IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper L

Difficulty Rating: 3.7267/1.7595

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$\cosh(A-B) \equiv \cosh A \cosh B - \sinh A \sinh B .$$

a) Prove the validity of the above hyperbolic identity by using the definitions of $\cosh x$ and $\sinh x$ in terms of exponentials. **(3)**

b) Hence solve the equation

$$\cosh(x - \ln 3) = \sinh x$$

giving the answer in terms of a natural logarithm. **(7)**

Question 2

$$f(x) = \ln(1+\cos 2x), \ 0 \le x < \frac{\pi}{2} .$$

a) Find an expression for $f'(x)$. **(1)**

b) Show clearly that

$$f''(x) = -2 - \frac{1}{2}\big(f'(x)\big)^2 . \quad \textbf{(5)}$$

c) Show further that the series expansion of the first three non zero terms of $f(x)$ is given by

$$\ln 2 - x^2 - \frac{1}{6}x^4 . \quad \textbf{(5)}$$

Question 3

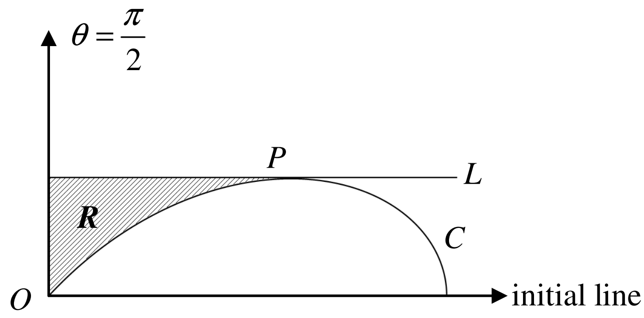
$$u_r = \frac{1}{6} r(r+1)(4r+11), \ r \in \mathbb{N}.$$

a) Simplify $u_r - u_{r-1}$ as far as possible. **(2)**

b) By using the method of differences, or otherwise, find the sum of the first 100 terms of the following series.

$$(1\times 5)+(2\times 7)+(3\times 9)+(4\times 11)+\ldots$$ **(6)**

Question 4



The figure above shows a curve C with polar equation

$$r^2 = 2\cos 2\theta, \ 0 \le \theta < \frac{\pi}{4}.$$

The straight line L is parallel to the initial line and is a tangent to C at the point P.

a) Show that the polar coordinates of P are $\left(1, \frac{\pi}{6}\right)$. **(7)**

The finite region R, shown shaded in the figure above, is bounded by C, L and the half line with equation $\theta = \frac{\pi}{2}$.

b) Show that the area of R is

$$\tfrac{1}{8}\left(3\sqrt{3} - 4\right).$$ **(7)**

Question 5

$$\left(2x - 4y^2\right)\frac{dy}{dx} + y = 0.$$

By reversing the role of x and y in the above differential equation, or otherwise, find its general solution. **(8)**

Question 6

$$f(x) \equiv \left(2x^2 - 1\right)\arcsin x + x\sqrt{1-x^2}\ ,\ -1 \le x \le 1\ .$$

a) Find a simplified expression for $f'(x)$. **(6)**

b) Hence find

$$\int_0^{\frac{\sqrt{2}}{2}} x\arcsin x \ \ dx\ .$$ **(3)**

Question 7

De Moivre's theorem states that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta,\ n \in \mathbb{Q}\ .$$

a) Use De Moivre's theorem to show that

$$\tan 5\theta \equiv \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}\ .$$ **(5)**

b) Use part **(a)** to find the solutions of the equation

$$t^4 - 10t^2 + 5 = 0\ ,$$

giving the answers in the form $t = \tan\varphi,\ 0 < \varphi < \pi$. **(5)**

c) Show further that

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5} = \sqrt{5}\ .$$ **(5)**
