IYGB - MATHEMATICAL METHODS 2 - PAPER B - QUESTION 1

STANDARD EVALUATION AFTER WRITING THE LIMITS EXPLICITLY

$$\int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=\cos\theta} r\sin\theta \ dr \, d\theta$$

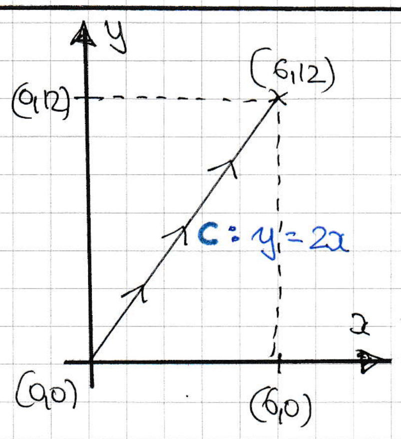
$$= \int_{\theta=0}^{\pi} \left[\frac{1}{2}r^2 \sin\theta \right]_{r=0}^{r=\cos\theta} d\theta = \int_{\theta=0}^{\pi} \frac{1}{2}\cos^2\theta \sin\theta - 0 \ d\theta$$

$$= \left[-\frac{1}{6}\cos^3\theta \right]_0^{\pi} = \frac{1}{6}\left[\cos^3\theta \right]_{\pi}^{0} = \frac{1}{6}\left[1 - (-1) \right]$$

$$= \frac{1}{6} \times 2 = \frac{1}{3}$$

IYGB - MATHEMATICAL METHODS 2 - PAPER B - QUESTION 2

START WITH A DIAGRAM - OBTAIN AN EXPRESSION FOR THE ARCLENGTH ELEMENT IN TERMS OF dx OR dy



- $y = 2x$

$\frac{dy}{dx} = 2$

- $ds^2 = dx^2 + dy^2$

$\frac{ds^2}{dx^2} = 1 + \left(\frac{dy}{dx}\right)^2$

$\frac{ds}{dx} = +\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$\frac{ds}{dx} = \sqrt{1+2^2}$

$\frac{ds}{dx} = \sqrt{5}$

$\boxed{ds = \sqrt{5}\, dx}$

$$\int_C (6x^2 - 2xy)\, ds$$

$$= \int_{x=0}^{6} \left[6x^2 - 2x(2x)\right](\sqrt{5}\, dx)$$

$$= \int_0^6 (6x^2 - 4x^2)\sqrt{5}\, dx$$

$$= \sqrt{5}\int_0^6 2x^2\, dx$$

$$= \sqrt{5}\left[\frac{2}{3}x^3\right]_0^6$$

$$= \sqrt{5}\left[\frac{2}{3}\times 216\right]$$

$$= 144\sqrt{5}$$

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START THE PROOF BY WRITING THE DOT & CROSS PRODUCT IN INDEX NOTATION

$$\underline{\nabla} \cdot \left[\underline{\nabla} f \wedge \nabla g \right] = \frac{\partial}{\partial x_k}\left[\varepsilon_{ijk} \frac{\partial f}{\partial x_i} \frac{\partial g}{\partial x_j} \right]$$

APPLY THE PRODUCT RULE

$$\dots = \varepsilon_{ijk} \frac{\partial^2 f}{\partial x_k \partial x_i} \frac{\partial g}{\partial x_j} + \varepsilon_{ijk} \frac{\partial f}{\partial x_i} \frac{\partial^2 g}{\partial x_k \partial x_j}$$

REWRITE AS FOLLOWS

$$\dots = \frac{\partial g}{\partial x_j}\left[\varepsilon_{ijk} \frac{\partial}{\partial x_k}\left(\frac{\partial f}{\partial x_i} \right) \right] + \frac{\partial f}{\partial x_i}\left[\varepsilon_{ijk} \frac{\partial}{\partial x_k}\left(\frac{\partial g}{\partial x_j} \right) \right]$$

NOW $\varepsilon_{ijk} = \varepsilon_{kij}$ AND $\varepsilon_{ijk} = -\varepsilon_{kji}$

$$\dots = \frac{\partial g}{\partial x_j}\left[\varepsilon_{kij} \frac{\partial}{\partial x_k}\left(\frac{\partial f}{\partial x_i} \right) \right] + \frac{\partial f}{\partial x_i}\left[-\varepsilon_{kji} \frac{\partial}{\partial x_k} \frac{\partial g}{\partial x_j} \right]$$

$$= \nabla g \cdot \left[\underline{\nabla} \wedge \underline{\nabla} f \right] + \underline{\nabla} f \cdot \left[-\underline{\nabla} \wedge \underline{\nabla} g \right]$$

$$= 0$$

SINCE $\underline{\nabla} \wedge \underline{\nabla} u = \underline{0}$

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MANIPULATE AS FOLLOWS

$$\nabla f(r) = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right] = \left[\frac{\partial f}{\partial r}\frac{\partial r}{\partial x}, \frac{\partial f}{\partial r}\frac{\partial r}{\partial y}, \frac{\partial f}{\partial r}\frac{\partial r}{\partial z}\right]$$

$$= \frac{\partial f}{\partial r}\left[\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial r}{\partial z}\right]$$

NOW WE HAVE

$\Rightarrow \underline{r} = (x, y, z)$

$\Rightarrow r = |\underline{r}| = (x^2+y^2+z^2)^{\frac{1}{2}}$

$\Rightarrow \frac{\partial r}{\partial x} = \frac{1}{2}(x^2+y^2+z^2)^{-\frac{1}{2}}(2x) = \frac{x}{(x^2+y^2+z^2)^{\frac{1}{2}}} = \frac{x}{r}$

AND SIMILARLY $\frac{\partial r}{\partial y}$ & $\frac{\partial r}{\partial z}$ ARE THERE IS CYCLIC SYMMETRY

RETURNING TO THE MAIN LINE WE OBTAIN

$$\dots = \frac{\partial f}{\partial r}\left[\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right] = f'(r) \times \frac{1}{r}(x, y, z) = \frac{\underline{r}}{r} f'(r)$$

AS REQUIRED

NOTE THE INITIAL MANIPULATION CAN BE THOUGHT OF A STANDARD CHAIN RULE

$$\nabla f(r) = f'(r)\,\nabla r = f'(r)\left[\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial r}{\partial z}\right] = \dots \text{ AS ABOVE}$$

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ALTHOUGH THIS IS NOT A FLUX INTEGRAL, IT CAN BE MANIPULATED AS FOLLOWS, SINCE THE SURFACE IS CLOSED AND THE DIVERGENCE THEOREM CAN BE USED

$$\oiint_S x^2+y+z \, dS = \oiint_S (x,1,1)\cdot(x,y,z)\, dS$$

NOW WE HAVE SINCE THE SURFACE IS A SPHERE

$$S: \ x^2+y^2+z^2=1$$

$$f(x,y,z) = x^2+y^2+z^2-1$$

$$\nabla f = (2x,2y,2z)$$

SCALED

$$\underline{n} = (x,y,z)$$

$$|\underline{n}| = \sqrt{x^2+y^2+z^2} = 1$$

$$\therefore \ \boxed{\hat{\underline{n}} = (x,y,z)}$$

RETURNING TO THE INTEGRAL, WE NOW HAVE

$$\ldots = \oiint_S (x,1,1)\cdot\hat{\underline{n}}\, dS = \oiint_S \underline{F}\cdot\hat{\underline{n}}\, dS$$

BY THE DIVERGENCE THEOREM

$$= \iiint_V \nabla\cdot\underline{F}\, dV = \iiint_V \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\cdot(x,1,1)\, dV$$

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$$= \iiint_V 1+0+0 \, dV = \iiint_V 1 \, dV$$

= VOLUME OF THE SPHERE OF RADIUS 1

$$= \frac{4}{3}\pi \times 1^3$$

$$= \frac{4}{3}\pi$$

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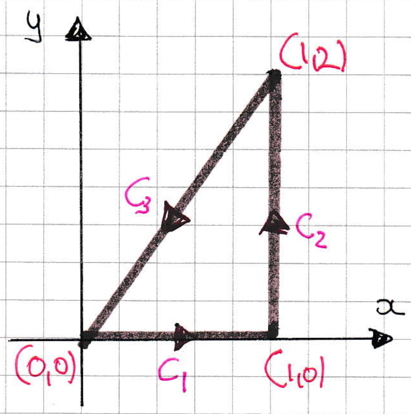
STARTING WITH THE LINE INTEGRAL



C_1: $y=0$, $dy=0$, x RUNS FROM 0 TO 1

C_2: $x=1$, $dx=0$, y RUNS FROM 0 TO 2

C_3: $y=2x$, $dy=2dx$, x RUNS FROM 1 TO 0

HENCE WE NOW HAVE

$$\oint_C (3x+4y)\,dx + (5x-2y)\,dy$$

$$= \underbrace{\int_{x=0}^{x=1} 3x\,dx}_{C_1} + \underbrace{\int_{y=0}^{y=2} 5-2y\,dy}_{C_2} + \underbrace{\int_{x=1}^{x=0} [3x+4(2x)]\,dx + [5x-2(2x)](2\,dx)}_{C_3}$$

$$= \int_0^1 3x\,dx + \int_0^2 5-2y\,dy + \int_1^0 13x\,dx$$

$$= \int_0^1 3x\,dx - \int_0^1 13x\,dx + \int_0^2 5-2y\,dy$$

$$= \int_0^2 5-2y\,dy - \int_0^1 10x\,dx$$

$$= \left[5y-y^2\right]_0^2 - \left[5x^2\right]_0^1$$

$$= \left[(10-4)-0\right] - \left[5-0\right]$$

$$= 6-5$$

$$= 1$$

NEXT GREEN'S THEOREM STATES

$$\boxed{\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy}$$

$$\Longrightarrow \iint_{\substack{\text{AREA OF}\\\text{TRIANGLE}}} \left[\frac{\partial}{\partial x}(5x-2y) - \frac{\partial}{\partial y}(3x+4y)\right] dx\,dy$$

$$= \iint_{\substack{\text{AREA OF}\\\text{TRIANGLE}}} (5-4)\,dy\,dx$$

$$= \iint_{\substack{\text{AREA OF}\\\text{TRIANGLE}}} 1\,dx\,dy$$

$$= \text{AREA OF THE TRIANGLE}$$

$$= \frac{1}{2}\times 1\times 2$$

$$= 1$$

AND THE THEOREM IS VERIFIED

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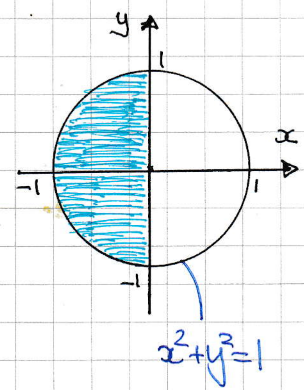
LOOKING AT THE REGION OF INTEGRATION AND THE STRUCTURE OF THE INTEGRAND IT IS EVIDENT THAT PLANE POLAR COORDINATES ARE NEEDED

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{0} \frac{\sqrt{16x^2+16y^2}}{x^2+y^2+1}\,dx\,dy$$

$x = -\sqrt{1-y^2}$

$x^2 = 1-y^2$

$x^2+y^2=1, \ x \leq 0$



$$= \int_{\theta=\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_{r=0}^{1} \frac{\sqrt{16r^2}}{r^2+1}\,(r\,dr\,d\theta)$$

$$= \int_{\theta=\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_{r=0}^{1} \frac{4r^2}{r^2+1}\,dr\,d\theta$$

CARRY OUT THE INTEGRATION WITH RESPECT TO θ FIRST

$$= \left(\frac{3\pi}{2}-\frac{\pi}{2}\right)\int_0^1 \frac{4r^2}{r^2+1}\,dr$$

$$= \pi\int_0^1 \frac{4r^2}{r^2+1}\,dr$$

MANIPULATE THE INTEGRAND AS FOLLOWS

$$= \pi\int_0^1 \frac{4(r^2+1)-4}{r^2+1}\,dr$$

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$$= \pi \int_0^1 4 - \frac{4}{r^2+1} \, dr$$

$$= \pi \Big[4r - 4\arctan r \Big]_0^1$$

$$= \pi \Big[(4 - 4\arctan 1) - (0) \Big]$$

$$= \pi \left(4 - 4 \times \frac{\pi}{4}\right)$$

$$= \pi(4 - \pi)$$

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IF THE "BUBBLE DENSITY" IS GIVEN BY

$\rho(z) = kz$, THEN

$$\text{TOTAL BUBBLES} = \int_V \rho(z)\, dV$$

$$= \int_{\text{HEMISPHERE}} kz \; dV$$

WORKING IN SPHERICAL POLARS

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\frac{\pi}{2}}\int_{r=0}^{a} k\,\underbrace{(r\cos\theta)}_{z}\,\underbrace{(r^2\sin\theta\, dr\, d\theta\, d\phi)}_{dV}$$

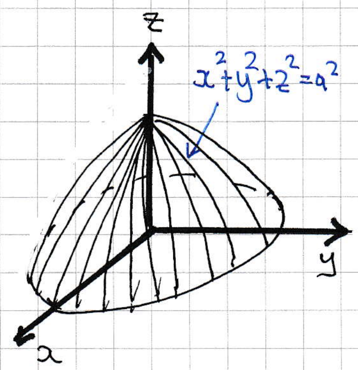
$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\frac{\pi}{2}}\int_{r=0}^{a} k r^3 \cos\theta\sin\theta \; dr\, d\theta\, d\phi$$

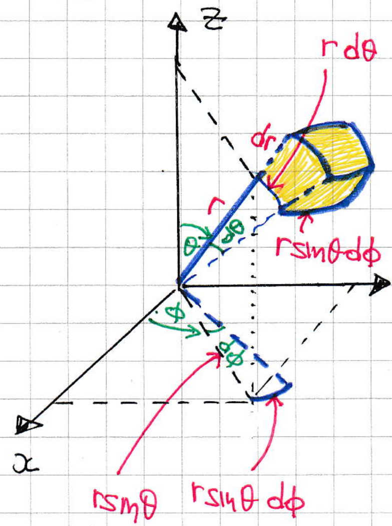
$$= \left[\int_{\phi=0}^{2\pi} k \; d\phi\right]\left[\int_{\theta=0}^{\frac{\pi}{2}} \cos\theta\sin\theta\; d\theta\right]\left[\int_{r=0}^{a} r^3\, dr\right]$$

$$= 2\pi k \times \left[\tfrac{1}{2}\sin^2\theta\right]_0^{\frac{\pi}{2}} \times \left[\tfrac{1}{4}r^4\right]_0^{a}$$

$$= 2\pi k \times \tfrac{1}{2} \times \tfrac{1}{4}a^4$$

$$= \tfrac{1}{4}\pi k a^4$$





$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

$x^2+y^2+z^2=a^2$

FOR THE HEMISPHERE

$0 \le r \le a$
$0 \le \theta \le \frac{\pi}{2}$
$0 \le \phi \le 2\pi$

$dV = r^2\sin\theta\, dr\, d\theta\, d\phi$

IYGB - MATHEMATICAL METHODS 2 - PAPER B - QUESTION 9

$$r(\theta,\phi) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \qquad \begin{matrix} 0 \leqslant \theta \leqslant \frac{\pi}{2} \\ 0 \leqslant \phi \leqslant \frac{\pi}{2} \end{matrix}$$

$$\iint_S z\hat{\underline{k}} \cdot d\underline{S} = \iint z\hat{\underline{k}} \cdot \hat{\underline{n}} \, dS$$

FIND THE UNIT NORMAL TO THE PARAMETRIZED SURFACE & SWITCH THE INTEGRAND INTO PARAMETRIC

$$\frac{\partial r}{\partial \theta} = \begin{bmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\theta \end{bmatrix} \quad \& \quad \frac{\partial r}{\partial \phi} = \begin{bmatrix} -\sin\theta\sin\phi \\ \sin\theta\cos\phi \\ 0 \end{bmatrix}$$

$$\therefore \quad \underline{n} = \left| \frac{\partial r}{\partial \theta} \wedge \frac{\partial r}{\partial \phi} \right| = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$$

$$= \left[\sin^2\theta\cos\phi, \sin^2\theta\sin\phi, \cos\theta\sin\theta\cos^2\phi + \cos\theta\sin\theta\sin^2\phi\right]$$

$$= \left[\sin^2\theta\cos\phi, \sin^2\theta\sin\phi, \cos\theta\sin\theta(\cos^2\phi + \sin^2\phi)\right]$$

$$= \left[\sin^2\theta\cos\phi, \sin^2\theta\sin\phi, \cos\theta\sin\theta\right]$$

IYGB - MATHEMATICAL METHODS 2 - PAPER B - QUESTION 9

RETURNING TO THE INTEGRAL

$$\ldots \iint_S z\underline{\hat{k}} \cdot \underline{\hat{n}}\, dS = \int_{\phi=0}^{\frac{\pi}{2}}\int_{\theta=0}^{\frac{\pi}{2}} \underbrace{\cos\theta}_{z}\, \underbrace{(0,0,1)}_{\underline{\hat{k}}} \cdot \underline{\hat{n}}\, dS$$

$$= \int_{\phi=0}^{\frac{\pi}{2}}\int_{\theta=0}^{\frac{\pi}{2}} (0,0,\cos\theta) \cdot \underbrace{\frac{\underline{n}}{|\underline{n}|}}_{\underline{\hat{n}}} \underbrace{\left|\frac{\partial \underline{r}}{\partial \theta} \wedge \frac{\partial \underline{r}}{\partial \phi}\right| d\phi\, d\theta}_{dS \text{ IN PARAMETRIC}}$$

$$= \int_{\phi=0}^{\frac{\pi}{2}}\int_{\theta=0}^{\frac{\pi}{2}} (0,0,\cos\theta) \cdot \frac{\frac{\partial \underline{r}}{\partial \theta} \wedge \frac{\partial \underline{r}}{\partial \phi}}{\cancel{\left|\frac{\partial \underline{r}}{\partial \theta} \wedge \frac{\partial \underline{r}}{\partial \phi}\right|}} \cancel{\left|\frac{\partial \underline{r}}{\partial \theta} \wedge \frac{\partial \underline{r}}{\partial \phi}\right|}\, d\phi\, d\theta$$

$$= \int_{\phi=0}^{\frac{\pi}{2}}\int_{\theta=0}^{\frac{\pi}{2}} (0,0,\cos\theta) \cdot (\sin^2\theta\cos\phi, \sin^2\theta\sin\phi, \cos\theta\sin\theta)\, d\theta\, d\phi$$

$$= \int_{\phi=0}^{\frac{\pi}{2}}\int_{\theta=0}^{\frac{\pi}{2}} \cos^2\theta\sin\theta\, d\theta\, d\phi$$

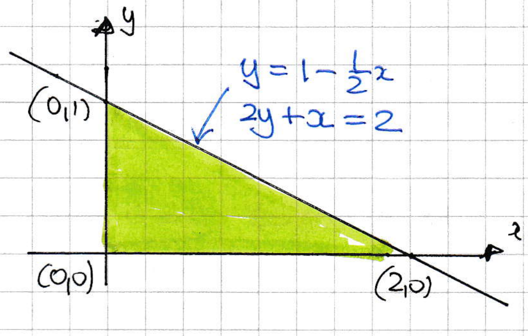
$$= \left[\int_{\phi=0}^{\frac{\pi}{2}} 1\, d\phi\right]\left[\int_{\theta=0}^{\frac{\pi}{2}} \cos^2\theta\sin\theta\, d\theta\right]$$

$$= \frac{\pi}{2} \times \left[-\frac{1}{3}\cos^3\theta\right]_0^{\frac{\pi}{2}} = \frac{\pi}{6}\left[\cos^3\theta\right]_{\frac{\pi}{2}}^{0}$$

$$= \frac{\pi}{6}(1-0) = \frac{\pi}{6}$$

IYGB - MATHEMATICAL METHODS 2 - PAPER B - QUESTION 10

THE ARGUMENT OF THE EXPONENTIAL & THE INTEGRATION AREA SUGGEST THE FOLLOWING TRANSFORMATIONS



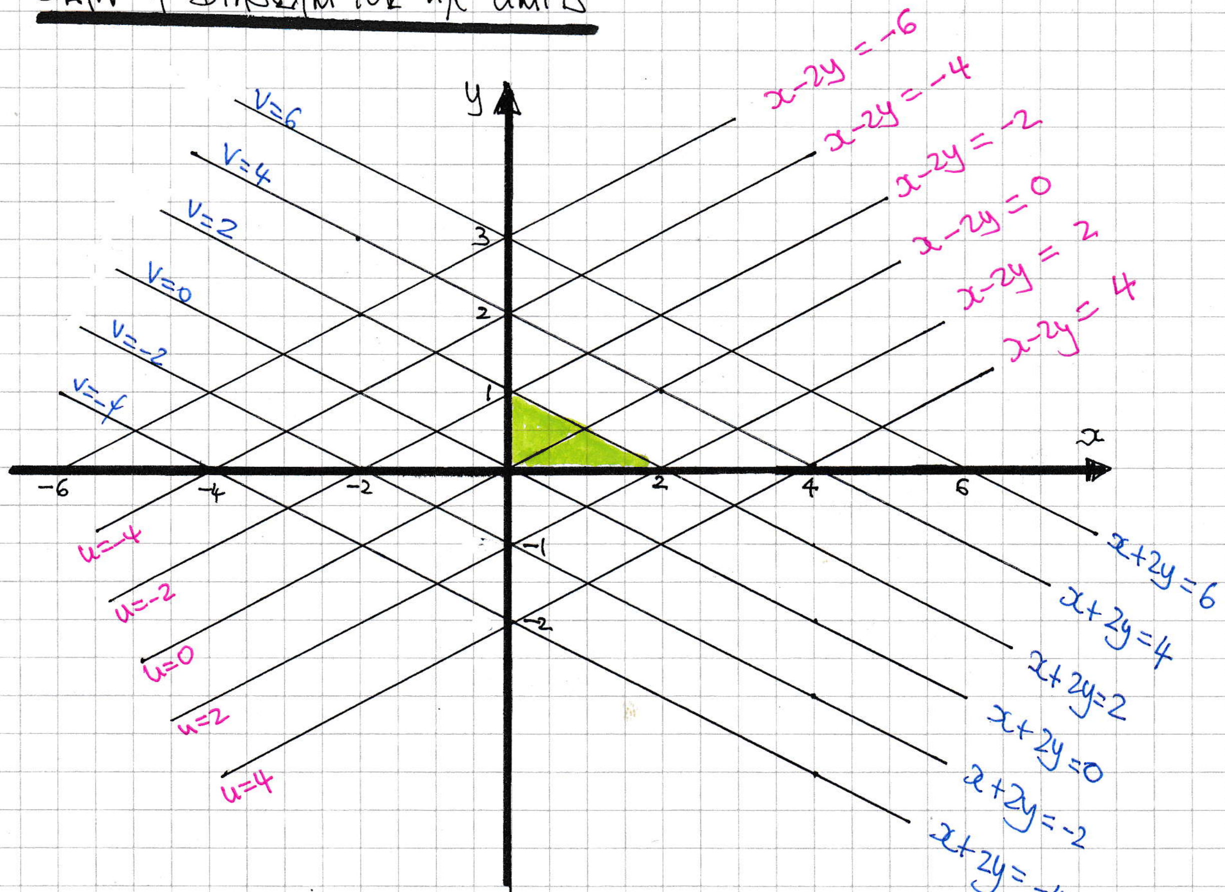
- $u = x - 2y$
- $v = x + 2y$

- $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 4$

$$dx\,dy = \frac{\partial(x,y)}{\partial(u,v)}\,du\,dv$$

$$\boxed{dx\,dy = \frac{1}{4}\,du\,dv}$$

DRAW A DIAGRAM FOR THE LIMITS



IYGB – MATHEMATICAL METHODS 2 – PAPER B – QUESTION 10

- y AXIS $\Rightarrow x=0$

$\Rightarrow \begin{cases} u=-2y \\ v=2y \end{cases}$

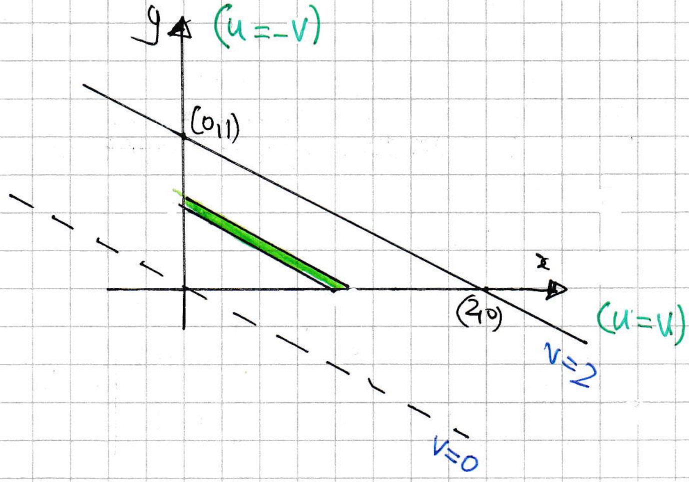
$\Rightarrow v+u=0$

$\Rightarrow \underline{\underline{u=-v}}$

- x AXIS $\Rightarrow y=0$

$\Rightarrow \begin{cases} u=x \\ v=x \end{cases}$

$\Rightarrow \underline{\underline{u=v}}$



FINALLY WE HAVE

$$\int_R e^{\frac{x-2y}{x+2y}}\,dx\,dy = \int_{v=0}^{v=2}\int_{u=-v}^{u=v} e^{\frac{u}{v}}\left(\tfrac{1}{4}\,du\,dv\right) = \int_{v=0}^{v=2}\int_{u=-v}^{u=v} \tfrac{1}{4}e^{\frac{u}{v}}\,du\,dv$$

$$= \int_{v=0}^{2}\left[\tfrac{v}{4}e^{\frac{u}{v}}\right]_{u=-v}^{u=v} dv = \int_{v=0}^{v=2} \tfrac{v}{4}e - \tfrac{v}{4}e^{-1}\,dv$$

$$= \int_0^2 \tfrac{v}{4}\left(e-e^{-1}\right)dv = \int_0^2 \tfrac{1}{2}v\left(\tfrac{1}{2}e^{1}-\tfrac{1}{2}e^{-1}\right)dv$$

$$= \int_0^2 \tfrac{1}{2}v(\sinh 1)\,dv = (\sinh 1)\left[\tfrac{1}{4}v^2\right]_0^2$$

$$= \sinh 1$$

$$= \tfrac{1}{2}\left(e-\tfrac{1}{e}\right) = \frac{e^2-1}{2e}$$

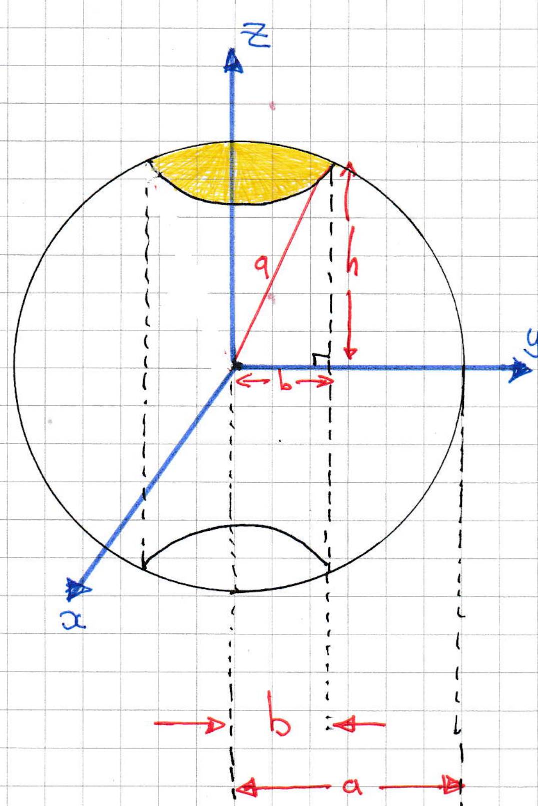
IYGB - MATHEMATICAL METHODS 2 - PAPER B - QUESTION 11

STARTING WITH A DIAGRAM

SPHERE: $x^2+y^2+z^2=a^2$

CYLINDER: $x^2+y^2=b^2$

$(a>b)$



AREA OF THE INNER CYLINDRICAL FACE IS GIVEN BY

$$"2\pi r H" = 2\pi b(2h)$$
$$= 4\pi b h$$
$$= 4\pi b(a^2-b^2)^{\frac{1}{2}}$$

NEXT WE FIND THE AREA OF ONE OF THE SPHERICAL CAPS, SHOWN IN YELLOW — PROJECT THE "TOP" CAP ($z>0$) ONTO THE xy PLANE

$$\Rightarrow z = +(a^2-x^2-y^2)^{\frac{1}{2}}$$

$$\frac{\partial z}{\partial x} = -x(a^2-x^2-y^2)^{-\frac{1}{2}} \quad \& \quad \frac{\partial z}{\partial y} = -y(a^2-x^2-y^2)^{-\frac{1}{2}}$$

$$\Rightarrow dS = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}\ dx\,dy$$

$$\Rightarrow dS = \sqrt{\frac{x^2}{a^2-x^2-y^2}+\frac{y^2}{a^2-x^2-y^2}+1}$$

IYGB - MATHEMATICAL METHODS 2 - PAPER B - QUESTION 11

$$\Longrightarrow dS = \sqrt{\frac{x^2+y^2+a^2-x^2-y^2}{a^2-x^2-y^2}}$$

$$\Longrightarrow \boxed{dS = \frac{a}{\sqrt{a^2-x^2-y^2}}}$$

HENCE THE AREA OF THE TWO CAPS IS GIVEN BY

$$\Longrightarrow A = 2\iint_S 1 \, dS = 2\iint_R \frac{a}{\sqrt{a^2-(x^2+y^2)}} \, dx\, dy$$

(CIRCLE $x^2+y^2=b^2$)

SWITCH INTO PLANE POLARS

$$= 2\iint_R \frac{a}{\sqrt{a^2-r^2}} \, (r\, dr\, d\theta)$$

$$= 2a\int_{\theta=0}^{2\pi}\int_{r=0}^{b} \frac{r}{(a^2-r^2)^{\frac{1}{2}}} \, dr\, d\theta$$

$$= 2a\left[\int_0^{2\pi} 1 \, d\theta\right]\left[\int_{r=0}^{b} r(a^2-r^2)^{-\frac{1}{2}} \, dr\right]$$

$$= 2a \times 2\pi \times \left[-(a^2-r^2)^{\frac{1}{2}}\right]_{r=0}^{b}$$

$$= 4a\pi\left[(a^2-r^2)^{\frac{1}{2}}\right]_b^0$$

$$= 4a\pi\left[a - (a^2-b^2)^{\frac{1}{2}}\right]$$

FINALLY WE HAVE THE AREA OF THE BEAD

$$4\pi a^2 - 4\pi a\left[a - (a^2-b^2)^{\frac{1}{2}}\right] + 4\pi b(a^2-b^2)^{\frac{1}{2}}$$

(SPHERE) (TWO CAPS) (INNER CYLINDRICAL SURFACE)

$$= 4\pi\left[a^2 - a\left[a - (a^2-b^2)^{\frac{1}{2}}\right] + b(a^2-b^2)^{\frac{1}{2}}\right]$$

$$= 4\pi\left[a^2 - a^2 + a(a^2-b^2)^{\frac{1}{2}} + b(a^2-b^2)^{\frac{1}{2}}\right]$$

$$= 4\pi\left[a(a^2-b^2)^{\frac{1}{2}} + b(a^2-b^2)^{\frac{1}{2}}\right]$$

$$= 4\pi(a^2-b^2)^{\frac{1}{2}}(a+b)$$

AS REQUIRED