IYGB GCE

Core Mathematics C4

Advanced

Practice Paper E

Difficulty Rating: 3.2600 /1.4599

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Edexcel Syllabus.
The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Show clearly that

$$\int_0^{\frac{1}{3}} x\mathrm{e}^{3x}\ dx = \frac{1}{9}. \qquad (\mathbf{4})$$

Question 2

The volume, V cm^3, of a sphere is given by

$$V = \frac{4}{3}\pi r^3,$$

where r is its radius.

The radius of a sphere is increasing at the constant rate of $2.5\ \text{cm}\,\text{s}^{-1}$.

Determine the rate at which the volume of the sphere is increasing when its radius has reached 8 cm. $(\mathbf{4})$

Question 3

A curve C has implicit equation

$$x^2 - 4xy + y^2 = 13.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{x-2y}{2x-y}. \qquad (\mathbf{4})$$

The points A and B are the two points on C whose x coordinate is 2.

b) Find the y coordinates of A and B. $(\mathbf{3})$

The tangents to C at A and B, meet at the point P.

c) Find the exact coordinates of P. $(\mathbf{5})$

Question 4

In the series expansion of

$$(1+ax)^n,\ |ax|<1,\ \ a,n\in\mathbb{R},$$

the coefficient of x is 15 and the coefficients of x^2 and x^3 are equal.

a) Given that n is not a positive integer, show that $a=6$. **(6)**

b) Find the value of n. **(1)**

c) Determine the coefficient of x^4. **(1)**

Question 5

Solve the differential equation

$$\frac{dy}{dx}=\frac{5y}{(2+x)(1-2x)}$$

subject to the condition $y=2$ at $x=0$, giving the answer in the form $y=f(x)$. **(10)**

Question 6

The points with coordinates $A(8,0,12)$ and $B(9,-2,14)$ are given.

a) Find a vector equation of the straight line l_1 that passes through A and B. $(\mathbf{3})$

The straight line l_2 has equation

$$\mathbf{r} = \mathbf{i} + 9\mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j}),$$

where μ is a scalar parameter.

b) Show that l_1 and l_2 are perpendicular. $(\mathbf{2})$

c) Show further that l_1 and l_2 intersect at some point P and state the coordinates of P. $(\mathbf{6})$

The point $C(9,13,2)$ lies on l_2 and the point D is the reflection of C about l_1.

d) Determine the coordinates of D. $(\mathbf{2})$

Question 7

x	0	$\frac{2\pi}{5}$	$\frac{4\pi}{5}$	$\frac{6\pi}{5}$	$\frac{8\pi}{5}$	2π
y	0	0.2031	0.8602			0

The table above shows some tabulated values for the equation

$$y = \sin^3\left(\tfrac{1}{2}x\right),\ 0 \le x \le 2\pi .$$

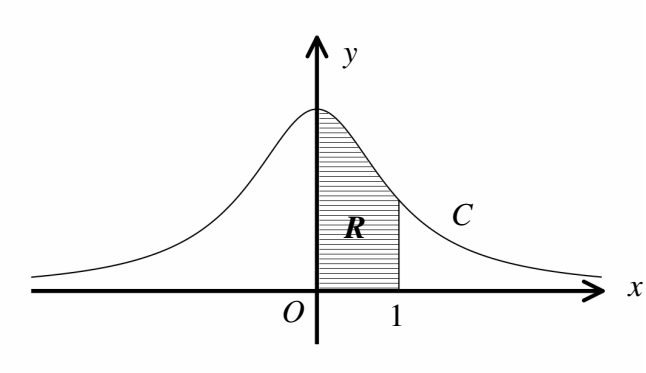
a) Complete the missing values in the table. **(1)**

b) Use the trapezium rule with all the values from the table to find an approximate value for

$$\int_0^{2\pi} \sin^3\left(\tfrac{1}{2}x\right)\ dx . \qquad \textbf{(3)}$$

c) By using the substitution $u = \cos\left(\tfrac{1}{2}x\right)$, or otherwise, find the value of the integral of part (b). **(7)**

Question 8



The figure above shows the curve C, defined by the parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta, \quad -\frac{\pi}{2} \leq \theta < \frac{\pi}{2}.$$

The finite region R is bounded by C, the coordinate axes and the straight line with equation $x = 1$.

a) Determine the area of R. $(\mathbf{5})$

The region R is revolved by 2π radians in the x axis, forming a solid S.

b) Show that the volume of S is

$$\frac{\pi}{8}(\pi + 2). \qquad (\mathbf{6})$$

c) Find a Cartesian equation of C, giving the answer in the form $y = f(x)$. $(\mathbf{2})$