1. $\int (x+1)e^{x+1}\,dx = \dots$ BY PARTS

$x+1$	1
e^{x+1}	e^{x+1}

$$= (x+1)e^{x+1} - \int e^{x+1}\,dx$$

$$= (x+1)e^{x+1} - e^{x+1} + C$$

$$= e^{x+1}\left[x+1-1\right] + C$$

$$= xe^{x+1} + C$$

2. a) $\sqrt{4-12x} = (4-12x)^{\frac{1}{2}} = 4^{\frac{1}{2}}(1-3x)^{\frac{1}{2}} = 2(1-3x)^{\frac{1}{2}}$

$$= 2\left[1 + \frac{\frac{1}{2}}{1}(-3x)^1 + \frac{\frac{1}{2}(-\frac{1}{2})}{1\times 2}(-3x)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\times 2\times 3}(-3x)^3 + O(x)\right]$$

$$= 2\left[1 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{27}{16}x^3 + O(x^4)\right]$$

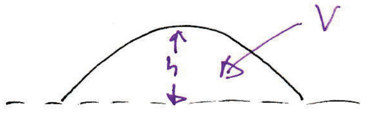
$$= 2 - 3x - \frac{9}{4}x^2 - \frac{27}{8}x^3 + O(x^4)$$

b) $(12x-4)(4-12)^{\frac{1}{2}} = (12x-4)\left(2-3x-\frac{9}{4}x^2+O(x^3)\right)$

$-36x^2$ $\quad +9x^2$

$\therefore -36+9 = -27$

3. $\boxed{\frac{dV}{dt} = 4}$



$$V = -8 + (h^4+64)^{\frac{1}{2}}$$

$$\frac{dV}{dh} = \frac{1}{2}(h^4+64)^{-\frac{1}{2}} \times 4h^3$$

$$\frac{dV}{dh} = \frac{2h^3}{\sqrt{h^4+64}}$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{\sqrt{h^4+64}}{2h^3} \times 4$$

$$\left.\frac{dh}{dt}\right|_{h=2} = \sqrt{5} \approx 2.34 \text{ cm}^2\text{s}^{-1}$$

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4. $\int \cos^3 x \, dx = \dots$ BY SUBSTITUTION

$u = \sin x$

$\frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

$$= \int \cos^3 x \frac{du}{\cos x} = \int \cos^2 x \, du$$

$$= \int (1 - \sin^2 x) \, du = \int 1 - u^2 \, du$$

$$= u - \frac{1}{3}u^3 + C = \sin x - \frac{1}{3}\sin^3 x + C$$

ALTERNATIVE, WITHOUT SUBSTITUTION

$$\int \cos^3 x \, dx = \int \cos x \cos^2 x \, dx = \int \cos x (1 - \sin^2 x) \, dx$$

$$= \int \cos x - \cos x \sin^2 x \, dx = \sin x - \frac{1}{3}\sin^3 x + C$$

BY RECOGNITION

5. a) $\left[1 + \frac{6}{2x+1}\right]^2 = 1^2 + 2 \times 1 \times \frac{6}{2x+1} + \left(\frac{6}{2x+1}\right)^2$

$$= 1 + \frac{12}{2x+1} + \frac{36}{(2x+1)^2}$$

IE $A = 12$
$B = 36$

b) $V = \pi \int_{x_1}^{x_2} [y(x)]^2 \, dx = \pi \int_0^1 \left(1 + \frac{6}{2x+1}\right)^2 dx$

$$= \pi \int_0^1 1 + \frac{12}{2x+1} + 36(2x+1)^{-2} \, dx$$

$$= \pi \left[x + 6\ln|2x+1| - 18(2x+1)^{-1}\right]_0^1$$

$$= \pi \left[x + 6\ln|2x+1| - \frac{18}{2x+1}\right]_0^1$$

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$$= \pi\left[\left(1 + 6\ln 3 - 6\right) - \left(0 + \cancel{6\ln 1} - 18\right)\right]$$

$$= \pi\left[13 + 6\ln 3\right]$$ AS REQUIRED

6. $\frac{dy}{dx} = \frac{y}{x(2-x)}$

$$\Rightarrow \frac{1}{y}\,dy = \frac{1}{x(2-x)}\,dx$$

$$\Rightarrow \frac{1}{y}\,dy = \frac{\frac{1}{2}}{x} + \frac{\frac{1}{2}}{2-x}$$

$$\Rightarrow \frac{2}{y}\,dy = \frac{1}{x} + \frac{1}{2-x}$$

PARTIAL FRACTIONS

$$\frac{1}{x(2-x)} \equiv \frac{A}{x} + \frac{B}{2-x}$$

$$1 \equiv A(2-x) + Bx$$

If $x=0 \Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$

If $x=2 \Rightarrow 1 = 2B \Rightarrow B = \frac{1}{2}$

INCORPORATE THE CONDITION INTO THE INTEGRAL (OR SOLVE TO A GENERAL SOLUTION)

$$\Rightarrow \int_{y=1}^{y} \frac{2}{y}\,dy = \int_{x=1}^{x} \frac{1}{x} + \frac{1}{2-x}$$

$$\Rightarrow \left[2\ln|y|\right]_1^y = \left[\ln|x| - \ln|2-x|\right]_1^x$$

$$\Rightarrow 2\ln|y| - \cancel{2\ln 1} = \left(\ln|x| - \ln|2-x|\right) - \cancel{\left(\ln 1 - \ln 1\right)}$$

$$\Rightarrow \ln(y^2) = \ln\left|\frac{x}{2-x}\right|$$

$$\Rightarrow y^2 = \frac{x}{2-x}$$

(P.T.O)

7. a) $\underline{a} = (4,-1,1)$, $\underline{b} = (-1,4,6)$ $\Big\}$ $\overrightarrow{AB} = \underline{b} - \underline{a} = (-1,4,6) - (4,-1,1)$

$= (-5,5,5)$

SCALE TO $(-1,1,1)$

$\therefore\ \underline{r} = (4,-1,1) + \lambda(-1,1,1)$

$(x,y,z) = (4-\lambda, \lambda-1, \lambda+1)$

b) $\underline{c} = (4,-2,-3)$, $\underline{d} = (p,q,-1)$ $\Longrightarrow$ MIDPOINT $= \left(\frac{p+4}{2}, \frac{q-2}{2}, -2\right)$

LIES ON l.

$4-\lambda = \frac{p+4}{2}$

$\lambda - 1 = \frac{q-2}{2}$

$\lambda + 1 = -2$

$\therefore\ \boxed{\lambda = -3}$

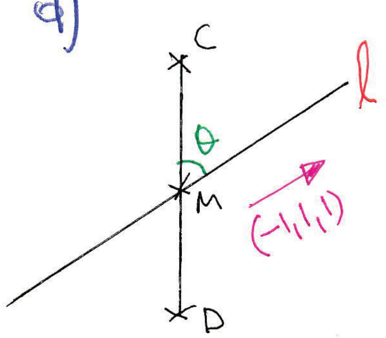
$\therefore\ M(7,-4,-2)$

c) $\frac{p+4}{2} = 7 \Longrightarrow p+4 = 14$

$\Longrightarrow p = 10$

$\frac{q-2}{2} = -4 \Longrightarrow q-2 = -8$

$q = -6$

d) 

$\overrightarrow{CM} = \underline{m} - \underline{c} = (7,-4,-2) - (4,-2,-3)$

$= (3,-2,1)$

DOTTING DIRECTION VECTOR

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$$(3,-2,1)\cdot(-1,1,1) = |3,-2,1||-1,1,1|\cos\theta$$

$$-3-2+1 = \sqrt{9+4+1}\sqrt{1+1+1}\cos\theta$$

$$-4 = \sqrt{14}\sqrt{3}\cos\theta$$

$$\cos\theta = \frac{-4}{\sqrt{42}}$$

$$\theta = 128.1$$

$\therefore$ ACUTE $180-128.1 \simeq 51.9$

8. a) $$\boxed{e^y = \frac{x^2+3}{x-1}}$$

Diff w.r.t x

$$e^y\frac{dy}{dx} = \frac{(x-1)(2x)-(x^2+3)\times 1}{(x-1)^2} = \frac{2x^2-2x-x^2-3}{(x-1)^2}$$

$$e^y\frac{dy}{dx} = \frac{x^2-2x-3}{(x-1)^2}$$

$$e^y\frac{dy}{dx} = \frac{(x+1)(x-3)}{(x-1)^2}$$

$$\frac{dy}{dx} = \frac{(x+1)(x-3)}{(x-1)^2}\times\frac{1}{e^y}$$

$$\frac{dy}{dx} = \frac{(x+1)(x-3)}{(x-1)^{\cancel{2}}}\times\frac{\cancel{x-1}}{x^2+3}$$

$$\frac{dy}{dx} = \frac{(x+1)(x-3)}{(x-1)(x^2+3)}$$ AS REQUIRED

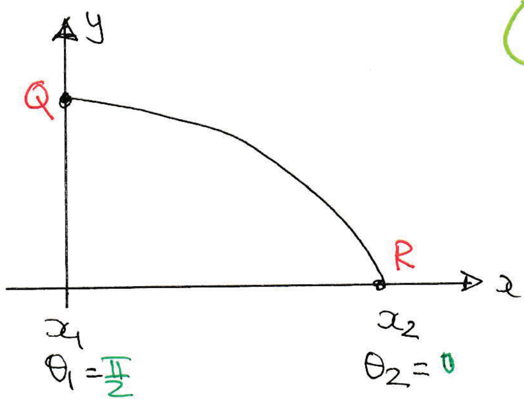
b) $\frac{dy}{dx}=0$ $\quad (x+1)(x-3)=0$

$x = \begin{cases} -1 \\ 3 \end{cases}$ $\quad e^y = \begin{cases} \cancel{-2} & (e^y>0) \\ 6 \end{cases}$

$\therefore (3, \ln 6)$

C4, IYGB, PAPER M

9. a)



$x = 12\cos\theta \qquad y = 6\sin\theta$

$x = 0$
$0 = 12\cos\theta$
$\cos\theta = 0$
$\theta = \frac{\pi}{2}$
↑ AT Q

$y = 0$
$0 = 6\sin\theta$
$\sin\theta = 0$
$\theta = 0$
↑ AT R

$$\underline{\text{TOTAL AREA}} = 4 \times \int_{x_1}^{x_2} y(x)\,dx = 4\int_{\theta_1}^{\theta_2} y(\theta)\frac{dx}{d\theta}\,d\theta$$

$$= 4\int_{\frac{\pi}{2}}^{0} 6\sin\theta\,(-12\sin\theta)\,d\theta$$

$$= 288\int_{0}^{\frac{\pi}{2}} \sin^2\theta\,d\theta = 288\int_{0}^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2}\cos 2\theta\,d\theta$$

$$= 288\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_{0}^{\frac{\pi}{2}}$$

$$= 288\left[\left(\frac{\pi}{4} - 0\right) - (0 - 0)\right]$$

$$= 72\pi$$

AS REQUIRED

b)

- $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{6\cos\theta}{-12\sin\theta} = -\frac{1}{2}\cot\theta = -\frac{1}{2\tan\theta}$

- $\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = -\frac{1}{2\tan\frac{\pi}{6}} = -\frac{\sqrt{3}}{2}$ ← tangent gradient

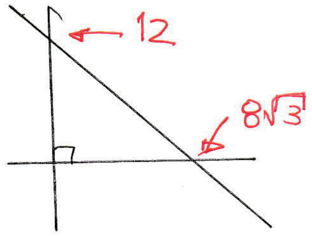
- $P(6\sqrt{3}, 3)$

- $y - y_0 = m(x - x_0) \Longrightarrow y - 3 = -\frac{\sqrt{3}}{2}(x - 6\sqrt{3})$

$2y - 6 = -\sqrt{3}x + 18$

$2y + \sqrt{3}x = 24$

c)



$2y + x\sqrt{3} = 24$

AREA OF TRIANGLE

$\frac{1}{2} \times 12 \times 8\sqrt{3} = 48\sqrt{3}$

AREA OF RHOMBUS

$48\sqrt{3} \times 4 = 192\sqrt{3}$

AREA OF "GOLD"

$192\sqrt{3} - 72\pi$

$\simeq 106$