IYGB – MI456 PAPER C – QUESTION 1

START BY FINDING $\vec{AB}$

$$\vec{AB} = \underline{b} - \underline{a} = (17\underline{i} - 5\underline{j}) - (2\underline{i} + 5\underline{j}) = 15\underline{i} - 10\underline{j}$$

FIND THE WORK DONE (IN OR OUT) BY THE FORCE

$$W = \underline{F} \cdot \underline{r} = (2.6\underline{i} - 0.1\underline{j}) \cdot (15\underline{i} - 10\underline{j})$$

$$= 39 + 1 = 40\ \text{J}$$

BY ENERGIES

$$\text{WORK IN} = \text{GAIN IN KINETIC ENERGY}$$

$$40 = \frac{1}{2}mv^2$$

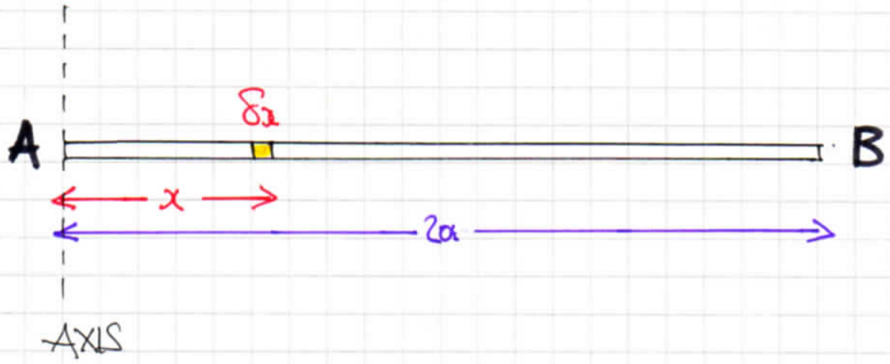
$$40 = \frac{1}{2} \times 0.2 \times v^2$$

$$v^2 = 400$$

$$v = 20\ \text{ms}^{-1}$$

IYGB - M456 PAPER C - QUESTION 2

LOOKING AT THE DIAGRAM BELOW



IF THE MASS OF THE ROD IS m, THEN $\rho = \frac{m}{2a}$

(MASS PER UNIT LENGTH)

- THE MASS OF INFINITESIMAL LENGTH δx IS $\rho \delta x$
- THE MOMENT OF INERTIA OF THE "INFINITESIMAL" ABOUT THE AXIS THROUGH A IS

$$(\rho \delta x) x^2$$

- SUMMING UP ALL THESE MOMENTS OF INERTIA FROM A TO B AND TAKING LIMITS

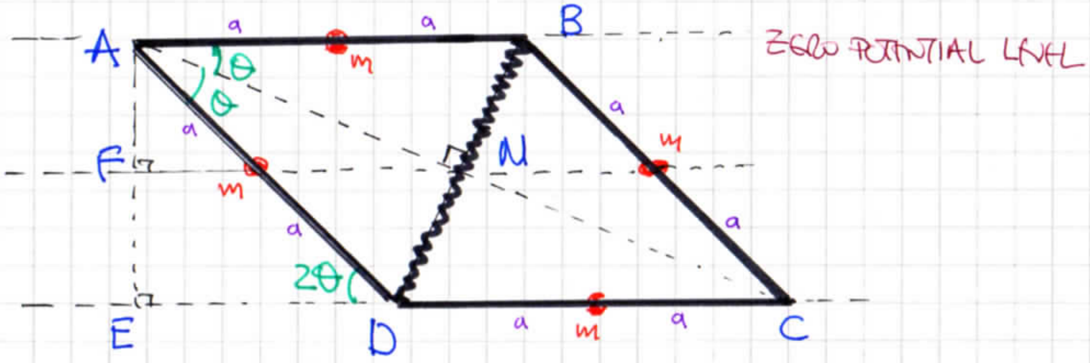
$$I = \int_{x=0}^{x=2a} \rho x^2 \, dx = \left[\frac{1}{3} \rho x^3 \right]_{x=0}^{x=2a} = \frac{1}{3}\rho (8a^3) - 0$$

$$= \frac{1}{3}\left(\frac{m}{2a}\right)(8a^3) = \frac{4}{3}ma^2$$

AS REQUIRED

IYGB - M456 PAPER C - QUESTION 3

START WITH A DETAILED DIAGRAM



CALCULATE SOME LENGTHS NEEDED, IN TERMS OF θ

- $|BM| = |AB|\sin\theta = 2a\sin\theta$
- $|BD| = 2|BM| = 4a\sin\theta$
- $|AE| = |AD|\sin 2\theta = 2a\sin 2\theta$
- $|AF| = |FE| = \frac{1}{2}|AE| = a\sin 2\theta$

TAKING THE LEVEL OF AB AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL

$$V_G(\theta) = -mg|AF| \times 2 - mg|AE|$$

(RODS AD, BC) (ROD DC)

$$V_G(\theta) = -2mg(a\sin 2\theta) - mg(2a\sin 2\theta)$$

$$V_G(\theta) = -4mga\sin 2\theta$$

ELASTIC POTENTIAL ENERGY NEXT

$$V_E(\theta) = \text{"}\frac{\lambda}{2l}x^2\text{"}$$

$$V_E(\theta) = \frac{mg}{2a}\left[|BD| - a\right]^2$$

$$V_E(\theta) = \frac{mg}{2a}\left[4a\sin\theta - a\right]^2$$

$$V_E(\theta) = \frac{1}{2}mga(4\sin\theta - 1)^2$$

IYGB - M456 PAPER C - QUESTION 3

TOTAL POTENTIAL ENERGY, AS A FUNCTION OF θ, IS GIVEN BY

$$V(\theta) = \frac{1}{2}mga(4\sin\theta - 1)^2 - 4mga\sin 2\theta + \text{CONSTANT}$$

LOOKING FOR EQUILIBRIUM POSITIONS

$$V'(\theta) = 4mga(4\sin\theta - 1)\cos\theta - 8mga\cos 2\theta$$

$$V'(\theta) = 4mga\left[(4\sin\theta - 1)\cos\theta - 2\cos 2\theta\right]$$

$$V'(\theta) = 4mga\left[4\sin\theta\cos\theta - \cos\theta - 2\cos 2\theta\right]$$

$$V'(\theta) = 4mga\left[2\sin 2\theta - \cos\theta - 2\cos 2\theta\right]$$

SOLVING FOR ZERO

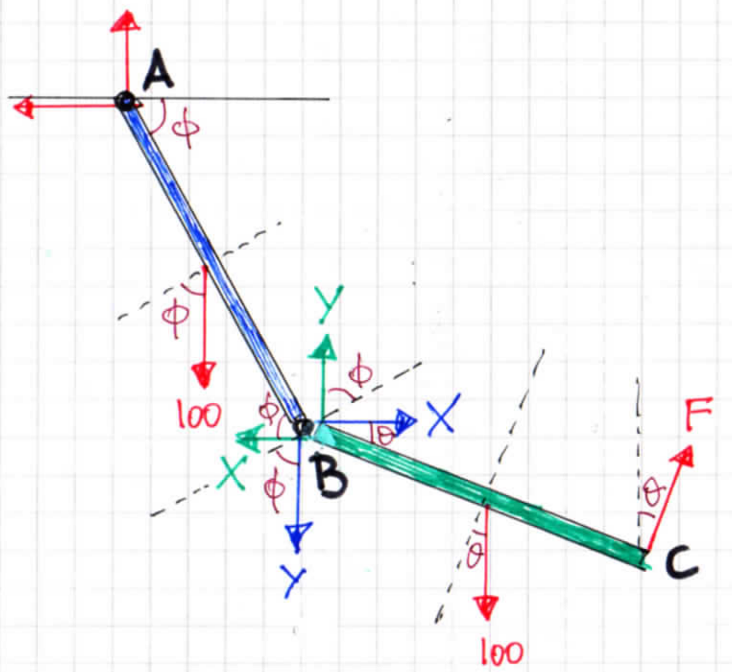
$$0 = \cancel{4mga}\left(2\sin 2\theta - \cos\theta - 2\cos 2\theta\right)$$

$$2\sin 2\theta - \cos\theta - 2\cos 2\theta = 0$$

AS REQUIRED

IYGB - M456 PAPER C - QUESTION 4

STARTING WITH A DETAILED DIAGRAM



$\tan\theta = \frac{3}{4}$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

LET WITHOUT LOSS OF GENERALITY $|AB| = |BC| = 2$

a) TAKING MOMENTS OF BC, ABOUT B

$$\Rightarrow F \times 2 = 100\cos\theta \times 1$$
$$\Rightarrow 2F = 100 \times \frac{4}{5}$$
$$\Rightarrow F = 40\text{ N}$$

b) RESOLVING FORCES ON THE ROD BC (GREEN FORCES)

$(\uparrow)$: $Y + F\cos\theta = 100$

$Y + 40 \times \frac{4}{5} = 100$

$Y = 68\text{ N}$

$(\rightarrow)$: $F\sin\theta = X$

$X = 40 \times \frac{4}{5}$

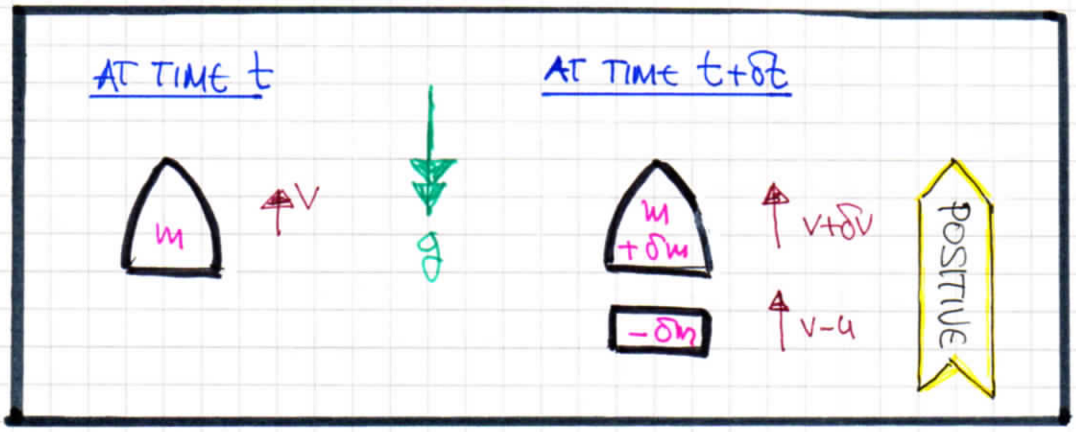
$X = 24\text{ N}$

c) TAKING MOMENTS OF AB ABOUT A (BLUE FORCES)

$$\Rightarrow 100\cos\phi \times 1 + Y\cos\phi \times 2 = X\sin\phi \times 2$$
$$\Rightarrow 100\cos\phi + 136\cos\phi = 48\sin\phi$$
$$\Rightarrow 236\cos\phi = 48\sin\theta$$
$$\Rightarrow \frac{\sin\phi}{\cos\phi} = \frac{236}{48}$$
$$\Rightarrow \tan\phi = \frac{59}{12}$$
$$\Rightarrow \phi \approx 78.5°$$

IYGB - M456 PAPER C - QUESTION 5

a) STARTING WITH THE USUAL MOMENTUM / IMPULSE DIAGRAM



BY THE IMPULSE MOMENTUM PRINCIPLE

$$\Longrightarrow -mg\,\delta t = \left[(m+\delta m)(v+\delta v) + (-\delta m)(v-u)\right] - [mv]$$

$$\Longrightarrow -mg\,\delta t = \cancel{mv} + m\delta v + \cancel{v\delta m} + \delta m\delta v - \cancel{v\delta m} + u\delta m - \cancel{mv}$$

$$\Longrightarrow -mg\,\delta t = m\delta v + u\,\delta m + \delta m\delta v$$

$$\Longrightarrow -mg = m\frac{\delta v}{\delta t} + u\frac{\delta m}{\delta t} + \frac{\delta m\,\delta v}{\delta t}$$

TAKING LIMITS AND REARRANGE FOR THE ACCELERATION

$$\Longrightarrow -mg = m\frac{dv}{dt} + u\frac{dm}{dt}$$

$$\Longrightarrow \frac{dv}{dt} = -g - \frac{u}{m}\frac{dm}{dt}$$

NEXT, AS THE EJECTION RATE OF THE FUEL IS CONSTANT

$$\Longrightarrow \frac{dm}{dt} = -\lambda\ , \quad \text{SUBJECT TO } t=0,\ m=M$$

$$\Longrightarrow m = M - \lambda t$$

IYGB - M456 PAPER C - QUESTION 5

COMBINING THE LAST TWO EXPRESSIONS

$$\Longrightarrow \frac{dv}{dt} = -g - \frac{u}{M-\lambda t}(-\lambda)$$

$$\Longrightarrow \frac{dv}{dt} = \frac{u\lambda}{M-\lambda t} - g$$

FINALLY FOR IMMEDIATE LIFT OFF $\frac{dv}{dt} > 0$, AT $t=0$

$$\Longrightarrow \frac{u\lambda}{M} - g > 0$$

$$\Longrightarrow u\lambda - gM > 0$$

$$\Longrightarrow u\lambda > Mg$$

$$\Longrightarrow \lambda > \frac{Mg}{u}$$

AS REQUIRED

b) SOLVING THE O.D.E BY DIRECT INTEGRATION - FIRST WE REQUIRE TO FIND THE TIME WHEN $m = \frac{3}{4}M$

$$\Longrightarrow m = M - \lambda t$$

$$\Longrightarrow \frac{3}{4}M = M - \left(\frac{3Mg}{u}\right)t$$

$$\Longrightarrow \frac{3\cancel{M}g}{u}t = \frac{1}{4}\cancel{M}$$

$$\Longrightarrow t = \frac{u}{12g}$$

SOLVING THE O.D.E, SUBJECT TO $t=0$, $v=0$,

$$\Longrightarrow 1\, dv = \left(\frac{u\lambda}{M-\lambda t} - g\right) dt$$

$$\Longrightarrow 1\, dv = \left(\frac{u\,\frac{3Mg}{u}}{M - \frac{3Mg}{u}t} - g\right) dt$$

IYGB - M456 PAPER C - QUESTION 5

$$\Rightarrow 1\,dv = \left(\frac{3Mg}{M - \frac{3Mgt}{u}} - g\right)dt$$

$$\Rightarrow 1\,dv = \left(\frac{3g}{1 - \frac{3gt}{u}} - g\right)dt$$

$$\Rightarrow 1\,dv = \left(\frac{3ug}{u - 3gt} - g\right)dt$$

$$\Rightarrow \int_{v=0}^{v} 1\,dv = \int_{t=0}^{t=\frac{u}{12g}} \frac{3ug}{u-3gt} - g \; dt$$

$$\Rightarrow \Big[v\Big]_0^v = \left[\frac{3ug}{-3g}\ln|u-3gt| - gt\right]_{t=0}^{t=\frac{u}{12g}}$$

$$\Rightarrow v = \Big[-u\ln|u-3gt| - gt\Big]_{t=0}^{t=\frac{u}{12g}}$$

$$\Rightarrow v = \Big[u\ln|u-3gt| + gt\Big]_{t=\frac{u}{12g}}^{t=0}$$

$$\Rightarrow v = \Big[u\ln u\Big] - \left[u\ln\left|u - 3g\left(\frac{u}{12g}\right)\right| + g\left(\frac{u}{12g}\right)\right]$$

$$\Rightarrow v = u\ln u - u\ln\left(\tfrac{3}{4}u\right) - \tfrac{1}{12}u$$

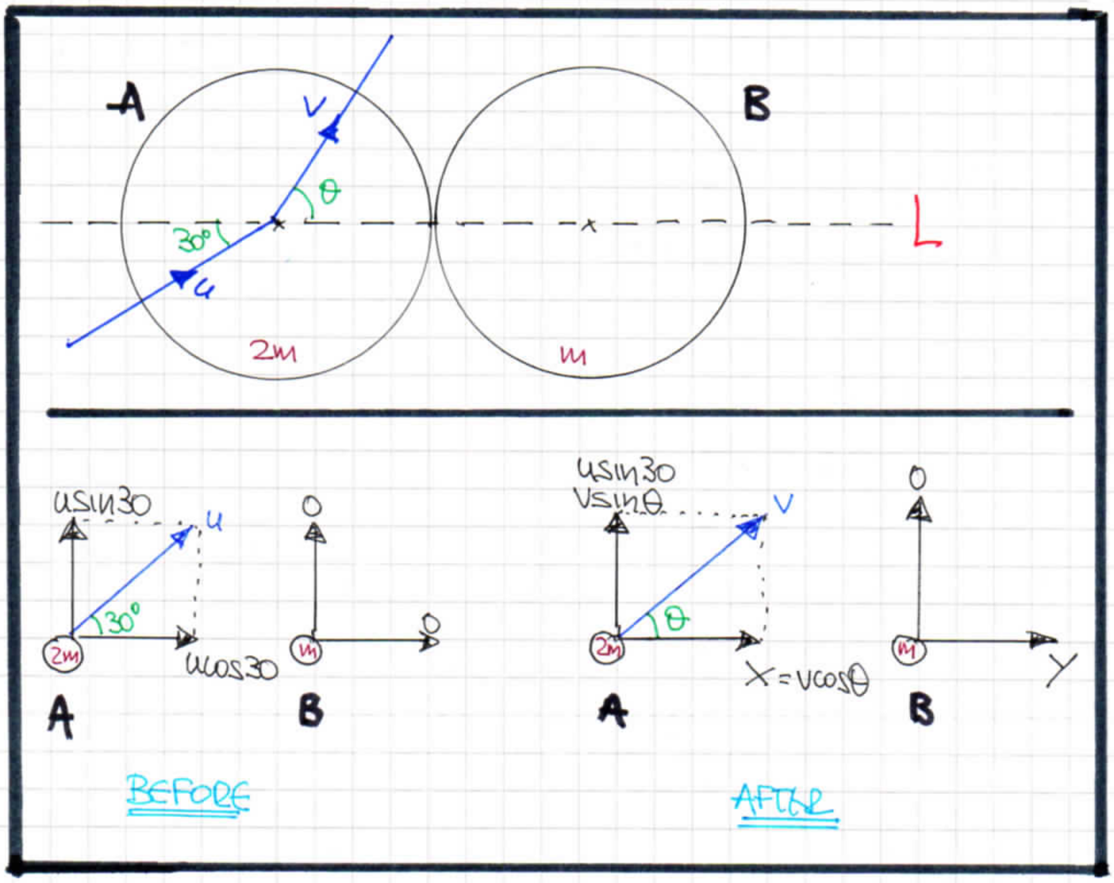
$$\Rightarrow v = u\ln\left(\frac{u}{\frac{3}{4}u}\right) - \frac{1}{12}u$$

$$\Rightarrow v = u\ln\frac{4}{3} - \frac{1}{12}u$$

$$\Rightarrow v = \left(\ln\frac{4}{3} - \frac{1}{12}\right)u$$

IYGB - M456 PAPER C - QUESTION 6

a) STARTING WITH A USUAL DIAGRAM FOR THE COLLISION



BY CONSERVATION OF MOMENTUM ALONG L

$\Rightarrow 2mu\cos30 + 0 = 2mX + mY$

$\Rightarrow 2X + Y = 2u\cos30°$

$\Rightarrow \boxed{2X + Y = \sqrt{3}u}$

BY RESTITUTION ALONG

$\Rightarrow \dfrac{Y - X}{u\cos30} = e$

$\Rightarrow -X + Y = eu\cos30°$

$\Rightarrow -X + Y = \frac{\sqrt{3}}{2}eu$

$\Rightarrow \boxed{-2X + 2Y = \sqrt{3}eu}$

IYGB - M456 PAPER C - QUESTION 6

SOLVING THE EQUATIONS YIELDS BY ADDITION

$$\Rightarrow 3Y = \sqrt{3}u + \sqrt{3}eu$$

$$\Rightarrow 3Y = \sqrt{3}u(1+e)$$

$$\Rightarrow Y = \frac{\sqrt{3}}{3}u(1+e)$$

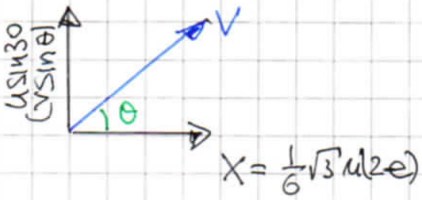
AND $X = Y - \frac{\sqrt{3}}{2}eu$

$$\Rightarrow X = \frac{\sqrt{3}}{3}u(1+e) - \frac{\sqrt{3}}{2}eu = \frac{\sqrt{3}}{3}u + \frac{\sqrt{3}}{3}eu - \frac{\sqrt{3}}{2}eu$$

$$\Rightarrow X = \frac{\sqrt{3}}{3}u - \frac{\sqrt{3}}{6}eu$$

$$\Rightarrow X = \frac{\sqrt{3}}{6}u(2-e)$$

SPEED OF A - NO MOMENTUM EXCHANGE $\perp$ TO L



$$|V|^2 = (u\sin 30)^2 + X^2$$

$$|V|^2 = \frac{1}{4}u^2 + \frac{1}{12}u^2(2-e)^2$$

$$|V|^2 = \frac{1}{12}u^2\left[3 + (2-e)^2\right]$$

$$|V|^2 = \frac{1}{12}u^2\left[e^2 - 4e + 7\right]$$

$$|V|_A = u\sqrt{\frac{e^2-4e+7}{12}}$$

SPEED OF B

SIMPLY FOUND ABOVE AS THE "PERPENDICULAR TO L" COMPONENT IS ZERO

$$|V_B| = \frac{1}{3}\sqrt{3}u(1+e)$$

IYGB - M456 PAPER C - QUESTION 6

b) K.E. BEFORE THE COLLISION

$$\frac{1}{2}(2m)u^2 = mu^2$$

K.E AFTER THE COLLISION

$$\frac{1}{2}(2m)V_4^2 + \frac{1}{2}mY^2$$

$$= m\left(\frac{u^2(e^2-4e+7)}{12}\right) + \frac{1}{2}m\left(\frac{u^2}{3}(1+e)^2\right)$$

$$= \frac{mu^2}{12}(e^2-4e+7) + \frac{mu^2}{6}(e^2+2e+1)$$

$$= \frac{mu^2}{12}\left[e^2-\cancel{4e}+7+2e^2+\cancel{4e}+2\right]$$

$$= \frac{mu^2}{12}\left[3e^2+9\right]$$

$$= \frac{1}{4}mu^2(e^2+3)$$

FINALLY WE HAVE

$$\Rightarrow \frac{\frac{1}{4}\cancel{mu^2}(e^2+3)}{\cancel{mu^2}} = \frac{4}{5} \quad \leftarrow \text{"20\% LOST"}$$

$$\Rightarrow \frac{1}{4}(e^2+3) = \frac{4}{5}$$

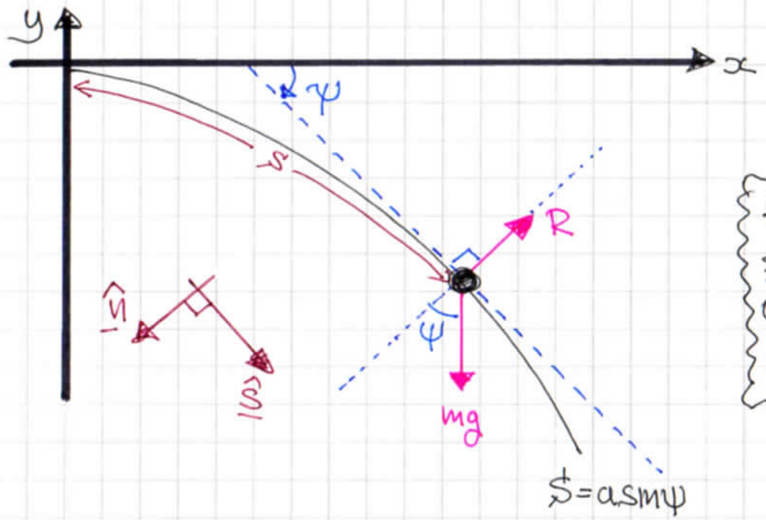
$$\Rightarrow e^2+3 = \frac{16}{5}$$

$$\Rightarrow e^2 = \frac{1}{5}$$

// AS REQUIRED

IYGB - M456 PAPER C - QUESTION 7

a) START BY PUTTING ALL INFO INTO A DIAGRAM



$t=0$
$s=0$
$\psi=0$
$\dot{s}=\sqrt{\frac{1}{2}ag}$

COLLECT/PREPARE THE USUAL AUXILLARIES

- $s = a\sin\psi \implies \rho = \frac{ds}{d\psi} = a\cos\psi$
- $\underline{a} = \ddot{s}\,\hat{\underline{s}} + \frac{\dot{s}^2}{\rho}\hat{\underline{n}}$

LOOKING AT THE TANGENTIAL MOTION ($\hat{\underline{s}}$)

$\implies m\ddot{s} = mg\sin\psi$

$\implies \ddot{s} = g\sin\psi$

$\implies \frac{1}{2}\frac{d}{ds}(\dot{s}^2) = g\left(\frac{s}{a}\right)$ ← EQUATION OF CURVE

$\implies \frac{d}{ds}(\dot{s}^2) = \frac{2g}{a}s$

$\implies \left[\dot{s}^2\right]_{\dot{s}=\sqrt{\frac{1}{2}ag}}^{\dot{s}=v} = \int_{s=0}^{s} \frac{2g}{a}s\ ds$

$\implies v^2 - \frac{1}{2}ag = \left[\frac{g}{a}s^2\right]_0^s$

$\implies v^2 - \frac{1}{2}ag = \frac{g}{a}s^2$

$\implies v^2 = \frac{g}{a}s^2 + \frac{1}{2}ag$

LOOKING NEXT AT THE NORMAL DIRECTION ($\hat{\underline{n}}$)

$\implies m\frac{\dot{s}^2}{\rho} = mg\cos\psi - R$

WHEN IT LEAVES, $R=0$

$\implies \cancel{m}\frac{\dot{s}^2}{\rho} = \cancel{m}g\cos\psi$

$\implies \dot{s}^2 = g\rho\cos\psi$

USING ABOVE EXPRESSION FOR $\dot{s}^2 = v^2$

IYGB - M456 PAPER C - QUESTION 7

$$\Rightarrow \frac{g}{a}s^2 + \frac{1}{2}ag = g(a\cos\psi)\cos\psi$$

$$\Rightarrow \frac{s^2}{a} + \frac{1}{2}a = a\cos^2\psi$$

$$\Rightarrow 2s^2 + a^2 = 2a^2\cos^2\psi$$

$$\Rightarrow 2s^2 + a^2 = 2a^2(1-\sin^2\psi)$$

$$\Rightarrow 2s^2 + a^2 = 2a^2\left(1-\left(\frac{s}{a}\right)^2\right)$$

$$\Rightarrow 2s^2 + a^2 = 2a^2\left(1-\frac{s^2}{a^2}\right)$$

$$\Rightarrow 2s^2 + a^2 = 2a^2 - 2s^2$$

$$\Rightarrow 4s^2 = a^2$$

$$\Rightarrow s^2 = \frac{1}{4}a^2$$

$$\Rightarrow s = \frac{1}{2}a$$

b) FIRSTLY WE HAVE

AT $s = \frac{1}{2}a \Rightarrow \frac{1}{2}a = a\sin\psi$

$$\Rightarrow \sin\psi = \frac{1}{2}$$

$$\Rightarrow \psi = \frac{\pi}{6}$$

NORMAL ACCELERATION ($\hat{n}$) IS SIMPLY $\ddot{s}$.

$$\Rightarrow \ddot{s} = g\sin\psi \quad \text{(FROM EARLIER)}$$

$$\Rightarrow \ddot{s} = g \times \sin\frac{\pi}{6}$$

$$\Rightarrow \ddot{s} = \frac{1}{2}g$$

TANGENTIAL ACCELERATION IS $\frac{\dot{s}^2}{\rho}$

$$\Rightarrow \frac{\dot{s}^2}{\rho} = \frac{\frac{g}{a}s^2 + \frac{1}{2}ag}{a\cos\psi}$$

$$\Rightarrow \frac{\dot{s}^2}{\rho} = \frac{\frac{g}{a}\left(\frac{1}{2}a\right)^2 + \frac{1}{2}ag}{a\cos\frac{\pi}{6}} = \frac{\frac{3}{4}ag}{a\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{2}g$$

HENCE THE MAGNITUDE OF THE ACCELERATION CAN BE FOUND AS

$$\sqrt{\left(\frac{1}{2}g\right)^2 + \left(\frac{\sqrt{3}}{2}g\right)^2} = \sqrt{\frac{1}{4}g^2 + \frac{3}{4}g^2} = g$$

IYGB - M456 PAPER C - QUESTION 8

$\underline{F}_1 = (3a+1)\underline{i} + 3\underline{j}$ AT $A(1,2)$

$\underline{F}_2 = (a-10)\underline{i} - 2\underline{j}$ AT $B(2,0)$

$\underline{F}_3 = \underline{i} + (1-a)\underline{j}$ AT $C(4,-1)$

a) IF THE SYSTEM IS TO REDUCE TO A COUPLE ABOUT O

$$\underline{F}_1 + \underline{F}_2 + \underline{F}_3 = (0,0)$$
$$(3a+1,3) + (a-10,-2) + (1,1-a) = (0,0)$$
$$(4a-8, 2-a) = (0,0)$$

THIS IN INDEED POSSIBLE IF $a=2$

FINDING THE MOMENT ABOUT O, WITH $a=2$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 2 & 0 \\ 7 & 3 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 0 & 0 \\ -8 & -2 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & -1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= [0,0,3-14] + [0,0,-4] + [0,0,-4+1]$$
$$= [0,0,-11] + [0,0,-4] + [0,0,-3]$$
$$= [0,0,-18]$$

HENCE, A MAGNITUDE OF 18 Nm, ANTICLOCKWISE

IYGB - M456 PAPER C - QUESTION 8

b) NOW FIND THE MOMENT OF THE FORCES ABOUT O, IN TERMS of a

$$\underline{G}_0 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 2 & 0 \\ 3a+1 & 3 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 0 & 0 \\ a-10 & -2 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & -1 & 0 \\ 1 & 1-a & 0 \end{vmatrix}$$

$$\underline{G}_0 = (0, 0, 3-6a-2) + (0, 0, -4) + (0, 0, 4-4a+1)$$

$$\underline{G}_0 = (0, 0, 1-6a) + (0, 0, -4) + (0, 0, 5-4a)$$

$$\underline{G}_0 = (0, 0, 2-10a)$$

THIS YIELDS ZERO MOMENT IF $a = \frac{1}{5}$

THEN $\underline{F}_1 + \underline{F}_2 + \underline{F}_3 = (4a-8, 2-a)$ ← FROM EARLIER

$$= \left(-\frac{36}{5}, \frac{9}{5}\right)$$

THE LINE PASSES THROUGH THE ORIGIN (ZERO MOMENT ABOUT O)

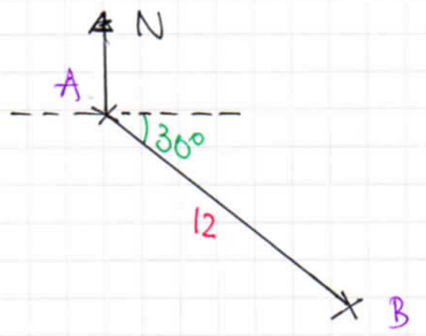
$$y = mx + \cancel{c}$$

$$m = \frac{\frac{9}{5} - 0}{-\frac{36}{5} - 0} = -\frac{1}{4}$$

∴ REDUCES TO $\underline{F} = -\frac{36}{5}\underline{i} + \frac{9}{5}\underline{j}$, WHICH ACTS ALONG THE LINE $y = -\frac{1}{4}x$

IYGB - M456 PAPER C - QUESTION 9

INITIAL CONFIGURATION



VELOCITY TRIANGLE

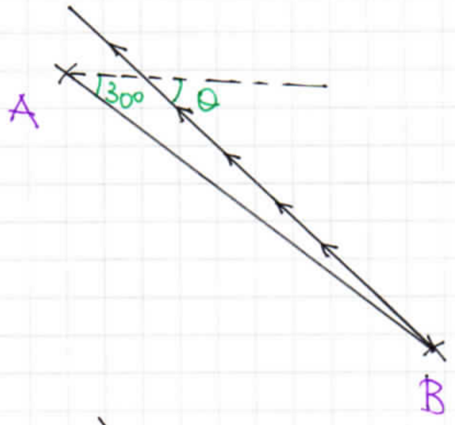
$$ {}_B\underline{V}_A = \underline{V}_B - \underline{V}_A $$

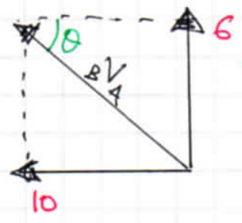
$$ \left(\underline{V}_B = \underline{V}_A + {}_B\underline{V}_A \right) $$

$$ {}_B\underline{V}_A = (-3\underline{i} + 9\underline{j}) - (7\underline{i} + 3\underline{j}) $$

$$ {}_B\underline{V}_A = -10\underline{i} + 6\underline{j} $$

FIXING A, AN OBSERVER ON A SEES THE FOLLOWING

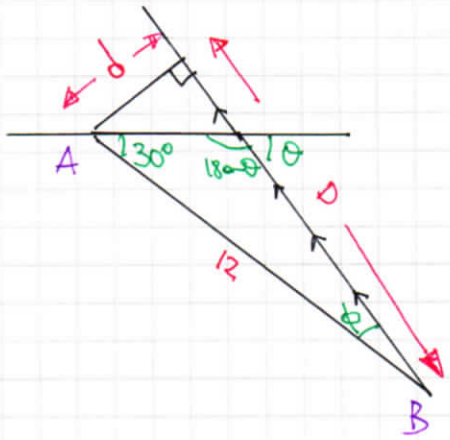




- $\tan\theta = \frac{6}{10}$

  $\theta \simeq 30.964...°$

- $|{}_B V_A| = \sqrt{10^2 + 6^2} = 2\sqrt{34}$



$\phi + 30 + 180 - \theta = 180$

$\phi = \theta - 30$

$\phi = 0.96375...°$

SHORTEST DISTANCE d IS

$d = 12 \sin\phi$

$d = 12 \sin(0.96375°...)$

$d \simeq 0.2018...$ km

$d \simeq 202$ m

IYGB - MU56 PAPER C - QUESTION 9

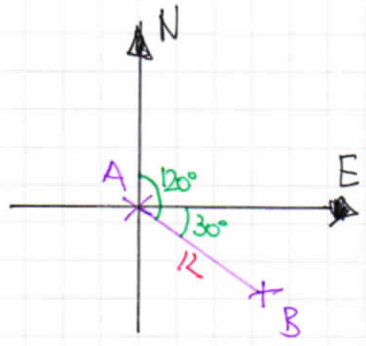
- $D = 12\cos\phi = 12\cos(0.96375...) = 11.9983024...$ km

- COVERED AT $2\sqrt{34}$ km h^{-1} WE OBTAIN

$$\frac{11.9983024...}{2\sqrt{34}} \simeq 1.0288... \text{ HOUR}$$
$$\simeq 1 \text{ HOUR} - 2 \text{ MINUTES}$$
$$\simeq 13:02$$

ALTERNATIVE BY VECTORS

- TAKE THE POSITION OF "A" AT NOON, TO BE THE ORIGIN → THE POSITION VECTOR OF "B" AT NOON WILL BE



$$\underline{r}_B = (12\cos 30)\underline{i} - (12\sin 30\underline{j})$$
$$6\sqrt{3}\underline{i} \qquad 6\underline{j}$$

- THE POSITION VECTORS OF THE TWO SHIPS, t HOURS AFTER NOON, IS

$$\underline{r}_A = (0,0) + (7,3)t = (7t, 3t)$$
$$\underline{r}_B = (6\sqrt{3}, -6) + (-3,9)t = (6\sqrt{3} - 3t, 9t - 6)$$

- THE POSITION VECTOR OF B, RELATIVE TO A IS GIVEN BY

$$\underline{r}_B - \underline{r}_A = (6\sqrt{3} - 10t, 6t - 6)$$

IYGB - M456 PAPER C - QUESTION 9

- THE DISTANCE BETWEEN THE SHIPS AT TIME t

$$d = |\underline{r}_B - \underline{r}_A|$$

$$d = |6\sqrt{3} - 10t, 6t - 6|$$

$$d = \sqrt{(6\sqrt{3} - 10t)^2 + (6t - 6)^2}$$

$$d = \sqrt{108 - 120\sqrt{3}t + 100t^2 + 36t^2 - 72t + 36}$$

$$d = \sqrt{136t^2 - (72 + 120\sqrt{3})t + 144}$$

$$d^2 = 136t^2 - 24(3 + 5\sqrt{3})t + 144$$

- LET $f(t) = 136t^2 - 24(3+5\sqrt{3})t + 144$
  BY COMPLETING THE SQUARE OR CALCULUS

$$f'(t) = 272t - 24(3 + 5\sqrt{3})$$

- SOLVING FOR ZERO YIELDS

$$t = \frac{24(3+5\sqrt{3})}{272} \simeq 1.0288\ldots$$

$\simeq$ 1 HOUR - 2 MINUTES

$\simeq$ 13:02

- AND TO FIND THE MINIMUM DISTANCE

$$d_{MIN} = \sqrt{136(1.0288\ldots)^2 - (72 + 120\sqrt{3})(1.0288\ldots) + 144}$$

$\simeq 0.201839\ldots$ km

$\simeq$ 202 m

IYGB - MM56 PAPER C - QUESTION 10

a) $F = \dfrac{25}{x^2} - \dfrac{50}{x^3}, \quad x > 0$

WORK BY F = -4

$$\Rightarrow \int_1^k \frac{25}{x^2} - \frac{50}{x^3}\,dx = -4$$

$$\Rightarrow \left[-\frac{25}{x} + \frac{25}{x^2}\right]_1^k = -4$$

$$\Rightarrow \left(-\frac{25}{k} + \frac{25}{k^2}\right) - \cancel{\left(-25 + 25\right)} = -4 \quad \times k^2$$

$$\Rightarrow -25k + 25 = -4k^2$$

$$\Rightarrow 4k^2 - 25k + 25 = 0$$

$$\Rightarrow (4k-5)(k-5) = 0$$

$$\Rightarrow k = \begin{cases} 5 \\ \frac{5}{4} \end{cases}$$

b) F = 0 YIELDS

$$\frac{25}{x^2} - \frac{50}{x^3} = 0$$

$$25x - 50 = 0$$

$$x = 2$$

NEXT THE WORK FROM $x=1$ TO $x=2$

$$W = \int_1^2 \frac{25}{x^2} - \frac{50}{x^3}\,dx = \left[-\frac{25}{x} + \frac{25}{x^2}\right]_1^2$$

IYGB - M456 PAPER C - QUESTION 10

$$= \left(-\frac{25}{2}+\frac{25}{4}\right) - \cancel{\left(-25+25\right)} = -\frac{25}{4} \quad \text{(1+ WORK OUT)}$$

BY ENERGIES

$$\Rightarrow KE_{x=1} - W_{OUT} = KE_{x=2}$$

$$\Rightarrow \frac{1}{2}mu^2 - \frac{25}{4} = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}(0.5)\times 13^2 - \frac{25}{4} = \frac{1}{2}(0.5)v^2$$

$$\Rightarrow \frac{169}{4} - \frac{25}{4} = \frac{1}{4}v^2$$

$$\Rightarrow 169 - 25 = v^2$$

$$\Rightarrow v^2 = 144$$

$$\Rightarrow |v| = 12 \text{ ms}^{-1}$$

IYGB – M456 PAPER C – QUESTION 11

$r = ke^{\theta \cot\alpha}$

$(k, \alpha \text{ CONSTANTS}, 0 < \alpha \leqslant \frac{\pi}{4})$

CONSTANT ANGULAR VELOCITY

$\dot{\theta} = \omega$

$\ddot{\theta} = 0$

DIFFERENTIATE THE EQUATION OF THE PATH TO OBTAIN $\dot{r}$ & $\ddot{r}$

$\Longrightarrow r = ke^{\theta \cot\alpha}$

$\Longrightarrow \dot{r} = ke^{\theta\cot\alpha} \times \dot{\theta}\cot\alpha$

$\Longrightarrow \boxed{\dot{r} = r\omega\cot\alpha}$

$\Longrightarrow \ddot{r} = \dot{r}\omega\cot\alpha = (r\omega\cot\alpha)\omega\cot\alpha$

$\Longrightarrow \boxed{\ddot{r} = r\omega^2\cot^2\alpha}$

NOW THE ACCELERATION IN POLARS IS GIVEN BY

$\Longrightarrow \underline{\ddot{r}} = (\ddot{r} - r\dot{\theta}^2)\underline{\hat{r}} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\underline{\hat{\theta}}$

$\Longrightarrow \underline{\ddot{r}} = (\ddot{r} - r\omega^2)\underline{\hat{r}} + \frac{1}{r}\frac{d}{dt}(r^2\omega)\underline{\hat{\theta}}$

$\Longrightarrow \underline{\ddot{r}} = (\ddot{r} - r\omega^2)\underline{\hat{r}} + 2\dot{r}\omega\,\underline{\hat{\theta}}$

$\Longrightarrow \underline{\ddot{r}} = (r\omega^2\cot^2\alpha - \omega^2 r)\underline{\hat{r}} + (2\omega^2 r\cot\alpha)\underline{\hat{\theta}}$

$\Longrightarrow \underline{\ddot{r}} = r\omega^2\Big[[\cot^2\alpha - 1]\underline{\hat{r}} + [2\cot\alpha]\underline{\hat{\theta}}\Big]$

NEXT THE MODULUS (MAGNITUDE OF ACCELERATION)

$\Longrightarrow |\underline{\ddot{r}}| = \Big| r\omega^2\left[(\cot^2\alpha - 1)\underline{\hat{r}} + (2\cot\alpha)\underline{\hat{\theta}}\right]\Big|$

$\Longrightarrow |\underline{\ddot{r}}| = r\omega^2\left[(\cot^2\alpha - 1)^2 + (2\cot\alpha)^2\right]^{\frac{1}{2}}$

IYGB - M456 PAPER C - QUESTION 11

$$\Rightarrow |\ddot{\underline{r}}| = r\omega^2\left[\cot^4\alpha - 2\cot^2\alpha + 1 + 4\cot^2\alpha\right]^{\frac{1}{2}}$$

$$\Rightarrow |\ddot{\underline{r}}| = r\omega^2\left[\cot^4\alpha + 2\cot^2\alpha + 1\right]^{\frac{1}{2}}$$

$$\Rightarrow |\ddot{\underline{r}}| = r\omega^2\sqrt{(\cot^2\alpha + 1)^2}$$

$$\Rightarrow \boxed{|\ddot{\underline{r}}| = r\omega^2\operatorname{cosec}^2\alpha}$$

NEXT WE REQUIRE A VELOCITY EXPRESSION, TO GET THE SPEED

$$\Rightarrow \underline{v} = \dot{r}\hat{\underline{r}} + r\dot{\theta}\hat{\underline{\theta}}$$

$$\Rightarrow \underline{v} = (r\omega\cot\alpha)\hat{\underline{r}} + r\omega\hat{\underline{\theta}}$$

$$\Rightarrow |\underline{v}| = |r\omega(\cot\alpha\,\hat{\underline{r}} + \hat{\theta})|$$

$$\Rightarrow |\underline{v}| = r\omega\sqrt{\cot^2\alpha + 1}$$

$$\Rightarrow |\underline{v}| = r\omega\sqrt{\operatorname{cosec}^2\alpha}$$

$$\Rightarrow \boxed{|\underline{v}| = r\omega\operatorname{cosec}\alpha}$$

FINALLY WE OBTAIN

$$|\ddot{\underline{r}}| = r\omega^2\operatorname{cosec}^2\alpha = \frac{1}{r}\left(r^2\omega^2\operatorname{cosec}^2\alpha\right) = \frac{|v|^2}{r}$$

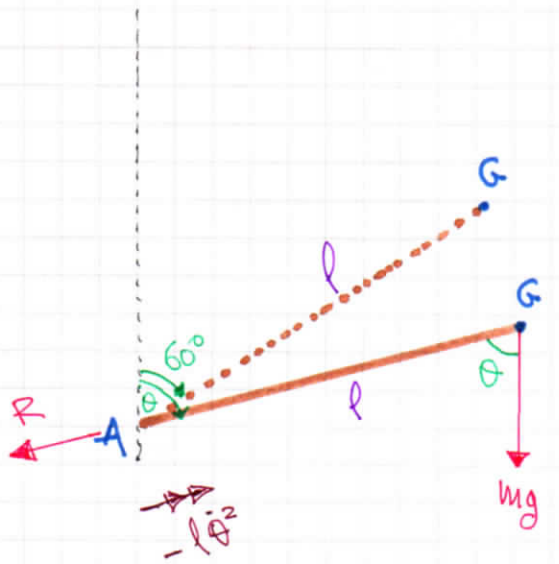
$$\therefore |\ddot{\underline{r}}| = \frac{|\underline{v}|^2}{r}$$

AS REQUIRED

IYGB - M456 PAPER C - QUESTION 12

a) STARTING WITH A DIAGRAM, WHERE G DENOTES THE MIDPOINT OF THE ROD

$$I_A = \frac{4}{3}ml^2$$
$$12\cancel{m}a^2 = \frac{4}{3}\cancel{m}l^2$$
$$l^2 = 9a^2$$
$$l = 3a$$



BY ENERGIES TAKING THE LEVEL OF "A", AS THE ZERO POTENTIAL LEVEL

$$\Rightarrow P.E_{60°} + \cancel{P.E_{60°}} = KE_\theta + P.E_\theta$$
$$\Rightarrow mgl\cos 60° = \frac{1}{2}I\dot{\theta}^2 + mgl\cos\theta$$
$$\Rightarrow mg(3a)\times\frac{1}{2} = \frac{1}{2}(12ma^2)\dot{\theta}^2 + mg(3a)\cos\theta$$
$$\Rightarrow \frac{3}{2}\cancel{mga} = 4\cancel{ma^2}\dot{\theta}^2 + 3\cancel{mga}\cos\theta$$
$$\Rightarrow 3g = 8a\dot{\theta}^2 + 6g\cos\theta$$
$$\Rightarrow 8a\dot{\theta}^2 = 3g - 6g\cos\theta$$
$$\Rightarrow 8a\dot{\theta}^2 = 3g(1-2\cos\theta)$$
$$\Rightarrow \dot{\theta}^2 = \frac{3g}{8a}(1-2\cos\theta)$$

IYGB - M456 PAPER C - QUESTION 12

b) LOOKING IN THE RADIAL DIRECTION

$\Rightarrow$ "$m\ddot{r}$ = RESULTANT FORCE"

$\Rightarrow m\left(-3a\dot{\theta}^2\right) = -R - mg\cos\theta$

$\Rightarrow R = 3ma\dot{\theta}^2 - mg\cos\theta$

$\Rightarrow R = 3ma\left(\frac{3g}{8a}\left(1-2\cos\theta\right)\right) - mg\cos\theta$

$\Rightarrow R = \frac{9}{8}mg\left(1-2\cos\theta\right) - mg\cos\theta$

$\Rightarrow R = \frac{1}{8}mg\left(9 - 18\cos\theta - 8\cos\theta\right)$

$\Rightarrow R = \frac{1}{8}mg\left(9 - 26\cos\theta\right)$

HENCE THE REQUIRED COMPONENT IS

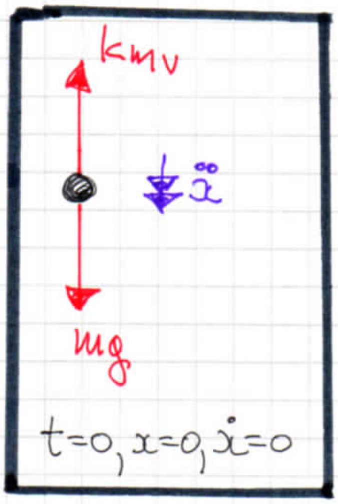
$\frac{1}{8}mg\left(9-26\cos\theta\right)$, RADIALLY INWARDS

OR

$\frac{1}{8}mg\left(26\cos\theta - 9\right)$, RADIALLY OUTWARDS

IYGB - MM56 PAPER C - QUESTION 13

a) SOLVING THE DIFFERENTIAL EQUATION



$$\Rightarrow m\ddot{x} = mg - kmv$$

$$\Rightarrow \ddot{x} = g - kv$$

$$\Rightarrow v\frac{dv}{dx} = g - kv$$

$$\Rightarrow \frac{v}{g-kv}\,dv = 1\,dx$$

$$\Rightarrow \frac{-kv}{g-kv}\,dv = -k\,dx$$

$$\Rightarrow \int_{v=0}^{v} \frac{-kv}{g-kv}\,dv = \int_{x=0}^{x} -k\,dx$$

$$\Rightarrow \int_0^v \frac{(g-kv)-g}{g-kv}\,dv = \int_0^x -k\,dx$$

$$\Rightarrow \int_0^v 1 - \frac{g}{g-kv}\,dv = \int_0^x -k\,dx$$

$$\Rightarrow \left[v + \frac{g}{k}\ln|g-kv|\right]_0^v = \left[-kx\right]_0^x$$

$$\Rightarrow \left[v + \frac{g}{k}\ln|g-kv|\right] - \left[\frac{g}{k}\ln g\right] = -kx - 0$$

$$\Rightarrow v + \frac{g}{k}\ln\left|\frac{g-kv}{g}\right| = -kx$$

$$\Rightarrow x = -\frac{1}{k}v - \frac{g}{k^2}\ln\left|\frac{g-kv}{g}\right|$$

$$\Rightarrow x = \frac{g}{k^2}\ln\left|\frac{g}{g-kv}\right| - \frac{v}{k}$$

AS REQUIRED

IYGB - MM56 PAPER C - QUESTION 13

b) LOOKING AT THE ORIGINAL O.D.E

$$\ddot{x} = g - kv$$

LIMITING SPEED $\Longrightarrow \ddot{x} = 0$

$$\Longrightarrow V = g/k$$

THUS WE HAVE IF $v = \frac{1}{2}V = g/2k$

$$x = \frac{g}{k^2}\ln\left|\frac{g}{g-kv}\right| - \frac{v}{k}$$

$$x = \frac{g}{k^2}\ln\left|\frac{g}{g-k\frac{g}{2k}}\right| - \frac{g/2k}{k}$$

$$x = \frac{g}{k^2}\ln 2 - \frac{g}{2k^2}$$

$$x = \frac{g}{2k^2}\left[2\ln 2 - 1\right]$$

$$x = \frac{1}{2g} \times \frac{g^2}{k^2}\left[\ln 4 - 1\right]$$

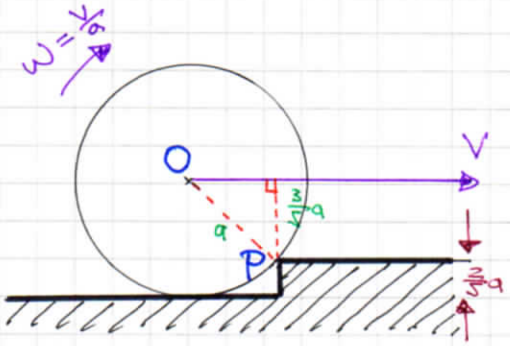
$$x = \frac{1}{2g}V^2\left[\ln 4 - 1\right]$$

$$x = \frac{V^2}{2g}\left[-1 + \ln 4\right]$$

AS REQUIRED

IYGB – M456 PAPER C – QUESTION 14

LOOKING AT THE DIAGRAM BELOW



MOMENT OF INERTIA OF THE SPHERE

$$I_O = \frac{2}{5}ma^2$$

$$I_P = \frac{2}{5}ma^2 + ma^2 = \frac{7}{5}ma^2$$

LET THE ANGULAR VELOCITY ABOUT P, BE Ω

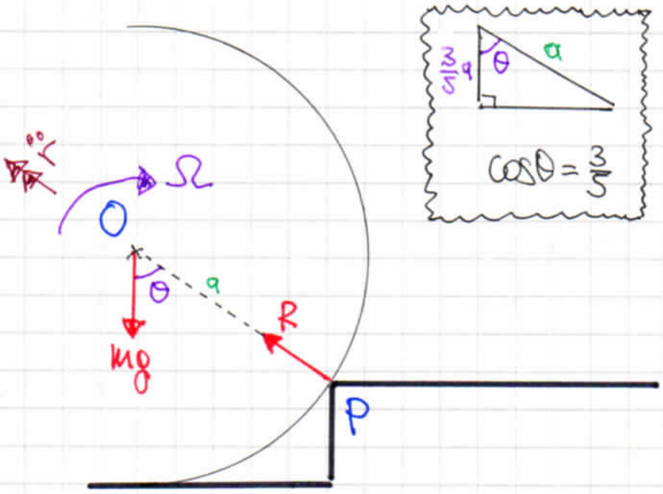
BY CONSERVATION OF MOMENTUM ABOUT P

$$\Longrightarrow \underbrace{(I_O\omega) + (mV) \times \frac{3}{5}a}_{\text{JUST BEFORE IMPACT}} = \underbrace{I_P\Omega}_{\text{AFTER IMPACT AT P}}$$

$$\Longrightarrow \frac{2}{5}ma^2\omega + \frac{3}{5}mVa = \frac{7}{5}ma^2\Omega \qquad \div \frac{1}{5}ma$$

$$\Longrightarrow \boxed{2a\omega + 3V = 7a\Omega}$$

NEXT WE CONSIDER THE INSTANT AFTER THE IMPACT



RADIALLY, AS IT ROTATES ABOUT P

$$\Longrightarrow m\ddot{r} = R - mg\cos\theta$$

$$\Longrightarrow m(-\Omega^2 a) = R - mg\cos\theta$$

$$\Longrightarrow R = mg\cos\theta - ma\Omega^2$$

IYGB - M456 PAPER C - QUESTION 14

FOR ROTATION ABOUT P, R>0

$$\Rightarrow \frac{3}{5}\cancel{m}g - \cancel{m}a\Omega^2 > 0$$

$$\Rightarrow a\Omega^2 < \frac{3}{5}g \quad \times 49a$$

$$\Rightarrow 49a^2\Omega^2 < \frac{147}{5}ag$$

IN ORDER TO "CREATE" $7a\omega = 2a\omega + 3V$ OBTAINED EARLIER

$$\Rightarrow (2a\omega + 3V)^2 < \frac{147}{5}ag$$

$$\Rightarrow \left(2a\frac{V}{a} + 3V\right)^2 < \frac{147}{5}ag$$

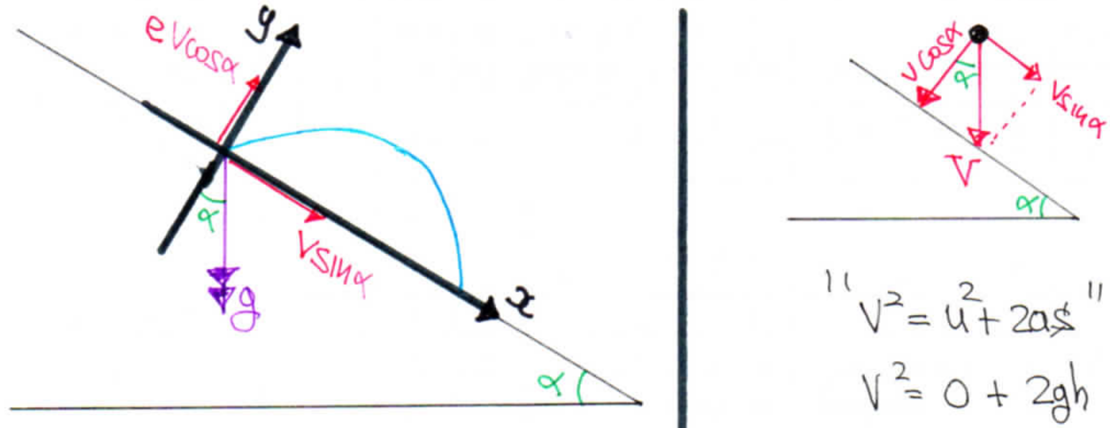
$$\Rightarrow (5V)^2 < \frac{147}{5}ag$$

$$\Rightarrow 25V^2 < \frac{147}{5}ag$$

$$\Rightarrow V^2 < \frac{147}{125}ag$$

// AS REQUIRED

IYGB – M456 PAPER C – QUESTION 15



" $v^2 = u^2 + 2as$ "

$V^2 = 0 + 2gh$

$V^2 = 2gh$

$V = \sqrt{2gh}$

- START BY COMPUTING THE COMPONENTS OF THE VELOCITY ON IMPACT, PARALLEL & PERPENDICULAR TO THE PLANE (TOP RIGHT). — NO MOMENTUM IS EXCHANGED PARALLEL TO THE PLANE HOWEVER PERPENDICULAR TO THE PLANE THE "REBOUNDING" VELOCITY COMPONENT IS $eV\cos\alpha$ (TOP LEFT)

- DERIVE THE EQUATIONS OF MOTION IN THE CO-ORDINATE SYSTEM SHOWN IN THE ABOVE DIAGRAM

  - $\ddot{x} = g\sin\alpha$
  - $\dot{x} = gt\sin\alpha + V\sin\alpha$
  - $x = \frac{1}{2}gt^2\sin\alpha + Vt\sin\alpha$

  - $\ddot{y} = -g\cos\alpha$
  - $\dot{y} = -gt\cos\alpha + eV\cos\alpha$
  - $y = -\frac{1}{2}gt^2\cos\alpha + eVt\cos\alpha$

- DETERMINE THE FLIGHT TIME TO THE "SECOND" IMPACT, I.E $y=0$

$\Rightarrow 0 = -\frac{1}{2}gt^2\cos\alpha + eVt\cos\alpha$

$\Rightarrow 0 = \frac{1}{2}t\cos\alpha\left[2eV - gt\right]$

$\Rightarrow t = \frac{2eV}{g} \quad (t \neq 0)$

IYGB - M456 PAPER C - QUESTION 15

- SUBSTITUTING INTO THE "x - EQUATION" TO FIND THE REQUIRED DISTANCE d

$$d = \frac{1}{2}g\left(\frac{2ev}{g}\right)^2 \sin\alpha + v\left(\frac{2ev}{g}\right)\sin\alpha$$

$$d = \frac{2e^2v^2}{g}\sin\alpha + \frac{2ev^2}{g}\sin\alpha$$

$$d = \left(\frac{2ev^2}{g}\sin\alpha\right)(e+1)$$

$$d = \frac{2e}{\cancel{g}}(2\cancel{g}h)(\sin\alpha)(e+1)$$

$$d = 4eh(e+1)\sin\alpha$$

$$\boxed{\begin{aligned} v &= \sqrt{2gh} \\ v^2 &= 2gh \end{aligned}}$$

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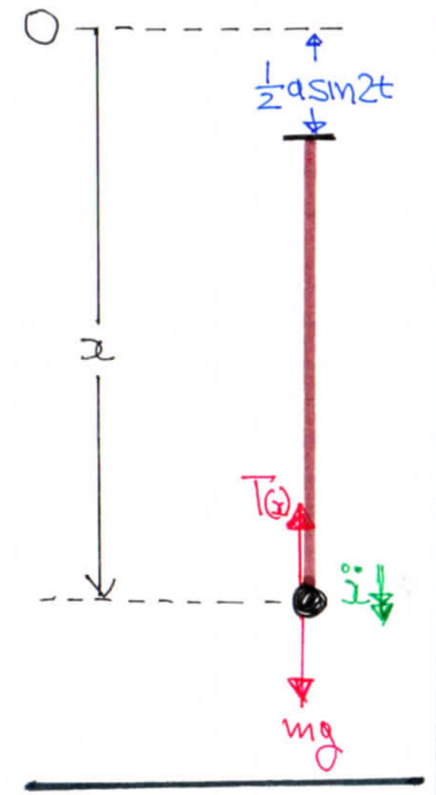

IN EQUILIBRIUM

$$mg = T = \frac{\lambda}{l}e$$

$$mg = \frac{\frac{1}{2}mg}{a}e$$

$$e = 2a$$

HENCE $t=0$

$$x = 2a + a = 3a$$

$$\dot{x} = 0$$

LOOKING AT THE DIAGRAM, FORM THE EQUATION OF MOTION

$$\Rightarrow m\ddot{x} = mg - T$$

$$\Rightarrow m\ddot{x} = mg - \frac{\lambda}{a}\left(x - a - \tfrac{1}{2}a\sin 2t\right)$$

$$\Rightarrow m\ddot{x} = mg - \frac{\frac{1}{2}mg}{a}\left(x - a - \tfrac{1}{2}a\sin 2t\right)$$

$$\Rightarrow \ddot{x} = g - \frac{g}{2a}\left(x - a - \tfrac{1}{2}a\sin 2t\right)$$

$$\Rightarrow \ddot{x} = g - \frac{gx}{2a} + \tfrac{1}{2}g + \tfrac{1}{4}g\sin 2t$$

$$\Rightarrow \ddot{x} + \frac{g}{2a}x = \tfrac{3}{2}g + \tfrac{1}{4}g\sin 2t$$

LET $\omega^2 = \frac{g}{2a} \Rightarrow g = 2a\omega^2$

$$\Rightarrow \ddot{x} + \omega^2 x = \tfrac{3}{2}(2a\omega^2) + \tfrac{1}{4}(2a\omega^2)\sin 2t$$

$$\Rightarrow \ddot{x} + \omega^2 x = 3a\omega^2 + \tfrac{1}{2}a\omega^2\sin 2t$$

THE COMPLEMENTARY FUNCTION IS THE STANDARD S.H.M SOLUTION

$$x = A\cos\omega t + B\sin\omega t$$

FOR PARTICULAR INTEGRAL WE TRY

- $x = P + Q\sin 2t$
- $\ddot{x} = -4Q\sin 2t$

$$\Rightarrow -4Q\sin 2t + \omega^2(P + Q\sin 2t) \equiv 3a\omega^2 + \tfrac{1}{2}a\omega^2\sin 2t$$

$$\Rightarrow P\omega^2 + (Q\omega^2 - 4Q)\sin 2t \equiv 3a\omega^2 + \tfrac{1}{2}a\omega^2\sin 2t$$

$$\Rightarrow P = 3a \quad \& \quad Q(\omega^2 - 4) = \tfrac{1}{2}a\omega^2$$

$$Q = \frac{a\omega^2}{2(\omega^2 - 4)}$$

IYGB - M456 PAPER C - QUESTION 16

HENCE THE GENERAL SOLUTION IS GIVEN BY

$$\boxed{x = A\cos\omega t + B\sin\omega t + 3a + \frac{a\omega^2}{2(\omega^2-4)}\sin 2t}$$

APPLY $t=0$, $x=3a$

$$3a = A + 3a$$

$$A = 0$$

$$\boxed{x = 3a + B\sin\omega t + \frac{a\omega^2}{2(\omega^2-4)}\sin 2t}$$

DIFFERENTIATE AND APPLY CONDITION $t=0$, $\dot{x}=0$

$$\dot{x} = B\omega\cos\omega t + \frac{a\omega^2}{\omega^2-4}\cos 2t$$

$$0 = B\omega + \frac{a\omega^2}{\omega^2-4}$$

$$B = -\frac{a\omega}{\omega^2-4}$$

$$x = 3a + \frac{a\omega^2}{2(\omega^2-4)}\sin 2t - \frac{a\omega}{\omega^2-4}\sin\omega t$$

$$x = 3a + \frac{a\omega}{2(\omega^2-4)}\left[\omega\sin 2t - 2\sin\omega t\right]$$