IYGB – FS2 PAPER M – QUESTION 1

OBTAIN SUMMARY STATISTICS WITH A CALCULATOR

- $\sum x = 490$
- $\sum y = 699$
- $\sum xy = 36144$
- $\sum x^2 = 27682$
- $\sum y^2 = 50313$
- $n = 10$

FIND S_{xx}, S_{yy} AND S_{xy}

$$S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 27682 - \frac{490 \times 490}{10} = 3672$$

$$S_{yy} = \sum y^2 - \frac{\sum y \sum y}{n} = 50313 - \frac{699 \times 699}{10} = 1452.9$$

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 36144 - \frac{490 \times 699}{10} = 1893$$

FINALLY WE HAVE THE P.M.C.C.

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{1893}{\sqrt{3672 \times 1452.9}} = 0.81956... \simeq 0.820$$

AS r IS WELL ABOVE 0.5 AND CLOSE TO 1, THERE IS A GOOD SUGGESTION THAT PLACING ADS TO THE LOCAL RADIO STATION HAS THE DESIRED EFFECT

IYGB - FS2 PAPER M - QUESTION 2

a) START BY FINDING THE SAMPLE MEANS FOR EACH VARIETY

21.9	23.0	23.9	22.0	24.5	23.4	25.1	24.2
22.0	22.5	24.0	20.5	22.4	23.5	21.9	22.2

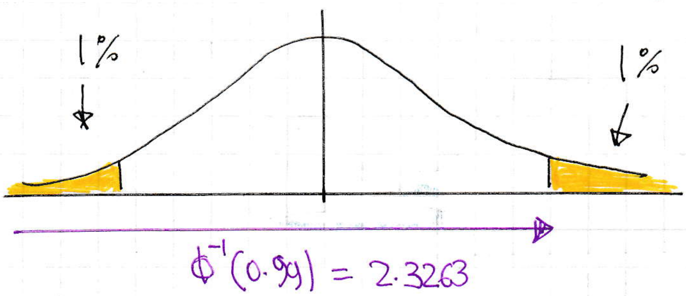
$$\bar{x}_A = \frac{21.9 + 23.0 + 23.9 + \dots + 24.2}{8} = \frac{188}{8} = 23.5$$

$$x_B = \frac{22.0 + 22.5 + 24.0 + \dots + 22.2}{8} = \frac{179}{8} = 22.375$$

OBTAIN THE STANDARD ERROR OF THE DIFFERENCE

$$\text{"}\frac{\sigma}{\sqrt{n}}\text{"} = \sqrt{\frac{1.5^2}{8} + \frac{1.5^2}{8}} = \sqrt{\frac{9}{16}} = \frac{3}{4} = 0.75$$

HENCE THE CONFIDENCE INTERVAL CAN NOW BE FOUND



$$(\mu_A - \mu_B) = (\bar{x}_A - \bar{x}_B) \pm \text{"}\frac{\sigma}{\sqrt{n}}\text{"}\,\Phi^{-1}(0.99)$$

$$= (23.5 - 22.375) \pm (0.75 \times 2.3263)$$

$$= 1.125 \pm 1.745$$

$$\therefore \text{CI} = (-0.617,\ 2.867)$$

IYGB - FS2 PAPER M - QUESTION 2

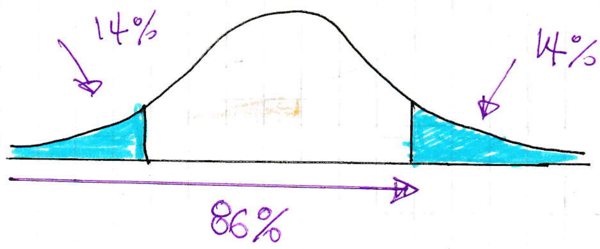
b) • NOTE THAT THE STANDARD ERROR WILL BE UNCHANGED

$$1.935 - 0.315 = 1.62$$
$$1.62 \div 2 = 0.81$$

• HENCE

$$\text{"}\frac{\sigma}{\sqrt{n}}\text{"} \times \Phi^{-1}(z) = 0.81$$
$$\frac{3}{4}\Phi^{-1}(z) = 0.81$$
$$\Phi^{-1}(z) = 1.08$$
$$z = \Phi(1.08)$$
$$z = 0.8599 \approx 86\%$$

• DRAWING A DIAGRAM



$\therefore \quad 100\% - 2 \times 14\% = 72\%$

IYGB - FS2 PAPER M - QUESTION 3

a) STARTING BY OBTAINING THE SAMPLE MEAN & STANDARD DEVIATION

$$\bar{x} = \frac{\sum x}{n} = \frac{2390}{10} = 239$$

$$s = \sqrt{\frac{1}{9}\left[\sum x^2 - \frac{\sum x \sum x}{n}\right]} = \sqrt{\frac{1}{9}\left[574495 - \frac{2390 \times 2390}{10}\right]} = \sqrt{365}$$

SETTING THE HYPOTHESES USING THE t DISTRIBUTION

$H_0 : \mu = 250$

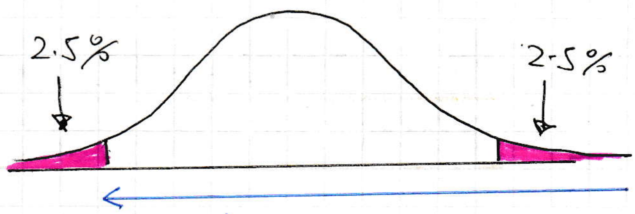
$H_1 : \mu \neq 250$

$\bar{x} = 239$

$n = 10$

$s = \sqrt{365}$

TWO TAILED t TEST AT 5%



$t_9(0.975) = -2.262$

$$t\text{-STAT} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{239 - 250}{\frac{\sqrt{365}}{\sqrt{10}}} = -1.821$$

AS $-1.821 > -2.262$ THERE IS NO SIGNIFICANT EVIDENCE TO SUPPORT THE MANAGER'S CLAIM – WE DO NOT HAVE SUFFICIENT EVIDENCE TO REJECT H_0

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b) ON THIS OCCASION THE HYPOTHESES WOULD BE

$H_0 : \mu = 250$

$H_1 : \mu < 250$

- THE CRITICAL VALUE NOW WILL BE -1.833.
- THE t-STAT WILL BE UNCHANGED AT -1.821

AS $-1.821 > -1.833$ THERE IS STILL NO SIGNIFICANT EVIDENCE TO SUPPORT THE MANAGER'S CLAIM - REJECT H_1

IYGB - FS2 PAPER N - QUESTION 4

a) REWRITE THE TABLE IN MORE USER FRIENDLY FORM

GYMNAST	A	B	C	D	E	F	G	H	I
JUDGE 1 RANK	6	4	2	1	3	5	9	8	7
JUDGE 2 RANK	7	6	4	1	2	3	8	9	5
d^2	1	4	4	0	1	4	1	1	4

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 20}{9 \times 80} = \frac{5}{6} = 0.8333$$

b) $H_0 : \rho_s = 0$ (JUDGES ARE NOT IN GENERAL AGREEMENT)

$H_1 : \rho_s > 0$ (JUDGES ARE IN GENERAL AGREEMENT)

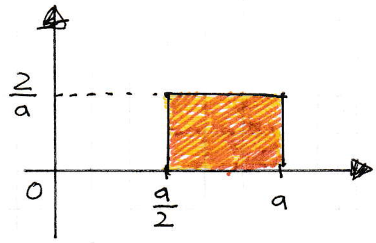
THE CRITICAL VALUE FOR $n=9$, AT 1% SIGNIFICANCE IS 0.7833

AS $0.8333 > 0.7833$, THERE IS EVIDENCE THAT THE JUDGES ARE IN GENERAL AGREEMENT — REJECT H_0

IYGB - FS2 PAPER M - QUESTION 5

a) • IF X REPRESENTS THE LENGTH OF THE LONGER PIECE THEN X CAN TAKE VALUES BETWEEN $\frac{a}{2}$ & a

$$f(x) = \begin{cases} \frac{2}{a} & \frac{1}{2}a \leqslant x \leqslant a \\ 0 & \text{OTHERWISE} \end{cases}$$



• $E(X) = \int_{\frac{a}{2}}^{a} x f(x)\, dx = \int_{\frac{a}{2}}^{a} \frac{2}{a}x\, dx = \left[\frac{1}{a}x^2\right]_{a/2}^{a} = \frac{1}{a}\left[a^2 - \frac{1}{4}a^2\right]$

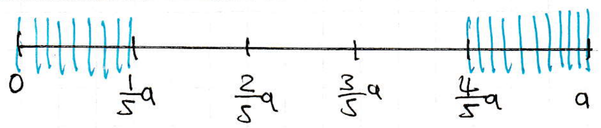
$= \frac{1}{a} \times \frac{3}{4}a^2 = \frac{3}{4}a$

• $E(X^2) = \int_{a/2}^{a} x^2 f(x)\, dx = \int_{a/2}^{a} \frac{2}{a}x^2\, dx = \left[\frac{2}{3a}x^3\right]_{a/2}^{a} = \frac{2}{3a}\left[a^3 - \frac{1}{8}a^3\right]$

$= \frac{2}{3a} \times \frac{7}{8}a^3 = \frac{7}{12}a^2$

• $\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{7}{12}a^2 - \left(\frac{3}{4}a\right)^2 = \frac{1}{48}a^2$ // AS REQUIRED

b) BY INSPECTION — DRAWING THE "ROPE" — THE CONDITION IS SATISFIED IF THE CUT IS MADE IN THE "BLUE" SECTION BELOW



∴ REQUIRED PROBABILITY IS $\frac{2}{5}$ //

ALTERNATIVE USING PART (a)

$P(X \geqslant 4(a-X)) = P(X \geqslant 4a - 4X)$

$= P(5X \geqslant 4a)$

$= P(X \geqslant \frac{4}{5}a)$

$= \left(a - \frac{4}{5}a\right) \times \frac{2}{a}$

$= \frac{2}{5}$ //

IYGB – FS2 PAPER M – QUESTION 6

a) • DEFINING VARIABLES & DISTRIBUTIONS

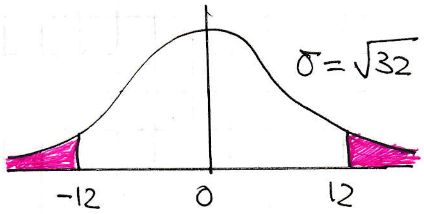
X = WEIGHT OF A BAG OF SUGAR
$X \sim N(1008, 4^2)$

• $X_1 - X_2 \sim N(1008 - 1008, 4^2 + 4^2)$
• $X_1 - X_2 \sim N(0, 32)$

• WE REQUIRE $P(X_1 - X_2 > 12)$ OR $P(X_2 - X_1 > 12)$

$P(X_1 - X_2 > 12) = 1 - P(X_1 - X_2 < 12)$

$= 1 - P\left(Z < \frac{12-0}{\sqrt{32}}\right)$



$= 1 - \Phi(2.1213)$

$= 1 - 0.9830$

$= 0.017$

• HENCE, THE REQUIRED PROBABILITY (BY SYMMETRY) IS $2 \times 0.017 = 0.034$

b) • LET $T = X_1 + X_2 + X_3 + \dots + X_{160} + Y$

WHERE Y = weight of the pallet/wrapping
$Y \sim N(9600, 500^2)$
(WORKING IN GRAMS)

IYGB - FS2 PAPER N - QUESTION 6

- $E(T) = (160 \times 1008) + 9600 = 170880$
- $Var(T) = 160 \times 4^2 + 500^2 = 252560$

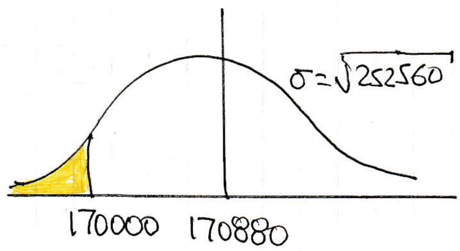
- HENCE WE NOW HAVE

$$\boxed{T \sim N(170880, 252560)}$$

$P(T < 170000)$

$= 1 - P(T > 170000)$

$= 1 - P\left(Z > \frac{170000 - 170880}{\sqrt{252560}}\right)$



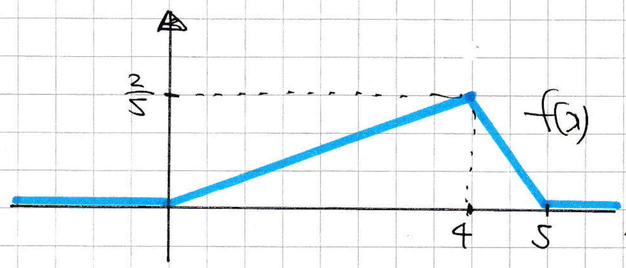
$= 1 - \Phi(-1.7511)$

$= 1 - 0.9600$

$= 0.0400$

IYGB - FS2 PAPER M - QUESTION 7

a) SKETCHING THE P.D.F CONSISTING OF TWO LINES



b) MODE IS 4

c) $E(X) = \int_a^b x f(x)\,dx$

$$E(X) = \int_0^4 x\left(\tfrac{1}{10}x\right)dx + \int_4^5 x\left(2-\tfrac{2}{5}x\right)dx = \int_0^4 \tfrac{1}{10}x^2\,dx + \int_4^5 2x - \tfrac{2}{5}x^2\,dx$$

$$= \left[\tfrac{1}{30}x^3\right]_0^4 + \left[x^2 - \tfrac{2}{15}x^3\right]_4^5 = \left(\tfrac{64}{30}-0\right) + \left(25-\tfrac{250}{15}\right) - \left(16 - \tfrac{128}{15}\right)$$

$$= \tfrac{32}{15} + 25 - \tfrac{50}{3} - 16 + \tfrac{128}{15} = 3$$

AS REQUIRED

d) FIRST COMPUTE $E(X^2) = \int_a^b x^2 f(x)\,dx$

$$E(X^2) = \int_0^4 x^2\left(\tfrac{1}{10}x\right)dx + \int_4^5 x^2\left(2-\tfrac{2}{5}x\right)dx = \int_0^4 \tfrac{1}{10}x^3\,dx + \int_4^5 2x^2 - \tfrac{2}{5}x^3\,dx$$

$$= \left[\tfrac{1}{40}x^4\right]_0^4 + \left[\tfrac{2}{3}x^3 - \tfrac{1}{10}x^4\right]_4^5$$

$$= \left(\tfrac{32}{5}-0\right) + \left(\tfrac{250}{3} - \tfrac{125}{2}\right) - \left(\tfrac{128}{3} - \tfrac{128}{5}\right) = \tfrac{61}{6}$$

USING $\text{Var}(X) = E(X^2) - [E(X)]^2$

$$\text{Var}(X) = \tfrac{61}{6} - 3^2$$

$$\text{Var}(X) = \tfrac{7}{6}$$

IYGB - FS2 PAPER M - QUESTION 7

$$F(x) = \int_a^x f(x)\,dx$$

$$F_1(x) = \int_0^x \frac{1}{10}x \, dx = \left[\frac{1}{20}x^2\right]_0^x = \frac{1}{20}x^2 - 0 = \frac{1}{20}x^2$$

$$F_1(4) = \frac{4}{5}$$

$$F_2(x) = \frac{4}{5} + \int_4^x 2 - \frac{2}{5}x \, dx = \frac{4}{5} + \left[2x - \frac{1}{5}x^2\right]_4^x$$

$$= \frac{4}{5} + \left[2x - \frac{1}{5}x^2\right] - \left[8 - \frac{16}{5}\right]$$

$$= \frac{4}{5} + 2x - \frac{1}{5}x^2 - 8 + \frac{16}{5}$$

$$= -\frac{1}{5}x^2 + 2x - 4$$

SPECIFYING

$$\begin{cases} 0 & x < 0 \\ \frac{1}{20}x^2 & \leq x < 4 \\ -\frac{1}{5}(x^2 - 10x + 20) & 4 \leq x < 5 \\ 1 & x > 5 \end{cases}$$