C4, IYGB, PAPER H

1. a) $$\frac{2x^2-3}{(x-1)^2} \equiv A + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$2x^2-3 \equiv A(x-1)^2 + B(x-1) + C$$

- If $x=1 \Rightarrow -1 = C \Rightarrow \boxed{C=-1}$
- If $x=0 \Rightarrow -3 = A - B + C \Rightarrow \boxed{A-B=-2}$
- If $x=2 \Rightarrow 5 = A+B+C \Rightarrow \boxed{A+B=6}$

ADDING
$2A=4$
$\boxed{A=2}$
$\therefore \boxed{B=4}$

$\therefore A=2,\ B=4,\ C=-1$

b) $$\int_2^3 \frac{2x^2-3}{x-1}\,dx = \int_2^3 2 + \frac{4}{x-1} - (x-1)^{-2}\,dx$$

$$= \Big[2x + 4\ln|x-1| + (x-1)^{-1}\Big]_2^3$$

$$= \Big[2x + 4\ln|x-1| + \frac{1}{x-1}\Big]_2^3$$

$$= \left(6 + 4\ln 2 + \tfrac{1}{2}\right) - \left(4 + \cancel{\ln 1} + 1\right)$$

$$= \frac{3}{2} + 4\ln 2 \quad \text{OR} \quad \frac{3}{2} + \ln 16$$

2.

x	0	0.25	0.5	0.75	1
y	1	0.9394	0.7788	0.5698	0.3679

$$\int_0^1 e^{-x^2}\,dx \simeq \frac{\text{THICKNESS}}{2}\Big[\text{FIRST} + \text{LAST} + 2\times\text{REST}\Big]$$

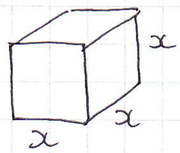
$$\simeq \frac{0.25}{2}\Big[1 + 0.3679 + 2(0.9394 + 0.7788 + 0.5698)\Big]$$

$$\simeq 0.743$$

b) $\displaystyle\int_0^1 e^{-x^2+3}\,dx = \int_0^1 e^{-x^2}\times e^3\,dx = e^3\int_0^1 e^{-x^2}\,dx$

$= e^3 \times 0.743\ldots \simeq 14.92$

3. a) $\boxed{\dfrac{dV}{dt} = 0.108}$



$V = x^3$

$\dfrac{dV}{dx} = 3x^2$

$\dfrac{dx}{dV} = \dfrac{1}{3x^2}$

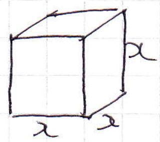
$\dfrac{dx}{dt} = \dfrac{dx}{dV}\times\dfrac{dV}{dt}$

$\dfrac{dx}{dt} = \dfrac{1}{3x^2}\times 0.108$

$\dfrac{dx}{dt} = \dfrac{9}{250x^2}$

$\left.\dfrac{dx}{dt}\right|_{x=3} = \dfrac{9}{250\times 3^2} = \dfrac{1}{250} = 0.004 \text{ cm s}^{-1}$

b)



$A = 6x^2$

$\dfrac{dA}{dx} = 12x$

$\dfrac{dA}{dt} = \dfrac{dA}{dx}\times\dfrac{dx}{dt}$

$\dfrac{dA}{dt} = 12x \times \dfrac{9}{250x^2}$

$\dfrac{dA}{dt} = \dfrac{54}{125x}$

$\left.\dfrac{dA}{dt}\right|_{x=3} = \dfrac{54}{125\times 3} = \dfrac{18}{125} = 0.144 \text{ cm}^2\text{s}^{-1}$

ALTERNATIVE SINCE $x=3$ IN BOTH PARTS

$\left.\dfrac{dA}{dt}\right|_{x=3} = \left.\dfrac{dA}{dx}\right|_{x=3}\times\left.\dfrac{dx}{dt}\right|_{x=3}$

$= (12\times 3)\times 0.004$

$= 0.144$ AS BEFORE

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4. • $4x^3 - 6xy + 3^y = 23$

$$\frac{d}{dx}(4x^3) - \frac{d}{dx}(6xy) + \frac{d}{dx}(3^y) = \frac{d}{dx}(23)$$

$$12x^2 - \left(6y + 6x\frac{dy}{dx}\right) + 3^y \ln 3 \times \frac{dy}{dx} = 0$$

• AT $(2,3)$

$$12\times 2^2 - 6\times 3 - 6\times 2\times\frac{dy}{dx} + 3^3\ln 3\,\frac{dy}{dx} = 0$$

$$48 - 18 - 12\frac{dy}{dx} + 27\ln 3\,\frac{dy}{dx} = 0$$

$$30 = (12 - 27\ln 3)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{30}{12 - 27\ln 3}$$

$$\frac{dy}{dx} = \frac{10}{4 - 9\ln 3}$$

$1 + d = 10$

5. $$\left.\begin{aligned} -5 &= \frac{a}{t} - 1 \\ 3 &= \frac{t+a}{t+1} \end{aligned}\right\} \Rightarrow \frac{a}{t} = -4 \Rightarrow \boxed{a = -4t}$$

Thus $3 = \dfrac{t - 4t}{t+1}$

$$3 = \frac{-3t}{t+1}$$

$$3t + 3 = -3t$$

$$6t = -3$$

$$\boxed{t = -\tfrac{1}{2}} \quad \& \quad \boxed{a = 2}$$

Thus $$\left.\begin{aligned} x &= \frac{2}{t} - 1 \\ y &= \frac{t+2}{t+1} \end{aligned}\right\} \Rightarrow \frac{2}{t} = x + 1$$

$$\frac{t}{2} = \frac{1}{x+1}$$

$$\boxed{t = \frac{2}{x+1}}$$

THUS $y = \frac{t-2}{t+1} = \frac{\frac{2}{x+1}+2}{\frac{2}{x+1}+1}$

MULTIPLY TOP & BOTTOM OF THE FRACTION BY $x+1$.

$y = \frac{2+2(x+1)}{2+1(x+1)} = \frac{2+2x+2}{2+x+1} = \frac{2x+4}{x+3}$ AS REQUIRED

6.

$x^2\frac{dy}{dx} = xy+y$

$\Rightarrow x^2\frac{dy}{dx} = y(x+1)$

$\Rightarrow \frac{1}{y}\,dy = \frac{x+1}{x^2}\,dx$

$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{1}{x}+\frac{1}{x^2}\,dx$

$\Rightarrow \ln|y| = \ln|x| - \frac{1}{x} + C$

$\Rightarrow y = e^{\ln|x| - \frac{1}{x} + C}$

$\Rightarrow y = e^{\ln|x|} \times e^{-\frac{1}{x}} \times e^{C}$

$\Rightarrow y = x \times e^{-\frac{1}{x}} \times A \quad (A = e^{C})$

$\Rightarrow y = Axe^{-\frac{1}{x}}$

7. a) $(1+\cos 2x)^2 = 1 + 2\cos 2x + \cos^2 2x$

$= 1 + 2\cos 2x + \left(\frac{1}{2}+\frac{1}{2}\cos 4x\right)$

$= \frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x$ AS REQUIRED

$\cos^2\theta \equiv \frac{1}{2}+\frac{1}{2}\cos 2\theta$

$\cos^2 2\theta \equiv \frac{1}{2}+\frac{1}{2}\cos 4\theta$

b) $V = \pi\int_{x_1}^{x_2} y(x)\,dx$

$V = \pi\int_0^{\frac{\pi}{2}} (1+\cos 2x)^2\,dx = \pi\int_0^{\frac{\pi}{2}} \frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\,dx$

$= \pi\left[\frac{3}{2}x + \sin 2x + \frac{1}{8}\sin 4x\right]_0^{\frac{\pi}{2}} = \pi\left[\left(\frac{3\pi}{4}+0+0\right) - (0)\right]$

$= \frac{3}{4}\pi^2$ AS REQUIRED

8. a) $\underline{r} = (a, b, 10) + \lambda(0, 1, 2) = (a, \lambda + b, 2\lambda + 10)$

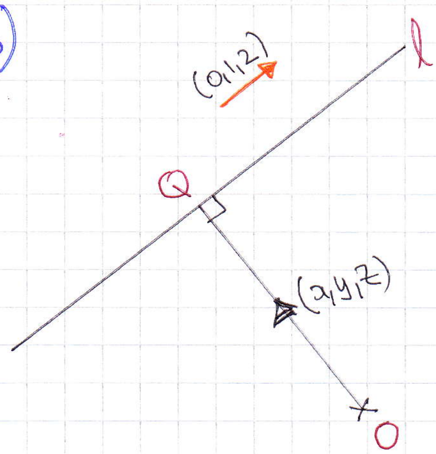
THUS $(7, 3, 6) = (a, \lambda + b, 2\lambda + 10)$

FROM $\underline{k}$: $2\lambda + 10 = 6 \Rightarrow \boxed{\lambda = -2}$

FROM $\underline{i}$: $a = 7$

FROM $\underline{j}$: $\lambda + b = 3 \Rightarrow -2 + b = 3 \Rightarrow b = 5$

b)



LET $Q(x, y, z)$
IE $\underline{q} = (x, y, z)$

- FIRSTLY OQ IS PERPENDICULAR TO l

$(x, y, z) \cdot (0, 1, 2) = 0$

$\boxed{y + 2z = 0}$

- Q LIES ON l

$(x, y, z) = (7, \lambda + 5, 2\lambda + 10)$

$$\boxed{\begin{array}{l} x = 7 \\ y = \lambda + 5 \\ z = 2\lambda + 10 \end{array}}$$

HENCE $(\lambda + 5) + 2(2\lambda + 10) = 0$

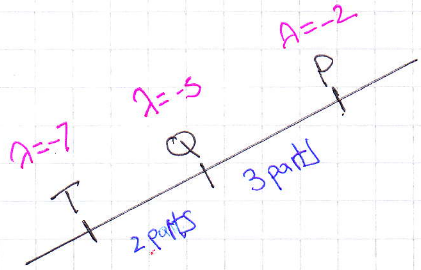
$\lambda + 5 + 4\lambda + 20 = 0$

$5\lambda = -25$

$\boxed{\lambda = -5}$

$\therefore Q(7, 0, 0)$

c)



$\therefore PQ : QT = 3 : 2$

9. a) $f(x) = (1+kx)^{-3} = 1 + \frac{-3}{1}(kx) + \frac{-3(-4)}{1\times 2}(kx)^2 + \frac{-3(-4)(-5)}{1\times 2\times 3}(kx)^3 + O(x^4)$

$= 1 - 3kx + 6k^2x^2 - 10k^3x^3 + O(x^4)$

b) $g(x) = \frac{6-x}{(1+kx)^3} = (6-x)(1 - 3kx + 6k^2x^2 - 10k^3x^3 + O(x^4))$

$3kx^2$

$36k^2x^2$

THUS $3kx^2 + 36k^2x^2 \equiv 3x^2$

$3k + 36k^2 = 3$

$36k^2 + 3k - 3 = 0$

$12k^2 + k - 1 = 0$

$(3k+1)(4k-1)$

$k = \frac{1}{4}$, $-\frac{1}{3}$

10. $\int_0^{\frac{\pi}{3}} 6x\sin 3x \, dx$ ... BY PARTS & IGNORING THE LIMITS

$6x$	6
$-\frac{1}{3}\cos 3x$	$\sin 3x$

$= -2x\cos 3x - \int -2\cos 3x \, dx$

$= -2x\cos 3x + \int 2\cos 3x \, dx$

$= -2x\cos 3x + \frac{2}{3}\sin 3x + C$

LIMITS ...

$= \left[-2x\cos 3x + \frac{3}{2}\sin 3x\right]_0^{\frac{\pi}{3}}$

$= \left(-\frac{2\pi}{3}\cos\pi + \cancel{\frac{3}{2}\sin\pi}\right) - \left(0 + \cancel{\frac{3}{2}\sin 0}\right) = \frac{2\pi}{3}$