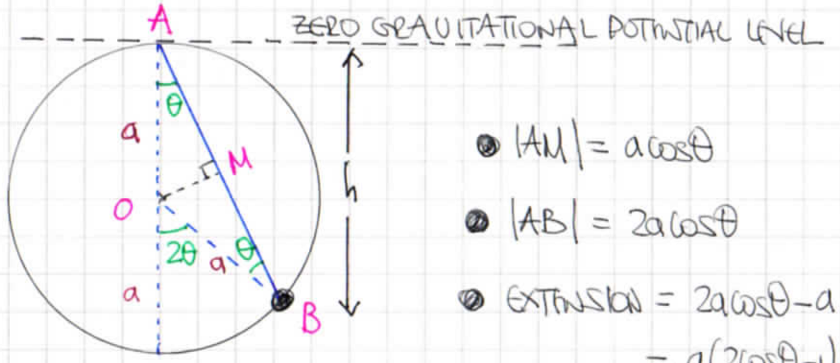
# IYGB - M456 PAPER D - QUESTION 1

a) LOOKING AT THE DIAGRAM BELOW




- $|AM| = a\cos\theta$
- $|AB| = 2a\cos\theta$
- EXTENSION $= 2a\cos\theta - a$
  $= a(2\cos\theta - 1)$

CALCULATE THE ELASTIC POTENTIAL ENERGY

$$\Rightarrow E.P.E = \text{"}\frac{\lambda}{2l}x^2\text{"} = \frac{3mg}{2a}\left[a(2\cos\theta - 1)\right]^2$$

$$\Rightarrow E.P.E = \frac{3mg}{2a} \times a^2\left[4\cos^2\theta - 4\cos\theta + 1\right]$$

$$\Rightarrow E.P.E. = \frac{3}{2}mga\left[4\cos^2\theta - 4\cos\theta\right] + \text{constant}$$

SIMILARLY FIND AN EXPRESSION FOR THE GRAVITATIONAL POTENTIAL ENERGY

$$\Rightarrow G.P.E = \text{"}-mgh\text{"} = -mg|AB|\cos\theta = -2mga\cos^2\theta + \text{constant}$$

TOTAL POTENTIAL ENERGY IS GIVEN BY

$$\Rightarrow V(\theta) = \frac{3}{2}mga\left[4\cos^2\theta - 4\cos\theta\right] - 2mga\cos^2\theta + \text{constant}$$

$$\Rightarrow V(\theta) = mga\left[6\cos^2\theta - 6\cos\theta - 2\cos^2\theta\right] + \text{constant}$$

$$\Rightarrow V(\theta) = mga\left[4\cos^2\theta - 6\cos\theta\right] + \text{constant}$$

$$\Rightarrow \boxed{V(\theta) = 2mga\left[2\cos^2\theta - 3\cos\theta\right] + \text{constant}}$$

// AS REQUIRED

b) DIFFERENTIATE THE POTENTIAL FUNCTION

$$\Rightarrow V(\theta) = 2mga\left[2\cos^2\theta - 3\cos\theta\right] + \text{constant}$$

$$\Rightarrow \boxed{V'(\theta) = 2mga\left[-4\cos\theta\sin\theta + 3\sin\theta\right]}$$

$$\Rightarrow V'(\theta) = 2mga\left[3\sin\theta - 2\sin 2\theta\right]$$

$$\Rightarrow V''(\theta) = 2mga\left[3\cos\theta - 4\cos 2\theta\right]$$

$$\Rightarrow V''(\theta) = 2mga\left[3\cos\theta - 4(2\cos^2\theta - 1)\right]$$

$$\Rightarrow \boxed{V''(\theta) = 2mga\left[4 + 3\cos\theta - 8\cos^2\theta\right]}$$

# IYGB - M456 PAPER D - QUESTION 1

SOLVING $V'(\theta)=0$, LOOKING FOR STATIONARY POINTS

$$\Rightarrow V'(\theta) = 0$$

$$\Rightarrow 2mga\left[-4\cos\theta\sin\theta + 3\sin\theta\right] = 0$$

$$\Rightarrow \sin\theta\left[3 - 4\cos\theta\right] = 0$$

EITHER $\sin\theta = 0$ OR $\cos\theta = \frac{3}{4}$

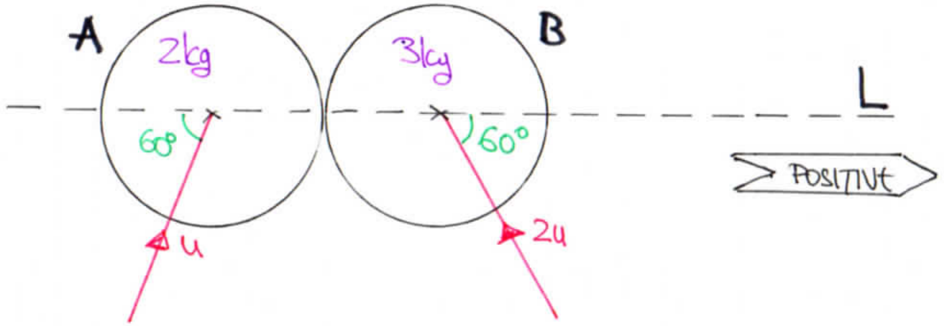
$\theta = 0$ ONLY

$\theta = \pm\arccos\frac{3}{4}$ (SYMMETRICALLY)

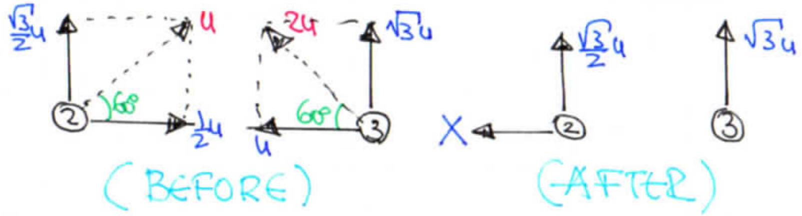
CHECKING THE STABILITY OF THESE POSITIONS

- $\theta=0$, $V''(0) = -2mga < 0$, IE LOCAL MAX $\Rightarrow$ UNSTABLE
- $\theta = \arccos\frac{3}{4}$, $V''(\arccos\frac{3}{4}) = \frac{7}{2}mga > 0$, I.E LOCAL MIN $\Rightarrow$ STABLE
- $\theta = -\arccos\frac{3}{4}$, $V''(-\arccos\frac{3}{4}) = \frac{7}{2}mga > 0$, IE LOCAL MIN $\Rightarrow$ STABLE

# IYGB - M456 PAPER D - QUESTION 2




- STARTING BY A BEFORE/AFTER DIAGRAM




- BY CONSERVATION OF MOMENTUM ALONG L

$\Rightarrow (2 \times \frac{1}{2}u) - (3 \times u) = -2X$

$\Rightarrow u - 3u = -2X$

$\Rightarrow 2X = 2u$

$\Rightarrow X = u$

- BY RESTITUTION CONSIDERATIONS ALONG L

$\Rightarrow e = \frac{SEP}{APP}$

$\Rightarrow e = \frac{X}{\frac{1}{2}u + u}$

$\Rightarrow e = \frac{\cancel{u}}{\frac{3}{2}\cancel{u}}$

$\Rightarrow e = \frac{2}{3}$

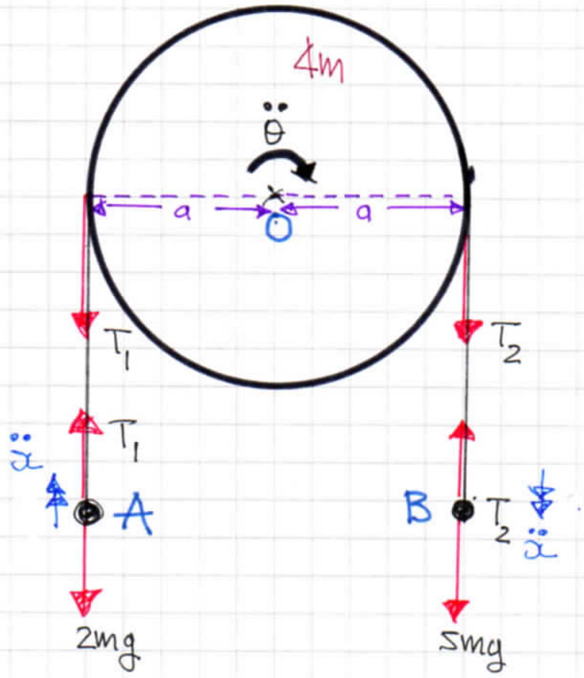
# IYGB - M456 PAPER D - QUESTION 3

## FORMING THE EQUATIONS OF MOTION

$$(A):\ T_1 - 2mg = 2m\ddot{x} \quad \text{— I}$$

$$(B):\ 5mg - T_2 = 5m\ddot{x} \quad \text{— II}$$

$$(\text{Pulley})\ I_0\ddot{\theta} = T_2 a - T_1 a \quad \text{— III}$$




$$I_0 = \tfrac{1}{2}(4m)a^2 = 2ma^2$$

NO SLIPPING $\Rightarrow x = a\theta$

$\Rightarrow \dot{x} = a\dot{\theta}$

$\Rightarrow \ddot{x} = a\ddot{\theta}$

## STARTING WITH EQUATION III

$$\Rightarrow 2ma^2\ddot{\theta} = (T_2 - T_1)a$$

$$\Rightarrow \boxed{2ma\ddot{\theta} = T_2 - T_1}$$

## ADDING I & II YIELDS

$$\Rightarrow T_1 - T_2 + 3mg = 7m\ddot{x}$$

$$\Rightarrow \boxed{3mg - 7m\ddot{x} = T_2 - T_1}$$

## COMBINING WE OBTAIN

$$\Rightarrow 2ma\ddot{\theta} = 3mg - 7m\ddot{x}$$

$$\Rightarrow 2a\ddot{\theta} = 3g - 7\ddot{x}$$

BUT $\ddot{x} = a\ddot{\theta}$

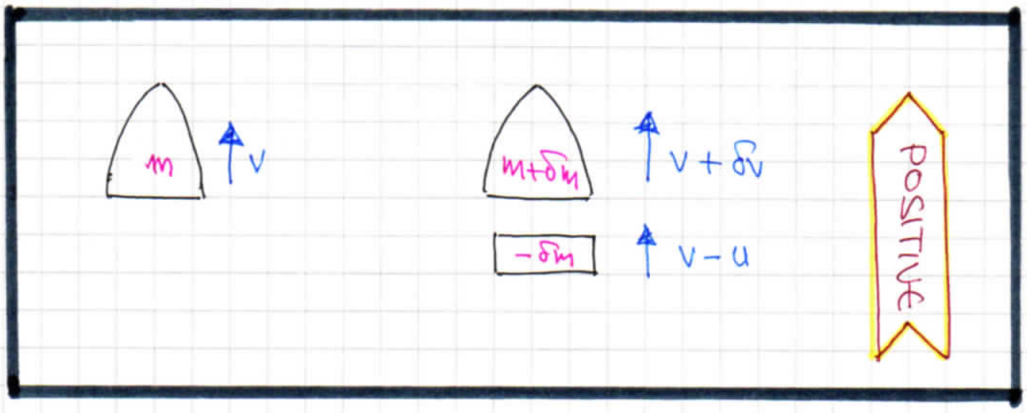
$$\Rightarrow 2\ddot{x} = 3g - 7\ddot{x}$$

$$\Rightarrow 9\ddot{x} = 3g$$

$$\Rightarrow \ddot{x} = \tfrac{1}{3}g$$

# IYGB - M456 PAPER D - QUESTION 4

a)




BY THE IMPULSE MOMENTUM PRINCIPLE

$\Longrightarrow$ MOMENTUM AFTER − MOMENTUM BEFORE = IMPULSE (OF EXTERNAL FORCES)

$\Longrightarrow \left[(m+\delta m)(v+\delta v)-\delta m(v-u)\right]-mv = 0$ ← NO EXTERNAL FORCES (DEEP SPACE)

$\Longrightarrow \cancel{mv}+m\delta v+\cancel{v\delta m}+\delta m\delta v-\cancel{v\delta m}+u\delta m-\cancel{mv}=0$

$\Longrightarrow m\delta v+u\delta m+\delta m\delta v=0$

$\Longrightarrow m\frac{\delta v}{\delta t}+u\frac{\delta m}{\delta t}+\frac{\delta m\delta v}{\delta t}=\frac{0}{\delta t}$

TAKING LIMITS & SOLVING THE EQUATION FOR THE ACCELERATION

$\Longrightarrow m\frac{dv}{dt}+u\frac{dm}{dt}=0$

$\Longrightarrow \boxed{\frac{dv}{dt}=-\frac{u}{m}\frac{dm}{dt}}$

USING THE "FUEL CONSUMPTION RATE" RELATIONSHIP

$\Longrightarrow \frac{dm}{dt}=-k$ (CONSTANT)

$\Longrightarrow m=M-kt$ (AS $t=0$, $m=M$)

$\Longrightarrow \frac{dv}{dt}=-\frac{u}{M-kt}(-k)$

$\Longrightarrow \frac{dv}{dt}=\frac{uk}{M-kt}$ AS REQUIRED

# IYGB – M456 PAPER D – QUESTION 4

b) SOLVING THE O.D.E. BY SEPARATION OF VARIABLES

$$\Longrightarrow \frac{dv}{dt} = \frac{uk}{M-kt}$$

$$\Longrightarrow \int_{v=2u}^{v} 1 \, dv = \int_{t=0}^{t} \frac{uk}{M-kt} \, dt$$

$$\Longrightarrow \Big[ v \Big]_{v=2u}^{v=v} = \left[ \frac{u\cancel{k}}{-\cancel{k}} \ln|M-kt| \right]_{t=0}^{t=t}$$

$$\Longrightarrow v - 2u = u \Big[ \ln|M-kt| \Big]_{t=t}^{t=0}$$

$$\Longrightarrow v = 2u + u \Big[ \ln M - \ln|M-kt| \Big]$$

$$\Longrightarrow \boxed{v = 2u + u \ln\left| \frac{M}{M-kt} \right|}$$

FINALLY WE NEED THE TIME WHEN $m = \frac{1}{3}M$

$$\Longrightarrow m = M - kt$$

$$\Longrightarrow \frac{1}{3}M = M - kt$$

$$\Longrightarrow kt = \frac{2}{3}M$$

$$\Longrightarrow v = 2u + u \ln\left| \frac{M}{M - \frac{2}{3}M} \right|$$

$$\Longrightarrow v = 2u + u \ln\left| \frac{1}{1/3} \right|$$

$$\Longrightarrow v = 2u + u \ln 3$$

$$\Longrightarrow v = (2 + \ln 3)u$$

# IYGB - M456 PAPER D - QUESTION 5

$$r = \frac{1}{4}e^{k\theta} \qquad t=0,\ \theta=0 \qquad \& \qquad \dot{\theta} = \omega = 2$$

- $\dot{\theta} = 2 \implies \ddot{\theta} = 0$

$$\Downarrow$$

$$\theta = 2t + C$$
$$\theta = 2t \qquad t=0, \theta=0$$

- REWRITE THE EQUATION AS

$$\implies r = \frac{1}{4}e^{2kt}$$

$$\implies \frac{dr}{dt} = \dot{r} = \frac{k}{2}e^{2kt}$$

$$\implies \frac{d^2r}{dt^2} = \ddot{r} = k^2 e^{2kt}$$

- RADIAL ACCELERATION ($\hat{\underline{r}}$)

$$\ddot{r} - r\dot{\theta}^2 = k^2e^{2kt} - \frac{1}{4}e^{2kt} \times 2^2 = k^2e^{2kt} - e^{2kt} = e^{2kt}(k^2-1)$$

- TRANSVERSE ACCELERATION $\hat{\underline{\theta}}$

$$\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = \frac{1}{\frac{1}{4}e^{2kt}}\frac{d}{dt}\left[\frac{1}{16}e^{4kt} \times 2\right] = 4e^{-2kt}\frac{d}{dt}\left[\frac{1}{8}e^{4kt}\right]$$

$$= 4e^{-2kt} \times \frac{k}{2}e^{4kt} = 2ke^{2kt}$$

# IYGB - M456 PAPER D - QUESTION 5

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\hat{\underline{r}} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\hat{\underline{\theta}}$$

- WHEN $t=0$ $\quad |\underline{a}| = 1.04$

$$\underline{a} = (k^2-1)\hat{\underline{r}} + 2k\hat{\underline{\theta}}$$

$$|a| = \sqrt{(k^2-1)^2 + (2k)^2}$$

$$1.04 = \sqrt{k^4 - 2k + 1 + 4k^2}$$

$$1.04 = \sqrt{k^4 + 2k + 1}$$

$$1.04 = \sqrt{(k^2+1)^2}$$

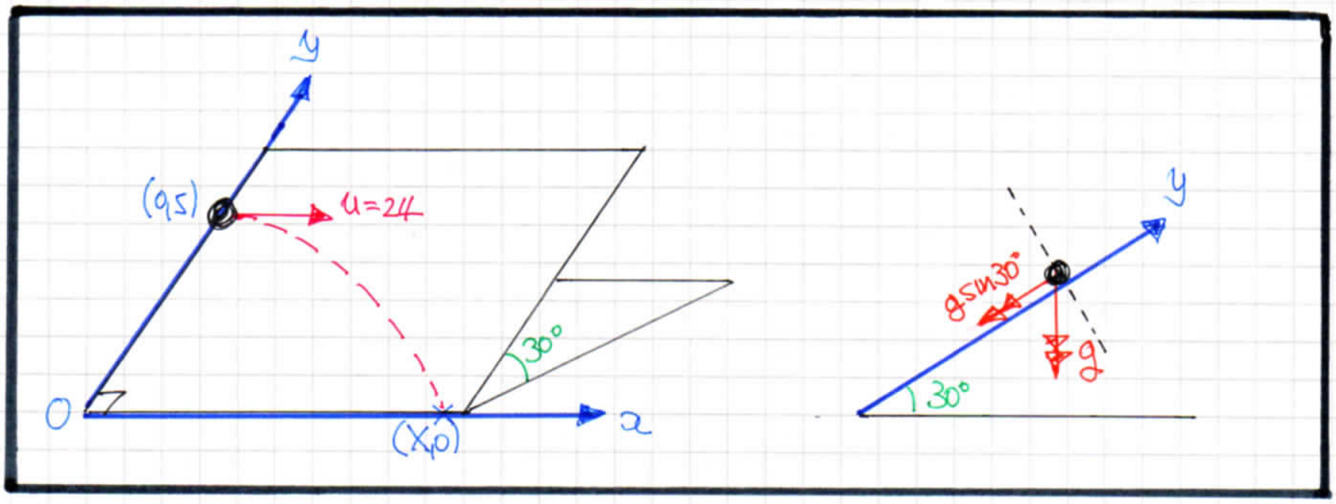
$$1.04 = |k^2+1|$$

$$k^2+1 = \left\langle \begin{matrix} 1.04 \\ -1.04 \end{matrix} \right.$$

$$k^2 = \left\langle \begin{matrix} 0.04 \\ \cancel{-2.04} \end{matrix} \right.$$

$$k = \pm 0.2$$

# IYGB – M456 PAPER D – QUESTION 6




EQUATIONS OF MOTION IN $x$ & $y$ CAN BE SIMPLIFIED, AS THE ACCELERATION IS ZERO IN $x$ & $-g\sin 30°$ IN $y$

- $\ddot{x} = 0$
- $\dot{x} = 24$
- $x = 24t$
- $\ddot{y} = -\frac{1}{2}g$
- $\dot{y} = -\frac{1}{2}gt \quad (v = \cancel{u} + at)$
- $y = 5 - \frac{1}{4}gt^2 \quad (\cancel{s} = s_0 + \cancel{ut} + \frac{1}{2}at^2)$

"JOURNEY TIME" OCCURS WHEN $y = 0$

$\Rightarrow 5 - \frac{1}{4}gt^2 = 0$

$\Rightarrow 20 = gt^2$

$\Rightarrow t = \sqrt{\frac{20}{g}}$

$\Rightarrow t = \frac{10}{7}$ OR $T = \frac{10}{7} = 1.43\text{ s}$

THE VALUE OF X IS SIMPLY

$X = 24T$

$X = 24 \times \frac{10}{7}$

$X = \frac{240}{7}$

$X = 34.3\text{ m}$

# IYGB - M456 PAPER D - QUESTION 6

FINALLY THE SPEED

$\dot{x} = 24$ (CONSTANT)

$\dot{y} = -\frac{1}{2}gt$

$\dot{y} = -\frac{1}{2}g \times \frac{10}{7}$

$\dot{y} = 7$ ("DOWNWARDS")

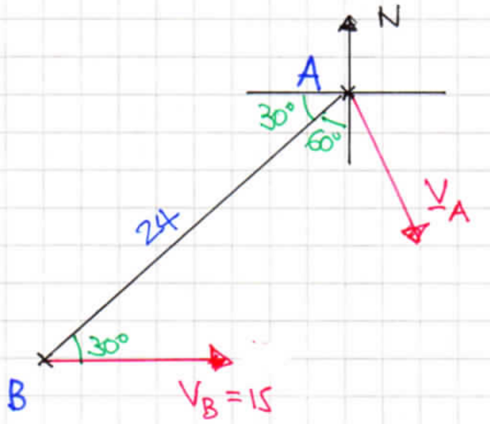
$\Longrightarrow$ SPEED $= \sqrt{\dot{x}^2 + \dot{y}^2}$

$= \sqrt{24^2 + 7^2}$

$= 25 \text{ ms}^{-1}$

# IYGB - M456 PAPER D - QUESTION 7

**a)** THE INITIAL CONFIGURATION IS SHOWN BELOW



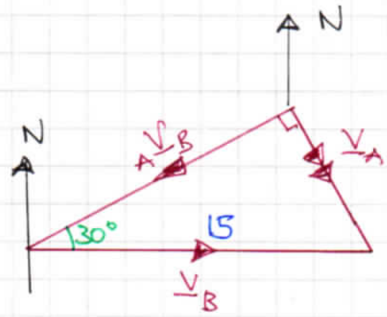

$$ {}_A\underline{V}_B = \underline{V}_A - \underline{V}_B $$

$$ \underline{V}_A = \underline{V}_B + {}_A\underline{V}_B $$

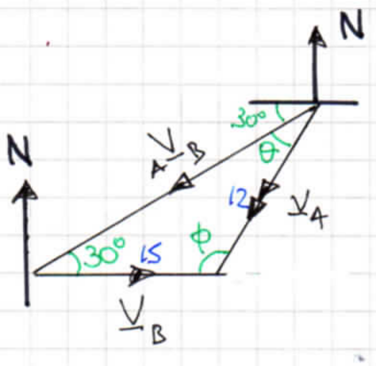
IF INTERCEPTION IS TAKING PLACE AT THE MINIMUM POSSIBLE SPEED

- AN OBSERVER ON B SEES A MOVING IN THE DIRECTION A TO B THROUGHOUT, IE ${}_A V_B$ IS IN THE DIRECTION OF THE ORIGINAL CONFIGURATION (IE BEARING 240°)
- $\underline{V}_A$ IS PERPENDICULAR TO ${}_A\underline{V}_B$
- $\sin 30 = \dfrac{|V_A|}{15}$

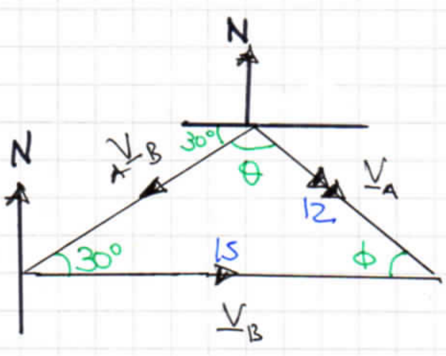
$$|\underline{V}_A| = 7.5 \text{ km h}^{-1}$$




**b)** DRAWING A VELOCITY DIAGRAM FOR EACH OF THE TWO CASES




"QUICKEST", θ ACUTE




"LONGEST", θ OBTUSE

# IYGB - M456 PAPER D - QUESTION 7

BY THE SINE RULE IN EACH OF THE TWO CASES

$$\frac{\sin\theta}{15} = \frac{\sin 30}{12}$$

$$\sin\theta = \frac{5}{8}$$

$$\underline{\theta = 38.68^\circ \text{ (ACUTE)}}$$

$\therefore$ BEARING $= 270 - 30 - \theta$

$= 201^\circ$

$$\underline{\theta = 141.32 \text{ (OBTUSE)}}$$

$\therefore$ BEARING $= 270 - 30 - \theta$

$= 99^\circ$

BY THE COSINE RULE IN EACH OF THE TWO CASES

- $|{}_A\underline{V}_B|^2 = 15^2 + 12^2 - 2\times15\times12\times\cos(111.32^\circ)$

$\phi = 180 - 30 - \theta$

$|{}_AV_B| \simeq \underline{22.358 \text{ kh}^{-1}}$

- TIME TAKEN TO COVER 24 km AT 22.358.. $\text{kmh}^{-1}$

$\approx \underline{1.073\ldots \text{ HOURS}}$

- ACTUAL DISTANCE COVERED

$1.073 \times 12 \approx 12.9$ km

$(\phi = 8.68^\circ)$

$|{}_AV_B| \approx \underline{3.623 \text{ kh}^{-1}}$

- TIME TAKEN TO COVER 24 km AT 3.623 ... $\text{kmh}^{-1}$

$\approx \underline{6.6245\ldots \text{ HOURS}}$

- ACTUAL DISTANCE COVERED

$6.6245\ldots \times 12 \simeq 79.5$ km

# IYGB - M456 PAPER D - QUESTION 8

LET THE MASS OF THE BOB BE $m$

THE EQUATION OF MOTION IS GIVEN BY

$\Longrightarrow \cancel{m}\ddot{s} = -\cancel{m}g\sin\theta$

$\Longrightarrow l\ddot{\theta} = -g\sin\theta$

$\Longrightarrow \ddot{\theta} = -\frac{g}{l}\sin\theta$

FOR "SMALL" OSCILLATIONS, $\sin\theta \simeq \theta$

$\Longrightarrow \ddot{\theta} = -\frac{g}{l}\theta$

$\Longrightarrow \ddot{\theta} = -\frac{9.8}{0.245}\theta$

$\Longrightarrow \ddot{\theta} = -40\theta$

IE S.H.M WITH $\omega^2 = 40$, "ANGULAR AMPLITUDE" OF $\frac{\pi}{9}$

USING EQUATION WITH $t=0$ AT THE "POSITIVE" ENDPOINT

$\Longrightarrow x = a\cos\omega t$

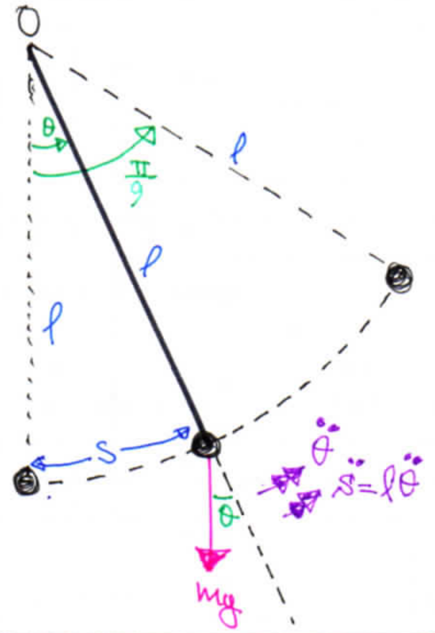
$\Longrightarrow \theta = \theta_0\cos\omega t$

$\Longrightarrow \theta = \frac{\pi}{9}\cos(\sqrt{40}\,t)$

SOLVING FOR $\theta = \frac{\pi}{18}$

$\Longrightarrow \frac{\pi}{18} = \frac{\pi}{9}\cos(\sqrt{40}\,t)$

$\Longrightarrow \frac{1}{2} = \cos(\sqrt{40}\,t)$




$l = 24.5$ cm

$l = 0.245$ m

ALSO WE HAVE

$s = l\theta$

$\dot{s} = l\dot{\theta}$

$\ddot{s} = l\ddot{\theta}$

# IYGB – M456 PAPER D – QUESTION 8

$$\Rightarrow \sqrt{40}\,t = \arccos\left(\tfrac{1}{2}\right) \quad \leftarrow \text{1ST POSITIVE SOLUTION}$$

$$\Rightarrow \sqrt{40}\,t = \frac{\pi}{3}$$

$$\Rightarrow t = \frac{\pi}{3\sqrt{40}} \approx 0.165576\ldots$$

SIMILARLY FOR $\theta = \pi/9$

$$\Rightarrow \frac{\pi}{36} = \frac{\pi}{4}\cos\left(\sqrt{40}\,t\right)$$

$$\Rightarrow \frac{1}{4} = \cos\left(\sqrt{40}\,t\right)$$

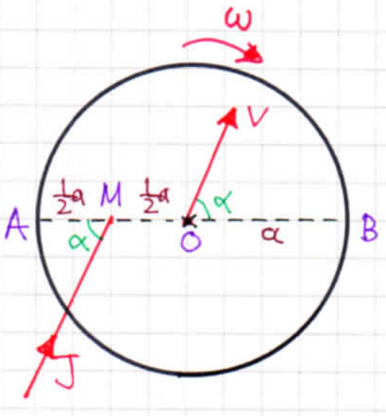
$$\Rightarrow \sqrt{40}\,t = \arccos\left(\tfrac{1}{4}\right) \quad \leftarrow \text{1ST POSITIVE SOLUTION}$$

$$\Rightarrow t = \frac{\arccos 1/4}{\sqrt{40}} \approx 0.208412\ldots$$

REQUIRED TIME IS

$$0.208412\ldots - 0.165576\ldots \simeq 0.0428\ \text{s}$$

# IYGB - M456 PAPER D - QUESTION 9




AS THE DISC IS UNCONSTRAINED, IT WILL AQUIRE

- LINEAR SPEED V, PARALLEL TO THE DIRECTION OF J
- ANGULAR SPEED ω, ABOUT THE CENTRE OF MASS

MOMENT OF INERTIA OF THE DISC IS

$$I_0 = \frac{1}{2}ma^2$$

## BY CONSERVATION OF LINEAR MOMENTUM

$$\Rightarrow J = m(v-u)$$

$$\Rightarrow J = m(v-0)$$

$$\Rightarrow J = mv$$

## BY CONSERVATION OF ANGULAR MOMENTUM ABOUT O

$$\Rightarrow (J\sin\alpha)\times\frac{1}{2}a = I_0(\omega-0)$$

[MOMENT OF MOMENTUM = CHANGE IN ANGULAR MOMENTUM ABOUT O]

$$\Rightarrow \frac{1}{2}Ja\sin\alpha = \left(\frac{1}{2}ma^2\right)\omega$$

$$\Rightarrow J\sin\alpha = ma\omega$$

## FINALLY ENERGIES

- K.E BEFORE $= 0$ (AT REST)
- KE AFTER $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}ma^2\omega^2\right)$$

IYGB - M456 PAPER D - QUESTION 9

$$= \frac{1}{2}mv^2 + \frac{1}{4}ma^2\omega^2$$

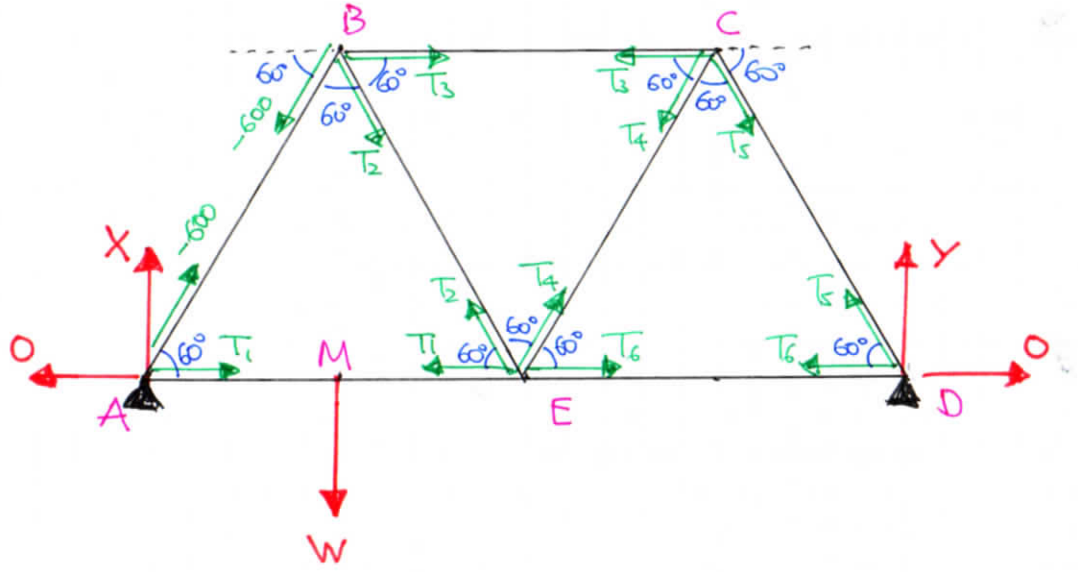
$$= \frac{1}{4m}\left[2m^2v^2 + m^2a^2\omega^2\right]$$

$$= \frac{1}{4m}\left[2J^2 + J^2\sin^2\alpha\right]$$

$$= \frac{J^2}{4m}\left[2 + \sin^2\alpha\right]$$

AS REQUIRED

# IYGB - M456 PAPER D - QUESTION 10




- START BY A GOOD DIAGRAM WHICH SHOWS EXTERNAL FORCES IN RED & INTERNAL FORCES (IN GREEN) AS TENSIONS

- LOOKING AT "A" HORIZONTALLY & NOTING THE EXTERNAL HORIZONTAL REACTION IS ZERO

$\Rightarrow T_1 + (-600\cos 60°) = 0$

$\Rightarrow T_1 = 300 \text{ N}$ // TENSION

- LOOKING AT "A" VERTICALLY

$\Rightarrow X + (-600\sin 60°) = 0$

$\Rightarrow X = 600\sin 60°$

$\Rightarrow X = 300\sqrt{3} \text{ N}$ //

- TAKING MOMENTS ABOUT "D"

$\Rightarrow X(4a) = W(3a)$

$\Rightarrow W = \frac{4}{3}X$

$\Rightarrow W = 400\sqrt{3} \text{ N}$ //

- RESOLVING EXTERNAL FORCES VERTICALLY

$\Rightarrow X + Y = W$

$\Rightarrow 300\sqrt{3} + Y = 400\sqrt{3}$

$\Rightarrow Y = 100\sqrt{3} \text{ N}$ //

# IYGB - M456 PAPER D - QUESTION 10

**LOOKING AT "B" VERTICALLY**

$\Rightarrow (-600 \cancel{\sin 60°}) + T_2 \cancel{\sin 60} = 0$

$\Rightarrow T_2 = 600\text{ N}$ (TENSION)

**LOOKING AT "B" HORIZONTALLY**

$\Rightarrow -600\cos 60° = T_3 + T_2 \cos 60$

$\Rightarrow -300 = T_3 + 600 \times \frac{1}{2}$

$\Rightarrow -600 = T_3$

$\Rightarrow T_3 = 600\text{ N}$ (THRUST)

**LOOKING AT "D" VERTICALLY**

$\Rightarrow Y + T_5 \sin 60° = 0$

$\Rightarrow 100\sqrt{3} + T_5 \frac{\sqrt{3}}{2} = 0$

$\Rightarrow 200 + T_5 = 0$

$\Rightarrow T_5 = -200$

$\Rightarrow T_5 = 200\text{ N}$ (THRUST)

**LOOKING AT "D" HORIZONTALLY**

$\Rightarrow T_5 + T_6 \cos 60° = 0$

$\Rightarrow -200 + T_6 \times \frac{1}{2} = 0$

$\Rightarrow T_6 = 400\text{ N}$ (TENSION)

**LOOKING AT "C" VERTICALLY**

$T_4 \cancel{\sin 60°} + T_5 \cancel{\sin 60°} = 0$

$T_4 = -T_5$

$T_4 = 200\text{ N}$ (TENSION)

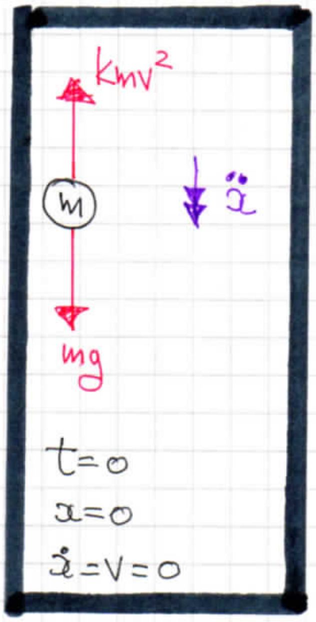
SUMMARIZING RESULTS

REACTION AT A: $300\sqrt{3}$ N, UPWARDS

REACTION AT D: $100\sqrt{3}$ N, UPWARDS

| | | |
|---|---|---|
| AE : | 300 N | TENSION |
| ED : | 400 N | TENSION |
| BC : | 600 N | THRUST |
| EB : | 600 N | TENSION |
| EC : | 200 N | TENSION |
| CD : | 200 N | THRUST |

# IYGB – M456 PAPER D – QUESTION 11




## FORMING THE EQUATION OF MOTION

$$\Rightarrow m\ddot{x} = mg - kmv^2$$

$$\Rightarrow \ddot{x} = g - kv^2$$

$$\Rightarrow v\frac{dv}{dx} = g - kv^2$$

## SOLVING BY SEPARATION OF VARIABLES, SUBJECT TO THE INITIAL CONDITIONS GIVEN

$$\Rightarrow v\,dv = (g - kv^2)\,dx$$

$$\Rightarrow \frac{v}{g-kv^2}\,dv = 1\,dx$$

$$\Rightarrow \int_{v=0}^{V} \frac{v}{g-kv^2}\,dv = \int_{x=0}^{x} 1\,dx$$

$\times(-2k)$

$$\Rightarrow \int_{v=0}^{V} \frac{-2kv}{g-kv^2}\,dv = \int_{x=0}^{x} -2k\,dx$$

$$\Rightarrow \Big[\ln|g-kv^2|\Big]_{v=0}^{v=V} = \Big[-2kx\Big]_{x=0}^{x}$$

$$\Rightarrow \ln|g-kV^2| - \ln g = -2kx - 0$$

$$\Rightarrow \ln\left|\frac{g-kV^2}{g}\right| = -2kx$$

$$\Rightarrow \frac{g-kV^2}{g} = e^{-2kx}$$

$$\Rightarrow 1 - \frac{k}{g}V^2 = e^{-2kx}$$

# IYGB - M456 PAPER D - QUESTION 11

$\Longrightarrow \; 1 - e^{-2kx} = \frac{k}{g}v^2$

$\Longrightarrow \; v^2 = \frac{g}{k}\left(1 - e^{-2kx}\right)$

AS REQUIRED

b) FIRSTLY THE TERMINAL VELOCITY IS $\sqrt{\frac{g}{k}}$

[ EITHER AS $x \to \infty \quad v^2 \longrightarrow \frac{g}{k}$

OR $\frac{dv}{dx} = 0$ (RATE OF CHANGE OF VELOCITY W.R.T $x$ IS ZERO)

$g - kv^2 = 0$

$v^2 = \frac{g}{k}$ ]

$\Longrightarrow$ WITH $v = \frac{1}{2}U = \frac{1}{2}\sqrt{\frac{g}{k}}$

$\Longrightarrow \; \frac{1}{4}\frac{g}{k} = \frac{g}{k}\left(1 - e^{-2kx}\right)$

$\Longrightarrow \; \frac{1}{4} = 1 - e^{-2kx}$

$\Longrightarrow \; e^{-2kx} = \frac{3}{4}$

$\Longrightarrow \; e^{2kx} = \frac{4}{3}$

$\Longrightarrow \; 2kx = \ln\frac{4}{3}$

$\Longrightarrow \; x = \frac{1}{2k}\ln\left(\frac{4}{3}\right)$

AS REQUIRED

# IYGB - M456 PAPER D - QUESTION 12

$\underline{F}_1 = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$    $A(12,-15,-2)$, SPEED $U$ ms$^{-1}$

$B(-8,5,2)$, SPEED 29 ms$^{-1}$

$\underline{F}_2 = \begin{pmatrix} k-2 \\ 2k+3 \\ 3k-1 \end{pmatrix}$    PARTICLE OF MASS, $m = 0.5$ kg

THE RESULTANT OF $\underline{F}_1$ & $\underline{F}_2$ IS $\underline{F}$

$$\underline{F} = \underline{F}_1 + \underline{F}_2 = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix} + \begin{pmatrix} k-2 \\ 2k+3 \\ 3k-1 \end{pmatrix} = \begin{pmatrix} k-4 \\ 2k+7 \\ 3k+4 \end{pmatrix}$$

THE VECTOR $\overrightarrow{AB}$ IS GIVEN BY

$$\overrightarrow{AB} = \underline{b} - \underline{a} = (-8,5,2) - (12,-15,-2) = (-20,20,4) = 4(-5,5,1)$$

$\underline{F}$ MUST BE IN THE DIRECTION OF $(-5,5,1)$

$$\Rightarrow \begin{pmatrix} k-4 \\ 2k+7 \\ 3k+4 \end{pmatrix} = \alpha \begin{pmatrix} -5 \\ 5 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{array}{l} -5\alpha = k-4 \\ 5\alpha = 2k+7 \end{array}$$

$$\Rightarrow 0 = 3k+3$$

$$\Rightarrow k = -1$$

WORK DONE BY $\underline{F}$ IS GIVEN BY

$$W = \underline{F} \cdot \overrightarrow{AB} = \begin{pmatrix} -5 \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -20 \\ 20 \\ 4 \end{pmatrix} = 100 + 100 + 4 = 204 \text{ J}$$

# IYGB - M456 PAPER D - QUESTION 12

BY ENERGIES

$\Rightarrow KE_A + WORK = KE_B$

$\Rightarrow \frac{1}{2}mu^2 + W = \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}\times\frac{1}{2}\times U^2 + 204 = \frac{1}{2}\times\frac{1}{2}\times 29^2$

$\Rightarrow \frac{1}{4}U^2 + 204 = \frac{841}{4}$

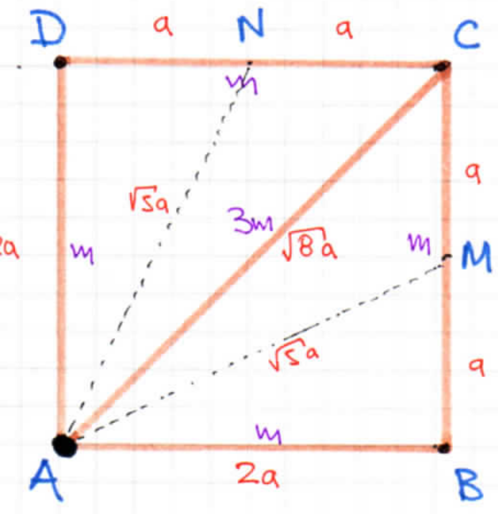
$\Rightarrow U^2 + 816 = 841$

$\Rightarrow U^2 = 25$

$\Rightarrow |U| = 5 \text{ ms}^{-1}$

# IYGB - M456 PAPER D - QUESTION 13

START BY A DIAGRAM




- LENGTH OF AC IS $2\sqrt{2}a$
- LENGTH OF AN OR AM IS $\sqrt{5}a$

MOMENT OF INERTIA OF THE ROD AB OR ROD AD ABOUT A

$$I = \frac{1}{3}ma^2 + ma^2 = \frac{4}{3}ma^2$$

MOMENT OF INERTIA OF THE ROD BC (OR DC) ABOUT A

$$I = \frac{1}{3}ma^2 + m(\sqrt{5}a)^2 = \frac{1}{3}ma^2 + 5ma^2 = \frac{16}{3}ma^2$$

MOMENT OF INERTIA OF THE ROD AC, ABOUT A

$$I = \frac{1}{3}(3m)(\sqrt{2}a)^2 + 3m(\sqrt{2}a)^2 = 2ma^2 + 6ma^2 = 8ma^2$$

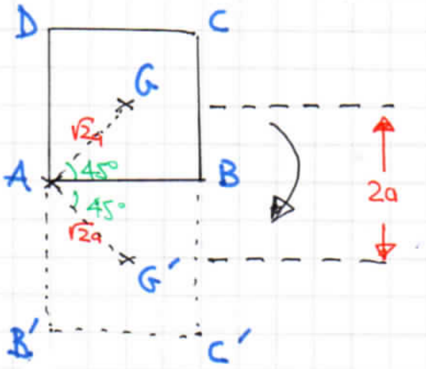
ADDING TOGETHER THE MOMENT OF INERTIA OF ALL THE RODS GIVES

$$I_{TOT} = \underset{(AB)}{\frac{4}{3}ma^2} + \underset{(AD)}{\frac{4}{3}ma^2} + \underset{(BC)}{\frac{16}{3}ma^2} + \underset{(DC)}{\frac{16}{3}ma^2} + \underset{(AC)}{8ma^2}$$

$$I_{TOTAL} = \frac{64}{3}ma^2$$

# IYGB - M456 PAPER D - QUESTION 13

WHEN "B" IS VERTICALLY BELOW "A", THE CENTRE OF MASS "G" OF THE SYSTEM WOULD HAVE "DROPPED" FROM a ABOVE THE LEVEL OF "A" TO $\sqrt{2}a\cos45 = a$ BELOW THE LEVEL OF A




BY ENERGIES

$$\Rightarrow \frac{1}{2}I\omega^2 = 7mg(2a)$$

$$\Rightarrow \frac{1}{2}\left(\frac{64}{3}ma^2\right)\omega^2 = 14mga$$

$$\Rightarrow \frac{32}{3}\cancel{m}a^{\cancel{2}}\omega^2 = 14\cancel{m}g\cancel{a}$$

$$\Rightarrow \frac{32}{3}a\omega^2 = 14g$$

$$\Rightarrow \frac{16}{3}a\omega^2 = 7g.$$

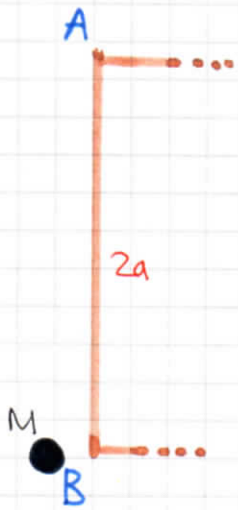
$$\Rightarrow \omega^2 = \frac{21g}{16a}$$

$$\Rightarrow \omega = \frac{1}{4}\sqrt{\frac{21g}{a}}$$

NOW BY CONSERVATION OF ANGULAR MOMENTUM ABOUT A

- $I_{\text{NEW TOTAL}} = \frac{64}{3}ma^2 + M(2a)^2$

$$= \frac{64}{3}ma^2 + 4Ma^2$$




- $I_{TOT} \times \omega = I_{\text{NEW TOT}} \times \Omega$

$$\left(\frac{64}{3}ma^2\right) \times \frac{1}{4}\sqrt{\frac{21g}{a}} = \left(\frac{64}{3}ma^2 + 4Ma^2\right)\left(\frac{2}{9}\sqrt{\frac{21g}{a}}\right)$$

# IYGB - M456 PAPER D - QUESTION 13

$$\Longrightarrow \frac{16}{3}\cancel{\sqrt{\frac{21g}{a}}}\,m\cancel{a^2} = \frac{2}{9}\cancel{\sqrt{\frac{21g}{a}}}\left(\frac{64}{3}m + 4M\right)\cancel{a^2}$$

$$\Longrightarrow \frac{16}{3}m = \frac{2}{9}\left(\frac{64}{3}m + 4M\right)$$

$$\Longrightarrow \frac{16}{3}m = \frac{128}{27}m + \frac{8}{9}M$$

$$\div 8$$

$$\Longrightarrow \frac{2}{3}m = \frac{16}{27}m + \frac{1}{9}M$$

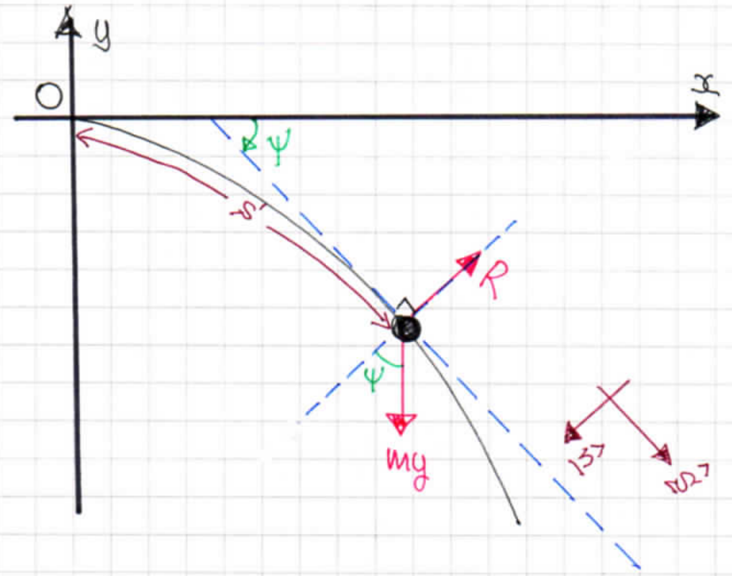
$$\times 27$$

$$\Longrightarrow 18m = 16m + 3M$$

$$\Longrightarrow 2m = 3M$$

$$\Longrightarrow M = \frac{2}{3}m$$

# IYGB - M456 PAPER D - QUESTION 14




$s = a\sin\psi$

$\rho = \frac{ds}{d\psi} = a\cos\psi$

ACCELERATION

$\underline{a} = \ddot{s}\,\hat{\underline{s}} + \frac{\dot{s}^2}{\rho}\,\hat{\underline{n}}$

a) I) LOOKING AT THE TANGENTIAL DIRECTION

$\Rightarrow m\ddot{s} = mg\sin\psi$

$\Rightarrow \ddot{s} = g\sin\psi$

$\Rightarrow \frac{1}{2}\frac{d}{ds}(\dot{s}^2) = g\sin\psi$

INTEGRATE SUBJECT TO CONDITIONS

$\Rightarrow \frac{1}{2}\left[\dot{s}^2\right]_{\dot{s}=\sqrt{\frac{1}{2}ag}}^{\dot{s}} = \int_{s=0}^{s} g\sin\psi\, ds$

$\Rightarrow \frac{1}{2}\left[\dot{s}^2 - \frac{1}{2}ag\right] = \int_{s=0}^{s=s} g\left(\frac{s}{a}\right) ds$

$\Rightarrow \frac{1}{2}\dot{s}^2 - \frac{1}{4}ag = \left[\frac{g}{2a}s^2\right]_{s=0}^{s=s}$

$\Rightarrow \frac{1}{2}\dot{s}^2 - \frac{1}{4}ag = \frac{g}{2a}s^2$

$\Rightarrow \dot{s}^2 - \frac{1}{2}ag = \frac{g}{a}s^2$

$\Rightarrow \dot{s}^2 = \frac{1}{2}ag + \frac{g}{a}s^2$

$\Rightarrow v^2 = \frac{g}{2a}\left[2s^2 + a^2\right]$

AS REQUIRED

ALTERNATIVE

$\Rightarrow m\ddot{s} = mg\sin\psi$

$\Rightarrow \ddot{s} = g\sin\psi$

$\Rightarrow v\frac{dv}{ds} = g\sin\psi$

$\Rightarrow v\,dv = g\sin\psi\, ds$

$\Rightarrow v\,dv = g\sin\psi\,\frac{ds}{d\psi}\,d\psi$

$\Rightarrow v\,dv = g\sin\psi\,(a\cos\psi)\,d\psi$

$\Rightarrow v\,dv = ag\sin\psi\cos\psi\,d\psi$

$\Rightarrow \int_{v=\sqrt{\frac{1}{2}ag}}^{v} v\,dv = \int_{\psi=0}^{\psi} ag\sin\psi\cos\psi\,d\psi$

$\Rightarrow \left[\frac{1}{2}v^2\right]_{v=\sqrt{\frac{1}{2}ag}}^{v} = \left[\frac{1}{2}ag\sin^2\psi\right]_{\psi=0}^{\psi}$

$\Rightarrow \frac{1}{2}v^2 - \frac{1}{4}ag = \frac{1}{2}ag\sin^2\psi - 0$

# IYGB - M456 PAPER D - QUESTION 14

$$\Rightarrow v^2 - \tfrac{1}{2}ag = ag\sin^2\psi$$

$$\Rightarrow v^2 = ag\sin^2\psi + \tfrac{1}{2}ag$$

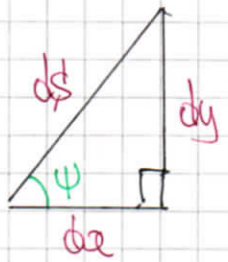
$$\Rightarrow v^2 = ag\left(\frac{s^2}{a^2}\right) + \tfrac{1}{2}ag$$

$$\Rightarrow v^2 = \frac{g}{a}s^2 + \tfrac{1}{2}ag$$

$$\Rightarrow v^2 = \frac{g}{2a}\left[2s^2 + a^2\right]$$

(AS BEFORE)

ALTERNATIVE VARIATION BY ENERGIES, TAKING THE LEVEL OF THE $x$ AXIS AS THE ZERO POTENTIAL LEVEL




$$\Rightarrow \frac{dy}{ds} = \sin\psi$$

$$\Rightarrow 1\,dy = \sin\psi\,ds$$

$$\Rightarrow 1\,dy = \sin\psi\,\frac{ds}{d\psi}\,d\psi$$

$$\Rightarrow 1\,dy = \sin\psi(a\cos\psi)\,d\psi$$

$$\Rightarrow 1\,dy = a\sin\psi\cos\psi\,d\psi$$

$$\Rightarrow \int_{y=0}^{y} 1\,dy = \int_{\psi=0}^{\psi} a\sin\psi\cos\psi\,d\psi$$

$$\Rightarrow \left[y\right]_0^y = \left[\tfrac{1}{2}a\sin^2\psi\right]_0^\psi$$

$$\Rightarrow \boxed{y = \tfrac{1}{2}a\sin^2\psi}$$

(WHERE $y$ IS MEASURED "DOWNWARDS")

# IYGB – M456 PAPER D – QUESTION 14

NOW $\quad K.E_0 + \cancel{P.E_0} = K.E_\psi + P.E_\psi$

$$\Rightarrow \tfrac{1}{2}mu^2 + 0 = \tfrac{1}{2}mv^2 - mgy$$

$$\Rightarrow u^2 = v^2 - 2gy$$

$$\Rightarrow v^2 = u^2 + 2gy$$

$$\Rightarrow v^2 = \left(\sqrt{\tfrac{1}{2}ag}\right)^2 + 2g\left(\tfrac{1}{2}a\sin^2\psi\right)$$

$$\Rightarrow v^2 = \tfrac{1}{2}ag + ag\sin^2\psi$$

$$\Rightarrow v^2 = \tfrac{1}{2}ag + ag\left(\frac{s^2}{a^2}\right)$$

$$\Rightarrow v^2 = \frac{g}{a}s^2 + \tfrac{1}{2}ag$$

$$\Rightarrow v^2 = \frac{g}{2a}\left(2s^2 + a^2\right) \quad \text{// AS BEFORE}$$

a) II) **LOOKING AT THE EQUATION OF MOTION IN THE NORMAL DIRECTION ($\hat{n}$)**

$$\Rightarrow m\frac{\dot{s}^2}{\rho} = mg\cos\psi - R$$

$$\Rightarrow R = mg\cos\psi - \frac{m\dot{s}^2}{\rho}$$

$$\Rightarrow R = mg\left[\cos\psi - \frac{\dot{s}^2}{g\rho}\right]$$

$$\Rightarrow R = mg\left[\cos\psi - \frac{1}{\cancel{g}(a\cos\psi)} \times \frac{\cancel{g}}{2a}\left(2s^2 + a^2\right)\right]$$

$$\Rightarrow R = mg\left[\cos\psi - \frac{2s^2 + a^2}{2a^2\cos\psi}\right]$$

$$\Rightarrow R = \frac{mg}{2\cos\psi}\left[2\cos^2\psi - \frac{2s^2 + a^2}{a^2}\right]$$

$$\Rightarrow R = \frac{mg}{2\cos\psi}\left[2(1 - \sin^2\psi) - 2\left(\frac{s^2}{a^2}\right) - 1\right]$$

# IYGB – M456 PAPER D – QUESTION 14

$$\Rightarrow R = \frac{mg}{2\cos\psi}\left[2 - 2\sin^2\psi - 2\sin^2\psi - 1\right]$$

$$\Rightarrow R = \frac{mg}{2\cos\psi}\left[1 - 4\sin^2\psi\right]$$

// AS REQUIRED

c) FINALLY IF $R = 0$

- $1 - 4\sin^2\psi = 0$

  $\sin^2\psi = \frac{1}{4}$

  $\sin\psi = +\frac{1}{2}$

  $\left(\psi = \frac{\pi}{6}\right)$

- $s = a\sin\psi$

  $s = \frac{1}{2}a$

ALTERNATIVE IF $R=0$

$\Rightarrow 1 - 4\sin^2\psi = 0$

$\Rightarrow 1 - 4\left(\frac{s}{a}\right)^2 = 0$

$\Rightarrow 1 - \frac{4s^2}{a^2} = 0$

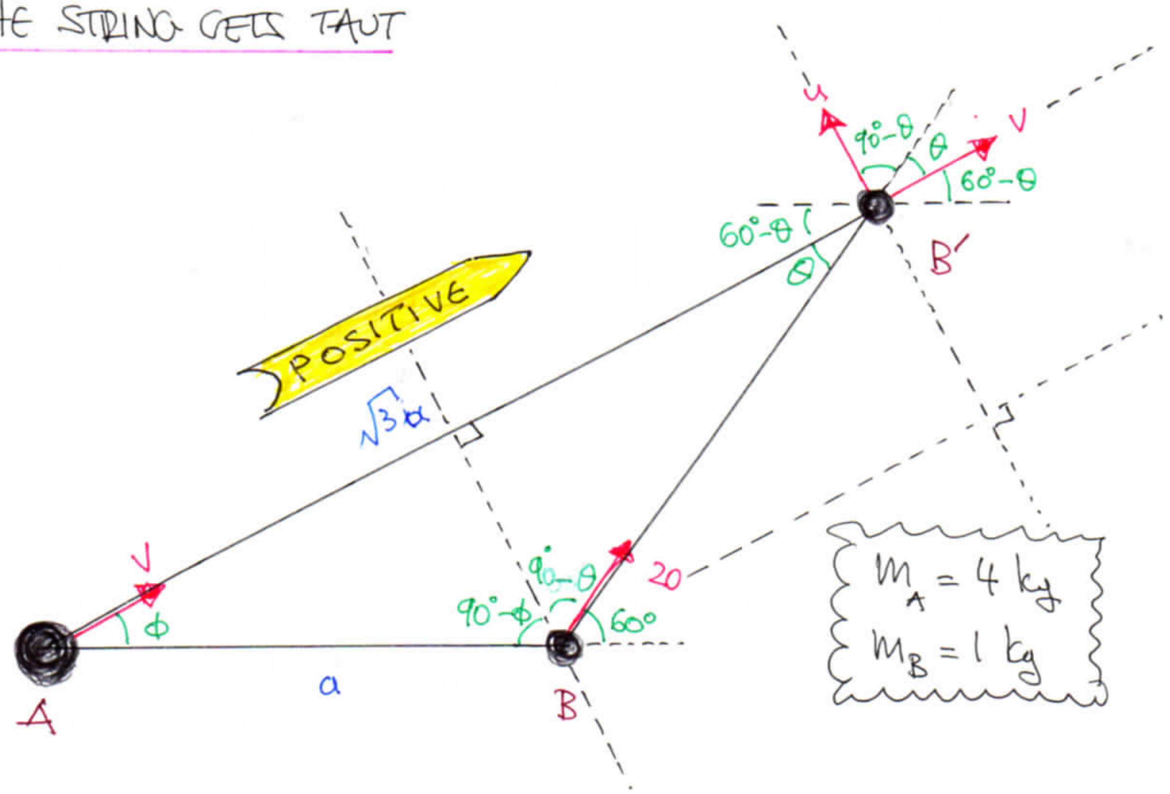
$\Rightarrow a^2 - 4s^2 = 0$

$\Rightarrow (a - 2s)(a + 2s)$

$\Rightarrow s = \frac{1}{2}a$ or $\cancel{-\frac{1}{2}a}$

# IYGB – PAPER D – QUESTION 15

- START WITH A HIGHLY GEOMETRICAL DIAGRAM JUST BEFORE THE STRING GETS TAUT




- BY THE SINE RULE ON $\overset{\triangle}{ABB'}$

$$\frac{\sin\theta}{a} = \frac{\sin 120°}{\sqrt{3}a} \implies \sin\theta = \frac{\sin 120°}{\sqrt{3}}$$

$$\implies \sin\theta = \frac{\sqrt{3}/2}{\sqrt{3}}$$

$$\implies \sin\theta = \frac{1}{2}$$

$$\implies \theta = 30°$$

& CONSEQUENTLY $\phi = 30°$

- BY CONSIDERING THE IMPULSE ON EACH PARTICLE AS THE STRING GETS TAUT

$$\left.\begin{array}{ll} \text{IMPULSE ON A:} & J = 4v - 4 \times 0 \\ \text{IMPULSE ON B:} & -J = 1v - 1(20\cos 30) \end{array}\right\} \implies \begin{array}{l} J = 4v \\ -J = v - 10\sqrt{3} \end{array}$$

ADDING EQUATIONS

IYGB - M456 PAPER D - QUESTION 15

$$\Rightarrow 0 = 5v - 10\sqrt{3}$$
$$\Rightarrow 5v = 10\sqrt{3}$$
$$\Rightarrow v = 2\sqrt{3} \text{ ms}^{-1}$$

- HENCE THE IMPULSIVE TENSION IN THE STRING IS

$$J = 4v$$
$$J = 8\sqrt{3} \simeq 13.86 \text{ Ns}$$

- NOW LOOKING AT THE DIRECTION PERPENDICULAR TO THE STRING AS IT GETS TAUT, NO MOMENTUM IS EXCHANGED

$$\therefore 1 \times u = 20\cos 60$$
$$u = 10 \text{ ms}^{-1}$$

- HENCE WE HAVE THE SPEED OF B

$$\text{SPEED} = \sqrt{u^2 + v^2} = \sqrt{10^2 + (2\sqrt{3})^2}$$
$$= \sqrt{100 + 12} = \sqrt{112}$$
$$= 4\sqrt{7} \text{ ms}^{-1}$$

AS REQUIRED