C1 IYGB PAPER W

1. a)

$$3^y = \frac{\sqrt{3}}{9}$$

$$3^y = \frac{3^{\frac{1}{2}}}{3^2}$$

$$3^y = 3^{-\frac{3}{2}}$$

$$y = -\frac{3}{2}$$

b)

$$\sqrt{525} = \sqrt{25 \times 21} = 5\sqrt{21} = 5\sqrt{7}\sqrt{3}$$

2. a)

$$\frac{y_2 - y_1}{x_2 - x_1} = -2$$

$$\Rightarrow \frac{-11-k}{-2-(-7)} = -2$$

$$\Rightarrow \frac{-11-k}{5} = -2$$

$$\Rightarrow -11-k = -10$$

$$\Rightarrow -1 = k$$

$$\Rightarrow k = -1$$

b)

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (a, b)$$

$$\left(\frac{a+4}{2}, \frac{-3+1}{2}\right) = (a, b)$$

$$\therefore \frac{a+4}{2} = a$$

$$a+4 = 2a$$

$$a = 4$$

$$b = -1$$

c)

$$\sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{17}$$

$$\Rightarrow \sqrt{(c-7)^2 + (-3-(-7))^2} = \sqrt{17}$$

$$\Rightarrow \sqrt{(c-7)^2 + 16} = \sqrt{17}$$

$$\Rightarrow (c-7)^2 + 16 = 17$$

$$\Rightarrow (c-7)^2 = 1$$

$$c - 7 = \begin{cases} 1 \\ -1 \end{cases}$$

$$c = \begin{cases} 8 \\ 6 \end{cases}$$

3. a)

$$f'(x) = (x-3)(3x-1)$$

$$f'(x) = 3x^2 - x - 9x + 3$$

$$f'(x) = 3x^2 - 10x + 3$$

$$f(x) = \int 3x^2 - 10x + 3 \, dx$$

$$\boxed{f(x) = x^3 - 5x^2 + 3x + C}$$

$$(2,3) \Rightarrow 3 = 2^3 - 5 \times 2^2 + 3 \times 2 + C$$

$$\Rightarrow 3 = 8 - 20 + 6 + C$$

$$\Rightarrow 9 = C$$

$$\therefore f(x) = x^3 - 5x^2 + 3x + 9$$

b) BY INSPECTION

$$(x+k)(x-3)^2$$

$$= (x+k)(x^2 - 6x + 9)$$

$$\therefore k = 1$$

$$\therefore (x+1)(x^2 - 6x + 9)$$

$$= x^3 - 6x^2 + 9x$$
$$\quad + x^2 - 6x + 9$$
$$x^3 - 5x^2 + 3x + 9$$

$$\therefore f(x) = (x+1)(x-3)^2$$

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c)

4. a) $\sum_{r=1}^{n} u_r = 7 + 3n^2$

$S_4 \rightarrow \sum_{r=1}^{4} u_r = 7 + 3\times 4^2 = 7 + (3\times 16) = 7 + 48 = 55$

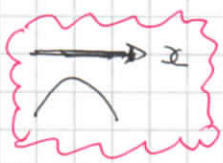
b) $S_5 \rightarrow \sum_{r=1}^{5} u_r = 7 + 3\times 5^2 = 7 + (3\times 25) = 7 + 75 = 82$

$\therefore u_5 = S_5 - S_4 = 82 - 55 = 27$

5. $y = k(2x^2 - x + 1) - 5x^2 + x - 2$

$\Rightarrow y = 2kx^2 - kx + k - 5x^2 + x - 2$

$\Rightarrow y = (2k-5)x^2 + (1-k)x + (k-2)$

BELOW THE x AXIS $\Rightarrow$ 

$\therefore$ $2k - 5 < 0$ AND $b^2 - 4ac < 0$

THUS $(1-k)^2 - 4(2k-5)(k-2) < 0$

$1 - 2k + k^2 - 4(2k^2 - 9k + 10) < 0$

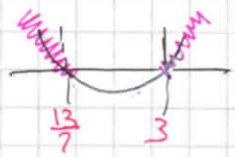
$1 - 2k + k^2 - 8k^2 + 36k - 40 < 0$

$-7k^2 + 34k - 39 < 0$

$7k^2 - 34k + 39 > 0$

$(7k - 13)(k - 3) > 0$

C.V. = $13/7$, 3



$k < \frac{13}{7}$ OR $k > 3$ BUT $2k - 5 < 0$

$2k < 5$

$k < \frac{5}{2}$

$\therefore k < \frac{13}{7}$ //

6. $t_{n+1} = at_n + b$ $\quad t_1 = 2$, $t_2 = 3$

- $t_2 = at_1 + b$

$3 = a \times 2 + b$

$\boxed{2a + b = 3}$

- $t_1 + t_2 + t_3 = 12$

$2 + 3 + [at_2 + b] = 12$

$5 + [3a + b] = 12$

$\boxed{3a + b = 7}$

$3a + b = 7$

$2a + b = 3$

$\therefore a = 4$

$b = -5$ //

7. $y = 2x^3 - 5x^2 + a$

$\frac{dy}{dx} = 6x^2 - 10x$

- $\left.\frac{dy}{dx}\right|_{x=2} = 6 \times 2^2 - 10 \times 2 = 24 - 20 = 4$
- $\left.y\right|_{x=2} = 2 \times 2^3 - 5 \times 2^2 + a = a - 4$
- $\left.\frac{dy}{dx}\right|_{x=1} = 6 \times 1^2 - 10 \times 1 = 6 - 10 = -4$
- $\left.y\right|_{x=1} = 2 \times 1^3 - 5 \times 1^2 + a = a - 3$

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- EQUATION OF TANGENT AT $(2, a-4)$

$y-(a-4)=4(x-2)$

$y-a+4=4x-8$

$\boxed{y=4x+a-12}$

- EQUATION OF NORMAL AT $(1, a-3)$

$y-(a-3)=\frac{1}{4}(x-1)$

$y-a+3=\frac{1}{4}(x-1)$

$4y-4a+12=x-1$

$\boxed{4y=x+4a-13}$

MEET ON THE x AXIS IMPLIES $y=0$

WHEN x & a IS THE SAME ON BOTH EQUATIONS

$4x+a=12$

$\boxed{a=12-4x}$

$x+4a=13$

$x+4(12-4x)=13$

$x+48-16x=13$

$35=15x$

$x=\frac{35}{15}=\frac{7}{3}$

$\therefore Q\left(\frac{7}{3},0\right)$

& $a=12-4\left(\frac{7}{3}\right)$

$a=12-\frac{28}{3}=\frac{36-28}{3}$

$a=\frac{8}{3}$

8. a) $a=10$, $d=2$, $h=12$

- $U_n=a+(n-1)d$

$U_{12}=10+11\times 2$

$U_{12}=32$

- $S_n=\frac{n}{2}[a+l]$

$S_{12}=\frac{12}{2}[10+32]$

$S_{12}=6\times 42$

$S_{12}=240+12=252$

$\therefore$ TOTAL $=252+125=377$

b)

WEEK	IN	OUT	NET
1	10	-3	7
2	12	-3	9
3	14	-3	11
4	16	-3	13
	ETC		

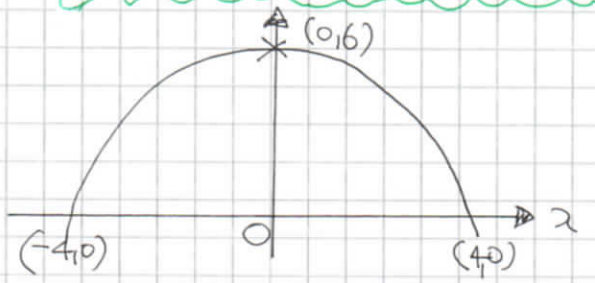
- NEW A.P WITH $a = 7$, $d = 2$
- $600 - 125 = 475$ ← NEW TOTAL
- $S_n = 475$

$$\frac{n}{2}[2\times 7 + 2(n-1)] = 475$$
$$\frac{n}{2}[14 + 2n - 2] = 475$$
$$\frac{n}{2}[12 + 2n] = 475$$
$$n(n+6) = 475$$
$$\therefore n = 19$$

BY TRIAL

$n=5$	$5\times 11 = 55$
$n=9$	$9\times 15 = 135$
$n=15$	$15\times 21 = 315$
$n=19$	$19\times 25 = 475$

9. MODEL BY $y = A - Bx^2$ $\quad A, B > 0$



- BY INSPECTION $A = 6$
- $y = 6 - Bx^2$
  $0 = 6 - B\times 4^2$
  $0 = 6 - 16B$
  $16B = 6$
  $B = \frac{3}{8}$

$\therefore \boxed{y = 6 - \frac{3}{8}x^2}$

WHEN $x = 3$, $y = 6 - \frac{3}{8}\times 3^2$
$y = 6 - \frac{27}{8} = \frac{21}{8} > 2$

$\therefore$ IT WILL PASS

THE LOAD MUST PASS THROUGH THE MIDDLE IN A SYMMETRICAL FASHION