IYGB - FS2 PAPER Q - QUESTION 1

SETTING HYPOTHESES

$H_0 : \sigma = 100$

$H_1 : \sigma < 100$

THE CRITICAL VALUE AT 10% SIGNIFICANCE & $\nu = 10-1 = 9$ IS 14.684

THE TEST STATISTIC IS

$$\frac{(n-1)s^2}{\sigma^2} = \frac{9 \times 78}{100} = 7.02$$

AS $7.02 < 14.684$ THERE IS NO SIGNIFICANT EVIDENCE THAT THE VARIANCE IS LESS THAN 100 — INSUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS2 PAPER Q - QUESTION 2

a) START BY GETTING SUMMARY STATISTICS FROM A STAT CALCULATOR

$\sum x = 114 \qquad \sum x^2 = 1446 \qquad \sum xy = 2337$

$\sum y = 192 \qquad \sum y^2 = 3872 \qquad n = 10$

OBTAIN THE VALUES OF S_{xx}, S_{yy} & S_{xy}

- $S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 1446 - \frac{114 \times 114}{10} = 146.4$
- $S_{yy} = \sum y^2 - \frac{\sum y \sum y}{n} = 3872 - \frac{192 \times 192}{10} = 185.6$
- $S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 2337 - \frac{114 \times 192}{10} = 148.2$

b) $r = \frac{S_{xy}}{\sqrt{S_{xx} \, S_{yy}}}$

$r = \frac{148.2}{\sqrt{146.4 \times 185.6}} = 0.899060007\ldots \simeq 0.9$ APPROX

AS REQUIRED

c) CALCULATE ALL THE AUXILIARIES

- $\bar{x} = \frac{\sum x}{n} = \frac{114}{10} = 11.4$ & $\bar{y} = \frac{\sum y}{n} = \frac{192}{10} = 19.2$
- $b = \frac{S_{xy}}{S_{xx}} = \frac{148.2}{146.4} = 1.01229508\ldots \simeq 1.012$
- $a = \bar{y} - b\bar{x} = 19.2 - (1.01229\ldots)(11.4) = 7.659836\ldots \simeq 7.660$

$\therefore \; y = 7.660 + 1.012x$

IYGB – FS2 PAPER Q – QUESTION 3

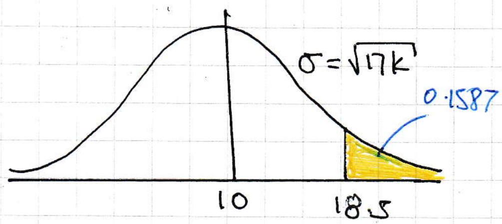
$$\boxed{A \sim N(30, k) \qquad \text{AND} \qquad B \sim N(40, 2k)}$$

DEFINE A NEW VARIABLE $X = 3A - 2B$

$$\text{MEAN} = E(X) = E(3A-2B) = 3E(A) - 2E(B)$$
$$= (3\times30) - (2\times40) = 10$$
$$\text{VARIANCE} = \text{Var}(X) = \text{Var}(3A-2B) = 3^2\text{Var}(A) + 2^2\text{Var}(B)$$
$$= 9\times k + 4\times 2k = 17k$$

THUS $X = 3A - 2B \sim N(10, 17k)$

$$P(X > 18.5) = 0.1587$$
$$P(X < 18.5) = 0.8413$$
$$P\left(Z < \frac{18.5-10}{\sqrt{17k}}\right) = 0.8413$$



$$\frac{8.5}{\sqrt{17k}} = +\Phi^{-1}(0.8413)$$
$$\frac{8.5}{\sqrt{17k}} = 1$$
$$8.5 = \sqrt{17k}$$
$$72.25 = 17k$$
$$k = \frac{17}{4}$$
$$k = 4.25$$

IYGB – FS2 PAPER Q – QUESTION 4

a) FORM A TABLE OF RANKS TO FIND $\sum d^2$

NAME	GINGE	PUSS	MOG	REX	LOULOU	RIRI	OLLIE
VET'S RANK	1	2	3	4	5	6	7
"ACTUAL" RANK	2	1	4	3	6	7	5
d^2	1	1	1	1	1	1	4

$$\sum d^2 = 10$$

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 10}{7 \times 48} = \frac{23}{28} = 0.8214$$

b) SETTING HYPOTHESES, WHERE ρ_s IS THE SPEARMAN CORRELATION FOR AN ENTIRE POPULATION, NOT OF THESE 7 CATS

$$\boxed{\begin{aligned} H_0 &: \rho_s = 0 \\ H_1 &: \rho_s > 0 \end{aligned}}$$

THE CRITICAL VALUE FOR $n=7$ AT 1% SIGNIFICANCE IS 0.8929

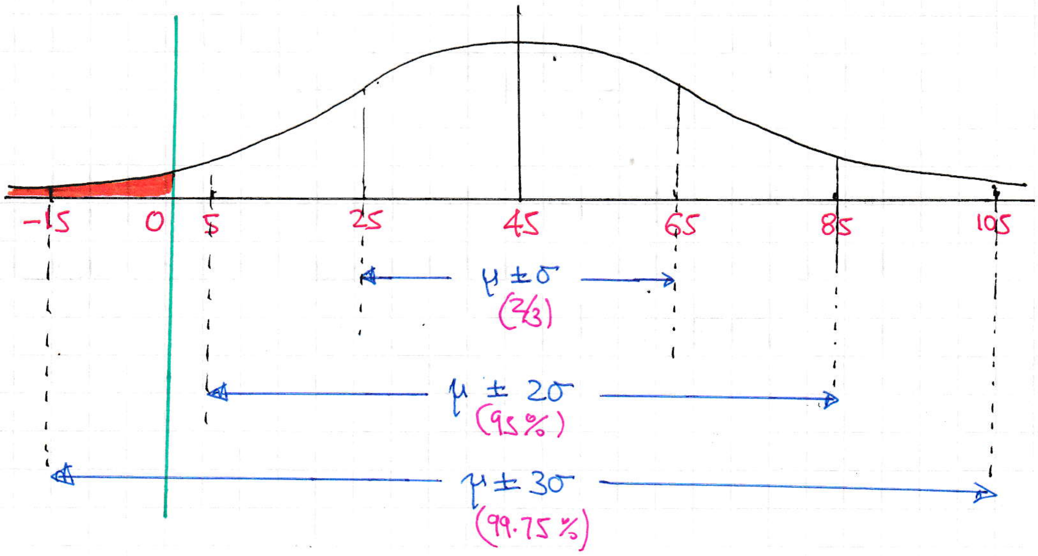
AS $0.8214 < 0.8929$ IT APPEARS THAT THE VET DOES NOT HAVE THE ABILITY TO IDENTIFY THE AGES OF CATS – SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS2 PAPER Q - QUESTION 5

a) T = TIME TO SERVICE CAR

AS $\bar{t} = 45$ BASED ON 100 OBSERVATIONS IT IS REASONABLE TO ASSUME THAT μ WILL BE CLOSE TO THAT FIGURE

SUPPOSE THAT $T \sim N(45, 20^2)$



THEN A SUBSTANTIAL PERCENTAGE OF DATA WOULD BE NEGATIVE, IE WILL NOT BE THE 3 STANDARD DEVIATION THRESHOLD //

$\bar{T}_{100} \sim N\left(\bar{x}, \frac{\sigma^2}{n}\right)$ (APPROXIMATELY BY THE C.L.T)

$\bar{T}_{100} \sim N\left(45, \frac{20^2}{100}\right)$

$\bar{T}_{100} \sim N(45, 2^2)$

HENCE $\bar{T}_{100}$ WILL HAVE AN APPROXIMATELY NORMAL DISTRIBUTION WITHOUT $\bar{T}_{100}$ GOING NEGATIVE (σ IS VERY SMALL NOW)

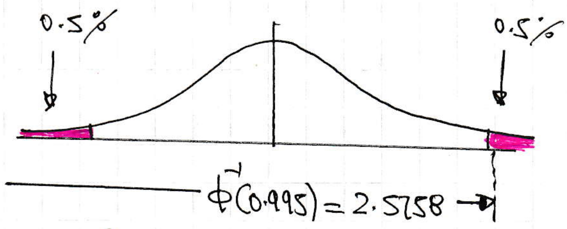
IYGB - FS2 PAPER Q - QUESTION 5

b)

$$\mu = \bar{x} \pm \frac{\sigma}{\sqrt{n}}\Phi^{-1}(0.995)$$

$$\mu = 45 \pm \frac{20}{\sqrt{100}} \times 2.5758$$

$$\mu = 45 \pm 5.1516$$



$\therefore$ CI = (39.85, 50.16)

CLAIM NOT JUSTIFIED AS 60 IS "WAY ABOVE" THE UPPER BOUND OF SUCH "HIGH CONFIDENCE" (99%) INTERVAL, BASED ON A LARGE n (100)

IYGB - FS2 PAPER Q - QUESTION 6

a) USING $\int_a^b f(x)\,dx = 1$

$$\Rightarrow \int_0^2 \frac{1}{4}x\,dx + k\int_2^4 x^3\,dx = 1$$

$$\Rightarrow \left[\frac{1}{8}x^2\right]_0^2 + k\left[\frac{1}{4}x^4\right]_2^4 = 1$$

$$\Rightarrow \frac{1}{2} + k(64-4) = 1$$

$$\Rightarrow 60k = \frac{1}{2}$$

$$\Rightarrow k = \frac{1}{120}$$

AS REQUIRED

b) USING $F(x) = \int_a^x f(x)\,dx$

$$\Rightarrow F_1(x) = \int_0^x \frac{1}{4}x\,dx = \left[\frac{1}{8}x^2\right]_0^x = \frac{1}{8}x^2 - 0 = \frac{1}{8}x^2$$

NOW $F_1(2) = \frac{1}{8}\times 2^2 = \frac{1}{2}$

$$\Rightarrow F_2(x) = \frac{1}{2} + \int_2^x \frac{1}{120}x^3\,dx = \frac{1}{2} + \frac{1}{480}\left[x^4\right]_2^x$$

$$= \frac{1}{2} + \frac{1}{480}x^4 - \frac{1}{30} = \frac{1}{480}x^4 + \frac{7}{15} = \frac{1}{480}(x^4+224)$$

$$\therefore F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{8}x^2 & 0 \leqslant x \leqslant 2 \\ \frac{1}{480}(x^4+224) & 2 < x \leqslant 4 \\ 1 & x > 4 \end{cases}$$

IYGB - FS2 PAPER Q - QUESTION 6

c) MEDIAN IS 2 $\qquad F_1(2) = \frac{1}{2}$

d) $E(X) = \int_a^b x f(x)\,dx$

$$E(X) = \int_0^2 x\left(\tfrac{1}{4}x\right)dx + \int_2^4 x\left(\tfrac{1}{120}x^3\right)dx = \int_0^2 \tfrac{1}{4}x^2\,dx + \int_2^4 \tfrac{1}{120}x^4\,dx$$

$$= \left[\tfrac{1}{12}x^3\right]_0^2 + \left[\tfrac{1}{600}x^5\right]_2^4$$

$$= \left(\tfrac{2}{3} - 0\right) + \left(\tfrac{128}{75} - \tfrac{4}{75}\right)$$

$$= \frac{58}{25} = 2.32$$

AS REQUIRED.

e) USING THE C.D.F

$F(x) = \frac{1}{4}$

$\frac{1}{8}x^2 = \frac{1}{4}$

$x^2 = 2$

$x = 1.4142..$

$F(x) = \frac{3}{4}$

$\frac{1}{480}(x^4 + 224) = \frac{3}{4}$

$x^4 + 224 = 360$

$x^4 = 136$

$x = 3.4150..$

$\therefore$ IQR = 3.41495297... − 1.414213562...

$= 2.0007....$

$\simeq 2$

IYGB – FS2 PAPER Q – QUESTION 7

OBTAIN UNBIASED ESTIMATORS FOR THE MEAN AND VARIANCE OF ALL THE DAILY MILEAGES (POPULATION)

$$\bar{x} = \frac{\sum x}{n} = \frac{8596}{56} = 153.5$$

$$s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{\sum x \sum x}{n}\right] = \frac{1}{55}\left[1409600 - \frac{8596^2}{56}\right] = \frac{90114}{55}$$

AS THE SAMPLE SIZE IS LARGE, THE DISTRIBUTION OF THE MEAN WILL BE APPROXIMATELY NORMAL

$H_0 : \mu = 145$

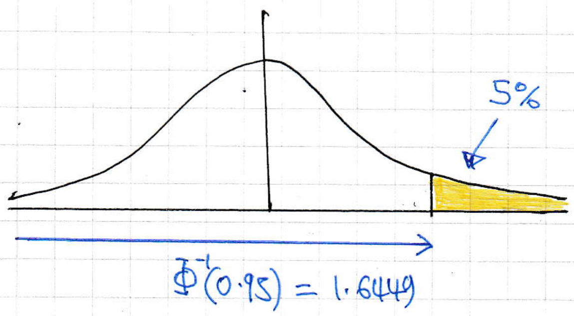
$H_1 : \mu > 145$

$n = 56$

$\bar{x}_{56} = 153.5$

$s^2 = \frac{90114}{55}$

5% SIGNIFICANCE



$$Z \text{ STAT} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{153.5 - 145}{\sqrt{\frac{\frac{90114}{55}}{56}}}$$

$$= 1.5714$$

AS $1.5714 < 1.6449$ THERE IS NO SIGNIFICANT EVIDENCE THAT THE DAILY MEAN DISTANCES HAVE INCREASED — INSUFFICIENT EVIDENCE TO REJECT H_0

IYGB – FS2 PAPER Q – QUESTION 8

OBTAIN SUMMARY STATISTICS FROM THE CALCULATOR

- $\sum x_B = 589$ • $\sum x_B^2 = 40189$ • $n_B = 9$
- $\sum x_G = 549$ • $\sum x_G^2 = 38947$ • $n_G = 8$

SETTING HYPOTHESES

$H_0 : \mu_B = \mu_G$

$H_1 : \mu_B = \mu_G$

WHERE μ REPRESENTS THE POPULATION MEANS (TWO TAILED AT 5% SIGNIFICANCE, IE TESTING AT 2.5% IN EACH TAIL)

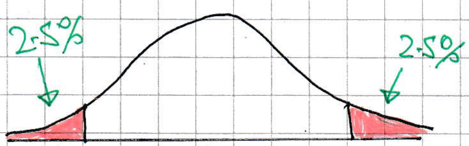
CALCULATING AUXILIARIES

- $\bar{x}_B = \dfrac{\sum x_B}{n_B} = \dfrac{589}{9} = 65.444\ldots$
- $\bar{x}_G = \dfrac{\sum x_G}{n_G} = \dfrac{549}{8} = 68.625$
- $s_B^{\prime 2} = \dfrac{1}{n_B - 1}\left[\sum x_B^2 - \dfrac{\sum x_B \sum x_B}{n_B}\right] = \dfrac{1}{8}\left[40189 - \dfrac{589 \times 589}{9}\right] = \dfrac{3695}{18}$
- $s_G^{\prime 2} = \dfrac{1}{n_G - 1}\left[\sum x_G^2 - \dfrac{\sum x_G \sum x_G}{n}\right] = \dfrac{1}{7}\left[38947 - \dfrac{549 \times 549}{8}\right] = \dfrac{10175}{56}$

NEXT THE POOLED ESTIMATE OF THE VARIANCE

$$s_p^{\prime 2} = \frac{(n_B - 1)s_B^{\prime 2} + (n_G - 1)s_G^{\prime 2}}{n_B + n_G - 2} = \frac{8 \times \frac{3695}{18} + 7 \times \frac{10175}{56}}{9 + 8 - 2} = \frac{41963}{216}$$

USING A t DISTRIBUTION WITH 15 DEGREES OF FREEDOM



(5% TWO TAILED)

$t_{15}(2.5\%) = \pm 2.131$

IYGB - FS2 PAPER Q - QUESTION 8

t - STATISTIC $= \dfrac{(\bar{x}_B - \bar{x}_G) - (\mu_B - \mu_G)}{S_p \sqrt{\frac{1}{n_B} + \frac{1}{n_G}}}$

$= \dfrac{(65.444... - 68.625) - (0)}{\sqrt{\frac{41963}{216}} \sqrt{\frac{1}{9} + \frac{1}{8}}}$

$= -0.46961...$

AS $-2.131 < -0.470 < 2.131$ THERE IS NO SIGNIFICANT EVIDENCE THAT THE POPULATION MEAN OF THE BOYS IS DIFFERENT TO THAT OF THE GIRLS — INSUFFICIENT EVIDENCE TO REJECT H_0

ASSUMPTIONS MADE

- SAMPLES ARE RANDOM
- SAMPLES COME FROM NORMAL DISTRIBUTIONS
- POPULATION VARIANCES ARE THE SAME FOR BOYS & GIRLS