to win 3 sets.

Andrew is assumed to have constant probability $\frac{3}{5}$ of winning any set.

a) Find the probability that Andrew will win the match in …

i. … 3 sets.

ii. … 4 sets.

b) Hence determine the probability that Andrew will win the match.

c) If Andrew wins a match, find the probability that more than 3 sets were played.

$\boxed{\frac{27}{125}}, \boxed{\frac{162}{625}}, \boxed{\frac{2133}{3125}}, \boxed{\frac{54}{79}}$

(a) (I) $AAA = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{27}{125}$

(II) $\left.\begin{matrix} BAAA \\ ABAA \\ AABA \end{matrix}\right\} = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5} \times 3 \text{ ways} = \frac{162}{625}$

(b) $\left.\begin{matrix} AABB\vdots A \\ ABBA\vdots A \\ BBAA\vdots A \\ BAAB\vdots A \\ ABAB\vdots A \\ BABA\vdots A \end{matrix}\right\} = \left(\frac{3}{5}\right)^3 \times \left(\frac{2}{5}\right)^2 \times 6 \text{ ways} = \frac{648}{3125}$

$\therefore P(\text{Andrew wins}) = \frac{27}{125} + \frac{162}{625} + \frac{648}{3125} = \frac{2133}{3125}$

(c) $P(\text{4 or 5 GAMES PLAYED} \mid \text{ANDREW WINS}) = \frac{P(\text{4 or 5 GAMES PLAYED} \cap \text{ANDREW WINS})}{P(\text{ANDREW WINS})}$

$= \frac{\frac{162}{625} + \frac{648}{3125}}{\frac{2133}{3125}}$

$= \frac{54}{79}$

Question 20 (****)

Tupac and Lutac are two friends that go weigh training on Mondays and Thursdays.

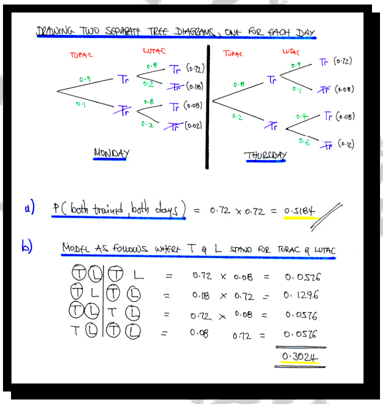
- The probability that Tupac trains on a Monday is 0.9 and on a Thursday 0.8.
- If Tupac trains on a Monday the probability that Lutac also trains is 0.8.
- If Tupac does not train on a Monday the probability that Lutac also does not train is 0.2.
- If Tupac trains on a Thursday the probability that Lutac also trains is 0.9.
- If Tupac does not train on a Thursday the probability that Lutac also does not train is 0.6.

Determine the probability that on a given week …

a) … they both trained on both days.

b) … one of them trained on both days and the other only once.

$\boxed{}$, $\boxed{0.5184}$, $\boxed{0.3024}$



Question 21 (****)

The possible scores and their corresponding probabilities obtained by a spinner are summarized in the table below.

Score	0	1	2	3	5
Probability	0.2	0.1	0.25	0.2	0.25

The spinner is spun twice and the product of their scores is recorded.

Determine the probability of the product of these scores will be less than 5.

$$\boxed{\frac{209}{400} = 0.5225}$$

FIRST THROW	SECOND THROW		
0	0,1,2,3,5	$\Rightarrow 0.2 \times 1$	$= 0.2$
1	0,1,2,3	$\Rightarrow 0.1 \times 0.75$	$= 0.075$
2	0,1,2	$\Rightarrow 0.25 \times 0.55$	$= 0.1375$
3	0,1	$\Rightarrow 0.2 \times 0.3$	$= 0.06$
5	0	$\Rightarrow 0.25 \times 0.2$	$= 0.05$

$0.5225 = \frac{209}{400}$

Question 22 (****)

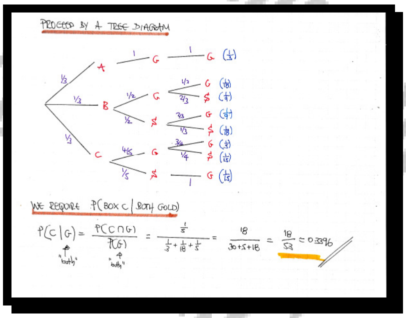
Three boxes A, B and C contain coins.

- Box A contains 3 gold coins.
- Box B contains 2 gold coins and 2 silver coins.
- Box C contains 4 gold coins and 1 silver coin.

A box is selected at random and 2 coins are selected.

Find the probability that box C was selected, if both coins selected were gold.

$\boxed{}$, $\boxed{\frac{18}{53}}$



Question 23 (****)

A market research company is conducting a survey on health, diet and exercise on the staff of a large factory.

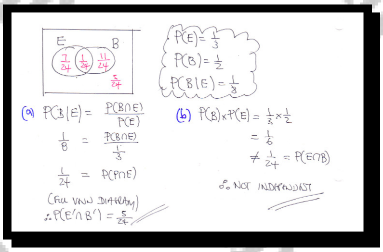
They found that $\frac{1}{3}$ of the factory workers took regular exercise and $\frac{1}{2}$ of the factory workers ate breakfast. They further found that of those factory workers who took regular exercise $\frac{1}{8}$ also ate breakfast.

A factory worker is selected at random.

a) Find the probability that the worker does not eat breakfast and does not take regular exercise.

b) Determine, with a reason, whether the events "take regular exercise" and "eat breakfast" are statistically independent.

$\boxed{\frac{5}{24}}$, $\boxed{\text{not independent}}$



Question 24 (****)

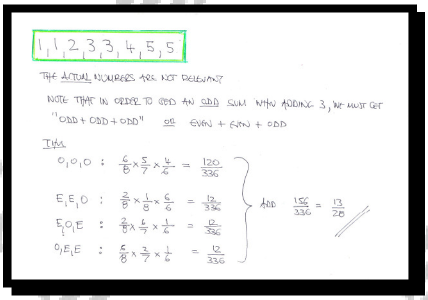
$$1,\ 1,\ 2,\ 3,\ 3,\ 4,\ 5,\ 5$$

The above eight single digit numbers are written on eight separate pieces of card.

Three cards are picked at random and the numbers they show are added together.

Find the probability that the sum of the three numbers will be an odd number.

$\boxed{\dfrac{13}{28}}$



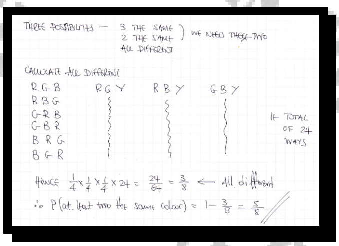
Question 25 (****)

A bag contains a **large** number of red, green, blue and yellow discs. The probability of drawing a disc of any colour is $\frac{1}{4}$

Dean picks three discs at random from the bag.

Determine the probability Dean will pick **at least** two discs of the same colour.

$\boxed{\dfrac{5}{8}}$



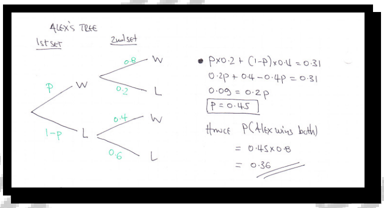
Question 26 (****)

Alex and Ben play each other **two** sets of tennis every weekend.

- If Alex wins the first set the probability he wins the second set is 0.8.
- If Ben wins the first set the probability he wins the second set is 0.6.
- The probability they win one set each is 0.31.

Determine the probability that Alex will win both sets on a given weekend.

$\boxed{0.36}$



Question 27 (****)

The probability of a biased coin showing heads is $\frac{1}{7}$.

Aaron, Benjamin, Caleb and David toss this coin repeatedly.

They start this experiment alphabetically and continue alphabetically until a head is obtained. The winner is the first person to obtain a head and once a winner is declared the experiment is over.

Determine the probability David …

a) … wins in his first toss.

b) … wins in his second toss.

c) … gets a third toss.

d) … gets exactly three tosses.

$\boxed{0.0900}$, $\boxed{0.0486}$, $\boxed{0.1835}$, $\boxed{0.0844}$

(a) $\cancel{A} \times \cancel{B} \times \cancel{C} \times D = \frac{6}{7} \times \frac{6}{7} \times \frac{6}{7} \times \frac{1}{7} = 0.0900$

(b) $\cancel{A}\cancel{B}\cancel{C}\cancel{D}$
$\cancel{A}\cancel{B}\cancel{C}D$ $= \left(\frac{6}{7}\right)^7 \times \frac{1}{7} \simeq 0.0486$

(c) $\cancel{A}\cancel{B}\cancel{C}\cancel{D}$
$\cancel{A}\cancel{B}\cancel{C}\cancel{D}$ $= \left(\frac{6}{7}\right)^{11} = 0.1835$
$\cancel{A}\cancel{B}\cancel{C}$

(d)
$\cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}\cancel{C}D \qquad \left(\frac{6}{7}\right)^{11} \times \frac{1}{7}$
$\cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}\cancel{C}\cancel{D}\ A \qquad \left(\frac{6}{7}\right)^{12} \times \frac{1}{7}$
$\cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}B \qquad \left(\frac{6}{7}\right)^{13} \times \frac{1}{7}$
$\cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}\cancel{C}\cancel{D}\ \cancel{A}\cancel{B}C \qquad \left(\frac{6}{7}\right)^{14} \times \frac{1}{7}$
Add

$$\frac{1}{7} \times \left(\frac{6}{7}\right)^{11} \left[\left(\frac{6}{7}\right)^{0} + \left(\frac{6}{7}\right)^{1} + \left(\frac{6}{7}\right)^{2} + \left(\frac{6}{7}\right)^{3}\right] = 0.0844$$

Question 28 (****)

The two way table below summarizes the sales of three different types of number of smart phones classified by their contract length in months.

	Length of Contract			
	12 month	24 month	36 month	TOTAL
Hamsung	17	16	17	50
Mokia	29	26	15	70
Ephone	30	27	23	80
TOTAL	76	69	55	200

Let A be the event that a customer chose a Hamsung smart phone.

Let B be the event that a customer chose a 24 month contract.

Let C be the event that a customer chose a Mokia smart phone.

A customer is selected at random.

a) Determine the values of …

i. … $\text{P}(B)$.

ii. … $\text{P}(A \cap B)$.

iii. … $\text{P}(A|B)$.

iv. … $\text{P}(B' \cup C')$.

[continues overleaf]

[continued from overleaf]

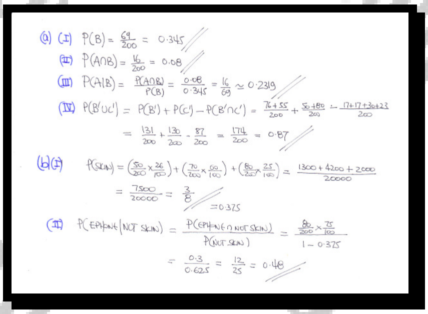
Of the customers who bought a Hamsung phone 26% bought a styling skin. The respective percentages of hose customers who bought Mokia or EPhone smart phones are 60% and 25%.

b) Find the probability that a customer selected at random bought …

i. … a skin.

ii. … a Ephone given he did **not** buy a skin.

$\boxed{\mathrm{P}(B)=\frac{69}{200}=0.345}$, $\boxed{\mathrm{P}(A\cap B)=\frac{16}{200}=0.08}$, $\boxed{\mathrm{P}(A|B)=\frac{16}{69}\approx 0.2319}$, $\boxed{\mathrm{P}(B'\cup C')=\frac{174}{200}=0.87}$, $\boxed{\mathrm{P}(\text{skin})=\frac{3}{8}=0.375}$, $\boxed{\mathrm{P}(\text{Ephone}|\text{not skin})=\frac{12}{25}=0.48}$



(a) (I) $P(B)=\frac{69}{200}=0.345$

(II) $P(A\cap B)=\frac{16}{200}=0.08$

(III) $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0.08}{0.345}=\frac{16}{69}\simeq 0.2319$

(IV) $P(B'\cup C')=P(B')+P(C')-P(B'\cap C')=\frac{76+55}{200}+\frac{50+80}{200}-\frac{17+17+30+23}{200}$
$=\frac{131}{200}+\frac{130}{200}-\frac{87}{200}=\frac{174}{200}=0.87$

(b)(I) $P(\text{SKIN})=\left(\frac{50}{200}\times\frac{26}{100}\right)+\left(\frac{70}{200}\times\frac{60}{100}\right)+\left(\frac{80}{200}\times\frac{25}{100}\right)=\frac{1300+4200+2000}{20000}$
$=\frac{7500}{20000}=\frac{3}{8}$ $=0.375$

(II) $P(\text{EPHONE}|\text{NOT SKIN})=\frac{P(\text{EPHONE}\cap\text{NOT SKIN})}{P(\text{NOT SKIN})}=\frac{\frac{80}{200}\times\frac{75}{100}}{1-0.375}$
$=\frac{0.3}{0.625}=\frac{12}{25}=0.48$

Question 29 (****)

A market research company is conducting a survey on the education of married couples, on the members of a sports club.

The probability that a husband has a degree is $\frac{7}{12}$ and the probability that a wife has a degree is $\frac{8}{15}$.

The probability that a husband does not have a degree given that his wife has, is $\frac{3}{8}$.

A couple is selected at random.

Find the probability that…

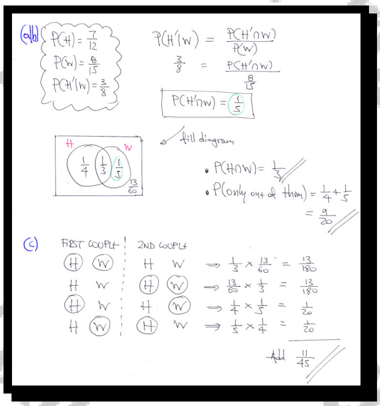
a) … both husband and wife have a degree.

b) … only one of the husband or wife has a degree.

Two couples are next selected at random.

c) Find the probability that only one of the husbands and only one of the wives will have a degree.

$\boxed{\frac{1}{3}}$, $\boxed{\frac{9}{20}}$, $\boxed{\frac{11}{45}}$



Question 30 (****)

A bag contains 25 keys of which 15 are of silver appearance and the rest are of bronze appearance. **Four** keys are to be drawn at random from the bag and placed on a mat next to each other.

Find the probability that…

a) … the **fourth** key to be drawn will be of silver appearance.

b) … two silver keys and two bronze keys will be drawn.

$\boxed{\frac{3}{5}}$, $\boxed{\frac{189}{506}}$

(a)

BBBS $\leftarrow \frac{10}{25}\times\frac{9}{24}\times\frac{8}{23}\times\frac{15}{22} = \frac{10800}{303600}$

BSSS, SBSS, SSBS $\leftarrow \frac{10}{25}\times\frac{15}{24}\times\frac{14}{23}\times\frac{13}{22}\times(3 \text{ WAYS}) = \frac{81900}{303600}$

BBSS, BSBS, SBBS $\leftarrow \frac{10}{25}\times\frac{9}{24}\times\frac{15}{23}\times\frac{14}{22}\times(3 \text{ WAYS}) = \frac{56700}{303600}$

SSSS $\leftarrow \frac{15}{25}\times\frac{14}{24}\times\frac{13}{23}\times\frac{12}{22} = \frac{32760}{303600}$

ADD TO GET $\frac{3}{5}$

(b)

BBSS, BSBS, BSSB, SBBS, SBSB, SSBB

$\frac{10}{25}\times\frac{9}{24}\times\frac{15}{23}\times\frac{14}{22}\times(6 \text{ WAYS}) = \frac{189}{506}$

Question 31 (****)

Two numbers are to be chosen at random from each of the two sets

$$\{1,2,3,4,5,6,7,8,9\} \quad \text{and} \quad \{1,2,3,4,5,6,7,8,9,11\}$$

Find the probability that…

a) … all **four** numbers to be drawn will be even.

b) … the **sum of all four** numbers will be even.

$\boxed{\frac{1}{45}}$, $\boxed{\frac{68}{135}}$

(a) $P(\text{All four even}) = \frac{4}{9} \times \frac{3}{8} \times \frac{4}{10} \times \frac{3}{9} = \frac{1}{45}$

	ODD	EVEN
1st	5	4
2nd	6	4

(b) $P(\text{sum is even})$

$$\left.\begin{aligned}
\text{All odd} &= \frac{5}{9} \times \frac{4}{8} \times \frac{6}{10} \times \frac{5}{10} = \frac{600}{6480} \\
\text{All even} &= \frac{4}{9} \times \frac{3}{8} \times \frac{4}{10} \times \frac{3}{9} = \frac{144}{6480} \\
OO-EE &= \frac{5}{9} \times \frac{4}{8} \times \frac{4}{10} \times \frac{3}{9} = \frac{240}{6480} \\
EE-OO &= \frac{4}{9} \times \frac{3}{8} \times \frac{6}{10} \times \frac{5}{9} = \frac{360}{6480} \\
(EO)-(EO) &= \left(\frac{4}{9} \times \frac{5}{8} \times 2\right) \times \left(\frac{4}{10} \times \frac{6}{9} \times 2\right) = \frac{1920}{6480}
\end{aligned}\right\} \text{ADD } \frac{3264}{6480} = \frac{68}{135}$$

2 WAYS 2 WAYS

Question 32 (****)

A spinner is numbered with 0, 1, 2, 4, 5 and X represents the number indicated by the spinner.

Adam and Ben play a game with this spinner and the object of the game is to score more than ten points. Once ten or more points are collected the game is over.

The spinner is spun successively and is not important who spins the spinner, but the score shown on it.

If the score is odd Adam collects the number of points shown and if the score is even then Ben collects the number of points. If a zero is spun nobody wins.

The spinner is biased as follows.

The $\mathrm{P}(X=1)=\mathrm{P}(X=2)=0.3$ and $\mathrm{P}(X=4)=\mathrm{P}(X=5)=0.1$.

Find the probability that …

a) … Adam wins in two spins.

b) … Ben wins in three spins.

c) … Adam wins in three spins.

$\boxed{0.01}$, $\boxed{0.01}$, $\boxed{0.018}$

	NONE	ADAM	BEN	BEN	ADAM.
x	0	1	2	4	5
$P(x)$	0.2	0.3	0.3	0.1	0.1

(a) $P(\text{ADAM WINS IN TWO}) = P(5,5) = 0.1 \times 0.1 = 0.01$

(b) $P(\text{BEN WINS IN THREE}) = P(4,4,4) + P(4,4,2) \times 3 \text{ WAYS}$
$= (0.1)^3 + 3 \times (0.1)^2 \times 0.3$
$= 0.001 + 0.009$
$= 0.01$

(c) $P(\text{ADAM WINS IN } 3) = P(\text{NO ADAM}, 5, 5) \quad 0.6 \times 0.1 \times 0.1 = 0.006$
$+ P(5, \text{NO ADAM}, 5) \quad 0.1 \times 0.6 \times 0.1 = 0.006$
$+ P(1, 5, 5) = 0.3 \times 0.1 \times 0.1 = 0.003$
$+ P(5, 1, 5) = 0.1 \times 0.3 \times 0.1 = 0.003$
0.018

OR BETTER

$P(\text{ADAM IN } 3) = P(\cancel{5}, 5, 5) + P(5, \cancel{5}, 5)$
$= (0.9 \times 0.1 \times 0.1) \times 2 \text{ WAYS}$
$= 0.009 \times 2$
$= 0.018$

Question 33 (****)

A box contains 5 balls of which 2 are white.

Balls are drawn from the box one after the other, without being replaced, until both the white balls are picked.

If the second ball picked is white, determine the probability that exactly 4 balls were picked out of the box.

$\boxed{}$, $\boxed{0.25}$

START BY RECORDING ALL THE OUTCOMES IN AN ORGANIZED WAY, INCLUDING PROBABILITIES

WW $\qquad \frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$

~~W~~WW, W~~W~~W $\qquad \left(\frac{3}{5}\times\frac{2}{4}\times\frac{1}{3}\right)\times 2=\frac{2}{10}$ OR $\frac{1}{10}$ EACH "BRANCH"

~~WW~~WW, ~~W~~W~~W~~W, W~~WW~~W $\qquad \left(\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2}\right)\times 3=\frac{3}{10}$ OR $\frac{1}{10}$ EACH "BRANCH"

W~~WWW~~W, ~~W~~W~~WW~~W, ~~WW~~W~~W~~W, ~~WWW~~WW $\qquad \left(\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2}\times 1\right)\times 4=\frac{4}{10}$ OR $\frac{1}{10}$ EACH "BRANCH"

NOW WE PICK OUT OF ALL THE BRANCHES THAT HAVE W IN THE SECOND SLOT / NOTE ALL BRANCHES TURNED OUT TO HAVE PROBABILITY OF $\frac{1}{10}$

$$\Longrightarrow \text{REQUIRED PROBABILITY} = \frac{\frac{1}{10}}{\frac{1}{10}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}} = \frac{1}{4}$$

Question 34 (****)

Two players, A and B, are taking penalty shots in turn, with A starting first.

The "winner" is the first player to score 2 goals.

The probability of A scoring a goal is 0.4.

The probability of B scoring a goal is 0.7.

You may assume all probabilities are independent of one another.

a) Determine the probability that A wins with his second shot.

b) Determine the probability that B wins with his second shot.

$\boxed{}$, $\boxed{0.16}$, $\boxed{0.4116}$

MODEL AS FOLLOWS AND NOTE THE RULE IS NOT UP TO TWO PENALTIES EACH BUT TAKE SHOTS IN TURN UNTIL YOU SCORE TWO GOALS

a) "A WINS" (on 2nd shot) $\Longrightarrow$ $\overset{\checkmark}{A}\ \overset{\checkmark\times}{B}\ \overset{\checkmark}{A} = 0.4 \times 1 \times 0.4 = 0.16$

b) "B WINS" (on 2nd shot) $\Longrightarrow$

$\overset{\checkmark}{A}\ \overset{\checkmark}{B}\ \overset{\times}{A}\ \overset{\checkmark}{B} = 0.4 \times 0.7 \times 0.6 \times 0.7 = 0.1176$

$\overset{\times}{A}\ \overset{\checkmark}{B}\ \overset{\checkmark}{A}\ \overset{\checkmark}{B} = 0.6 \times 0.7 \times 0.4 \times 0.7 = 0.1176$

$\overset{\times}{A}\ \overset{\checkmark}{B}\ \overset{\times}{A}\ \overset{\checkmark}{B} = 0.6 \times 0.7 \times 0.6 \times 0.7 = 0.1764$

$\therefore$ REQUIRED PROBABILITY $= 0.4116$

Question 35 (****+)

A box contains red and blue cards only, in the ratio $3 : 2$.

Two cards are selected from the box, without replacement.

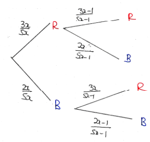
If the probability of selecting different colour cards is $\frac{1}{2}$, use algebra to find the total number of cards in the box.

$\boxed{}$, $\boxed{25}$

AS THE RATIO RED : BLUE IS 3:2, LET THE NUMBER OF...

RED BALLS BE $3x$
BLUE BALLS BE $2x$
TOTAL BALLS BE $5x$

THUS WE NOW HAVE



THUS WE NOW HAVE

$$\Rightarrow \frac{3x}{5x} \times \frac{2x}{5x-1} + \frac{2x}{5x} \times \frac{3x}{5x-1} = \frac{1}{2}$$

$$\Rightarrow \frac{3}{5} \times \frac{2x}{5x-1} + \frac{2}{5} \times \frac{3x}{5x-1} = \frac{1}{2}$$

$$\Rightarrow \frac{12x}{5x-1} + \frac{12x}{5x-1} = 5$$

$$\Rightarrow \frac{24x}{5x-1} = 5$$

$$\Rightarrow 24x = 25x - 5$$

$$\Rightarrow 5 = x$$

$\therefore$ TOTAL NUMBER IS $5x$

$\therefore$ 25 IN TOTAL

Question 36 (****+)

At a college course 75% of the students are male and 25% are female.

It is further known that 60% of the male students own a bike and 40% of the female students own a bike.

A student is selected at random.

Given that the student selected owns a bike determine the probability that this students is female ...

a) ... by a two way table.

b) ... by a tree diagram.

c) ... by a Venn diagram.

$\boxed{}$, $\boxed{\frac{2}{11}}$

- BEST METHOD TO APPROACH THE PROBLEM IS BY A TWO WAY TABLE

	BIKE	NO BIKE	TOTAL
MALE	45	30	75
FEMALE	10	15	25
TOTAL	55	45	100

75% ARE MALE

25% ARE FEMALE

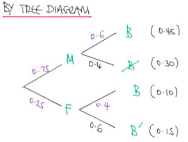
60% OF 75

40% OF 25

SAY THERE WERE 100 STUDENTS IN TOTAL

HENCE $P(\text{FEMALE}|\text{BIKE}) = \frac{10}{55} = \frac{2}{11}$

- BY TREE DIAGRAM



- $P(\text{FEMALE}|\text{BIKE})$

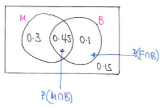
$= \frac{P(\text{FEMALE} \cap \text{BIKE})}{P(\text{BIKE})}$

$= \frac{0.10}{0.10+0.45}$

$= \frac{0.10}{0.55} = \frac{2}{11}$

- BY A VENN DIAGRAM

- $P(M) = 0.75$
- $P(F) = 0.25$
- $P(B|M) = 0.6 \Longrightarrow \frac{P(B \cap M)}{P(M)} = 0.6 \Longrightarrow P(B \cap M) = 0.6 \times 0.75 = 0.45$
- $P(B|F) = 0.4 \Longrightarrow \frac{P(B \cap F)}{P(F)} = 0.4 \Longrightarrow P(B \cap F) = 0.4 \times 0.25 = 0.1$



$\therefore P(F|B) = \frac{P(F \cap B)}{P(B)} = \frac{0.1}{0.45+0.1} = \frac{0.1}{0.55} = \frac{2}{11}$

Question 37 (****+)

A random generator of numbers is programmed to produce **three positive real numbers** X_1, X_2 and X_3, such that $X_1 + X_2 + X_3 = 180$.

Joel uses these numbers to represent the three angles of a triangle.

Given that $X_1 = 10$, determine the probability that the remaining two numbers will be such, so that the resulting triangle will have an obtuse angle.

$\boxed{\frac{16}{17}}$

$X_1 + X_2 + X_3 = 180$

$\boxed{X_2 + X_3 = 170}$

NOW IF $X_2 < 80 \implies X_3 > 90$ IF TRIANGLE HAS OBTUSE ANGLE

IF $X_2 > 90$ TRIANGLE HAS OBTUSE ANGLE

$P(X_2 < 80) = \frac{80}{170} = \frac{8}{17}$

$P(X_2 > 90) = \frac{170-90}{170} = \frac{80}{170} = \frac{8}{17}$

ADD $\frac{16}{17}$

Question 38 (****+)

A fair six sided die is rolled repeatedly and number of "*sixes*" is recorded.

Determine the probability that the **third** "*six*" will be obtained in the 7^{th} attempt.

$\boxed{\frac{3125}{93312} \approx 0.0335}$

- START WITH AN ORGANIZED TABLE OF OUTCOMES WHERE $V=6$, $X=1,2,3,4,5$

1st	2ND	3rd	4th	5th	6th	7th.
V	V	×	×	×	×	V
V	×	V	×	×	×	V
V	×	×	V	×	×	V
V	×	×	×	V	×	V
V	×	×	×	×	V	V
×	V	V	×	×	×	V
×	V	×	V	×	×	V
×	V	×	×	V	×	V
×	V	×	×	×	V	V
×	×	V	V	×	×	V
×	×	V	×	V	×	V
×	×	V	×	×	V	V
×	×	×	V	V	×	V
×	×	×	V	×	V	V
×	×	×	×	V	V	V

15 OUTCOMES

- AS ALL THE PROBABILITIES ARE THE SAME EXCEPT THE ORDER, WE HAVE

$\left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^4 \times 15 \text{ WAYS} = \frac{3125}{93312} \approx 0.0335$

Question 39 (****+)

A regular daily bus service leaves the Voder bus Station and arrives in the Hostend bus station some time later on the same day.

From its scheduled departure time from Voder, the bus will either leave on time or late but never early.

From its scheduled arrival time to Hostend, the bus will either arrive early, on time or late.

If the bus leaves Voder on time, the probability it arrives in Hostend early is 4%, on time 52% and late 21%.

The probability that the bus arrives in Hostend early is 6% and on time 69%.

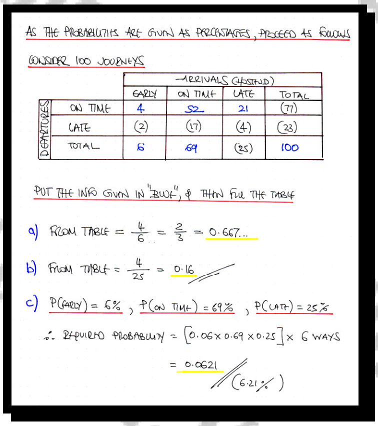
a) Given that the bus arrives early in Hostend, determine the probability that it left Voder on time.

b) Given that the bus arrives late in Hostend, find the probability that it left Voder late.

Three consecutive days for this bus service are monitored.

c) Find the probability that these three days, the bus arrives early once, late once and on time once.

$\boxed{\quad}$, $\boxed{\frac{2}{3}}$, $\boxed{0.16}$, $\boxed{0.0621}$



AS THE PROBABILITIES ARE GIVEN AS PERCENTAGES, PROCEED AS FOLLOWS

CONSIDER 100 JOURNEYS

DEPARTURES	ARRIVALS (HOSTEND) EARLY	ON TIME	LATE	TOTAL
ON TIME	4	52	21	(77)
LATE	(2)	(17)	(4)	(23)
TOTAL	6	69	(25)	100

PUT THE INFO GIVEN IN "BLUE", & THEN FILL THE TABLE

a) FROM TABLE $= \frac{4}{6} = \frac{2}{3} = 0.667\ldots$

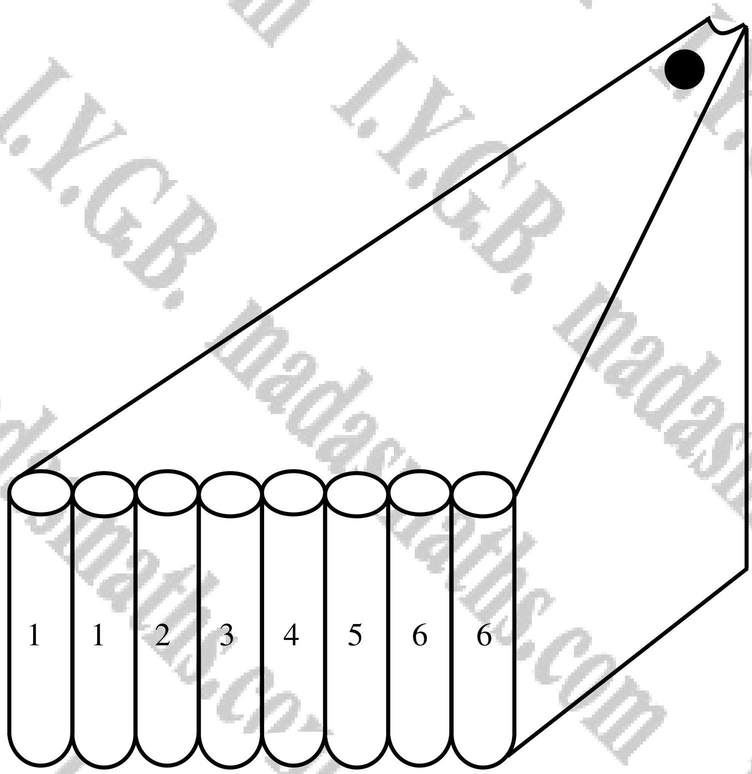
b) FROM TABLE $= \frac{4}{25} = 0.16$

c) $P(\text{EARLY}) = 6\%$, $P(\text{ON TIME}) = 69\%$, $P(\text{LATE}) = 25\%$

$\therefore$ REQUIRED PROBABILITY $= [0.06 \times 0.69 \times 0.25] \times 6$ WAYS

$= 0.0621$ (6.21%)

Question 40 (*****)



The figure below shows a contraption where balls are rolling down a slope and fall into one of 8 vertical tubes. There is **equal chance** of a ball falling into any of these 8 vertical tubes. The ball scores according to which tube it falls in, and these scores are marked clearly in the figure below.

Two balls are rolled in succession.

Determine the probability that …

a) … their sum is even.

b) … the second ball will score higher than the first.

c) … their sum is even and the second ball will score higher than the first.

[continues overleaf]

[continued from overleaf]

Three balls are rolled in succession.

Determine the probability that …

d) … they will fall into different tubes.

e) … their scores will be different.

$\boxed{}$, $\boxed{\frac{1}{2}}$, $\boxed{\frac{13}{32}}$, $\boxed{\frac{5}{32}}$, $\boxed{\frac{21}{32}}$, $\boxed{\frac{33}{64}}$

a) SORTING OUT THE OUTCOMES

$$P(\text{SUM OF TWO IS EVEN}) = P(\text{EVEN},\text{EVEN}) + P(\text{ODD},\text{ODD})$$
$$= \left(\tfrac{1}{2}\times\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\times\tfrac{1}{2}\right)$$
$$= \tfrac{1}{2}$$

b) LISTING THE DESIRED OUTCOMES

$$P(2^{\text{ND}} > 1^{\text{ST}}) = P(6, 1\text{ to }5) + P(5, 1\text{ to }4) + P(4, 1\text{ to }3) + P(3, 1\text{ or }2) + P(2,1)$$
$$= \left(\tfrac{1}{4}\times\tfrac{3}{4}\right) + \left(\tfrac{1}{8}\times\tfrac{5}{8}\right) + \left(\tfrac{1}{8}\times\tfrac{1}{2}\right) + \left(\tfrac{1}{8}\times\tfrac{3}{8}\right) + \left(\tfrac{1}{8}\times\tfrac{1}{4}\right)$$
$$= \tfrac{13}{32}$$

ALTERNATIVE FOR THIS PART

$$P(\text{"SAME"}) = \underbrace{P(1,1) + P(6,6)}_{\left(\frac{1}{4}\times\frac{1}{4}\right)\times 2\text{ WAYS}} + \underbrace{P(2,2) + P(3,3) + P(4,4) + P(5,5)}_{\frac{1}{8}\times\frac{1}{8}\times 4\text{ WAYS}}$$
$$= \tfrac{1}{8} + \tfrac{1}{16}$$
$$= \tfrac{3}{16}$$

BY SYMMETRY AS $P(2^{\text{ND}} > 1^{\text{ST}}) = P(1^{\text{ST}} > 2^{\text{ND}})$ THE REQUIRED ANSWER WILL BE

$$\tfrac{1}{2}\left(1 - \tfrac{3}{16}\right) = \tfrac{13}{32} \quad \text{AS BEFORE}$$

c) SENSIBLE TO LIST AGAIN

$$P(\text{SUM IS EVEN \& HIGHER ON THE } 2^{\text{ND}}) = P(1,3) + P(1,5) + P(2,4) + P(2,6) + P(3,5) + P(4,6)$$
$$= \frac{(2\times1)+(2\times1)+(1\times1)+(1\times2)+(1\times1)+(1\times2)}{8\times8}$$
$$= \frac{2+2+1+2+1+2}{64} = \frac{10}{64} = \frac{5}{32}$$

d)
$$P(\text{DIFFERENT TUBES}) = \underset{\substack{\text{CHOICES}\\\text{OUT OF 8}}}{\cancel{\tfrac{8}{8}}} \times \underset{\substack{\text{CHOICES}\\\text{OUT OF 8}}}{\tfrac{7}{8}} \times \underset{\substack{\text{CHOICES}\\\text{OUT OF 8}}}{\tfrac{6}{8}} = \frac{42}{64} = \frac{21}{32}$$

e) ORGANISED OUTCOMES AGAIN – BEST TO WORK WITH COMPLEMENT

1,1 WITH 2,3,4,5	$\left(\frac{2}{8}\times\frac{2}{8}\times\frac{1}{8}\times 3\text{ WAYS}\right)\times 4 =$	$\frac{48}{512}$
2,2 WITH 3,4,5 3,3 WITH 2,4,5 4,4 WITH 2,3,5 5,5 WITH 2,3,4	$\left[\left(\frac{1}{8}\times\frac{1}{8}\times\frac{1}{8}\times 3\text{ WAYS}\right)\times 3\right]\times 4 =$	$\frac{36}{512}$
6,6 WITH 2,3,4,5	$\left(\frac{2}{8}\times\frac{2}{8}\times\frac{1}{8}\times 3\text{ WAYS}\right)\times 4 =$	$\frac{48}{512}$
6,6,1 OR 6,1,1	$\left(\frac{2}{8}\times\frac{2}{8}\times\frac{2}{8}\times 3\text{ WAYS}\right)\times 2 =$	$\frac{48}{512}$
2,2 WITH 1 OR 6 3,3 WITH 1 OR 6 4,4 WITH 1 OR 6 5,5 WITH 1 OR 6	$\left(\frac{1}{8}\times\frac{1}{8}\times\frac{2}{8}\times 3\text{ WAYS}\right)\times 2\times 4 =$	$\frac{48}{512}$

THE ABOVE OUTCOMES ARE "TWO THE SAME" – WE ALSO HAVE ALL 3 THE SAME

1,1,1 OR 6,6,6	$\frac{2}{8}\times\frac{2}{8}\times\frac{2}{8}\times 2 =$	$\frac{16}{512}$
2,2,2 TO 5,5,5	$\frac{1}{8}\times\frac{1}{8}\times\frac{1}{8}\times 4 =$	$\frac{4}{512}$

ADDING ALL THE PROBABILITIES FOUND

$$\frac{48+36+48+48+48+16+4}{512} = \frac{248}{512}$$

HENCE WE HAVE

$$P(\text{ALL 3 DIFFERENT SCORES}) = 1 - \frac{248}{512}$$
$$= \frac{264}{512}$$
$$= \frac{33}{64}$$