IYGB - M456 PAPER B - QUESTION 1

- USING THE DEFINITION OF IMPULSE IN INTEGRAL FORM

$$\boxed{I = \int_{t_1}^{t_2} F(t)\, dt}$$

- HERE WE HAVE $m = 0.5\text{kg}$, $t_1 = 1$ $u = -6$ AND WE REQUIRE THE TIME t_2 WHEN $V = +30$

$$\Longrightarrow \quad I = \int_{t_1=1}^{t_2} 4t - 9 \, dt$$

$$\Longrightarrow \quad m(v-u) = \int_{1}^{t_2} 4t - 9 \, dt$$

$$\Longrightarrow \quad \tfrac{1}{2}(30 - (-6)) = \left[2t^2 - 9t\right]_1^{t_2}$$

$$\Longrightarrow \quad 18 = (2t_2^2 - t_2) - (2-9)$$

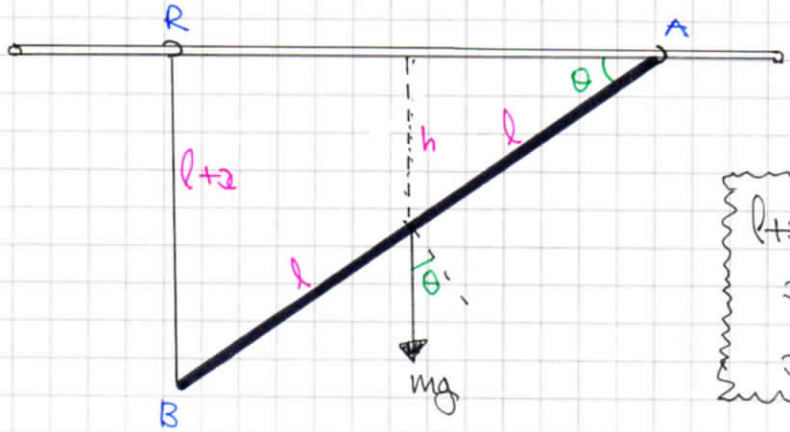
$$\Longrightarrow \quad 0 = 2t_2^2 - 9t_2 - 11$$

$$\Longrightarrow \quad (2t_2 - 11)(t_2 + 1) = 0$$

$$\Longrightarrow \quad t_2 = \begin{cases} 11/2 \\ \cancel{-1} \end{cases}$$

IYGB – M456 PAPER B – QUESTION 2

LOOKING AT THE DIAGRAM BELOW



$$l+x = 2l\sin\theta$$
$$x = 2l\sin\theta - l$$
$$x = l(2\sin\theta - 1)$$

DETERMINE THE ELASTIC ENERGY

$$V_{ELASTIC} = \frac{\lambda}{2l}x^2 = \frac{mg}{2l}\left[l(2\sin\theta-1)\right]^2$$
$$= \tfrac{1}{2}mgl(2\sin\theta-1)^2$$
$$= \tfrac{1}{2}mgl(4\sin^2\theta - 4\sin\theta + 1)$$

FIND THE POTENTIAL ENERGY TAKING THE LEVEL OF THE WIRE AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL

$$V_{GRAV} = -mgh = -mg(l\sin\theta) = -mgl\sin\theta + C$$

CONSTRUCTING THE POTENTIAL FUNCTION

$$V_{TOTAL}(\theta) = \tfrac{1}{2}mgl(4\sin^2\theta - 4\sin\theta + 1) - mgl\sin\theta + C$$

$$V_{TOT.}(\theta) = mgl(2\sin^2\theta - 2\sin\theta) + \tfrac{1}{2}mgl - mgl\sin\theta + C$$

$$V_{TOT}(\theta) = mgl(2\sin^2\theta - 3\sin\theta) + \text{constant}$$

IYGB - M456 PAPER B - QUESTION 2

LOOK FOR STATIONARY VALUES AND THEIR NATURE

$$\boxed{\begin{aligned} V'(\theta) &= mgl\left[4\sin\theta\cos\theta - 3\cos\theta\right] \\ V''(\theta) &= mgl\left[-4\sin^2\theta + 4\cos^2\theta + 3\sin\theta\right] \end{aligned}}$$

$$V'(\theta) = 0 \implies 4\sin\theta\cos\theta - 3\cos\theta = 0$$
$$\implies \cos\theta(4\sin\theta - 3) = 0$$
$$\implies \cos\theta = 0 \quad \text{OR} \quad \sin\theta = \frac{3}{4}$$
$$\implies \theta = \frac{\pi}{2} \quad \text{OR} \quad \theta = \arcsin\frac{3}{4}$$

INVESTIGATING THE STABILITY

- IF $\theta = \frac{\pi}{2}$ $\quad V''\left(\frac{\pi}{2}\right) = mgl(-4+0+3) = -mgl < 0$

IE A LOCAL MAX

$\theta = \frac{\pi}{2}$ IS UNSTABLE

- IF $\theta = \arcsin\frac{3}{4}$

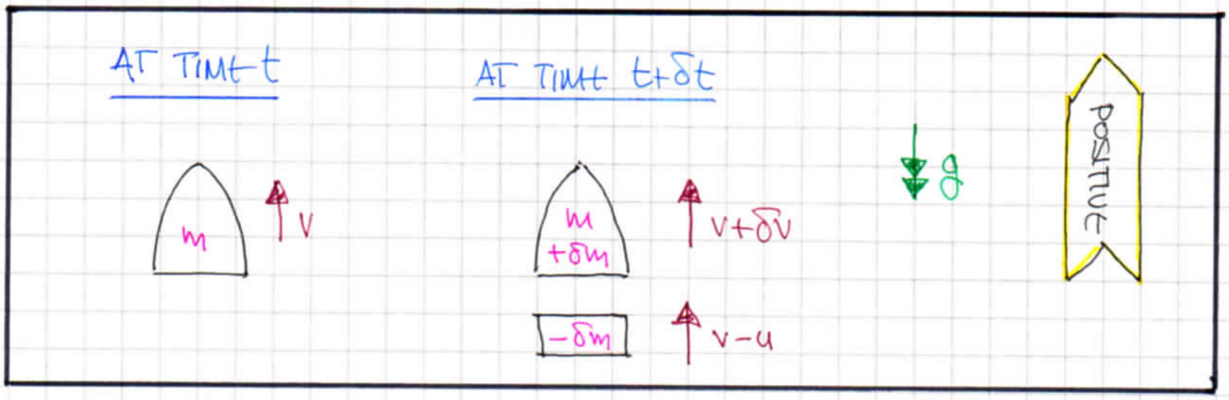
$$V''(\theta) = mgl\left[-4\sin^2\theta + 4(1-\sin^2\theta) + 3\sin\theta\right]$$
$$V''(\theta) = mgl\left[4 - 8\sin^2\theta + 3\sin\theta\right]$$
$$V''\left(\arcsin\frac{3}{4}\right) = mgl\left[4 - 8\times\frac{9}{16} + 3\times\frac{3}{4}\right]$$
$$V''\left(\arcsin\frac{3}{4}\right) = \frac{7}{4}mgl > 0$$

I.E A LOCAL MIN

$\theta = \arcsin\frac{3}{4}$ IS STABLE

IYGB - M456 PAPER B - QUESTION 3

STARTING WITH THE USUAL MOMENTUM/IMPULSE DIAGRAM



BY THE IMPULSE MOMENTUM PRINCIPLE

$$\Rightarrow -mg \times \delta t = \left[(m+\delta m)(v+\delta v) - \delta m(v-u)\right] - mv$$

$$\Rightarrow -mg\,\delta t = \cancel{mv} + m\delta v + \cancel{v\delta m} + \delta m\delta v - \cancel{v\delta m} + u\delta m - \cancel{mv}$$

$$\Rightarrow -mg\,\delta t = m\delta v + u\delta m + \delta m\delta v$$

$$\Rightarrow -mg = m\frac{\delta v}{\delta t} + u\frac{\delta m}{\delta t} + \frac{\delta m\delta v}{\delta t}$$

TAKING LIMITS AND REARRANGING FOR THE ACCELERATION

$$\Rightarrow -mg = m\frac{dv}{dt} + u\frac{dm}{dt}$$

$$\Rightarrow \frac{dv}{dt} = -g - \frac{u}{m}\frac{dm}{dt}$$

USING THE MASS CONSUMPTION RELATIONSHIP

$$\Rightarrow m = M(1-kt)$$

$$\Rightarrow \frac{dm}{dt} = -Mk$$

IYGB - M456 PAPER B - QUESTION 3

RETURNING TO THE MAIN O.D.E.

$$\Longrightarrow \frac{dv}{dt} = -g - \frac{u}{M(1-kt)}(-Mk)$$

$$\Longrightarrow \frac{dv}{dt} = -g + \frac{uk}{1-kt}.$$

SOLVING THE O.D.E BY DIRECT INTEGRATION, SUBJECT TO THE CONDITION $t=0,\ v=0$

$$\Longrightarrow \int_{v=0}^{v} 1\ dv = \int_{t=0}^{t} -g + \frac{uk}{1-kt}\ dt$$

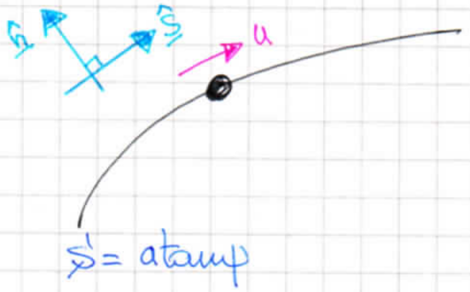
$$\Longrightarrow \Big[v\Big]_0^v = \Big[-gt - u\ln|1-kt|\Big]_0^t$$

$$\Longrightarrow v - 0 = \Big[gt + u\ln|1-kt|\Big]_t^0$$

$$\Longrightarrow v = \cancel{u\ln 1} - (gt + u\ln|1-kt|$$

$$\Longrightarrow v = -gt - u\ln|1-kt|$$

IYGB - M456 PAPER B - QUESTION 4



ACCELERATION IN INTRINSICS

$$\underline{a} = \ddot{s}\,\hat{\underline{s}} + \frac{\dot{s}^2}{\rho}\,\hat{\underline{n}}$$

- HERE $\dot{s} = u = \text{CONSTANT}$
- $\rho = \frac{ds}{d\psi} = a\sec^2\psi$

THE NORMAL COMPONENT ($\hat{\underline{n}}$) OF THE ACCELERATION IS GIVEN BY

$$\frac{\dot{s}^2}{\rho} = \frac{u^2}{a\sec^2\psi} = \frac{u^2}{a(1+\tan^2\psi)} = \frac{u^2}{a\left(1+\frac{s^2}{a^2}\right)} = \frac{u^2}{a+\frac{s^2}{a}}$$

$$= \frac{au^2}{a^2+s^2}$$

WE NEED TO FULLY ELIMINATE s - THE SPEED $\dot{s}$ ALONG THE CURVE IS CONSTANT

$$\Longrightarrow \frac{ds}{dt} = u$$

$$\Longrightarrow 1\,ds = u\,dt$$

$$\Longrightarrow \int_{s=0}^{s} 1\,ds = \int_{t=0}^{t} u\,dt$$

$t=0$, $\psi = 0$
$s = a\tan 0$
$s = 0$

$$\Longrightarrow \left[s\right]_0^s = \left[ut\right]_0^t$$

$$\Longrightarrow s = ut$$

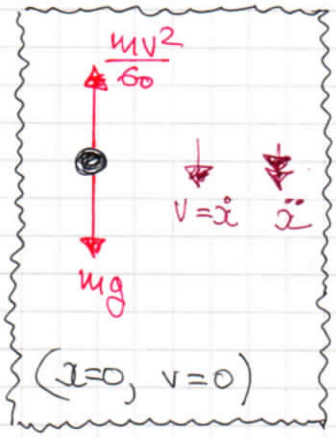
HENCE THE RESULT FOLLOWS

$$\frac{\dot{s}^2}{\rho} = \dots = \frac{au^2}{a^2+s^2} = \frac{au^2}{a^2+u^2t^2}$$

AS REQUIRED

IYGB - M456 PAPER B - QUESTION 5

FORMING A DIFFERENTIAL EQUATION FOR THE MOTION



$$\Rightarrow m\ddot{x} = mg - \frac{mv^2}{60}$$

$$\Rightarrow \ddot{x} = g - \frac{1}{60}v^2$$

$$\Rightarrow 60\ddot{x} = 60g - v^2$$

$$\Rightarrow 60\frac{dv}{dt} = 600 - v^2 \qquad (g = 10)$$

$$\Rightarrow 60\frac{dv}{dx}\frac{dx}{dt} = 600 - v^2$$

$$\Rightarrow 60\frac{dv}{dx}v = 600 - v^2$$

SOLVING BY SEPARATION OF VARIABLES SUBJECT TO CONDITIONS

$$\Rightarrow \frac{60v}{600 - v^2}\,dv = 1\,dx$$

$$\Rightarrow \int_{v=0}^{14} \frac{60v}{600 - v^2}\,dv = \int_{x=0}^{x} 1\,dx$$

$$\Rightarrow -30\int_{v=0}^{14} \frac{-2v}{600 - v^2}\,dv = \int_{x=0}^{x} 1\,dx$$

$$\Rightarrow -30\Big[\ln|600 - v^2|\Big]_0^{14} = \Big[x\Big]_0^{x}$$

$$\Rightarrow -30\Big[\ln 404 - \ln 600\Big] = x$$

$$\Rightarrow x = 30\ln\frac{150}{101} \approx 11.8654\ldots$$

APPROX 11.90 m

IYGB – M456 PAPER B – QUESTION 6

a) DIFFERENTIATE USING THE STANDARD RESULTS

$$\frac{d}{d\theta}(\hat{\underline{r}}) = \hat{\underline{\theta}} \quad \& \quad \frac{d}{d\theta}(\hat{\underline{\theta}}) = -\hat{\underline{r}}$$

$$\Longrightarrow \underline{r} = r\hat{\underline{r}}$$

$$\Longrightarrow \frac{d\underline{r}}{dt} = \frac{d}{dt}(r\hat{\underline{r}}) = \dot{r}\hat{\underline{r}} + r\frac{d\hat{\underline{r}}}{dt}$$

$$\Longrightarrow \underline{v} = \dot{r}\hat{\underline{r}} + r\frac{d\hat{\underline{r}}}{d\theta}\frac{d\theta}{dt}$$ ← RELATED RATES/CHAIN RULE

$$\Longrightarrow \underline{v} = \dot{r}\hat{\underline{r}} + r\hat{\underline{\theta}}\dot{\theta}$$

$$\Longrightarrow \underline{v} = \dot{r}\hat{\underline{r}} + r\dot{\theta}\hat{\underline{\theta}}$$

DIFFERENTIATE AGAIN TO OBTAIN THE ACCELERATION VECTOR

$$\Longrightarrow \frac{d\underline{v}}{dt} = \frac{d}{dt}(\dot{r}\hat{\underline{r}}) + \frac{d}{dt}(r\dot{\theta}\hat{\underline{\theta}})$$

$$\Longrightarrow \underline{a} = \ddot{r}\hat{\underline{r}} + \dot{r}\frac{d\hat{\underline{r}}}{dt} + \dot{r}\dot{\theta}\hat{\underline{\theta}} + r\ddot{\theta}\hat{\underline{\theta}} + r\dot{\theta}\frac{d\hat{\underline{\theta}}}{dt}$$

$$\Longrightarrow \underline{a} = \ddot{r}\hat{\underline{r}} + \dot{r}\frac{d\hat{\underline{r}}}{d\theta}\frac{d\theta}{dt} + \dot{r}\dot{\theta}\hat{\underline{\theta}} + r\ddot{\theta}\hat{\underline{\theta}} + r\dot{\theta}\frac{d\hat{\underline{\theta}}}{d\theta}\frac{d\theta}{dt}$$

$$\Longrightarrow \underline{a} = \ddot{r}\hat{\underline{r}} + \dot{r}\hat{\underline{\theta}}\dot{\theta} + \dot{r}\dot{\theta}\hat{\underline{\theta}} + r\ddot{\theta}\hat{\underline{\theta}} + r\dot{\theta}(-\hat{\underline{r}})\dot{\theta}$$

$$\Longrightarrow \underline{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\underline{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\underline{\theta}}$$

WHICH CAN ALSO BE WRITTEN AS

$$\Longrightarrow \underline{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\underline{r}} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

IYGB - M456 PAPER B - QUESTION 6

b) IF $r^2\frac{d\theta}{dt}$ IS CONSTANT $\Longrightarrow \frac{d}{dt}(r^2\dot{\theta}) = 0$

$\Longrightarrow$ NO TRANSVERSE ACCELERATION

$\Longrightarrow$ RESULTANT FORCE IS RADIAL (CENTRAL)

c) $r = 2 + \cos\theta$ AND ANGULAR SPEED $= \dot{\theta} = \sqrt{5}$

$\Longrightarrow r = 2 + \cos\theta$

$\Longrightarrow \frac{dr}{dt} = -\sin\theta \times \dot{\theta}$

$\Longrightarrow \dot{r} = -\sqrt{5}\sin\theta$ $\quad \boxed{\dot{r}\big|_{\theta=\frac{\pi}{2}} = -\sqrt{5}}$

DIFFERENTIATE AGAIN

$\Longrightarrow \ddot{r} = -\sqrt{5}\cos\theta \times \dot{\theta}$

$\Longrightarrow \ddot{r} = -5\cos\theta$ $\quad \boxed{\ddot{r}\big|_{\theta=\frac{\pi}{2}} = 0}$

- $\underline{v} = \dot{r}\hat{\underline{r}} + r\dot{\theta}\hat{\underline{\theta}} = -\sqrt{5}\hat{\underline{r}} + 2\sqrt{5}\hat{\underline{\theta}}$ $\quad \left[r\big|_{\frac{\pi}{2}} = 2 \right]$

$|\underline{v}| = \sqrt{(-\sqrt{5})^2 + (2\sqrt{5})^2} = 5 \text{ ms}^{-1}$

- $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\underline{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\underline{\theta}}$

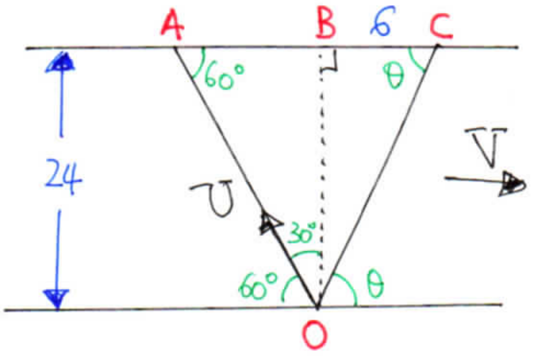
$\underline{a} = [0 - 2(\sqrt{5})^2]\hat{\underline{r}} + [2(-\sqrt{5})\sqrt{5} + 0]\hat{\underline{\theta}}$ $\quad [\ddot{\theta} = 0]$

$\underline{a} = -10\hat{\underline{r}} - 10\hat{\underline{\theta}}$

$|\underline{a}| = \sqrt{(-10)^2 + (-10)^2} = 10\sqrt{2} \text{ ms}^{-2}$

IYGB - MM56 PAPER B - QUESTION 7

- STARTING WITH A DIAGRAM



$$_{B}\underline{V}_{W} = \underline{V}_{B} - \underline{V}_{W}$$

$$\underline{V}_{B} = {_{B}\underline{V}_{W}} + \underline{V}_{W}$$

- USING STANDARD KINEMATICS

$\Longrightarrow \text{SPEED} = \dfrac{\text{DISTANCE}}{\text{TIME}}$

$\Longrightarrow U\sin 60 = \dfrac{24}{18}$

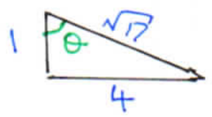
$\Longrightarrow U \times \dfrac{\sqrt{3}}{2} = \dfrac{4}{3}$

$\Longrightarrow U = \dfrac{8}{3\sqrt{3}}$

- BY SIMPLE GEOMETRY ON $\overset{\triangle}{OBC}$

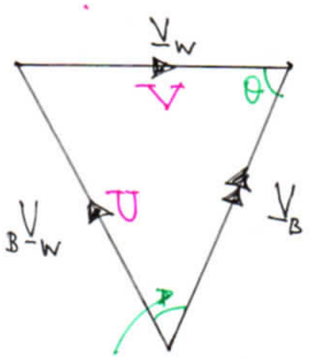
$\tan\theta = \dfrac{24}{6}$

$\tan\theta = 4$



$$\boxed{\sin\theta = \frac{4}{\sqrt{17}} \qquad \cos\theta = \frac{1}{\sqrt{17}}}$$

- FINALLY WE HAVE BY THE SINE RULE ON THE TRIANGLE BELOW



$30 + (90-\theta) = 120-\theta$

$\Longrightarrow \dfrac{V}{\sin(120-\theta)} = \dfrac{U}{\sin\theta}$

$\Longrightarrow V = \dfrac{U\sin(120-\theta)}{\sin\theta}$

$\Longrightarrow V = U\left[\dfrac{\sin 120\cos\theta - \cos 120\sin\theta}{\sin\theta}\right]$

$\Longrightarrow V = \dfrac{8}{3\sqrt{3}}\left[\dfrac{\sin 120}{\tan\theta} - \cos 120\right]$

$\Longrightarrow V = \dfrac{8}{3\sqrt{3}}\left[\dfrac{\sqrt{3}}{8} + \dfrac{1}{2}\right]$

$\Longrightarrow V = \dfrac{1}{3} + \dfrac{4}{3\sqrt{3}} = \dfrac{3+4\sqrt{3}}{9}$ //

IYGB - M456 PAPER B - QUESTION 8

a)

$$\underline{r}_1 = (-1,1,0) \qquad \underline{F}_1 = (3,-2,0)$$
$$\underline{r}_2 = (2,0,5) \qquad \underline{F}_2 = (4,-1,2)$$
$$\underline{r}_3 = (-6,2,1) \qquad \underline{F}_3 = (0,3,-4)$$

FIRSTLY COLLECT THE REQUIRED INFORMATION

- $$\sum_{i=1}^{3} \underline{F}_i = (3,-2,0) + (4,-1,2) + (0,3,-4)$$
$$= (7,0,-2)$$
$$\neq (0,0,0)$$

- $$\underline{G}_0 = \sum_{i=1}^{3} (\underline{r}_i \wedge \underline{F}_i)$$

$$\underline{G}_0 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 1 & 0 \\ 3 & -2 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 0 & 5 \\ 4 & -1 & 2 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & -1 & 2 \\ 0 & 3 & -4 \end{vmatrix}$$

$$\underline{G}_0 = (0,0,-1) + (5,16,-2) + (-11,-24,-18)$$

$$\underline{G}_0 = (-6,-8,-21)$$

- THUS WE HAVE

$$\left(\sum_{i=1}^{3} \underline{F}_i\right) \cdot \left(\sum_{i=1}^{3} (\underline{r}_i \wedge \underline{F}_i)\right) = (7,0,-2) \cdot (-6,-8,-21)$$
$$= -42 + 0 + 42$$
$$= 0$$

AS $\sum_{i=1}^{3} \underline{F}_i \neq 0$ AND $\left(\sum_{i=1}^{3} \underline{F}_i\right) \cdot \left(\sum_{i=1}^{3} (\underline{r}_i \wedge \underline{F}_i)\right) = 0$ WITHOUT $\sum_{i=1}^{3} \underline{r}_i \wedge \underline{F}_i = 0$, THEN THE SYSTEM REDUCES TO A FORCE

IYGB - M456 PAPER B - QUESTION 8

b) THE SYSTEM REDUCES TO A COUPLE — LET THE RESULTANT FORCE $\sum_{i=1}^{3} F_i$ ACT THROUGH $P(x, y, z)$

$$\Longrightarrow \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ x & y & z \\ 7 & 0 & -2 \end{vmatrix} = \underline{G}_0$$

$$\Longrightarrow (-2y, 7z + 2x, -7y) = (-6, -8, -21)$$

THUS WE HAVE BY EQUATING COMPONENTS

$-2y = -6$ & $7z + 2x = -8$

$y = 3$

LET $z = 0$

$2x = -8$

$x = -4$

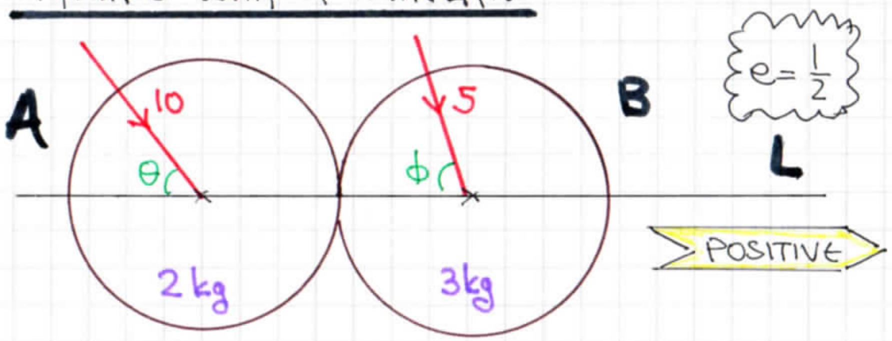
I.E THE EQUATIONS CAN BE SATISFIED BY

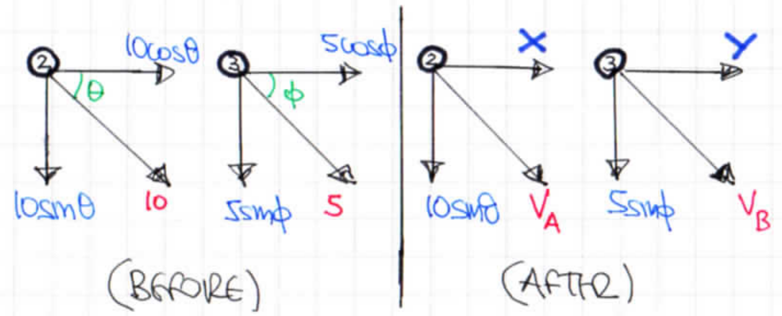
$x = -4$
$y = 3$
$z = 0$

$$\therefore \quad \underline{r} = (-4, 3, 0) + \lambda(7, 0, -2)$$

IYGB – M456 PAPER B – QUESTION 9

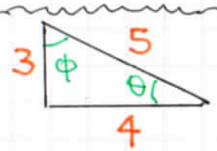
- STARTING WITH A DIAGRAM





- NO MOMENTUM IS EXCHANGED "PERPENDICULARLY" TO L
- BY CONSERVATION OF MOMENTUM OF L

$$\Rightarrow 2(10\cos\theta) + 3(5\cos\phi) = 2X + 3Y$$
$$\Rightarrow 20\cos\theta + 15\cos\phi = 2X + 3Y$$
$$\Rightarrow 20\left(\tfrac{4}{5}\right) + 15\left(\tfrac{3}{5}\right) = 2X + 3Y$$
$$\Rightarrow 2X + 3Y = 16 + 9$$
$$\Rightarrow 2X + 3Y = 25$$



$\sin\theta = \cos\phi = \frac{3}{5}$

$\cos\theta = \sin\phi = \frac{4}{5}$

- BY CONSIDERING RESTITUTION ALONG L

$$\Rightarrow e = \frac{SEP}{APP}$$
$$\Rightarrow \frac{1}{2} = \frac{Y - X}{10\cos\theta - 5\cos\phi}$$
$$\Rightarrow \frac{1}{2} = \frac{Y - X}{10\left(\frac{4}{5}\right) - 5\left(\frac{3}{5}\right)}$$
$$\Rightarrow -X + Y = \frac{5}{2}$$
$$\Rightarrow -2X + 2Y = 5$$

- SOLVING SIMULTANEOUSLY

$5Y = 30$ (ADDING)

$Y = 6$

& $2X + 3Y = 25$

$2X + 18 = 25$

$2X = 7$

$X = \frac{7}{2}$

IYGB – M456 PAPER B – QUESTION 9

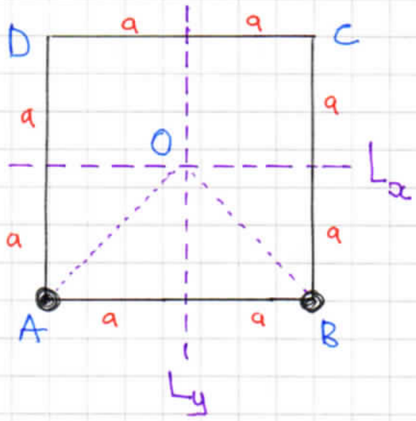
- FINALLY THE "AFTER" SPEEDS CAN BE OBTAINED

$$V_A = \sqrt{(10\sin\theta)^2 + X^2} = \sqrt{\left[10\left(\tfrac{3}{5}\right)\right]^2 + 3.5^2} = \sqrt{48.25} \simeq 6.95 \text{ ms}^{-1}$$

$$V_B = \sqrt{(5\sin\phi)^2 + Y^2} = \sqrt{5\left(\tfrac{4}{5}\right)^2 + 6^2} = \sqrt{52} \approx 7.21 \text{ ms}^{-1}$$

IYGB - M456 PAPER B - QUESTION 10

START BY DETERMINING THE MOMENT OF INERTIA OF THE LOADED LAMINA, ABOUT O



- BY THE "STRETCH RULE" BACKWARDS

$$I_{L_x} = I_{L_y} = \frac{1}{3}ma^2,$$

AS THE LAMINA COMPRESSES TO A ROD OF LENGTH $2a$ & MASS m, WITH ITS CENTRE AT O

- BY THE PERPENDICULAR AXES THEOREM, I_O, PERPENDICULAR TO THE PLANE OF THE LAMINA IS GIVEN BY

$$I_O = \frac{1}{3}ma^2 + \frac{1}{3}ma^2 = \frac{2}{3}ma^2$$

- $|OA| = |OB| = \sqrt{2}a$ (BY PYTHAGORAS)

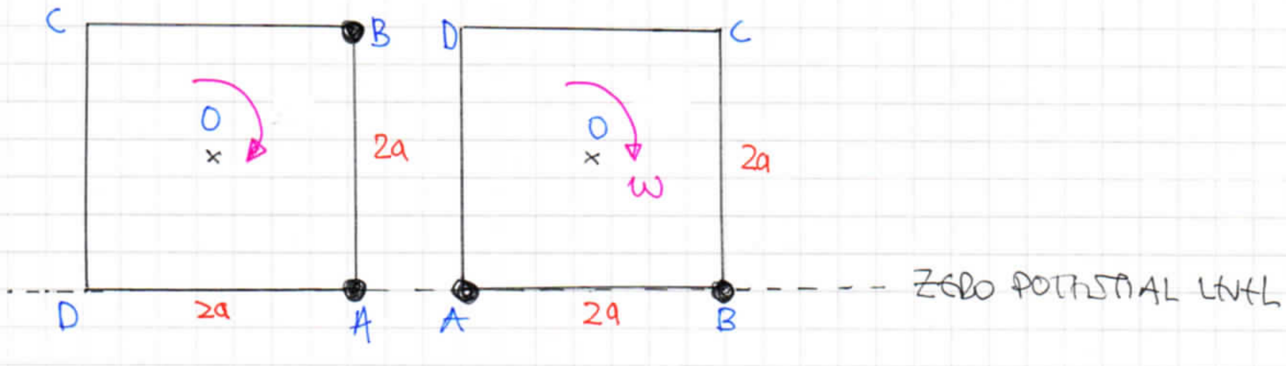
- CONTRIBUTION OF EACH PARTICLE, TO THE MOMENT OF INERTIA ABOUT O, IS GIVEN BY

$$m(\sqrt{2}a)^2 = 2ma^2$$

- TOTAL MOMENT OF INERTIA ABOUT O

$$I_{TOT} = \frac{2}{3}ma^2 + 2(2ma^2) = \frac{14}{3}ma^2$$

LOOKING AT THE DIAGRAM BELOW AND BY CONSIDERING ENERGY



IYGB – M456 PAPER B – QUESTION 10

NOTE THAT THE CENTRE OF MASS OF THE LAMINA (MASS m) DOES NOT MOVE AND HENCE WE MAY IGNORE IT

$$\Longrightarrow KE_{BEFORE} + PE_{BEFORE} = KE_{AFTER} + PE_{AFTER}$$

$$\Longrightarrow 0 + mg(2a) = \frac{1}{2}I_{TOT}\,\omega^2 + 0$$

$$\Longrightarrow 2mga = \frac{1}{2}\left(\frac{14}{3}ma^2\right)\omega^2$$

$$\Longrightarrow 2\cancel{m}g\cancel{a} = \frac{7}{3}\cancel{m}a^{\cancel{2}}\omega^2$$

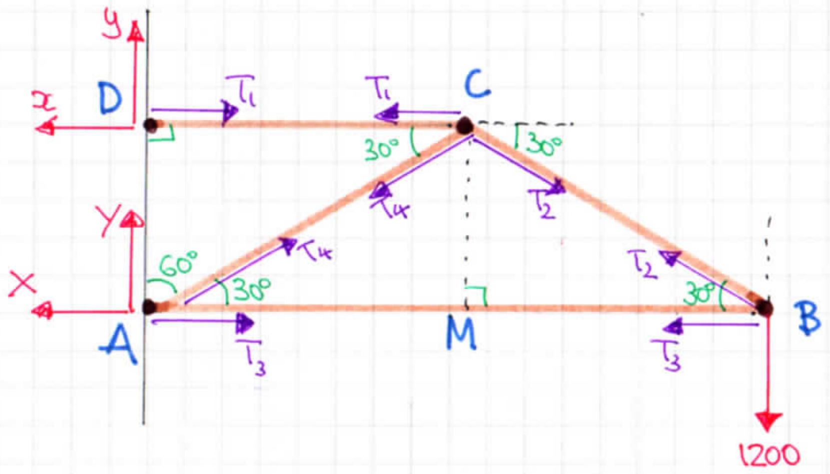
$$\Longrightarrow 2g = \frac{7}{3}a\omega^2$$

$$\Longrightarrow \frac{6g}{7a} = \omega^2$$

$$\Longrightarrow \omega = \sqrt{\frac{6g}{7a}}$$

IYGB - MMS6 PAPER B - QUESTION 11

- DRAW A DIAGRAM - MARK ALL INTERNAL FORCES AS TENSIONS



- LOOKING AT "B" VERTICALLY

$$T_2 \sin 30° = 1200$$
$$\tfrac{1}{2}T_2 = 1200$$
$$T_2 = 2400$$

- LOOKING AT "B" HORIZONTALLY

$$T_3 + T_2 \cos 30 = 0$$
$$T_3 = -T_2 \cos 30°$$
$$T_3 = -2400 \times \frac{\sqrt{3}}{2}$$
$$T_3 = -1200\sqrt{3}$$

- LOOKING AT "C" VERTICALLY

$$T_4 \cancel{\sin 30°} + T_2 \cancel{\sin 30°} = 0$$
$$T_4 = -T_2$$
$$T_4 = -2400$$

- LOOKING AT "C" HORIZONTALLY

$$T_1 + T_4 \cos 30° = T_2 \cos 30$$
$$T_1 + (-2400)\frac{\sqrt{3}}{2} = 2400 \times \frac{\sqrt{3}}{2}$$
$$T_1 = 2400\sqrt{3}$$

- LOOKING AT "D" HORIZONTALLY

$$x = T_1$$
$$x = 2400\sqrt{3}$$

IYGB - M456 PAPER B - QUESTION 11

- LOOKING AT "A" HORIZONTALLY

$X = T_3 + T_4\cos 30$

$X = -1200\sqrt{3} - 2400\left(\frac{\sqrt{3}}{2}\right)$

$X = -1200\sqrt{3} - 1200\sqrt{3}$

$X = -2400\sqrt{3}$

(OPPOSITE DIRECTION TO THAT MARKED)

- LOOKING AT "A" VERTICALLY

$Y + T_4\sin 30 = 0$

$Y + (-2400)\times\frac{1}{2} = 0$

$Y = 1200$

- LOOKING AT "D" VERTICALLY

$y = 0$

HENCE WE HAVE IN SUMMARY

- REACTION AT D, $2400\sqrt{3}$
- REACTION AT A, $1200\sqrt{13}$ (≈ 4327)

$\uparrow$

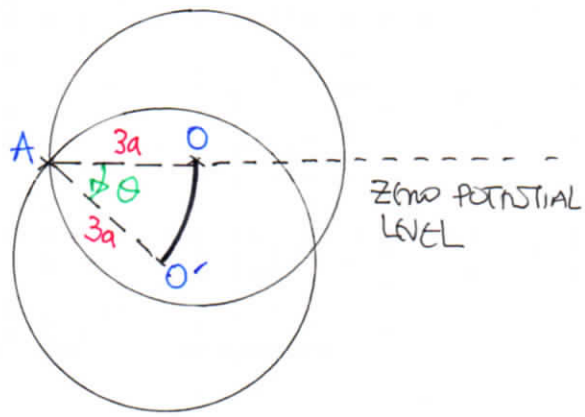
$\sqrt{X^2+Y^2}$

-

ROD	FORCE / STATUS
AB	$1200\sqrt{3}$, THRUST
BC	2400, TENSION
CA	2400, THRUST
CD	$2400\sqrt{3}$, TENSION

IYGB - M456 PAPER B - QUESTION 12

FIRSTLY THE MOMENT OF INERTIA OF THE DISC ABOUT A IS GIVEN BY

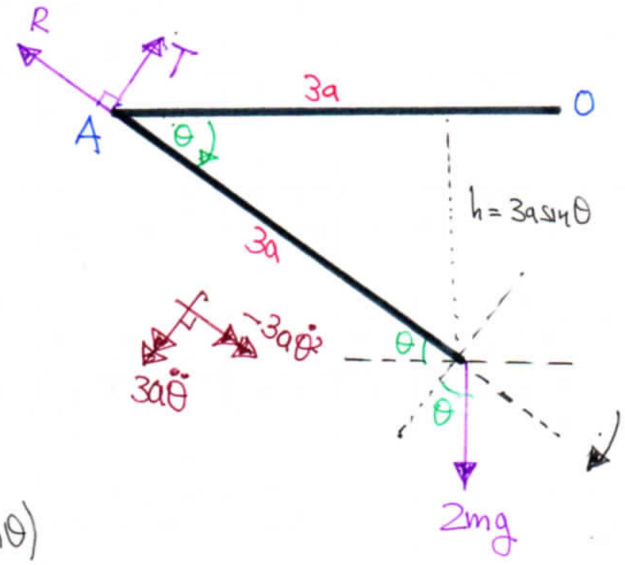


$$\frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$$

$$= \frac{3}{2}(2m)(3a)^2$$

$$= 27ma^2$$

BY ENERGIES TAKING THE INITIAL POSITION AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL



$$KE_{INIT} + PE_{INIT} = KE_{\theta} + P.E_{\theta}$$

$$0 + 0 = \frac{1}{2}I\dot{\theta}^2 - Mgh$$

$$0 = \frac{1}{2}(27ma^2)\dot{\theta}^2 - (2m)g(3a\sin\theta)$$

$$0 = \frac{27}{2}a\dot{\theta}^2 - 6g\sin\theta$$

$$\frac{27}{2}a\dot{\theta}^2 = 6g\sin\theta$$

$$\dot{\theta}^2 = \frac{4g}{9a}\sin\theta$$

EQUATION OF ROTATIONAL MOTION ALSO YIELDS

$$I\ddot{\theta} = L$$

$$(27ma^2)\ddot{\theta} = (2mg\cos\theta) \times 3a$$

$$27a\ddot{\theta} = 6g\cos\theta$$

$$\ddot{\theta} = \frac{2g}{9a}\cos\theta$$

IYGB - M456 PAPER B - QUESTION 12

THE EQUATION OF MOTION GIVES

RADIALLY

$\Rightarrow 2mg\sin\theta - R = 2m(-3a\dot{\theta}^2)$

$\Rightarrow 2mg\sin\theta + 6ma\dot{\theta}^2 = R$

$\Rightarrow R = 2mg\sin\theta + 6ma\left(\frac{4g}{9a}\sin\theta\right)$

$\Rightarrow R = 2mg\sin\theta + \frac{8}{3}mg\sin\theta$

$\Rightarrow R = \frac{14}{3}mg\sin\theta$

TRANSVERSLY

$\Rightarrow 2mg\cos\theta - T = 2m(3a\ddot{\theta})$

$\Rightarrow 2mg\cos\theta - 6ma\ddot{\theta} = T$

$\Rightarrow T = 2mg\cos\theta - 6ma\left(\frac{2g}{9a}\cos\theta\right)$

$\Rightarrow T = 2mg\cos\theta - \frac{4}{3}mg\cos\theta$

$\Rightarrow T = \frac{2}{3}mg\cos\theta$

FINALLY THE MAGNITUDE OF THIS FORCE IS GIVEN BY

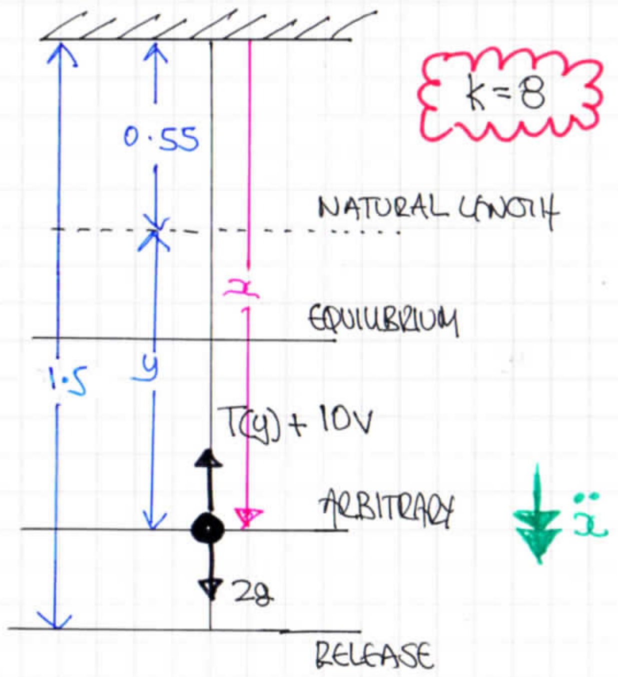
$$F = \sqrt{R^2 + T^2}$$

$$F = \frac{2}{3}mg\sqrt{(7\sin\theta)^2 + \cos^2\theta}$$

$$F = \frac{2}{3}mg\sqrt{49\sin^2\theta + \cos^2\theta}$$

$$F = \frac{2}{3}mg\sqrt{1 + 48\sin^2\theta}$$

IYGB - M456 PAPER B - QUESTION 13



- THE EQUATION OF MOTION, LOOKING AT THE ABOVE DIAGRAM IS GIVEN BY

$$\Rightarrow m\ddot{x} = 2g - T(y) - 10v$$
$$\Rightarrow 2\ddot{x} = 2g - ky - 10v$$
$$\Rightarrow 2\ddot{x} = 2g - 8(x-0.55) - 10\dot{x}$$
$$\Rightarrow \ddot{x} = g - 4(x-0.55) - 5\dot{x}$$
$$\Rightarrow \ddot{x} = g - 4x + 2.2 - 5\dot{x}$$
$$\Rightarrow \ddot{x} + 5\dot{x} + 4x = g + 2.2$$
$$\Rightarrow \ddot{x} + 5\dot{x} + 4x = 12$$

- AUXILIARY EQUATION

$$\lambda^2 + 5\lambda + 4 = 0$$
$$(\lambda+1)(\lambda+4) = 0$$
$$\lambda = \begin{cases} -1 \\ -4 \end{cases}$$

- COMPLEMENTARY FUNCTION

$$x = Ae^{-t} + Be^{-4t}$$

- PARTICULAR INTEGRAL BY INSPECTION

$$x = 3$$

- GENERAL SOLUTION

$$x = Ae^{-t} + Be^{-4t} + 3$$

IYGB - M456 PAPER B - QUESTION 13

- DIFFERENTIATE AND APPLY CONDITIONS

$$x = Ae^{-t} + Be^{-4t} + 3$$

$$\dot{x} = -Ae^{-t} - 4Be^{-4t}$$

- $t=0,\ x=1.5 \implies 1.5 = A + B + 3$

$\implies A + B = -1.5$

- $t=0,\ \dot{x}=0 \implies 0 = -A - 4B$

$\implies A = -4B$

$\implies -4B + B = -1.5$

$\implies -3B = -1.5$

$\implies B = \frac{1}{2}$

$\implies A = -2$

$$\therefore x = \frac{1}{2}e^{-4t} - 2e^{-t} + 3$$

IYGB - M456 PAPER B - QUESTION 14

$A(-10,-1,3)$ & $B(8,11,9)$

$$\overrightarrow{AB} = \underline{b} - \underline{a} = (8,11,9) - (-10,-1,3) = (18,12,6) = \lambda(3,2,1)$$

THE RESULTANT OF THE 3 FORCES MUST ACT IN THAT DIRECTION

$$\underline{F}_1 + \underline{F}_2 + \underline{F}_3 = \alpha(3,2,1)$$

$$(1,2,6) + (7,-2,4) + (2k-1,\, 2k+2,\, 3-2k) = (3\alpha,\, 2\alpha,\, \alpha)$$

$$(2k+7,\, 2k+2,\, 13-2k) = (3\alpha,\, 2\alpha,\, \alpha)$$

$$\left.\begin{array}{l} \alpha = 13-2k \\ 2\alpha = 2k+2 \end{array}\right\} \Longrightarrow \begin{array}{l} 26-4k = 2k+2 \\ 24 = 6k \\ k=4 \quad \& \quad \alpha = 5 \end{array}$$

THUS $\underline{F}$ (RESULTANT) IS $(15,10,5)$ & $\overrightarrow{AB} = (18,12,6)$

$$W = \underline{F} \cdot \underline{r} = (15,10,5)\cdot(18,12,6)$$

$$= 5(3,2,1) \cdot 6(3,2,1)$$

$$= 30(3,2,1)\cdot(3,2,1)$$

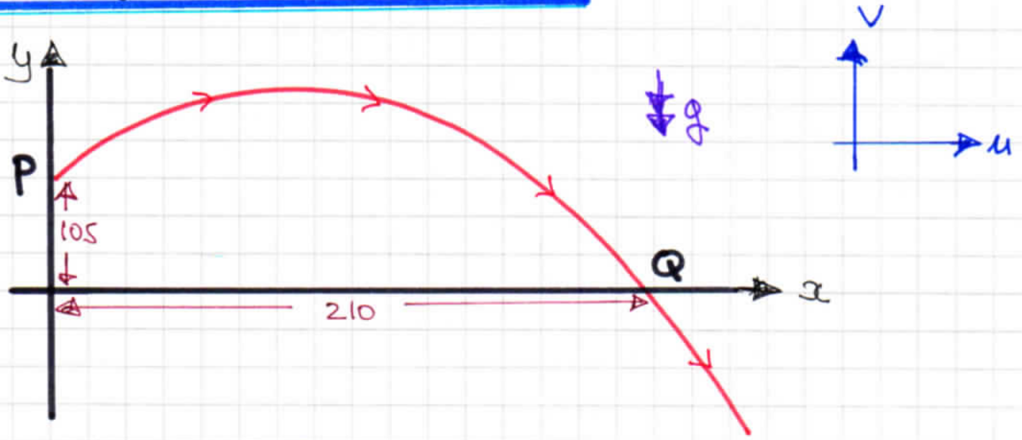
$$= 30(9+4+1)$$

$$= 30 \times 14$$

$$= 420\text{ J}$$

IYGB – MM56 PAPER B – QUESTION 15

a) LOOKING AT THE DIAGRAM BELOW



HORIZONTALLY
(DISTANCE = SPEED × TIME)

$210 = uT$

$T = \frac{210}{u}$

VERTICALLY
$(s = s_0 + ut + \frac{1}{2}at^2)$

$0 = 105 + vT + \frac{1}{2}(-10)T^2$

$5T^2 - vT - 105 = 0$

$\Rightarrow 5\left(\frac{210}{u}\right)^2 - v\left(\frac{210}{u}\right) - 105 = 0$

$\Rightarrow \frac{220500}{u^2} - \frac{210v}{u} - 105 = 0$ $\quad \div 105$

$\Rightarrow \frac{2100}{u^2} - \frac{2v}{u} - 1 = 0$ $\quad \times u^2$

$\Rightarrow 2100 - 2uv - u^2 = 0$

$\Rightarrow u^2 + 2uv = 2100$

// AS REQUIRED

b) "MOVING PARALLEL TO $\mathbf{i}$ $\Rightarrow$ NO VERTICAL SPEED WHEN $t = 2$

"$v = u + at$" $\Rightarrow 0 = v - 10 \times 2$

$\Rightarrow v = 20$

IYGB - M456 PAPER B - QUESTION 15

AND USING $u^2 + 2uv = 2100$

$\Rightarrow u^2 + 40u = 2100$

$\Rightarrow u^2 + 40u - 2100$

$\Rightarrow (u-30)(u+70) = 0$

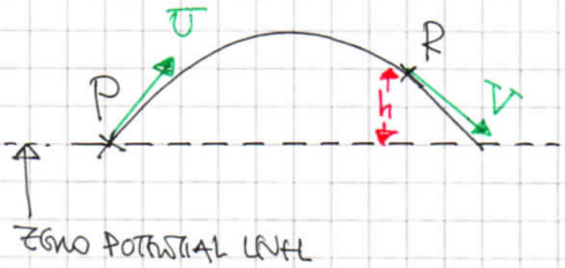
$\Rightarrow u = 30 \quad (u > 0)$

FINALLY USING $210 = uT$

$\Rightarrow 210 = 30T$

$\Rightarrow T = 7$

c) BY CONSIDERING ENERGY & TAKING THE LEVEL OF P AS THE ZERO POTENTIAL LEVEL



$\Rightarrow KE_P + \cancel{PE_P} = KE_Q + PE_Q$

$\Rightarrow \frac{1}{2}mU^2 = \frac{1}{2}mV^2 + mgh$

$\Rightarrow U^2 = V^2 + 2gh$

$\Rightarrow \left[\sqrt{\underset{u^2}{30^2} + \underset{v^2}{20^2}}\right]^2 = (10\sqrt{29})^2 + 2gh$

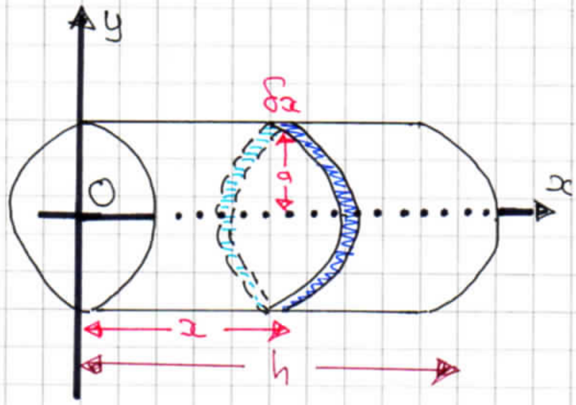
$\Rightarrow 1300 = 2900 + 2 \times 10 \times h$

$\Rightarrow h = -80$

IT 80m BELOW THE LEVEL OF PROJECTION

IYGB – M456 PAPER B – QUESTION 16

LOOKING AT THE DIAGRAM



- SURFACE AREA OF CURVED SURFACE IS $\pi a^2 h$
- ρ; (MASS PER UNIT AREA) IS
$$\rho = \frac{M}{\pi a^2 h}$$
- MASS OF INFINITESIMAL HOOP OF THICKNESS δx IS GIVEN BY
$$\delta m = (2\pi a\, \delta x)\rho$$
- THE MOMENT OF INERTIA OF THE INFINITESIMAL HOOP ABOUT THE x AXIS IS GIVEN BY
$$\delta I = \delta m\, a^2$$
- MOMENT OF INERTIA OF THE HOOP ABOUT A DIAMETER (BY USING THE PERPENDICULAR AXES THEOREM) IS GIVEN BY
$$\frac{1}{2}\delta I \quad \text{(USING THE THEOREM "BACKWARDS")}$$
- FINALLY, BY THE PARALLEL AXES THEOREM, THE MOMENT OF INERTIA OF THE INFINITESIMAL HOOP, ABOUT THE y AXIS IS GIVEN BY
$$\frac{1}{2}\delta I + \delta m\, x^2$$

SUMMING UP AND TAKING LIMITS

$$J = \int_{x=0}^{x=h} \left(\frac{1}{2}dI + x^2\,dm\right) = \int_{x=0}^{x=h} \frac{1}{2}a^2 dm + x^2\,dm$$

$$= \int_{x=0}^{x=h} \left(\frac{1}{2}a^2 + x^2\right) dm$$

IYGB - M456 PAPER B - QUESTION 16

$$\Rightarrow I = \int_{x=0}^{x=h} \left(\tfrac{1}{2}a^2 + x^2\right)\left(2\pi a \rho \, dx\right)$$

$$\Rightarrow I = \int_{x=0}^{x=h} 2\pi a \rho \left(\tfrac{1}{2}a^2 + x^2\right) \, dx$$

$$\Rightarrow I = 2\pi a \rho \int_0^h \tfrac{1}{2}a^2 + x^2 \, dx$$

$$\Rightarrow I = \cancel{2\pi a} \times \frac{M}{\cancel{2\pi a} h} \left[\tfrac{1}{2}a^2 x + \tfrac{1}{3}x^3\right]_0^h$$

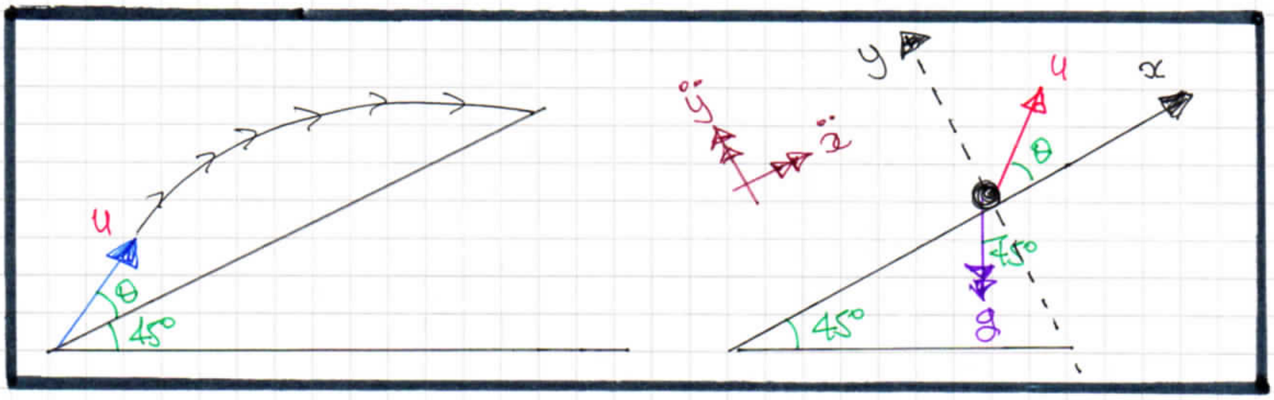
$$\Rightarrow I = \frac{M}{h}\left[\left(\tfrac{1}{2}a^2 h + \tfrac{1}{3}h^3\right) - 0\right]$$

$$\Rightarrow I = M\left(\tfrac{1}{2}a^2 + \tfrac{1}{3}h^2\right)$$

$$\Rightarrow I = \tfrac{1}{6}M\left(3a^2 + 2h^2\right)$$

AS REQUIRED

IYGB - M456 PAPER B - QUESTION 17



DETERMINE EQUATIONS FOR DISPLACEMENTS & VELOCITIES IN x & y BY SUCCESSIVE INTEGRATIONS

$$\Rightarrow \ddot{x} = -g\sin 45°$$
$$\Rightarrow \ddot{x} = -g\frac{\sqrt{2}}{2}$$
$$\Rightarrow \dot{x} = -gt\frac{\sqrt{2}}{2} + u\cos\theta$$
$$\Rightarrow x = ut\cos\theta - \frac{1}{4}\sqrt{2}gt^2$$
$$[t=0,\ x=0,\ \dot{x}=u\cos\theta]$$

$$\Rightarrow \ddot{y} = -g\cos 45°$$
$$\Rightarrow \ddot{y} = -g\frac{\sqrt{2}}{2}$$
$$\Rightarrow \dot{y} = -gt\frac{\sqrt{2}}{2} + u\sin\theta$$
$$\Rightarrow y = ut\sin\theta - \frac{1}{4}\sqrt{2}gt^2$$
$$[t=0,\ y=0,\ \dot{y}=u\sin\theta]$$

NEXT FIND THE FLIGHT TIME BY SOLVING $y=0$

$$\Rightarrow ut\sin\theta - \frac{1}{4}\sqrt{2}gt^2 = 0$$
$$\Rightarrow \frac{1}{4}t\left[4u\sin\theta - \sqrt{2}gt\right] = 0$$
$$\Rightarrow t = \begin{cases} \cancel{0} & \text{(INITIALLY)} \\ \dfrac{4u\sin\theta}{\sqrt{2}g} = \dfrac{2\sqrt{2}u\sin\theta}{g} \end{cases}$$

NEXT WE FIND THE RANGE UP THE PLANE (x), USING THE FLIGHT TIME

$$\Rightarrow x = u\left(\frac{2\sqrt{2}u\sin\theta}{g}\right)\cos\theta - \frac{1}{4}\sqrt{2}g\left(\frac{2\sqrt{2}u\sin\theta}{g}\right)^2$$
$$\Rightarrow x = \frac{2\sqrt{2}u^2\sin\theta\cos\theta}{g} - \frac{2\sqrt{2}u^2\sin^2\theta}{g}$$

IYGB - M456 PAPER B - QUESTION 17

$$\Rightarrow x = \frac{2\sqrt{2}u^2}{g}\left[\sin\theta\cos\theta - \sin^2\theta\right]$$

$$\Rightarrow x = \frac{2\sqrt{2}u^2}{g}\left[\frac{1}{2}\sin 2\theta - \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)\right]$$

$$\Rightarrow x = \frac{2\sqrt{2}u^2}{g}\left[\frac{1}{2}\sin 2\theta + \frac{1}{2}\cos 2\theta - \frac{1}{2}\right]$$

$$\Rightarrow x = \frac{\sqrt{2}u^2}{g}\left[\sin 2\theta + \cos 2\theta - 1\right]$$

MANIPULATE THE TRIGONOMETRIC EXPRESSION DIRECTLY OR BY THE "R-TRANSFORMATION" METHOD

$$\Rightarrow x = \frac{\sqrt{2}u^2}{g} \times \sqrt{2} \times \left[\frac{1}{\sqrt{2}}\sin 2\theta + \frac{1}{\sqrt{2}}\cos 2\theta - \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow x = \frac{2u^2}{g}\left[\cos 45 \sin 2\theta + \sin 45 \cos 2\theta - \frac{1}{\sqrt{2}}\right]$$

$$\Rightarrow x = \frac{2u^2}{g}\left[\sin(2\theta + 45) - \frac{1}{\sqrt{2}}\right]$$

TO MAXIMIZE x, WE REQUIRE

$$\Rightarrow \sin(2\theta + 45) = 1$$

$$\Rightarrow 2\theta + 45 = 90$$

$$\Rightarrow 2\theta = 45$$

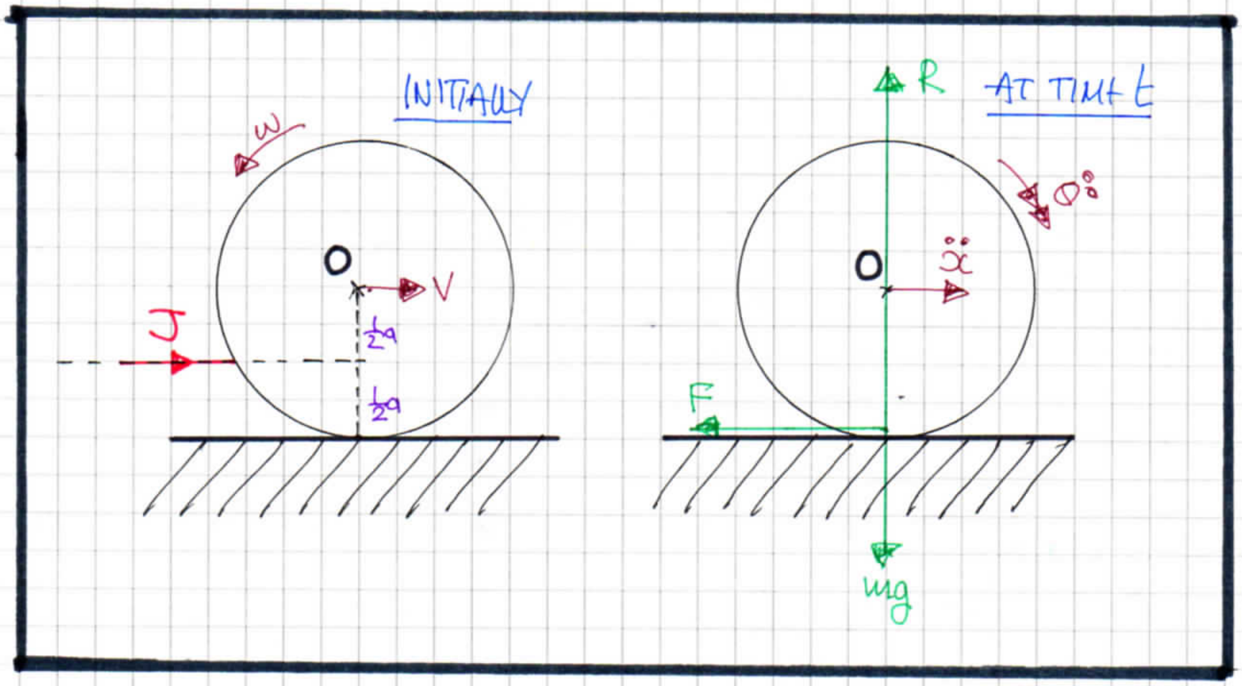
$$\Rightarrow \theta = 22.5$$

$\therefore$ PROJECTION ANGLE IS $22.5°$ TO THE PLANE

OR

$67.5°$ TO THE HORIZONTAL

IYGB - M456 PAPER B - QUESTION 18



LOOKING AT THE INITIAL CONFIGURATION

- $I_0 = \frac{2}{5}ma^2$ (STANDARD RESULT)
- BY THE CONSERVATION OF LINEAR MOMENTUM : $J = mV$
  $V = \frac{J}{m}$
  (INITIAL SPEED)
- BY CONSERVATION OF ANGULAR MOMENTUM ABOUT O
  $$J \times \frac{1}{2}a = I\omega$$
  $$J \times \frac{1}{2}a = \frac{2}{5}ma^2\omega$$
  $$\omega = \frac{5J}{4ma}$$ (INITIAL ANGULAR SPEED, "BACKWARDS")

EQUATIONS OF MOTION, AT TIME t

$$\left\{\begin{array}{l} m\ddot{x} = -F \\ I\ddot{\theta} = Fa \end{array}\right\} \Rightarrow \left\{\begin{array}{l} m\ddot{x} = -F \\ \frac{2}{5}ma^2\ddot{\theta} = Fa \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} m\ddot{x} = -F \\ \frac{2}{5}ma\ddot{\theta} = F \end{array}\right\}$$

[ADDING EQUATIONS]

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$$\Rightarrow m\ddot{x} + \tfrac{2}{5}ma\ddot{\theta} = 0$$

$$\Rightarrow \ddot{x} + \tfrac{2}{5}a\ddot{\theta} = 0$$

INTEGRATE WITH RESPECT TO t

$$\Rightarrow \boxed{\dot{x} + \tfrac{2}{5}a\dot{\theta} = C}$$

- $t=0$
- $\theta=0$, $x=0$
- $\dot{x}=V$, $\dot{\theta}=\omega$ "BACKWARDS"

$$V + \tfrac{2}{5}a(-\omega) = C$$

$$C = \frac{J}{m} - \frac{2}{5}a\left(\frac{5J}{4ma}\right)$$

$$C = \frac{J}{m} - \frac{J}{2m} = \frac{J}{2m}$$

$$\Rightarrow \boxed{\dot{x} + \tfrac{2}{5}a\dot{\theta} = \frac{J}{2m}} \quad \text{OR} \quad \boxed{\dot{x} + \tfrac{2}{5}a\dot{\theta} = \tfrac{1}{2}V}$$

WHEN THE SPHERE BEGINS TO ROLL $\dot{x} = a\dot{\theta}$

$$\Rightarrow \dot{x} + \tfrac{2}{5}\dot{x} = \tfrac{1}{2}V$$

$$\Rightarrow \tfrac{7}{5}\dot{x} = \tfrac{1}{2}V$$

$$\Rightarrow \dot{x} = \tfrac{5}{14}V$$

$\therefore$ WHEN IT BEGINS TO ROLL WHEN IT REACHES $\frac{5}{14}$ OF ITS INITIAL SPEED V