IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 1

USING THE DEFINITION OF THE PRODUCT OPERATOR

$$\prod_{r=3}^{16}\left[1+\frac{4}{r-2}\right] = \prod_{r=3}^{16}\left[\frac{r-2+4}{r-2}\right]$$

$$= \prod_{r=3}^{16}\left[\frac{r+2}{r-2}\right]$$

AS THERE ARE NOT TOO MANY TERMS, WE MAY WRITE THEM OUT

$$= \frac{\cancel{5}}{1}\times\frac{\cancel{6}}{2}\times\frac{\cancel{7}}{3}\times\frac{\cancel{8}}{4}\times\frac{\cancel{9}}{\cancel{5}}\times\frac{\cancel{10}}{\cancel{6}}\times\frac{\cancel{11}}{\cancel{7}}\times\frac{\cancel{12}}{\cancel{8}}\times\frac{\cancel{13}}{\cancel{9}}\times\frac{\cancel{14}}{\cancel{10}}\times\frac{15}{\cancel{11}}\times\frac{16}{\cancel{12}}\times\frac{17}{\cancel{13}}\times\frac{18}{\cancel{14}}$$

$$= \frac{15\times16\times17\times18}{1\times2\times3\times4}$$

$$= \frac{(5\times\cancel{3})(\cancel{2}\times\cancel{4}\times2)\times17\times18}{1\times\cancel{2}\times\cancel{3}\times\cancel{4}}$$

$$= 5\times2\times17\times18$$

$$= \underline{3060}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 2

a)

$$\underline{C} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 4 & 2 \end{bmatrix}$$

MATRIX OF MINORS $= \begin{bmatrix} -2 & 3 & 7 \\ 0 & 1 & 2 \\ 1 & -1 & -3 \end{bmatrix}$

MATRIX OF COFACTORS $= \begin{bmatrix} -2 & -3 & 7 \\ 0 & 1 & -2 \\ 1 & +1 & -3 \end{bmatrix}$

ADJUGATE MATRIX $= \begin{bmatrix} -2 & 0 & 1 \\ -3 & 1 & 1 \\ 7 & -2 & -3 \end{bmatrix}$

$$|\underline{C}| = 1\times(-2) + 2(-3) + 1\times 7 = -2-6+7 = -1$$

$$\underline{C}^{-1} = \frac{1}{|C|}(\text{ADJUGATE}) = \frac{1}{-1}\begin{bmatrix} -2 & 0 & 1 \\ -3 & 1 & 1 \\ 7 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -1 & -1 \\ -7 & 2 & 3 \end{bmatrix}$$

b) NOW THE INVERSE BY ROW OPERATIONS

$$\begin{array}{c} \quad\; C \qquad\qquad I \\ \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 4 & 2 & 0 & 0 & 1 \end{array}\right] \end{array} \xRightarrow[r_{13}(-1)]{r_{12}(-2)} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -3 & -1 & -2 & 1 & 0 \\ 0 & 2 & 1 & -1 & 0 & 1 \end{array}\right]$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 2

$$\xrightarrow{r_{23}} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -1 & 0 & 1 \\ 0 & -3 & -1 & -2 & 1 & 0 \end{array}\right] \xrightarrow{r_2\left(\frac{1}{2}\right)} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & -3 & -1 & -2 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{r_{23}(3)} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{7}{2} & 1 & \frac{3}{2} \end{array}\right] \xrightarrow{r_3(2)} \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -7 & 2 & 3 \end{array}\right]$$

$$\xrightarrow{r_{21}(-2)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & -7 & 2 & 3 \end{array}\right] \xrightarrow{r_{32}\left(-\frac{1}{2}\right)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 1 & -7 & 2 & 3 \end{array}\right]$$

$$\underline{I} \qquad \underline{C^{-1}}$$

//

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 3

$$z = x^4 + y^4 - 4xy \qquad x \in \mathbb{R},\ y \in \mathbb{R}$$

- START BY GETTING EXPRESSIONS FOR THE FIRST AND SECOND DERIVATIVES OF z

- $\frac{\partial z}{\partial x} = 4x^3 - 4y$
- $\frac{\partial z}{\partial y} = 4y^3 - 4x$
- $\frac{\partial^2 z}{\partial x^2} = 12x^2$
- $\frac{\partial^2 z}{\partial y^2} = 12y^2$
- $\frac{\partial^2 z}{\partial x \partial y} = -4$

- FOR STATIONARY POINTS $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0$

$$\left.\begin{array}{l} 4x^3 - 4y = 0 \Rightarrow y = x^3 \\ 4y^3 - 4x = 0 \Rightarrow x = y^3 \end{array}\right\} \Rightarrow x = (x^3)^3$$

$$\Rightarrow x = x^9$$

$$\Rightarrow x^9 - x = 0$$

$$\Rightarrow x(x^8 - 1) = 0$$

$$\Rightarrow x(x^4 - 1)(x^4 + 1) = 0$$

$$\Rightarrow x(x^2 - 1)(x^2 + 1)(x^4 + 1) = 0$$

$$\Rightarrow x(x-1)(x+1)(x^2+1)(x^4+1) = 0$$

IRREDUCIBLE OVER THE REALS

- HENCE POTENTIALLY STATIONARY POINTS AT

$$x = \begin{cases} 0 \\ -1 \\ 1 \end{cases} \qquad y = \begin{cases} 0 \\ -1 \\ 1 \end{cases} \qquad z = \begin{cases} 0 \\ -2 \\ -2 \end{cases}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C QUESTION 3

- NOW TO CHECK THE NATURE

$$\begin{vmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial x \partial y} & \frac{\partial^2 z}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 12x^2 & -4 \\ -4 & 12y^2 \end{vmatrix} \quad \text{SCALED TO} \quad \begin{vmatrix} 3x^2 & -1 \\ -1 & 3y^2 \end{vmatrix}$$

- CHECKING EACH POINT SEPARATELY

$(0,0,0)$	$(-1,-1,-2)$	$(1,1,-2)$
$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$	$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$	$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$
$\begin{vmatrix} -\lambda & -1 \\ -1 & -\lambda \end{vmatrix} = 0$ $\lambda^2 - 1 = 0$ $\lambda = \pm 1$ MIXED SIGNS IN THE EIGENVALUES $\therefore (0,0,0)$ IS A SADDLE	$\begin{vmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{vmatrix} = 0$ $\Rightarrow (3-\lambda)^2 - 1 = 0$ $\Rightarrow (\lambda-3)^2 - 1 = 0$ $\Rightarrow \lambda - 3 = \begin{cases} 1 \\ -1 \end{cases}$ $\Rightarrow \lambda = \begin{cases} 4 \\ 2 \end{cases}$ BOTH EIGENVALUES ARE POSITIVE $\therefore (-1,-1,-2)$ IS A LOCAL MINIMUM	IDENTICAL WORKINGS TO $(-1,-1,-2)$ $\therefore (1,1,-2)$ IS A LOCAL MINIMUM

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 4

STARTING FROM THE DEFINITION OF THE LAPLACE TRANSFORM

$$\mathcal{L}[f(t)] \equiv \int_0^\infty f(t)\,e^{-st}\,dt$$

LET $f(t) = \cosh at$

$$\mathcal{L}[\cosh at] = \int_0^\infty (\cosh at)\,e^{-st}\,dt = \int_0^\infty \left(\tfrac{1}{2}e^{at} + \tfrac{1}{2}e^{-at}\right)e^{-st}\,dt$$

$$= \int_0^\infty \tfrac{1}{2}e^{(a-s)t} + \tfrac{1}{2}e^{-(a+s)t}\,dt$$

$$= \left[\tfrac{1}{2}\times\frac{1}{a-s}e^{(a-s)t} + \tfrac{1}{2}\times\frac{1}{-(a+s)}e^{-(a+s)t}\right]_0^\infty$$

$$= \frac{1}{2}\left[\frac{e^{(a-s)t}}{a-s} - \frac{e^{-(a+s)t}}{a+s}\right]_0^\infty$$

s IS SUFFICIENTLY LARGE FOR THE INTEGRAL TO CONVERGE

$$= \frac{1}{2}\left[(0-0) - \left(\frac{1}{a-s} - \frac{1}{a+s}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \frac{1}{2}\left[\frac{s+a+s-a}{(s-a)(s+a)}\right]$$

$$= \frac{1}{2}\times\frac{2s}{s^2-a^2}$$

$$= \frac{s}{s^2-a^2}$$

NOTE THAT $\mathcal{L}[\cosh at] = \mathcal{L}[\cos(iat)] = \ldots$ STANDARD RESULTS

$$= \frac{s}{s^2+(ia)^2} = \frac{s}{s^2-a^2}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 5

- START BY INTRODUCING A PARAMETER K AS FOLLOWS

$$I(k) = \int_0^1 \frac{x^k - 1}{\ln x}\, dx$$

- DIFFERENTIATING W.R.T k

$$\frac{\partial I}{\partial k} = \frac{\partial}{\partial k}\left[\int_0^1 \frac{x^k-1}{\ln x}\, dx\right] = \int_0^1 \frac{1}{\ln x} \frac{\partial}{\partial k}(x^k - 1)\, dx$$

$$\frac{\partial I}{\partial k} = \int_0^1 \frac{1}{\cancel{\ln x}}\left[x^k \cancel{\ln x}\right] dx = \int_0^1 x^k\, dx$$

$$\frac{\partial I}{\partial k} = \left[\frac{1}{k+1} x^{k+1}\right]_0^1 = \frac{1}{k+1}\left[1 - 0\right]$$

$$\frac{\partial I}{\partial k} = \frac{1}{k+1}$$

- INTEGRATING W.R.T k

$$I(k) = \ln|k+1| + C$$

$$\int_0^1 \frac{x^k - 1}{\ln x}\, dx = \ln|k+1| + C$$

- LET $k=0$

$$0 = \cancel{\ln 1} + C \qquad \text{i.e. } C = 0$$

- THUS WE HAVE

$$\int_0^1 \frac{x^k - 1}{\ln x}\, dx = \ln|k+1|$$

$$\int_0^1 \frac{x-1}{\ln x}\, dx = \ln 2$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 6

USING STANDARD TECHNIQUES

$$u_{n+1} = 3u_n - 1 \quad , \quad u_1 = 2$$

"AUXILIARY EQUATION"

$$\Rightarrow u_{n+1} - 3u_n = -1$$

$$\Rightarrow \lambda - 3 = 0$$

$$\Rightarrow \lambda = 3$$

GENERAL SOLUTION OF $u_{n+1} = 3u_n$ IS GIVEN BY

$$u_n = A \times 3^n$$

"PARTICULAR INTEGRAL" - TRY $u_n = C$ (CONSTANT)

$$\Rightarrow C - 3C = -1$$

$$\Rightarrow -2C = -1$$

$$\Rightarrow C = \frac{1}{2}$$

GENERAL SOLUTION OF $u_{n+1} = 3u_n - 1$ IS GIVEN BY

$$\Rightarrow u_{n+1} = \frac{1}{2} + A \times 3^n$$

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USING THE CONDITION, $u_1 = 2$, IF $n=1$, $u_n = 2$

$$\Longrightarrow u_1 = \frac{1}{2} + A \times 3^1$$

$$\Longrightarrow 2 = \frac{1}{2} + 3A$$

$$\Longrightarrow 4 = 1 + 6A$$

$$\Longrightarrow A = \frac{1}{2}$$

$$\therefore \quad u_n = \frac{1}{2} + \frac{1}{2}(3^n)$$

SUMMING UP, RECALLING THAT FOR GEOMETRIC PROGRESSIONS THE SUM OF THE FIRST n TERMS IS GIVEN BY "$\frac{a(r^n-1)}{r-1}$"

$$\Longrightarrow S_n = \sum_{r=1}^{n}\left(\frac{1}{2} + \frac{1}{2}(3^r)\right) = \frac{1}{2}\sum_{r=1}^{n} 1 + \frac{1}{2}\sum_{r=1}^{n} 3^r$$

$$\Longrightarrow S_n = \frac{1}{2} \times n + \frac{1}{2}\left(3 + 3^2 + 3^3 + 3^4 + \dots + 3^n\right)$$

$$\Longrightarrow S_n = \frac{n}{2} + \frac{1}{2}\left(\frac{3(3^n-1)}{3-1}\right) = \frac{n}{2} + \frac{1}{4}\left(3^{n+1} - 3\right)$$

$$\Longrightarrow S_n = \frac{n}{2} + \frac{3^{n+1} - 3}{4}$$

$$\Longrightarrow S_n = \frac{3^{n+1} + 2n - 3}{4}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 7

a) STATING THE "FOURIER SERIES THEOREM"

If $f(x)$ is piecewise continuous on $(-L, L)$, $L>0$, then

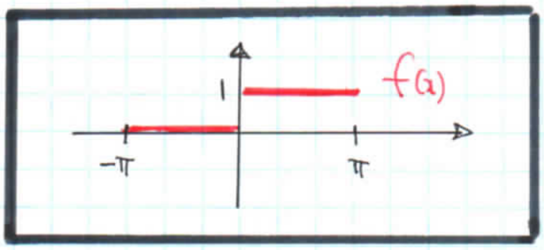
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

WHERE

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx \qquad n = 0,1,2,3,\ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx \qquad n = 1,2,3,4,\ldots$$

b) $f(x) = \frac{1}{2} + \frac{1}{2}\operatorname{sign} x$



NOTE $\operatorname{sign} x = \begin{cases} 1 & x \geqslant 0 \\ -1 & x < 0 \end{cases}$

- $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{0}^{\pi} 1\,dx = \frac{1}{\pi}\times\pi = \underline{1}$

- $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{0}^{\pi} 1\times\cos nx\,dx = \frac{1}{n\pi}\left[\sin nx\right]_0^{\pi}$

$$= \frac{1}{n\pi}\left[\sin n\pi - 0\right] = \underline{0}$$

- $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{0}^{\pi} 1\times\sin nx\,dx = \frac{1}{-\pi n}\left[\cos nx\right]_0^{\pi}$

$$= -\frac{1}{n\pi}\left[\cos n\pi - 1\right] = \frac{1-\cos n\pi}{n\pi} = \frac{1-(-1)^n}{n\pi}$$

IYGB - MATHEMATICAL METHODS I - PAPER C - QUESTION 7

$$= \begin{cases} 2/n\pi & \text{IF } n \text{ IS ODD} \\ 0 & \text{IF } n \text{ IS EVEN} \end{cases} \qquad \therefore b_n = \frac{2}{(2n-1)\pi}$$

$$\therefore f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \left[\frac{2}{(2n-1)\pi} \sin\left[(2n-1)x\right] \right]$$

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left[(2n-1)x\right]}{2n-1}$$

c) START FROM THE FOURIER THEOREM STATEMENT

$$\Rightarrow f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L} \right]$$

MULTIPLY THROUGH BY $\frac{1}{L}f(x)$ & INTEGRATE W.R.T x, BETWEEN $-L$ & L

$$\Rightarrow \frac{1}{L}\int_{-L}^{L} \left[f(x)\right]^2 dx = \frac{a_0}{2L}\int_{-L}^{L} f(x)\,dx + \frac{1}{L}\int_{-L}^{L} f(x) \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right] dx$$

INTERCHANGE INTEGRATION AND SUMMATION

$$\Rightarrow \frac{1}{L}\int_{-L}^{L} \left[f(x)\right]^2 dx = \frac{a_0}{2}\boxed{\frac{1}{L}\int_{-L}^{L} f(x)\,dx} + \sum_{n=1}^{\infty}\left[a_n \boxed{\frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}} + b_n \boxed{\frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx}\right]$$

$$\Rightarrow \frac{1}{L}\int_{-L}^{L} \left[f(x)\right]^2 dx = \frac{a_0}{2} \times a_0 + \sum_{n=1}^{\infty}\left[a_n \times a_n + b_n \times b_n\right]$$

$$\Rightarrow \frac{1}{L}\int_{-L}^{L} \left[f(x)\right]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left[a_n^2 + b_n^2\right]$$

WHICH IS PARSEVAL'S IDENTITY

IYGB - MATHEMATICAL METHODS I - PAPER C - QUESTION 7

d) USING PARSEVAL'S IDENTITY WITH $f(x)=\frac{1}{2}+\frac{1}{2}\text{sign}(x)$ IN THE INTERVAL $(-\pi,\pi)$

$$\Rightarrow \frac{1}{\pi}\int_{-\pi}^{\pi}[f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2+b_n^2)$$

$$\Rightarrow \frac{1}{\pi}\int_{0}^{\pi}1^2\,dx = \frac{1^2}{2} + \sum_{n=1}^{\infty}\left[0^2+\left[\frac{2}{(2n-1)\pi}\right]^2\right]$$

$$\Rightarrow \frac{1}{\pi}\int_{0}^{\pi}1\,dx = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{4}{\pi^2(2n-1)^2}$$

$$\Rightarrow \frac{1}{\pi}\times\pi = \frac{1}{2} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$$\Rightarrow 1 = \frac{1}{2} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$$\Rightarrow \frac{1}{2} = \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

AS REQUIRED

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$$\boxed{\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = \frac{(5x-2)e^{4x}}{x^3}}$$

AUXILIARY EQUATION FOR THE LHS

$$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$$
$$\Rightarrow (\lambda - 4)(\lambda + 1) = 0$$
$$\Rightarrow \lambda = \begin{cases} 4 \\ -1 \end{cases}$$

C.F: $y = Ae^{4x} + Be^{-x}$

FIND THE PARTICULAR INTEGRAL BY VARIATION OF PARAMETERS

WRONSKIAN WITH $e_1 = e^{4x}$ & $e_2 = e^{-x}$

$$W = \begin{vmatrix} e_1 & e_2 \\ e_1' & e_2' \end{vmatrix} = \begin{vmatrix} e^{4x} & e^{-x} \\ 4e^{4x} & -e^{-x} \end{vmatrix} = -e^{3x} - 4e^{3x} = -5e^{3x}$$

THE PARTICULAR INTEGRAL IS GIVEN BY

$$y_p = -e_1 \int \frac{e_2 f}{aW}\,dx + e_2 \int \frac{e_1 f}{aW}\,dx$$

WHERE $f = f(x) = \dfrac{(5x-2)e^{4x}}{x^3}$

$a = a(x) = 1 \leftarrow$ COEFFICIENT OF $\dfrac{d^2y}{dx^2}$

$$\Rightarrow y_p = -e^{4x} \int \frac{\cancel{e^{-x}} \times \frac{(5x-2)\cancel{e^{4x}}}{x^3}}{1 \times (-5\cancel{e^{3x}})}\,dx + e^{-x} \int \frac{e^{4x} \times \frac{(5x-2)e^{4x}}{x^3}}{1 \times (-5e^{3x})}\,dx$$

$$\Rightarrow y_p = -e^{4x} \int \frac{5x-2}{-5x^3}\,dx + e^{-x} \int \frac{(5x-2)e^{5x}}{-5x^3}\,dx$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 8

$$\Rightarrow y_p = \frac{1}{5}e^{4x}\int \frac{5}{x^2} - \frac{2}{x^3}\,dx - \frac{1}{5}e^{-x}\int \frac{(5x-2)e^{5x}}{x^3}\,dx$$

$$\frac{d}{dx}\left(\frac{e^{5x}}{x^2}\right) = \frac{(5x-2)e^{5x}}{x^3}$$

$$\Rightarrow y_p = \frac{1}{5}e^{4x}\left[-\frac{5}{x} + \frac{1}{x^2}\right] - \frac{1}{5}e^{-x}\left[\frac{e^{5x}}{x^2}\right]$$

$$\Rightarrow y_p = \frac{1}{5}e^{4x}\left[\frac{1}{x^2} - \frac{5}{x}\right] - \frac{1}{5}e^{4x}\left[\frac{1}{x^2}\right]$$

$$\Rightarrow y_p = \frac{1}{5}e^{4x}\left[\cancel{\frac{1}{x^2}} - \frac{5}{x} - \cancel{\frac{1}{x^2}}\right]$$

$$\Rightarrow y_p = -\frac{e^{4x}}{x}$$

$$\therefore\ y = Ae^{4x} + Be^{-x} - \frac{e^{4x}}{x}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 9

IT IS GIVEN THAT

$$z = f(u,v) \quad \text{WHERE} \quad u(x,y) = 2xy \quad \& \quad v(x,y) = x^2 - y^2$$

- DIFFERENTIATE USING THE CHAIN RULE

- $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x} = \frac{\partial f}{\partial u}(2y) + \frac{\partial f}{\partial v}(2x)$

- $\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} = \frac{\partial f}{\partial u}(2x) + \frac{\partial f}{\partial v}(-2y)$

- TIDYING UP

$$\left.\begin{array}{l} \frac{\partial f}{\partial x} = 2y\frac{\partial f}{\partial u} + 2x\frac{\partial f}{\partial v} \\ \frac{\partial f}{\partial y} = 2x\frac{\partial f}{\partial u} - 2y\frac{\partial f}{\partial v} \end{array}\right\} \Longrightarrow$$

$$\left.\begin{array}{l} x\frac{\partial f}{\partial x} = 2xy\frac{\partial f}{\partial u} + 2x^2\frac{\partial f}{\partial v} \\ y\frac{\partial f}{\partial y} = 2xy\frac{\partial f}{\partial u} - 2y^2\frac{\partial f}{\partial v} \end{array}\right\} \Longrightarrow$$

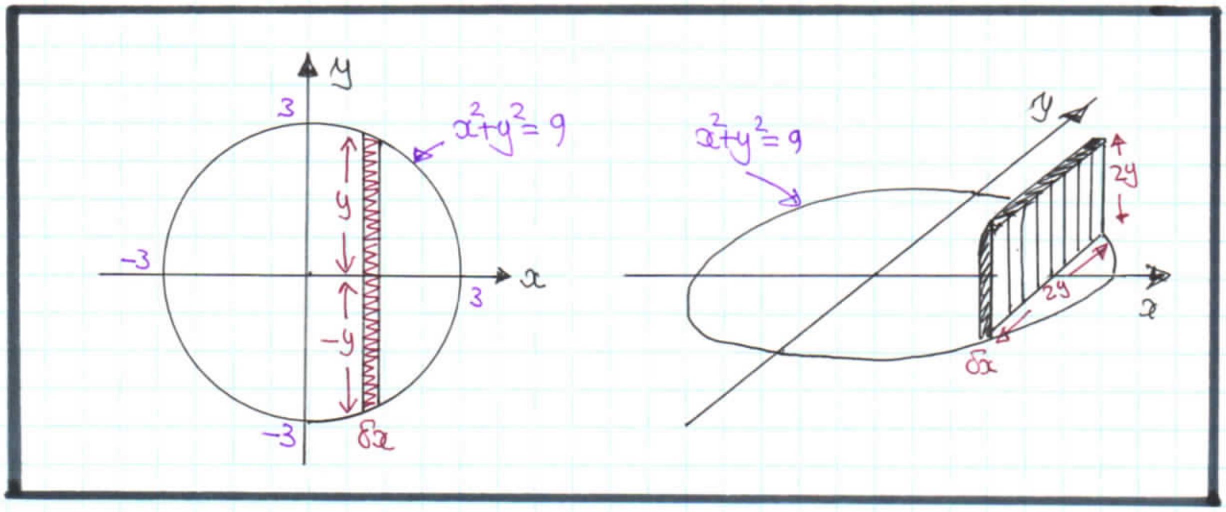
- ADDING THE EQUATIONS

$$\Rightarrow x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 4xy\frac{\partial f}{\partial u} + (2x^2 - 2y^2)\frac{\partial f}{\partial v}$$

$$\Rightarrow x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2\left[2xy\frac{\partial f}{\partial u} + (x^2 - y^2)\frac{\partial f}{\partial v}\right]$$

$$\Rightarrow x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2\left[u\frac{\partial f}{\partial u} + v\frac{\partial f}{\partial v}\right]$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 10



LOOKING AT THE FIGURES ABOVE

- $x^2+y^2=9$
  $y^2=9-x^2$
  $y=\pm\sqrt{9-x^2}$
- THE LENGTH OF THE SQUARE IS $2y=+2\sqrt{9-x^2}$
- THE AREA OF THE SQUARE IS $4y^2=4(9-x^2)$
- THE VOLUME OF THE "INFINITESIMAL SLICE" IS $4(9-x^2)\delta x$

SUMMING ALL "INFINITESIMAL SLICES" OF THICKNESS δx, FROM $x=-3$ TO $x=3$, & TAKE LIMITS

$$\Rightarrow V=\int_{-3}^{3}4(9-x^2)\,dx$$

EVEN INTEGRAND

$$\Rightarrow V=8\int_{0}^{3}9-x^2\,dx$$

$$\Rightarrow V=8\left[9x-\tfrac{1}{3}x^3\right]_0^3$$

$$\Rightarrow V=8\left[(27-9)-0\right]$$

$$\Rightarrow V=144$$

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$$4x^2\frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + 9y = 0 \qquad y(1)=4 \quad y'(1)=10$$

IT IS GIVEN THAT A SOLUTION IS $y_1 = Ax^{\frac{3}{2}}$

TRY A SOLUTION OF THE FORM $y_2 = x^{\frac{3}{2}}V(x)$

DIFFERENTIATE TWICE

$$\frac{dy_2}{dx} = \frac{3}{2}x^{\frac{1}{2}}V(x) + x^{\frac{3}{2}}V'(x)$$

$$\frac{d^2y_2}{dx^2} = \frac{3}{4}x^{-\frac{1}{2}}V(x) + \frac{3}{2}x^{\frac{1}{2}}V'(x) + \frac{3}{2}x^{\frac{1}{2}}V'(x) + x^{\frac{3}{2}}V''(x)$$

$$= \frac{3}{4}x^{-\frac{1}{2}}V(x) + 3x^{\frac{1}{2}}V'(x) + x^{\frac{3}{2}}V''(x)$$

SUB INTO THE O.D.E.

$$\Rightarrow 4x^2\left[x^{\frac{3}{2}}V''(x) + 3x^{\frac{1}{2}}V'(x) + \frac{3}{4}x^{-\frac{1}{2}}V(x)\right] - 8x\left[\frac{3}{2}x^{\frac{1}{2}}V(x) + x^{\frac{3}{2}}V'(x)\right] + 9x^{\frac{3}{2}}V(x) = 0$$

$$\Rightarrow 4x^{\frac{7}{2}}V''(x) + 12x^{\frac{5}{2}}V'(x) + \cancel{3x^{\frac{3}{2}}V(x)} - \cancel{12x^{\frac{3}{2}}}V(x) - 8x^{\frac{5}{2}}V'(x) + \cancel{9x^{\frac{3}{2}}}V(x) = 0$$

$$\Rightarrow 4x^{\frac{7}{2}}V''(x) + 4x^{\frac{5}{2}}V'(x) = 0$$

$$\left(4x^{\frac{5}{2}} \neq 0\right)$$

$$\Rightarrow x\frac{d^2V}{dx^2} + \frac{dV}{dx} = 0$$

$$\Rightarrow x\frac{dP}{dx} + P = 0$$

WHERE $P = \frac{dV}{dx}$

$$\Rightarrow x\frac{dP}{dx} = -P$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 11

$$\Longrightarrow \frac{1}{p}\,dp = -\frac{1}{x}\,dx$$

$$\Longrightarrow \ln|p| = \ln B - \ln|x|$$

$$\Longrightarrow \ln|p| = \ln\left|\frac{B}{x}\right|$$

$$\Longrightarrow p = \frac{B}{x}$$

$$\Longrightarrow \frac{dv}{dx} = \frac{B}{x}$$

$$\Longrightarrow v = B\ln|x| + C$$

HENCE THE GENERAL SOLUTION IS

$$y = Ax^{\frac{3}{2}} + x^{\frac{3}{2}}\left[B\ln x + C\right] \qquad x>0$$

$$y = Ax^{\frac{3}{2}} + Bx^{\frac{3}{2}}\ln x$$

$$y = x^{\frac{3}{2}}(A + B\ln x)$$

APPLY CONDITION $y(1) = 4$

$$4 = 1(A + B\times 0) \Longrightarrow A = 4$$

DIFFERENTIATE AND APPLY $y'(1) = 10$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}(A + B\ln x) + x^{\frac{3}{2}}\left(\frac{B}{x}\right)$$

$$10 = \frac{3}{2}(4) + B$$

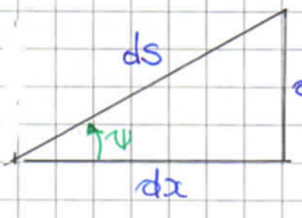
$$B = 4$$

$$\therefore\ y = 4x^{\frac{3}{2}}(1 + \ln x)$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 12

$$\boxed{s = 8\left(\sec^3\psi - 1\right) \quad 0 \leq \psi < \frac{\pi}{2} \qquad x=0,\ y=0,\ s=0,\ \psi=0}$$

FORMING TWO SEPARATE DIFFERENTIAL EQUATIONS BASED ON THE TRIANGLE SHOWN BELOW



$$\frac{ds}{d\psi} = 8\left(3\sec^3\psi \tan\psi\right)$$

$$\rho = 24\sec^3\psi \tan\psi$$

$$\Rightarrow \frac{dx}{ds} = \cos\psi$$

$$\Rightarrow dx = \cos\psi \, ds$$

$$\Rightarrow dx = \cos\psi \frac{ds}{d\psi} \, d\psi$$

$$\Rightarrow dx = \cos\psi \left(24\sec^3\psi \tan\psi\right) d\psi$$

$$\Rightarrow dx = 24\sec^2\psi \tan\psi \, d\psi$$

$$\Rightarrow \int_{x=0}^{x} 1 \, dx = \int_{\psi=0}^{\psi} 24\sec^2\psi \tan\psi \, d\psi$$

$$\Rightarrow \Big[x\Big]_0^x = \Big[12\sec^2\psi\Big]_0^\psi$$

$$\Rightarrow \boxed{x = 12\sec^2\psi - 12} \quad \text{OR} \quad \boxed{x = 12\tan^2\psi}$$

SIMILARLY WE HAVE

$$\Rightarrow \frac{dy}{ds} = \sin\psi$$

$$\Rightarrow dy = \sin\psi \, ds$$

$$\Rightarrow dy = \sin\psi \frac{ds}{d\psi} \, d\psi$$

$$\Rightarrow dy = \sin\psi \left(24\sec^3\psi \tan\psi\right) d\psi$$

$$\Rightarrow dy = 24\sec^2\psi \tan^2\psi \, d\psi$$

IYGB - MATHEMATICAL METHODS I - PAPER C - QUESTION 12

$$\Rightarrow \int_{y=0}^{y} 1 \, dy = \int_{\psi=0}^{\psi} 24\sec^2\psi \tan^2\psi \, d\psi$$

$$\Rightarrow \Big[y\Big]_0^y = \Big[8\tan^3\psi\Big]_0^\psi$$

$$\Rightarrow \boxed{y = 8\tan^3\psi}$$

TREATING ψ AS A PARAMETER ELIMINATE INTO CARTESIAN

$$\Rightarrow \left\{\begin{array}{l} x = 12\tan^2\psi \\ y = 8\tan^3\psi \end{array}\right\}$$

$$\Rightarrow \left\{\begin{array}{l} x^3 = 1728\tan^6\psi \\ y^2 = 64\tan^6\psi \end{array}\right\}$$

$$\Rightarrow \frac{x^3}{y^2} = \frac{1728}{64}$$

$$\Rightarrow \frac{x^3}{y^2} = 27$$

$$\Rightarrow y^2 = \frac{x^3}{27}$$

// AS REQUIRED

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 13

a) WRITE THE O.D.E IN D OPERATOR FORM

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 10e^{-2x}$$

$$\Rightarrow D^2y - Dy - 6y = 10e^{-2x}$$

$$\Rightarrow (D^2 - D - 6)y = 10e^{-2x}$$

$$\Rightarrow (D+2)(D-3)y = 10e^{-2x}$$

HENCE WE OBTAIN

$$\Rightarrow y = Ae^{-2x} + Be^{3x} + \frac{1}{D^2 - D - 6}\left\{10e^{-2x}\right\}$$

-2 MAKES THIS ZERO

$$\Rightarrow y = Ae^{-2x} + Be^{3x} + \frac{1}{D^2 - D - 6}\left\{10 \times e^{-2x}\right\}$$

$f(x)$

USING THE THEOREM

$$\boxed{[f(D)]\left\{e^{kx}V(x)\right\} = e^{kx}[f(D+k)]\left\{V(x)\right\}}$$

$$\Rightarrow y = Ae^{-2x} + Be^{3x} + \frac{e^{-2x}}{(D-2)^2 - (D-2) - 6}\left\{10\right\}$$

$$\Rightarrow y = Ae^{-2x} + Be^{3x} + \frac{e^{-2x}}{D^2 - 5D}\left\{10\right\}$$

$$\Rightarrow y = Ae^{-2x} + Be^{3x} + \frac{1}{D} \times \frac{e^{-2x}}{D-5}\left\{10e^{0x}\right\}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 13

$$\Rightarrow y = Ae^{-2x} + Be^{3x} + \frac{e^{-2x}}{D} \times \left[\frac{10}{0-5} e^{0x}\right]$$

$$\Rightarrow y = Ae^{-2x} + Be^{3x} + \frac{e^{-2x}}{D}\left[-2\right]$$

$$\Rightarrow y = Ae^{-2x} + Be^{3x} + e^{-2x}(-2x)$$

$$\Rightarrow y = Ae^{-2x} + Be^{3x} - 2xe^{-2x}$$

b) • AFTER MANIPULATING THE O.D.E INTO D OPERATOR FORM

$$(D+2)(D-3)y = 10e^{-2x}$$

• LET $Y = (D+2)y$ SO THE O.D.E BECOMES

$$\Rightarrow (D-3)Y = 10e^{-2x}$$

$$\Rightarrow Y = Ae^{3x} + \frac{1}{D-3}\left\{10e^{-2x}\right\}$$

$$\Rightarrow Y = Ae^{3x} + \frac{1}{-2-3} \times \left(10e^{-2x}\right)$$

$$\Rightarrow Y = Ae^{3x} - 2e^{-2x}$$

$$\Rightarrow (D+2)y = Ae^{3x} - 2e^{-2x}$$

FROM C.F.

$$\Rightarrow y = \frac{1}{D+2}\left\{Ae^{3x}\right\} - \frac{1}{D+2}\left\{2e^{-2x}\right\} + Be^{-2x}$$

$$\Rightarrow y = \frac{1}{-3+2}\left(Ae^{3x}\right) - \frac{e^{-2x}}{(D-2)+2}\left\{2\right\} + Be^{-2x}$$

$$\Rightarrow y = Ae^{3x} - e^{-2x}\frac{1}{D}\left\{2\right\} + Be^{-2x}$$

$$\Rightarrow y = Ae^{3x} + Be^{-2x} - 2xe^{-2x}$$

AS BEFORE

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 14

$$\boxed{x^2+y^2+z^2+xy+xz+yz=2}$$

START BY WRITING THE QUADRIC IN THE USUAL NOTATION

$$(x\ \ y\ \ z)\begin{bmatrix} 1\,{\scriptstyle x^2} & \frac{1}{2}\,{\scriptstyle xy} & \frac{1}{2}\,{\scriptstyle xz} \\ \frac{1}{2}\,{\scriptstyle yx} & 1\,{\scriptstyle y^2} & \frac{1}{2}\,{\scriptstyle yz} \\ \frac{1}{2}\,{\scriptstyle zx} & \frac{1}{2}\,{\scriptstyle zy} & 1\,{\scriptstyle z^2} \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2$$

SOLVE THE CHARACTERISTIC EQUATION OF THE SYMMETRIC MATRIX

$$\begin{vmatrix} 1-\lambda & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1-\lambda & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1-\lambda \end{vmatrix} = 0 \implies \cancel{\left(\frac{1}{2}\right)^3}\begin{vmatrix} 2-2\lambda & 1 & 1 \\ 1 & 2-2\lambda & 1 \\ 1 & 1 & 2-2\lambda \end{vmatrix} = 0$$

$$\implies C_{32}(-1)\begin{vmatrix} 2-2\lambda & 0 & 1 \\ 1 & 1-2\lambda & 1 \\ 1 & 2\lambda-1 & 2-2\lambda \end{vmatrix} = 0$$

$$\implies (2\lambda-1)\begin{vmatrix} 2-2\lambda & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 2-2\lambda \end{vmatrix} = 0$$

$$\implies r_{23}(1)\quad (2\lambda-1)\begin{vmatrix} 2-2\lambda & 0 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & 3-2\lambda \end{vmatrix} = 0$$

EXPAND BY MIDDLE COLUMN $\implies (2\lambda-1)\times\left\{-\begin{vmatrix} 2-2\lambda & 1 \\ 2 & 3-2\lambda \end{vmatrix}\right\} = 0$

$$\implies (2\lambda-1)\big(2-(2-2\lambda)(3-2\lambda)\big) = 0$$

$$\implies (2\lambda-1)\big(2-(2\lambda-2)(2\lambda-3)\big) = 0$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 14

$$\Rightarrow (2\lambda-1)(2-4\lambda^2+10\lambda-6)=0$$
$$\Rightarrow (2\lambda-1)(-4\lambda^2+10\lambda-4)=0$$
$$\Rightarrow (2\lambda-1)(2\lambda^2-5\lambda+2)=0 \quad \div(-2)$$
$$\Rightarrow (2\lambda-1)(2\lambda-1)(\lambda-2)=0$$
$$\Rightarrow \lambda = \begin{cases} \frac{1}{2} \\ \frac{1}{2} \text{ (REPEATED)} \\ 2 \end{cases}$$

FIND EIGENVECTORS FOR EACH EIGENVALUE

- IF $\lambda = 2$

$$\left.\begin{array}{l} x+\frac{1}{2}y+\frac{1}{2}z=2x \\ \frac{1}{2}x+y+\frac{1}{2}z=2y \\ \frac{1}{2}x+\frac{1}{2}y+z=2z \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x+\frac{1}{2}y+\frac{1}{2}z=0 \\ \frac{1}{2}x-y+\frac{1}{2}z=0 \\ \frac{1}{2}x+\frac{1}{2}y-z=0 \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{ll} (\text{I}) & -2x+y+z=0 \\ (\text{II}) & x-2y+z=0 \\ (\text{III}) & x+y-2z=0 \end{array}\right\} \Rightarrow z = 2x-y \quad \text{— (I)}$$

SUB INTO THE OTHER TWO EQUATIONS

$$\Rightarrow \begin{cases} x-2y+2x-y=0 \\ x+y-4x+2y=0 \end{cases}$$

$$\Rightarrow y = x \quad \& \quad z = x$$

$$\Rightarrow \alpha\begin{pmatrix}1\\1\\1\end{pmatrix} \text{ NORMALIZED TO } \boxed{\begin{pmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{pmatrix}}$$

- IF $\lambda = \frac{1}{2}$

$$\left.\begin{array}{l} x+\frac{1}{2}y+\frac{1}{2}z=\frac{1}{2}x \\ \frac{1}{2}x+y+\frac{1}{2}z=\frac{1}{2}y \\ \frac{1}{2}x+\frac{1}{2}y+z=\frac{1}{2}z \end{array}\right\} \text{ ALL REDUCE TO } \quad \begin{array}{l} \frac{1}{2}x+\frac{1}{2}y+\frac{1}{2}z=0 \\ x+y+z=0 \end{array}$$

PICK ANY TWO INDEPENDENT EIGENVECTORS THAT SATISFY ABOVE RELATIONSHIP AND ARE PERPENDICULAR TO ONE ANOTHER (ALL 3)

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 14

SAY $\beta\begin{pmatrix}1\\-1\\0\end{pmatrix}$ & $\gamma\begin{pmatrix}-1\\1\\0\end{pmatrix}$ NOTE $\alpha\begin{pmatrix}1\\1\\1\end{pmatrix}$

NORMALIZED WE HAVE

$$\begin{bmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{bmatrix}\begin{bmatrix}1/\sqrt{2}\\-1/\sqrt{2}\\0\end{bmatrix}\begin{bmatrix}-1/\sqrt{2}\\1/\sqrt{2}\\0\end{bmatrix}$$

I.E
$$P=\begin{bmatrix}\frac{1}{\sqrt{3}}&\frac{1}{\sqrt{2}}&-\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{3}}&-\frac{1}{\sqrt{2}}&\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{3}}&0&0\end{bmatrix} \quad \& \quad D=\begin{bmatrix}2&0&0\\0&\frac{1}{2}&0\\0&0&\frac{1}{2}\end{bmatrix}$$

$$\begin{matrix}\uparrow&\uparrow&\uparrow\\\lambda=2&\lambda=\frac{1}{2}&\lambda=\frac{1}{2}\end{matrix}$$

HENCE THE DIAGONILIZED FORM OF THE QUADRIC IS

$$2X^2+\frac{1}{2}Y^2+\frac{1}{2}Z^2=2$$

$$X^2+\frac{1}{4}Y^2+\frac{1}{4}Z^2=1$$

$$\frac{X^2}{1}+\frac{Y^2}{4}+\frac{Z^2}{4}=1$$

$\therefore$ AN ELLIPSOID WITH ITS LINES OF SYMMETRY IN THE ORIGINAL COORDINATE SYSTEM IN THE DIRECTION OF THE EIGENVECTORS

$$\begin{pmatrix}1\\1\\1\end{pmatrix},\begin{pmatrix}1\\-1\\0\end{pmatrix},\begin{pmatrix}-1\\1\\0\end{pmatrix}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 15

FIRST METHOD (BY SURD CONJUGATION)

$$\lim_{n\to\infty}\left[\sqrt{n^2+3n}-n\right] = \lim_{n\to\infty}\left[\frac{\left[\sqrt{n^2+3n}-n\right]\left[\sqrt{n^2+3n}+n\right]}{\sqrt{n^2+3n}+n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{(\cancel{n^2}+3n)-\cancel{n^2}}{\sqrt{n^2+3n}+n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{3n}{|n|\sqrt{1+\frac{3}{n}}+n}\right]$$

AS n WILL BE POSITIVE $|n| = n$

$$= \lim_{n\to\infty}\left[\frac{3\cancel{n}}{\cancel{n}\sqrt{1+\frac{3}{n}}+\cancel{n}}\right]$$

$$= \lim_{n\to\infty}\left[\frac{3}{\sqrt{1+\frac{3}{n}}+1}\right]$$

$$= \frac{3}{1+1} = \frac{3}{2}$$

SECOND METHOD (BY BINOMIAL EXPANSION)

$$\lim_{n\to\infty}\left[\sqrt{n^2+3n}-n\right] = \lim_{n\to\infty}\left[|n|\left(1+\frac{3}{n}\right)^{\frac{1}{2}}-n\right]$$

AND HERE AS $n\to+\infty$, $|n| = n$

$$= \lim_{n\to\infty}\left[n\left(1+\frac{3}{n}\right)^{\frac{1}{2}}-n\right]$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 15

NOW EXPANDING BINOMIALLY WE HAVE

$$= \lim_{n \to \infty} \left[n \left[1 + \frac{\frac{1}{2}}{1}\left(\frac{3}{n}\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1 \times 2}\left(\frac{3}{n}\right)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1 \times 2 \times 3}\left(\frac{3}{n}\right)^3 + \dots \right] - n \right]$$

$$= \lim_{n \to \infty} \left[n \left[1 + \frac{3}{2n} - \frac{9}{8n^2} + \frac{27}{16n^3} + \dots \right] - n \right]$$

$$= \lim_{n \to \infty} \left[\cancel{n} + \frac{3}{2} - \frac{9}{8n} + \frac{27}{16n^2} + \dots \cancel{-n} \right]$$

$$= \lim_{n \to \infty} \left[\frac{3}{2} + O\left(\frac{1}{n}\right) \right]$$

$$= \frac{3}{2}$$

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 16

- START BY SKETCHING THE CURVE WITH EQUATION $y = \dfrac{2-x}{x^2-9x+18}$

$$y = \frac{2-x}{(x-3)(x-6)} = \frac{\frac{1}{3}}{x-3} - \frac{\frac{4}{3}}{x-6} \quad \text{(BY INSPECTION)}$$

- FROM THE ABOVE FORMS WE HAVE

$x=0,\ y=\frac{1}{9}$ $\quad (0,\frac{1}{9})$

$y=0,\ x=2$ $\quad (2,0)$

VERTICAL ASYMPTOTES $x=3,\ x=6$

HORIZONTAL ASYMPTOTE $y=0$ (AS $x \to \pm\infty$)

- LOOK FOR STATIONARY POINTS

$$\frac{dy}{dx} = -\frac{1}{3}(x-3)^{-2} + \frac{4}{3}(x-6)^{-2} = \frac{1}{3}\left[\frac{4}{(x-6)^2} - \frac{1}{(x-3)^2}\right]$$

$$= \frac{1}{3}\left[\frac{4(x-3)^2-(x-6)^2}{(x-6)^2(x-3)^2}\right]$$

$$= \frac{1}{3}\left[\frac{[2(x-3)-(x-6)][2(x-3)+(x-6)]}{(x-6)^2(x-3)^2}\right]$$

$$= \frac{1}{3}\left[\frac{x(3x-12)}{(x-6)^2(x-3)^2}\right] = \frac{x(x-4)}{(x-6)^2(x-3)^2}$$

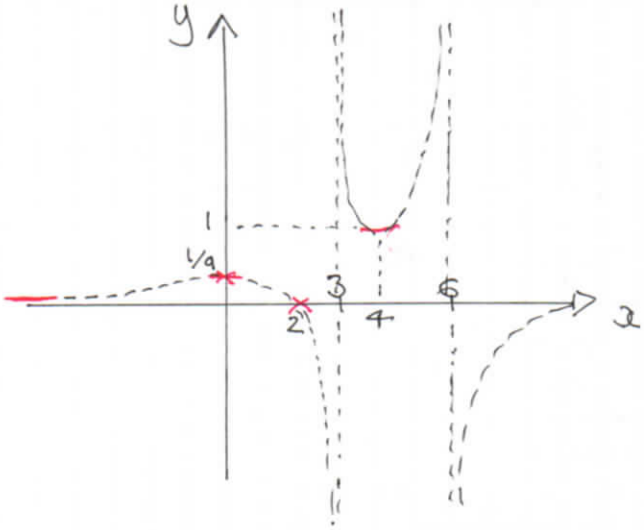
SOLVING FOR ZERO YIELDS

$$x = \begin{cases} 0 \\ 4 \end{cases} \qquad y = \begin{cases} \frac{1}{9} \\ \frac{2-4}{(4-3)(4-6)} = \frac{-2}{-2} = 1 \end{cases}$$

$\therefore\ (0,\frac{1}{9})$ & $(4,1)$ ARE STATIONARY

IYGB - MATHEMATICAL METHODS 1 - PAPER C - QUESTION 16

- PUT THE INFORMATION SO FAR ONTO A PRELIMINARY GRAPH



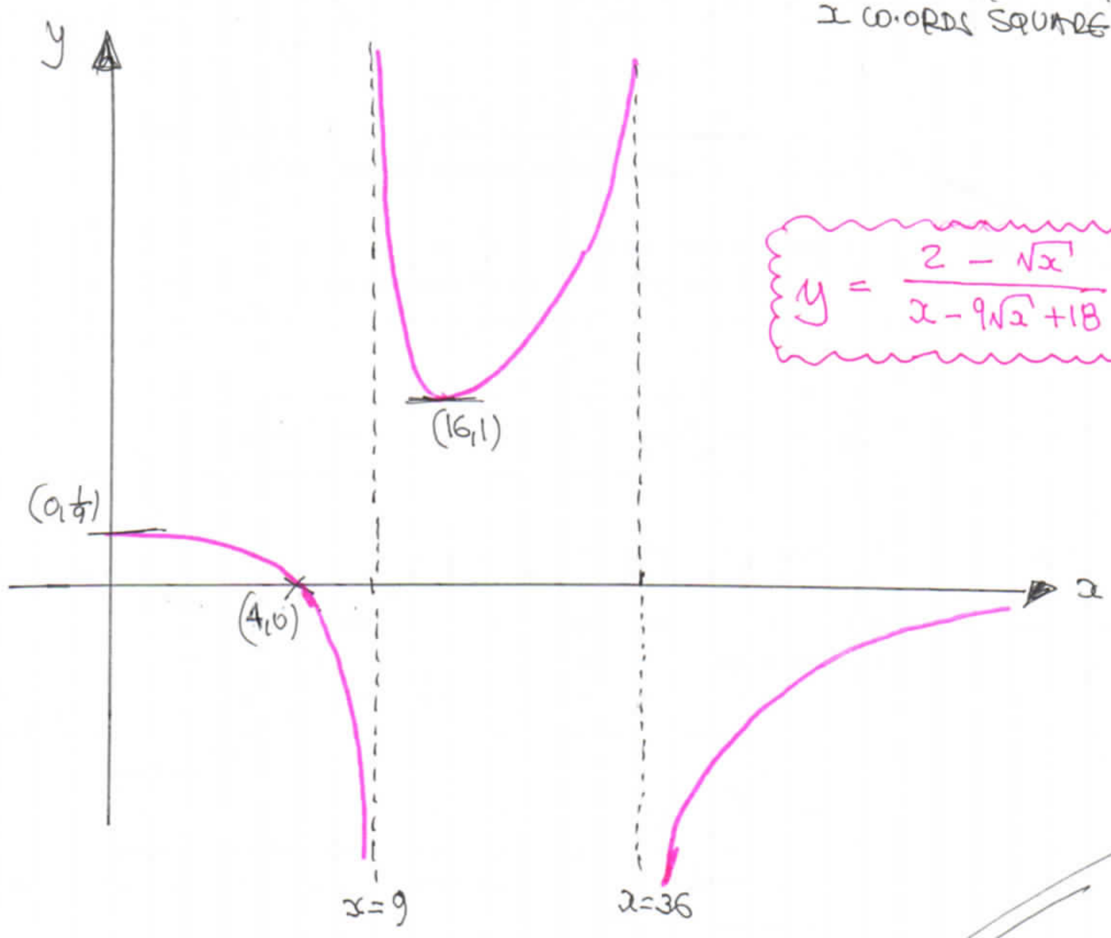
AS THERE IS ONLY ONE x INTERCEPT AND NO SQUARED BRACKETS IN THE DENOMINATOR, (SO THE CURVE MUST "ALTERNATE" AS IT APPEARS AND REAPPEARS ON THE VERTICAL ASYMPTOTES), THE CURVE CAN BE SKETCHED

- THUS WE CAN NOW SKETCH ITS TRANSFORMATION $x \mapsto \sqrt{x}$

CURVE ONLY EXISTS FOR $x \geq 0$

y COORDS ARE UNCHANGED

x CO-ORDS "SQUARE"



$$y = \frac{2-\sqrt{x}}{x-9\sqrt{x}+18}$$

IYGB - MATHEMATICAL METHODS I - PAPER C - QUESTION 17

AS ALL THE TERMS ARE POSITIVE WE MAY IGNORE MODULI IN THE RATIO TEST

$$\frac{u_{n+1}}{u_n} = \frac{\frac{5^{n+1}+1}{(n+1)^{n+1}+8}}{\frac{5^n+1}{n^n+8}} = \frac{5^{n+1}+1}{(n+1)^{n+1}+8} \times \frac{n^n+8}{5^n+1}$$

$$< \frac{5^{n+1}+5}{(n+1)^{n+1}+8} \times \frac{n^n+8}{5^n+1}$$

$$= \frac{5\cancel{(5^n+1)}}{(n+1)^{n+1}+8} \times \frac{n^n+8}{\cancel{5^n+1}}$$

$$= \frac{5(n^n+8)}{(n+1)^{n+1}+8}$$

$$< \frac{5 \times n^n}{(n+1)^{n+1}+8}$$

$$< \frac{A \times n^n}{(n+1)^{n+1}} \quad \text{(FOR SUFFICIENTLY LARGE } A\text{)}$$

$$= A \frac{n^n}{(n+1)^n(n+1)}$$

$$= A\left(\frac{n}{n+1}\right)^n \times \frac{1}{n+1}$$

$$= \frac{A}{n+1} \times \left(\frac{n+1}{n}\right)^{-n} = \frac{A}{n+1} \times \left(1+\frac{1}{n}\right)^{-n}$$

$$= \frac{A}{n+1} \times e^{-1} \longrightarrow 0 \quad \text{AS } n \to \infty$$

$$\therefore \sum_{n=1}^{\infty}\left[\frac{5^n+1}{n^n+8}\right] \text{ CONVERGES}$$