# IYGB — FS2 PAPER O — QUESTION 1

OBTAIN SUMMARY STATISTICS

$$\sum x = 84 \qquad \sum x^2 = 920 \qquad n = 8$$

$$s^2 = \frac{1}{n-1} S_{xx} = \frac{1}{n-1}\left[\sum x^2 - \frac{\sum x \sum x}{n}\right] = \frac{1}{7}\left[920 - \frac{84 \times 84}{8}\right] = \frac{38}{7}$$

NOW WE HAVE FROM TABLES

- $$\frac{(n-1)s^2}{\chi^2_{n-1}(0.025)} = \frac{7 \times \frac{38}{7}}{\chi^2_7(0.025)} = \frac{38}{16.013} = 2.373\ldots$$
- $$\frac{(n-1)s^2}{\chi^2_{n-1}(0.975)} = \frac{7 \times \frac{38}{7}}{\chi^2_7(0.975)} = \frac{38}{1.690} = 22.485\ldots$$

$$\therefore \ \text{C.I} = (2.37, 22.5)$$

# IYGB - FS2 PAPER D - QUESTION 2

| | |
|---|---|
| $X$ = weight of female bodybuilder<br>$X \sim N(60, 4^2)$ | $Y$ = weight of male bodybuilder<br>$Y \sim N(100, 6^2)$ |

## DEFINE A NEW VARIABLE W

- $W = 4Y - (X_1 + X_2 + X_3 + X_4 + X_5 + X_6)$

- $E(W) = (4 \times 100) - (6 \times 60) = 40$

- $\text{Var}(W) = 4^2 \times \text{Var}(Y) + 6 \times \text{Var}(X)$
$= (16 \times 36) + (6 \times 16)$
$= 672$

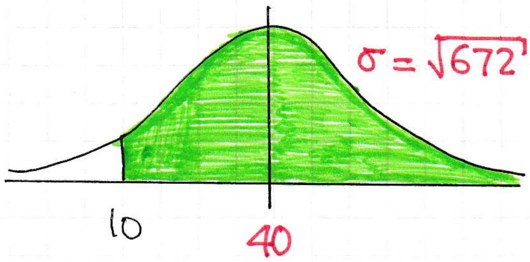
## THE WAY W IS DEFINED, WE MODEL AS "4 TIMES MALE WEIGHT MORE THAN 10 kg THAT OF 6 FEMALES"

$P(W > 10)$

$= P\left(Z > \frac{10 - 40}{\sqrt{672}}\right)$

$= \Phi(-1.1573)$

$= 0.8764$



# IYGB - FS2 PAPER O - QUESTION 3

a) START BY REWRITING THE TABLE IN RANKS

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| LAP TIME RANK | 4 | 6 | 3 | 1 | 7 | 5 | 8 | 2 |
| FINISH ORDER RANK | 5 | 6 | 1 | 3 | 7 | 4 | 8 | 2 |
| $d^2$ | 1 | 0 | 4 | 4 | 0 | 1 | 0 | 0 |

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 10}{8 \times 63} = 1 - \frac{5}{42} = \frac{37}{42} \simeq 0.8810$$

b) $\boxed{\begin{array}{ll} H_0 : \rho_s = 0 & \text{(NO ASSOCIATION, "POSITIVE" OR "NEGATIVE")} \\ H_1 : \rho_s \neq 0 & \text{(ASSOCIATION EXISTS)} \end{array}}$

THE CRITICAL VALUE FOR $n=8$, AT 5%, TWO TAILED, IS $\pm 0.7381$

AS $0.8810 > 0.7381$, THERE IS SIGNIFICANT EVIDENCE OF (POSITIVE) ASSOCIATION BETWEEN THE FASTEST QUALIFYING LAP TIME AND THE RACE FINISHING POSITION. — REJECT $H_0$

# IYGB - FS2 PAPER O - QUESTION 4

$\sum d = 385.5 \qquad \sum v = 22.5 \qquad \sum dv = 650.25$

$\sum d^2 = 11543.25 \qquad \sum v^2 = 38.25 \qquad n = 15$

a) DEPTH IS THE EXPLANATORY VARIABLE, I.E INDEPENDENT VARIABLE AND FREQUENCY IS THE RESPONSE VARIABLE (DEPENDENT VARIABLE)

THIS IS BECAUSE IT IS THE DEPTH WHICH AFFECTS FREQUENCY AND <u>NOT</u> THE OTHER WAY ROUND

b) CALCULATE $S_{dd}$ $S_{vv}$ & $S_{dv}$

$$S_{dd} = \sum d^2 - \frac{\sum d \sum d}{n} = 11543.25 - \frac{385.5 \times 385.5}{15} = 1635.9$$

$$S_{vv} = \sum v^2 - \frac{\sum v \sum v}{n} = 38.25 - \frac{22.5 \times 22.5}{15} = 4.5$$

$$S_{dv} = \sum dv - \frac{\sum d \sum v}{n} = 650.25 - \frac{385.5 \times 22.5}{15} = 72$$

c) FIND THE PMCC.

$$r = \frac{S_{dv}}{\sqrt{S_{dd} S_{vv}}} = \frac{72}{\sqrt{1635.9 \times 4.5}} \simeq 0.839$$

d) POSITIVE CORRELATION, I.E THE GREATER THE DEPTH, THE HIGHER THE FREQUENCY AND VISE VERSA

e) THE P.M.C.C IS REASONABLY HIGH TO SUGGEST A GOOD LINEAR MODEL MIGHT BE APPROPRIATE

f)

$$b = \frac{S_{dv}}{S_{dd}}$$

$$b = \frac{72}{1635.9}$$

$$d = \frac{240}{5453} \approx 0.0440$$

3 s.f.

# IYGB - FS2 PAPER O - QUESTION 4

"$a = \bar{y} - b\bar{x}$", HERE $a = \bar{v} - b\bar{d}$

$\frac{\sum d}{n} = \frac{385.5}{15} = 25.7$

$\frac{\sum v}{n} = \frac{22.5}{15} = 1.5$

$a = 1.5 - 0.0440\ldots \times 25.7$

$a = 0.369$ 3 sf

g) $a =$ "y INTERCEPT"

THIS REPRESENTS THE "FREQUENCY OF DOLPHIN" WHEN IT IS AT THE SURFACE (ZERO DEPTH)

$b =$ "gradient"

INCREASE IN THE FREQUENCY PER METRE DEPTH, FOR EVERY METRE FOR WHICH THE DOLPHIN DIVES, THE FREQUENCY INCREASES BY 0.044 KHz

# IYGB – FS2 PAPER O – QUESTION 5

## SETTING HYPOTHESES

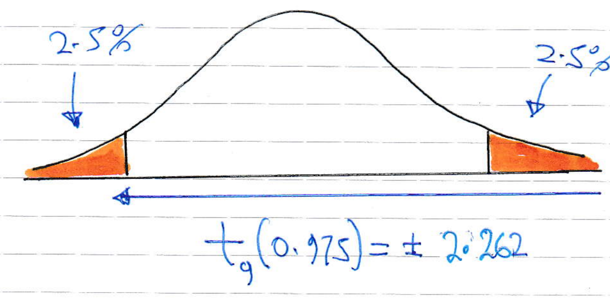
$H_0 : \mu = 40$

$H_1 : \mu \neq 40$ WHERE $\mu$ IS THE MEAN AMOUNT OF ALL CRISP PACKETS (POPULATION MEAN)

## OBTAINING SAMPLE STATISTICS

- $\bar{x} = \frac{\Sigma x}{n} = \frac{387.5}{10} = 38.75$
- $s = \sqrt{\frac{1}{n-1}\left[\Sigma x^2 - \frac{\Sigma x \Sigma x}{n}\right]} = \sqrt{\frac{1}{9}\left[15096.25 - \frac{387.5^2}{10}\right]} = 2.99304...$

## LOOKING AT A t-DISTRIBUTION DIAGRAM




$$t\text{-stat} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{38.75 - 40}{\frac{2.99304...}{\sqrt{10}}} = -1.3206...$$

AS $-2.262 < -1.3206 < 2.262$, THERE IS NO SIGNIFICANT EVIDENCE THAT THE MEAN WEIGHT IS NOT 40

WE DO NOT HAVE SUFFICIENT EVIDENCE TO REJECT $H_0$

# IYGB - FS2 PAPER O - QUESTION 6

a) $\bar{x} = \frac{\sum x}{n} = \frac{1350}{60} = 22.5$

$$s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{\sum x \sum x}{n}\right] = \frac{1}{59}\left[30685 - \frac{1350 \times 1350}{60}\right]$$

$$= \frac{310}{59} \simeq 5.254$$

b)

$H_0 : \mu_x = \mu_y$

$H_1 : \mu_y > \mu_x$

$\bar{x} = 22.5$

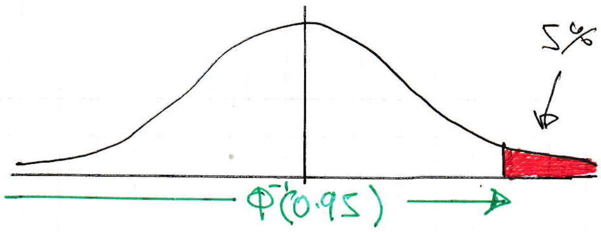
$\bar{y} = 24.1$

$n_x = n_y = 60$

$s_x^2 = 5.254...$

$s_y^2 = 5.48$

5% SIGNIFICANCE




$$\text{Z-STAT} = \frac{(\bar{y} - \bar{x}) - (\mu_y - \mu_x)}{\sqrt{\frac{s_y^2}{n_y} + \frac{s_x^2}{n_x}}}$$

$$\text{Z-STAT} = \frac{(24.1 - 22.5) - (0)}{\sqrt{\frac{5.48^2}{60} + \frac{5.254^2}{60}}}$$

$\text{Z-STAT} = 1.5304$

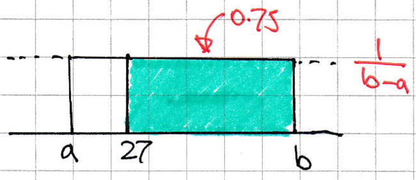
CRITICAL VALUE $\Phi^{-1}(0.95) = 1.6449$

AS $1.5304 < 1.6449$ THERE IS NO SIGNIFICANT EVIDENCE THAT THE MEAN TIME OF THE SOLDIERS IS GREATER — THERE IS NO SUFFICIENT EVIDENCE TO REJECT $H_0$

- VALIDATION — ALTHOUGH THERE IS NO EVIDENCE OF NORMALITY THE SAMPLING DISTRIBUTION OF THE DIFFERENCE OF THE MEANS WILL BE APPROXIMATELY NORMAL (BY THE CENTRAL LIMIT THEOREM)
- ASSUMPTION — $s_x^2 = \sigma_x^2$ FOR THE 1200 & $s_y^2 = \sigma_y^2$ FOR THE 1500

# IYGB - FS2 PAPER O - QUESTION 7

## DEFINE AN INTERVAL $(a,b)$, $b>a$ AND DRAW A SKETCH




$$\Rightarrow P(X>27) = 0.75$$
$$\Rightarrow \frac{1}{b-a} \times (b-27) = \frac{3}{4}$$
$$\Rightarrow \frac{b-27}{b-a} = \frac{3}{4}$$

## NEXT USE THE VARIANCE

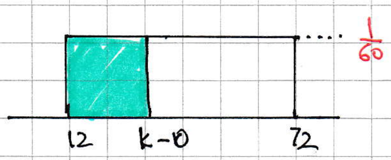
$$\Rightarrow \text{Var}(X) = 300$$
$$\Rightarrow \frac{(b-a)^2}{12} = 300$$
$$\Rightarrow (b-a)^2 = 3600$$
$$\Rightarrow b-a = +60$$

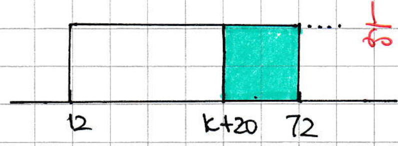
## COMBINE EQUATIONS NEXT

$$\Rightarrow \frac{b-27}{b-a} = \frac{3}{4}$$
$$\Rightarrow b-27 = \frac{3}{4}(b-a)$$
$$\Rightarrow b-27 = \frac{3}{4} \times 60$$
$$\Rightarrow b-27 = 45$$
$$\Rightarrow b = 72$$

AND EVIDENTLY $a = 12$

## FINALLY DRAWING TWO SEPARATE DIAGRAMS




$$P(X<k-10) = \frac{k-10-12}{60} = \frac{k-22}{60}$$

$$P(X>k+20) = \frac{72-(k+20)}{60} = \frac{52-k}{60}$$

## FINALLY WE HAVE

$$\Rightarrow 4P(X<k-10) = P(X>k+20)$$
$$\Rightarrow 4\left(\frac{k-22}{60}\right) = \frac{52-k}{60} \quad \times 60$$
$$\Rightarrow 4(k-22) = 52-k$$
$$\Rightarrow 4k-88 = 52-k$$
$$\Rightarrow 5k = 140$$

$$\therefore k = 28$$

# IYGB - FS2 PAPER O - QUESTION 8

a) $E(X) = \int_a^b x f(x)\, dx$

$$E(X) = \int_2^4 x\left(\frac{1}{60}x^3\right) dx = \int_2^4 \frac{1}{60}x^4 = \left[\frac{1}{300}x^5\right]_2^4$$

$$= \frac{1}{300}\left[1024 - 32\right] = \frac{248}{75} \simeq 3.31$$

b) START BY FINDING $E(X^2) = \int_a^b x^2 f(x)\, dx$

$$E(X^2) = \int_2^4 x^2\left(\frac{1}{60}x^3\right) dx = \int_2^4 \frac{1}{60}x^5\, dx = \frac{1}{360}\left[x^6\right]_2^4$$

$$= \frac{1}{360}\left[4096 - 64\right] = \frac{56}{5}$$

NOW USING $Var(X) = E(X^2) - [E(X)]^2$

$$Var(X) = \frac{56}{5} - \left(\frac{248}{75}\right)^2$$

$$Var(X) = 0.2659555....$$

$\therefore$ STANDARD DEVIATION $= \sqrt{0.2659555...}$

$\simeq 0.516$ 3 d.p

c) $F(x) = \int_a^x f(x)\, dx$

$$F(x) = \int_2^x \frac{1}{60}x^3\, dx = \left[\frac{1}{240}x^4\right]_2^x = \frac{1}{240}\left(x^4 - 16\right)$$

$$\therefore F(x) = \begin{cases} 0 & x < 2 \\ \frac{1}{240}(x^4 - 16) & 2 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

# IYGB – FS2 PAPER O – QUESTION 8

d) $\underline{P(X>3.5)} = 1 - P(X<3.5)$

$= 1 - F(3.5)$

$= 1 - \frac{1}{240}(3.5^4 - 16)$

$= \frac{113}{256}$

e) SOLVING $F(x) = \frac{1}{2}$

$\frac{1}{240}(x^4 - 16) = \frac{1}{2}$

$x^4 - 16 = 120$

$x^4 = 136$

$x = +\sqrt[4]{136}$

$x \simeq 3.41$ 2 d.p.