IYGB – FS2 PAPER P – QUESTION 1

a) OBTAIN SUMMARY STATISTICS FROM A CALCULATOR

$\sum x = 286$ $\qquad \sum x^2 = 10784$ $\qquad \sum xy = 8466$

$\sum y = 231$ $\qquad \sum y^2 = 6831$ $\qquad n = 8$

OBTAIN THE VALUES OF S_{xx}, S_{yy}, S_{xy}

- $S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 10784 - \frac{286 \times 286}{8} = 559.5$
- $S_{yy} = \sum y^2 - \frac{\sum y \sum y}{n} = 6831 - \frac{231 \times 231}{8} = 160.875$
- $S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 8466 - \frac{286 \times 231}{8} = 207.75$

CALCULATE THE P.M.C.C.

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{207.75}{\sqrt{559.5 \times 160.875}} = 0.6924632\ldots$$

$$\simeq 0.692$$

b) POSITIVE CORRELATION, I.E. THE HIGHER THE NUMBER OF TEACHERS (x) THE HIGHER THE NUMBER OF BURGLARIES (y), AND VISE VERSA

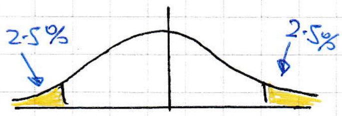
c) CORRELATION DOES NOT IMPLY CAUSATION – THEY MAY BE ANOTHER VARIABLE (OR VARIABLES) THAT x & y ARE BOTH CONNECTED TO

$\therefore$ STATEMENT IS NOT LIKELY TO BE VALID

IYGB - FS2 PAPER P - QUESTION 2

THE CONFIDENCE INTERVAL FOR A POPULATION MEAN WILL BE

$$\mu = \bar{x} \pm \frac{s}{\sqrt{n}} \Phi^{-1}(0.975)$$



BY SYMMETRY THE SAMPLE MEAN WILL BE THE MIDPOINT

$$\bar{x} = \frac{150.66 + 166.34}{2} = \frac{317}{2} = 158.5$$

OR BY ALGEBRA BY SUBTRACTING THESE EXPRESSIONS

$$150.66 = \bar{x} - \frac{s}{\sqrt{n}} \Phi^{-1}(0.975)$$

$$166.34 = \bar{x} + \frac{s}{\sqrt{n}} \Phi^{-1}(0.975)$$

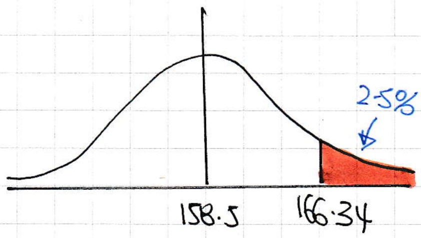
NOW LOOKING AT EITHER END, SAY THE TOP END

$$166.34 = 158.5 + \frac{s}{\sqrt{100}} \times 1.96$$

$$166.34 = 158.5 + \frac{1}{10} s \times 1.96$$

$$7.84 = 0.196 s$$

$$s = 40$$



$$\Phi^{-1}(0.975) = 1.96$$

IYGB - FS2 PAPER P - QUESTION 3

- FIRSTLY THE SAMPLE IS LARGE $n = 120$, SO
  - THE SAMPLING DISTRIBUTION OF THE MEAN WILL BE APPROX NORMAL
  - $s \approx \sigma$
  - WE CAN USE NORMAL DISTRIBUTION INSTEAD OF t - DISTRIBUTION

- SETTING THE HYPOTHESES

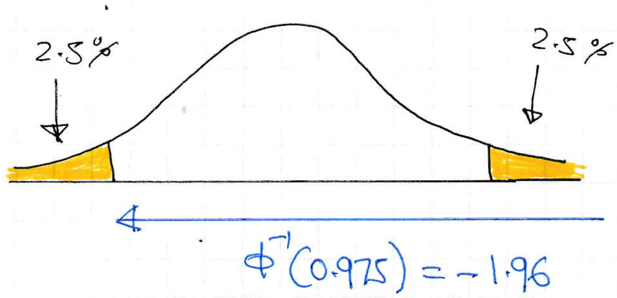
$$H_0 : \mu = 250$$
$$H_1 : \mu \neq 250$$

$n = 120$

$\bar{x}_{120} = 249$

$s \simeq \sigma = ?$

5% TWO TAILED



- $Z - \text{STAT} = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

- HERE $\dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} < -1.96$

$$\Rightarrow \frac{249 - 250}{\frac{\sigma}{\sqrt{120}}} < -1.96$$

$$\Rightarrow \frac{-\sqrt{120}}{\sigma} < -1.96$$

$$\Rightarrow -1.96\sigma > -\sqrt{120}$$

$$\Rightarrow \sigma < 5.589$$

$$\therefore \sigma \approx s \approx 5.58$$

IYGB – FS2 PAPER P – QUESTION 4

CANDIDATE	A	B	C	D	E	F	G	H	I	J
PAPER 1	91	85	81	90	78	82	71	88	75	94
PAPER 2	85	86	82	80	80	83	72	84	70	90

SETTING UP A PAIRED TWO TAILED TEST

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

OR

$$H_0 : \mu_{diff} = 0$$
$$H_0 : \mu_{diff} \neq 0$$

PREPARING ALL THE "AUXILIARIES" – LET "$d = 1 - 2$"

- $\sum \text{differences } (d) = 6 - 1 - 1 + 10 - 2 - 1 - 1 + 4 + 5 + 4$
- $\sum d = 23$
- $\sum d^2 = 201$

FINDING THE ESTIMATORS

$$\bar{d} = \frac{\sum d}{n} = \frac{23}{10}$$

$$s_d^2 = \frac{1}{n-1}\left[\sum d^2 - \frac{\sum d \sum d}{n}\right] = \frac{1}{9}\left[201 - \frac{23 \times 23}{10}\right] = 16.4555\ldots$$

USING THE t-DISTRIBUTION, WITH 9 DEGREES OF FREEDOM, AT 10%, TWO TAILED

$t_9(95\%) = 1.833$

IYGB - FS2 PAPER P - QUESTION 4

OBTAIN THE TEST STATISTIC TO COMPARE WITH THE CRITICAL VALUE OF 1.833

$$t\text{-stat} = \frac{\bar{d} - 0}{s/\sqrt{n}} = \frac{2.3}{\sqrt{\frac{16.555...}{10}}} = 1.7929$$

AS $-1.8333 < 1.7929 < 1.8333$, THERE IS NO SIGNIFICANT EVIDENCE (AT 10%) THAT THE PAPERS WERE OF DIFFERENT DIFFICULTY

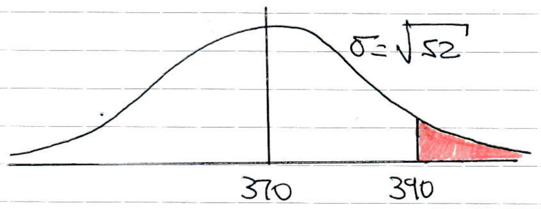
NO SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS2 PAPER P - QUESTION 5

$$X \sim N(100, 5^2) \quad \bullet \quad Y \sim N(90, 3^2)$$

a)
$$E\left(X + \sum_{i=1}^{3} Y_i\right) = 100 + 3 \times 90 = 370$$
$$Var\left(X + \sum_{i=1}^{3} Y_i\right) = 5^2 + 3 \times 3^2 = 52$$

- $P\left(X + \sum_{i=1}^{3} Y_i > 390\right)$



$= 1 - P\left(X + \sum_{i=1}^{3} Y_i < 390\right)$

$= 1 - P\left(z < \frac{390 - 370}{\sqrt{52}}\right)$

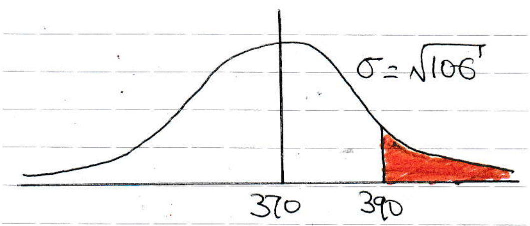
$= 1 - \Phi(2.7735)$

$= 1 - 0.99723$

$= 0.00277$

b)
$$E(X + 3Y) = 100 + 3 \times 90 = 370$$
$$Var(X + 3Y) = 5^2 + \underbrace{3^2 \times 3^2}_{3^2 Var(Y)} = 106$$

- $P(X + 3Y > 390)$



$= 1 - P(X + 3Y < 390)$

$= 1 - P\left(z < \frac{390 - 370}{\sqrt{106}}\right)$

$= 1 - \Phi(1.942571\ldots)$

IYGB - FS2 PAPER P - QUESTION 5

$= 1 - 0.97397...$

$= 0.02603....$

$= \underline{0.026}$

c) $\boxed{\begin{array}{l} E(X-Y) = 100 - 90 = 10 \\ Var(X-Y) = 5^2 + 3^2 = 34 \end{array}}$

- $P(|X-Y| < 4)$

$= P(-4 < X-Y < 4)$

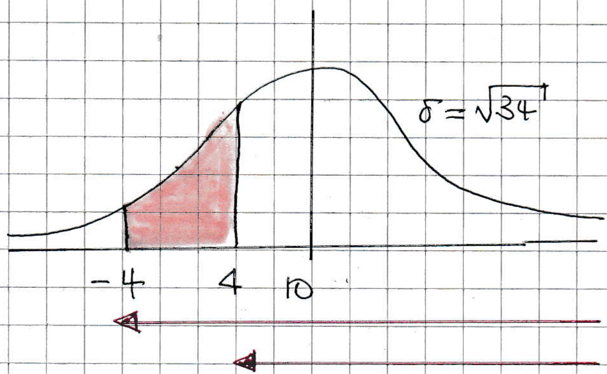
$= P(X-Y > -4) - P(X-Y > 4)$

$= P\left(Z > \frac{-4-10}{\sqrt{34}}\right) - P\left(Z > \frac{4-10}{\sqrt{34}}\right)$

$= \Phi(-2.4001) - \Phi(-1.0290)$

$= 0.99180 - 0.848$

$= \underline{0.1435}$



IYGB – FS2 PAPER P – QUESTION 6

a) $\int_a^b f(x)\,dx = 1$

$k\int_1^4 (x-1)(x-4)\,dx = 1$

$k\int_1^4 x^2-5x+4\,dx = 1$

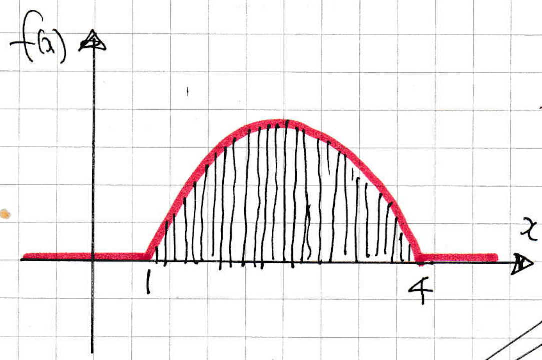
$k\left[\frac{1}{3}x^3-\frac{5}{2}x^2+4x\right]_1^4 = 1$

$k\left[\left(\frac{64}{3}-40+16\right)-\left(\frac{1}{3}-\frac{5}{2}+4\right)\right] = 1$

$-\frac{9}{2}k = 1$

$k = -\frac{2}{9}$

b) SKETCHING THE P.D.F



c) AS QUADRATICS ARE SYMMETRICAL

$E(X) = \frac{4+1}{2} = 2.5$

d) FIRST FIND THE $E(X^2) = \int_a^b x^2 f(x)\,dx$

$E(X^2) = \int_1^4 x^2\left[-\frac{2}{9}(x-1)(x-4)\right]dx = \frac{2}{9}\int_4^1 x^4-5x^3+4x^2\,dx$

$= \frac{2}{9}\left[\frac{1}{5}x^5-\frac{5}{4}x^4+\frac{4}{3}x^3\right]_4^1 = \frac{2}{9}\left[\left(\frac{1}{5}-\frac{5}{4}+\frac{4}{3}\right)-\left(\frac{1024}{5}-320+\frac{256}{3}\right)\right]$

$= 6.7$

USING $Var(X) = E(X^2) - [E(X)]^2$

$Var(X) = 6.7 - 2.5^2$

$Var(X) = 0.45$

IYGB – FS2 PAPER P – QUESTION 6

e) USING $\int_a^x f(x)\,dx = F(x)$

$$F(x) = \int_1^x -\frac{2}{9}(x-1)(x-4)\,dx = \frac{2}{9}\int_x^1 x^2 - 5x + 4\,dx$$

$$= \frac{2}{9}\left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x\right]_x^1 = \frac{2}{9}\left[\left(\frac{1}{3} - \frac{5}{2} + 4\right) - \left(\frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x\right)\right]$$

$$= \frac{2}{9}\left[\frac{11}{6} - \frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x\right] = \frac{11}{27} - \frac{8x}{9} + \frac{5}{9}x^2 - \frac{2}{27}x^3$$

$$\therefore \quad F(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{27}(11 - 24x + 15x^2 - 2x^3) & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

f) CLEARLY SYMMETRICAL AS MODE = MEDIAN = MEAN = 2.5

IYGB - FS2 PAPER P - QUESTION 7

TO INVESTIGATE THESE CLAIMS WE NEED THE REGRESSION LINE

- $S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 114000 - \frac{900 \times 900}{10} = 33000$
- $S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 6834 - \frac{900 \times 78.4}{10} = -222$

TO INVESTIGATE THE FIRST CLAIM WE NEED THE GRADIENT

$$b = \frac{S_{xy}}{S_{xx}} = \frac{-222}{33000} = -\frac{37}{5000} = -0.0067272...$$

REDUCTION IN HOURS PER 1 MILLIGRAM

$$\Longrightarrow \quad -0.00672... \times 60 = -0.4036...$$

REDUCTION IN HOURS PER 60 mg

$$= -0.4036... \times 60$$
$$= -24.218...$$

CLAIM NOT JUSTIFIED AS EVERY ADDITIONAL 60 mg REDUCE THE NIGHTLY SLEEP BY APPROXIMATELY 24 MINUTES

NEXT FIND THE Y INTERCEPT

- $\bar{x} = \frac{\sum x}{n} = \frac{900}{10} = 90$
- $\bar{y} = \frac{\sum y}{n} = \frac{78.4}{10} = 7.84$

$$a = \bar{y} - b\bar{x} = 7.84 - (-0.0067272...) \times 90 = 7.84 + 90 \times 0.0067272...$$
$$= \frac{929}{110} \simeq 8.45 \text{ hours}$$

CLAIM NOT JUSTIFIED AS $8.45 > 8$