IYGB GCE

Mathematics FP2

Advanced Level

Practice Paper U

Difficulty Rating: 4.1933/2.2140

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

It is given that the functions of x, f and g, satisfy the following coupled first order differential equations.

$$f'(x) - 5f(x) = 3g(x) \qquad \text{and} \qquad g'(x) + 4g(x) = -6f(x).$$

a) Show that

$$f''(x) - f'(x) - 2f(x) = 0. \qquad \textbf{(4)}$$

b) Given further that $f(0) = 1$ and $g(0) = 3$, solve the differential equation of part **(a)** to obtain simplified expressions for $f(x)$ and $g(x)$. **(7)**

Question 2

Use an appropriate substitution to find an exact value for the following integral.

$$\int_{\frac{1}{2}}^{\frac{1}{2}\sqrt{3}} \frac{1 - \sqrt{\arcsin x}}{\sqrt{1 - x^2}\ \arcsin x}\ dx. \qquad \textbf{(8)}$$

Question 3

Use the method of differences to find a simplified expression for the first n terms of the following series.

$$\frac{1}{1\times3} + \frac{2}{3\times5} + \frac{3}{5\times7} + \frac{4}{7\times9} + \ldots$$

Give your answer in the form $\frac{1}{4} - f(n)$, where $f(n)$ is a single simplified fraction. **(8)**

Question 4

The complex number w is defined as $w = e^{\frac{2}{5}\pi i}$.

a) Prove that

$$1 + w + w^2 + w^3 + w^4 = 0.$$ **(3)**

b) Derive a quadratic equation with integer coefficients whose roots are $\left(w + w^4\right)$ and $\left(w^2 + w^3\right)$, and hence show with full justification that

$$\cos\left(\tfrac{2}{5}\pi\right) = \frac{-1+\sqrt{5}}{4} \quad \text{and} \quad \cos\left(\tfrac{4}{5}\pi\right) = \frac{-1-\sqrt{5}}{4}.$$ **(7)**

Question 5

Find the first **four non zero** terms in the Maclaurin expansion of

$$y = \ln\left(1 + \cosh x\right).$$ **(12)**

Question 6

It is given that for suitable values of x

$$y = \ln\left[\tan\left(\tfrac{1}{4}\pi + \tfrac{1}{2}x\right)\right].$$

Show, with detailed workings, that

$$\sinh y = \tan x,$$

and hence deduce a simplified expression for e^y in terms of x. **(12)**

Question 7

$$y = \arccos x, \ x \in \mathbb{R}, \ -1 \le x \le 1.$$

a) By writing the above equation in the from $x = f(y)$, show that

$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}. \qquad \textbf{(4)}$$

A curve has equation

$$y = \arccos\left(1-x^2\right), \ \ x \in \mathbb{R}, \ 0 < x \le \sqrt{2}.$$

b) Show further that

$$\frac{d^2y}{dx^2} = \frac{2x}{\left(2-x^2\right)^{\frac{3}{2}}}. \qquad \textbf{(5)}$$

c) Show with detailed workings that

$$16\frac{d^3y}{dx^3} = 4x\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^2 + \left(2+x^2\right)\left(\frac{dy}{dx}\right)^5. \qquad \textbf{(5)}$$