IYGB – MMS PAPER A – QUESTION 1

a) WRITE THE DATA IN ORDER OF SIZE

110 140 172 ¦ 175 180 195 ¦ 205 241 256 ¦ 375 402 450

$$Q_1 = \frac{172+175}{2} \qquad Q_2 = \frac{195+205}{2} \qquad Q_3 = \frac{256+375}{2}$$

$$\underline{Q_1 = 173.5} \qquad \underline{Q_2 = 200} \qquad \underline{Q_3 = 315.5}$$

b) USING CALCULATOR

$$\underline{\sum x = 2901} \qquad \underline{\sum x^2 = 832705} \qquad \underline{n = 12}$$

- $\bar{x} = \dfrac{\sum x}{n} = \dfrac{2901}{12} = 241.75$ //

- $\sigma = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\dfrac{832705}{12} - 241.75^2} = 104.6375\ldots$

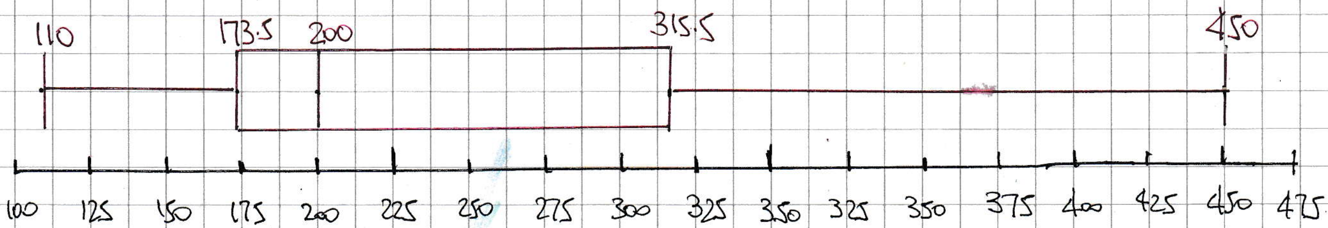
$\simeq \underline{104.64}$ //

c) USING THE FORMULA GIVEN

$$\frac{3(\text{MEAN} - \text{MEDIAN})}{\text{STANDARD DEVIATION}} = \frac{3(241.75 - 200)}{104.64} \approx \underline{1.2}$$ //

POSITIVE //

d)



//

IYGB, MMS PAPER A - QUESTION 2

a) $X \sim B(25, 0.35)$

$H_0 : p = 0.35$

$H_1 : p \neq 0.35$, WHERE p IS THE PROPORTION IN GENERAL

CRITICAL REGION REQUIRED, TWO TAILED, REJECTING IN EACH TAIL AS CLOSE AS POSSIBLE TO 2.5%

- $P(X \leq 3) = 0.0097 = 0.97\%$
- $P(X \leq 4) = 0.0320 = 3.20\%$ ← CLOSER TO 2.5%

⋮

NOT ACTUALLY NEEDED

- $P(X \geqslant 13) = 1 - P(X \leq 12) = 1 - 0.9396 = 0.0604 = 6.04\%$
- $P(X \geqslant 14) = 1 - P(X \leq 13) = 1 - 0.9745 = 0.0255 = 2.55\%$

CLOSER TO 2.5%

- $P(X \geqslant 15) = 1 - P(X \leq 14) = 1 - 0.9907 = 0.0093 = 0.93\%$

CRITICAL REGION IS $\{0, 1, 2, 3, 4\} \cup \{14, 15, 16, \ldots, 25\}$

b) "ACTUAL SIGNIFICANCE" $= 3.20\% + 2.55\% = 5.75\%$

IYGB - MMS PAPER A - QUESTION 3

a) FROM CALCULATOR IN STATISTICAL MODE

$$P.M.C.C. = r = 0.692$$

b) UNCHANGED AT 0.692

(INDEPENDENT OF SCALING/UNITS)

c) SETTING UP HYPOTHESES

$H_0: \rho = 0$
$H_1: \rho > 0$

WHERE ρ IS THE P.M.C.C OF THE ENTIRE POPULATION

THE CRITICAL VALUE FOR $n=8$, AT 1% SIGNIFICANCE IS 0.7887 (TABLES)

AS $0.692 < 0.7887$ THERE IS NO SIGNIFICANT EVIDENCE OF POSITIVE CORRELATION - INSUFFICIENT EVIDENCE TO REJECT H_0

d) THE CRITICAL VALUE FOR $n=8$ AT 5% IS NOW 0.6215 (SAME HYPOTHESES)

AS $0.692 > 0.6215$ THERE IS SIGNIFICANT EVIDENCE OF POSITIVE CORRELATION AT 5% - SUFFICIENT EVIDENCE TO REJECT H_0

e) USING A STATISTICAL CALCULATOR WE OBTAIN THE REGRESSION LINE

$$y = 29.2 + 0.595x$$

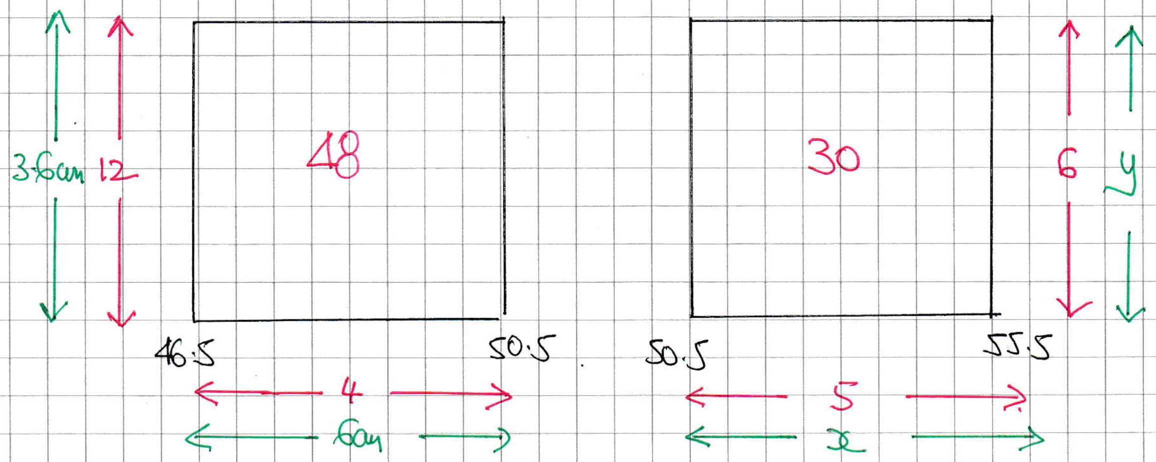
USING $x = 80$

$$y = 29.2 + 0.595 \times 80$$
$$y \simeq 76.8\ldots$$

$\therefore \simeq 77$ seconds

IYGB – MMS PAPER A – QUESTION 4

START WITH TWO RECTANGLES OF THE HISTOGRAM — NOT TO SCALE



- $\dfrac{3.6}{12} = \dfrac{y}{6}$

  $12y = 21.6$

  $y = 1.8$

- $\dfrac{4}{6} = \dfrac{5}{x}$

  $4x = 30$

  $x = 7.5$

$\therefore$ BASE 7.5 cm & HEIGHT 1.8 cm

IYGB - MMS PAPER A - QUESTION 5

$P(A) = 0.4$ • $P(A \cap B) = 0.12$ • A & B INDEPENDENT

a) IF INDEPENDENT $\Rightarrow$ $P(A) \times P(B) = P(A \cap B)$

$$0.4 \times P(B) = 0.12$$

$$P(B) = 0.3$$

b) USING THE "MAIN" PROBABILITY FORMULA

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.4 + 0.3 - 0.12$$

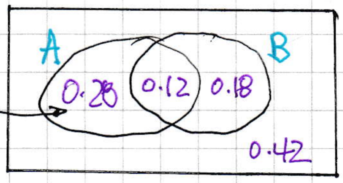
$$P(A \cup B) = 0.58$$

c) AS THE EVENTS ARE INDEPENDENT, $\cap$ IMPLIES MULTIPLICATION

$$P(A \cap B') = P(A) \times P(B') = 0.4 \times 0.7 = 0.28$$

OR BY A FULLY COMPLETED VENN DIAGRAM

$P(A \cap B')$



d) AS THE EVENTS ARE INDEPENDENT — NO DEPENDENCE ON A'

$$P(B' | A') = P(B') = 1 - P(B) = 1 - 0.3 = 0.7$$

OR USING "FORMULA" IN THE ABOVE VENN DIAGRAM

$$P(B' | A') = \frac{P(B' \cap A')}{P(A')} = \frac{0.42}{1 - 0.4} = \frac{0.42}{0.6} = 0.7$$

IYGB - MMS PAPER A - QUESTION 6

a)

X = NUMBER OF ORANGE SWEETS

$X \sim B(20, 0.2)$

$$P(3 \leqslant X \leqslant 7) = P(X \leqslant 7) - P(X \leqslant 2)$$

$$= 0.967857\ldots - 0.2060847\ldots$$

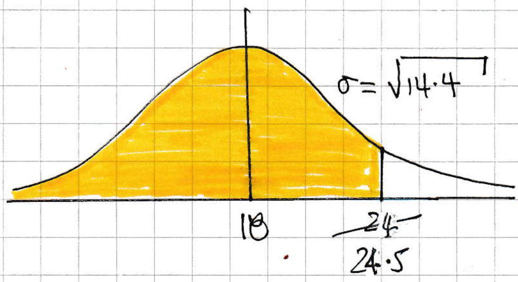
$$\simeq \underline{0.7618}$$

b)

NOW $X \sim B(90, 0.2)$

- MEAN $= E(X) = np = 90 \times 0.2 = 18$
- VARIANCE $= \text{Var}(X) = np(1-p) = 18 \times 0.8 = 14.4$

APPROXIMATE BY $Y \sim N(18, 14.4)$



$$P(X < 25)$$

$$= P(X \leqslant 24)$$

$$= P(Y < 24.5)$$

$$= P\left(Z < \frac{24.5 - 18}{\sqrt{14.4}}\right)$$

$$= \Phi(1.712900\ldots)$$

$$= \underline{0.9566}$$

IYGB - MMS PAPER A - QUESTION 7

a) LOOKING AT THE TABLE

$$P(Y=6) = \text{"6"} + \text{"1,5"} + \text{"5,1"} + \text{"2,4"} + \text{"4,2"} + \text{"3,3"}$$

$$= \frac{1}{2} + \left(\frac{1}{10}\times\frac{1}{10}\right)\times 5 \text{ WAYS}$$

$$= \frac{1}{2} + \frac{1}{20}$$

$$= 0.55$$

b) WE REQUIRE $P(Y<7 \mid Y>4)$

$$\therefore P(Y<7 \mid Y>4) = P(Y\leq 6 \mid Y\geq 5) = \frac{P(5,6)}{P(5,6,7,\dots,11)}$$

NEED OUT-COMES

- $P(Y=5)$

1,4
4,1
2,3
3,2

$\therefore \frac{1}{10}\times\frac{1}{10}\times 4$

$\frac{4}{100}$

- $P(Y=6)$

Part (a)

$\frac{55}{100}$

- $P(Y=7)$

1,6 $\cancel{6,1}$
3,4 4,3
2,5 ,2

$\frac{1}{10}\times\frac{1}{10}\times 4$
$+$
$\frac{1}{10}\times\frac{1}{2}$

$\frac{9}{100}$

- $P(Y=8)$

2,6 $\cancel{6,2}$
3,5 5,3
4,4

$\frac{1}{10}\times\frac{1}{10}\times 3$
$+$
$\frac{1}{10}\times\frac{1}{2}$

$\frac{8}{100}$

- $P(Y=9)$

3,6 $\cancel{6,3}$
4,5 5,4

$\frac{1}{10}\times\frac{1}{10}\times 2$

$\frac{1}{10}\times\frac{1}{2}$

$\frac{7}{100}$

- $P(Y=10)$

4,6 $\cancel{6,4}$
5,5

$\frac{1}{10}\times\frac{1}{10} + \frac{1}{10}\times\frac{1}{2} = \frac{6}{100}$

- $P(Y=11)$

5,6 $\cancel{6,5}$

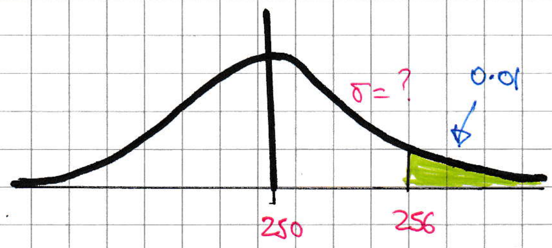
$\frac{1}{10}\times\frac{1}{2} = \frac{5}{100}$

THUS WE HAVE

$$P(Y<7 \mid Y>4) = \frac{\frac{4}{100}+\frac{55}{100}}{\frac{4}{100}+\frac{55}{100}+\frac{9}{100}+\frac{8}{100}+\frac{7}{100}+\frac{6}{100}+\frac{5}{100}} = \frac{59}{94} \approx 0.628$$

a) W = weight of marmalade Jar

$W \sim N(250, \sigma^2)$



$P(W > 256) = 0.01$

$P(W < 256) = 0.99$

$P\left(Z < \frac{256-250}{\sigma}\right) = 0.99$

INVERSION

$\frac{6}{\sigma} = +\Phi^{-1}(0.99)$

$\frac{6}{\sigma} = 2.3263$

$\sigma = 2.5792...$

$\sigma \approx 2.6$

b) $P(249 < W < 253)$

$= P(W < 253) - P(W < 249)$

$= P(W < 253) - [1 - P(W > 249)]$

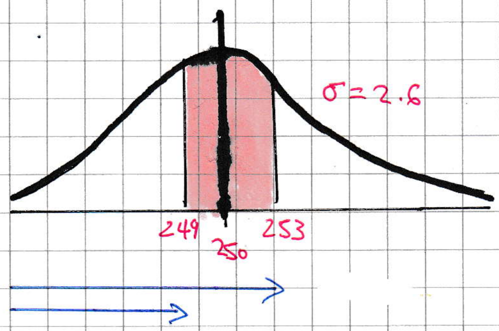
$= P(W < 253) + P(W > 249) - 1$

$= P\left(Z < \frac{253-250}{2.6}\right) + P\left(Z > \frac{249-250}{2.6}\right) - 1$

$= \Phi(1.16315...) + \Phi(-0.38777...) - 1$

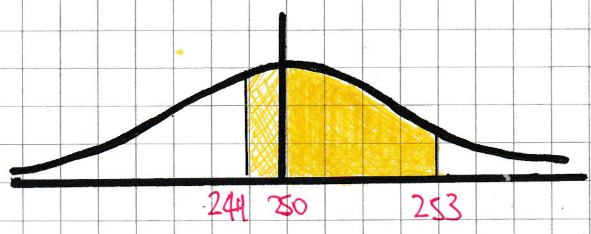
$= 0.87762 + 0.65089 - 1$

$= 0.5285$



c) $P(W > 250 \mid 249 < W < 253)$

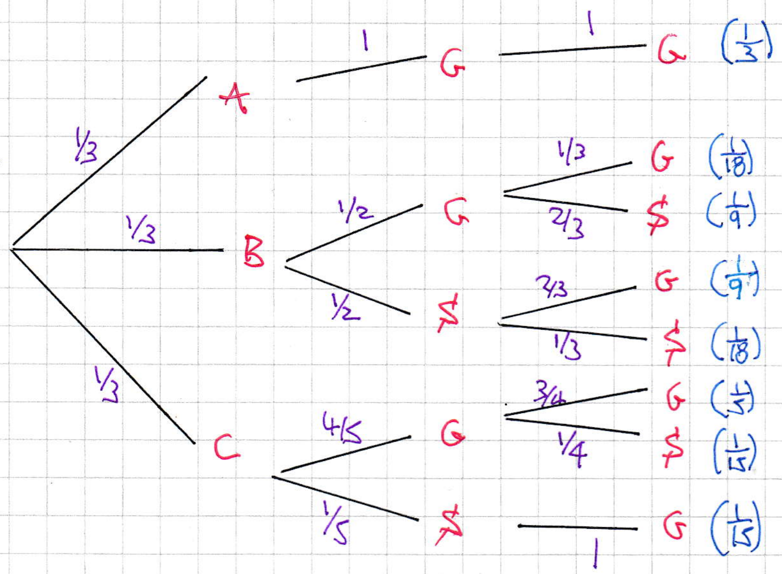
NOTE THAT $P(W < 253) = 0.8776$ FROM PART (b)



THE REQUIRED PROBABILITY IS GIVEN BY $= \frac{0.8776 - 0.5}{0.5285} \approx 0.7145$

IYGB - MMS PAPER A - QUESTION 9)

PROCEED BY A TREE DIAGRAM



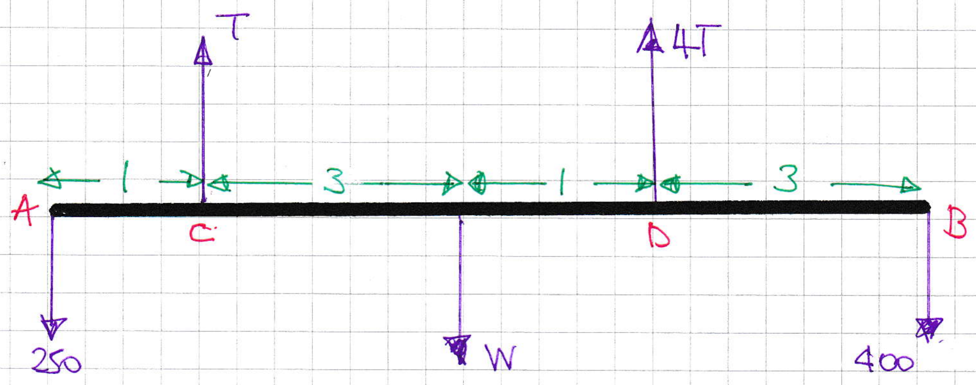
WE REQUIRE P(BOX C | BOTH GOLD)

$$P(C|G) = \frac{P(C \cap G)}{P(G)} = \frac{\frac{1}{5}}{\frac{1}{3}+\frac{1}{18}+\frac{1}{5}} = \frac{18}{30+5+18} = \frac{18}{53} \simeq 0.3396$$

"both" "both"

IYGB - MMS PAPER A - QUESTION 10

START WITH A DIAGRAM



TAKE MOMENTS ABOUT THE MIDPOINT

$$(250\times4)+(4T\times1)=(T\times3)+(400\times4)$$
$$1000+4T=3T+1600$$
$$\underline{T=600\text{ N}}$$

RESOLVING VERTICALLY

$$T+4T=250+W+400$$
$$600+2400=650+W$$
$$\underline{W=2350\text{ N}}$$

IYGB - MMS PAPER A - QUESTION 11

a) WHEN $t=0$ $\underline{v} = 3\underline{i} - 5\underline{j}$

$$\text{SPEED} = |\underline{v}| = |3\underline{i} - 5\underline{j}| = \sqrt{3^2+(-5)^2}$$
$$= \sqrt{9+25} = \sqrt{34} \simeq 5.83 \text{ ms}^{-1}$$

b) USING $\underline{v} = \underline{u} + \underline{a}t$

$$\Longrightarrow 11\underline{i} + 7\underline{j} = 3\underline{i} - 5\underline{j} + \underline{a} \times 4$$
$$\Longrightarrow 8\underline{i} + 12\underline{j} = 4\underline{a}$$
$$\Longrightarrow \underline{a} = 2\underline{i} + 3\underline{j}$$

$\underline{F} = m\underline{a}$

$$\Longrightarrow \underline{F} = 2(2\underline{i} + 3\underline{j})$$
$$\Longrightarrow \underline{F} = 4\underline{i} + 6\underline{j}$$

c) WRITE A GENERAL EXPRESSION FOR THE VELOCITY VECTOR IN TIME t

$$\Longrightarrow \underline{v} = \underline{u} + \underline{a}t$$
$$\Longrightarrow \underline{v} = (3\underline{i} - 5\underline{j}) + (2\underline{i} + 3\underline{j})t$$
$$\Longrightarrow \underline{v} = (2t+3)\underline{i} + (3t-5)\underline{j}$$

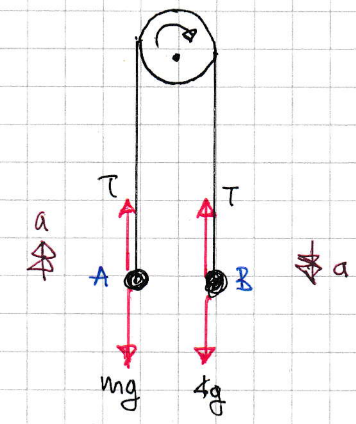
WHEN MOVING DUE EAST, $\underline{v}$ HAS THE FORM $\underline{v} = \lambda\underline{i}$

$\therefore$ NO $\underline{j}$ COMPONENT

$$\Longrightarrow 3t - 5 = 0$$
$$\Longrightarrow t = \tfrac{5}{3}$$

IYGB - MMS PAPER A - QUESTION 12

a) LOOKING AT THE DIAGRAM OPPOSITE



(A): $T - mg = ma$

(B): $4g - T = 4a$

ADDING THE EQUATIONS

$$4g - mg = (m+4)a$$
$$(4-m)g = (m+4)a$$
$$\therefore a = \frac{4-m}{4+m}g$$

b) USING RESULT FROM PART (a)

$$T = 4g - 4a$$
$$T = 4g - \frac{4-m}{4+m}4g$$
$$T = 4g\left[1 - \frac{4-m}{4+m}\right]$$
$$T = 4g\left[\frac{\cancel{4} + m - \cancel{4} + m}{4+m}\right]$$
$$T = 4g \times \frac{2m}{4+m}$$
$$T = \frac{8mg}{m+4}$$ AS REQUIRED

c) KINEMATICS WITH CONSTANT ACCELERATION

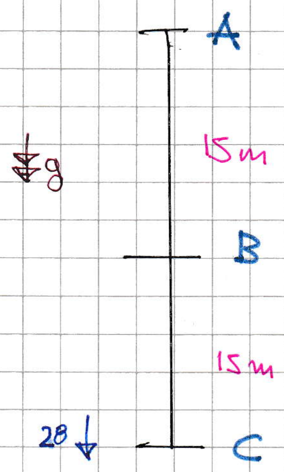
$u = 0$	$\Rightarrow v^2 = \cancel{u^2} + 2as$
$a = \frac{4-m}{4+m}$	$\Rightarrow 1.4^2 = 2 \times \frac{4-m}{4+m} \times 9.8 \times 0.7$
$s = 0.7$	$\Rightarrow \frac{4-m}{4+m} = \frac{1}{7}$
t	
$v = 1.4$	$\Rightarrow 28 - 7m = 4 + m$

$$\Rightarrow 24 = 8m$$
$$\Rightarrow m = 3$$

IYGB - MMS PAPER A - QUESTION 13

RECORDING THE GIVEN FOR THE PROBLEM - WORKING AT AC

$u = \text{N/A}$
$a = 9.8 \text{ ms}^{-1}$
$s = 15 \text{ m}$
$t = ?$
$v = 28 \text{ ms}^{-1}$



$$s = vt - \frac{1}{2}at^2$$

$$15 = 28t - \frac{1}{2}(9.8)t^2$$

$$15 = 28t - 4.9t^2$$

$$4.9t^2 - 28t + 15 = 0$$

$$49t^2 - 280t + 150 = 0$$

$$t = \frac{280 \pm \sqrt{49000}}{2 \times 49}$$

$$t = \begin{cases} 0.598 \\ 5.116 \end{cases}$$

WE CAN ONLY REJECT ONE OF THESE ANSWERS BY COMMON SENSE - SO WE PROCEED AS FOLLOWS

$$v^2 = u^2 + 2as$$

$$28^2 = u^2 + 2(9.8) \times 15$$

$$784 = u^2 + 294$$

$$u^2 = 490$$

$$u = +7\sqrt{10}$$

$$v = u + at$$

$$28 = 7\sqrt{10} + 9.8t$$

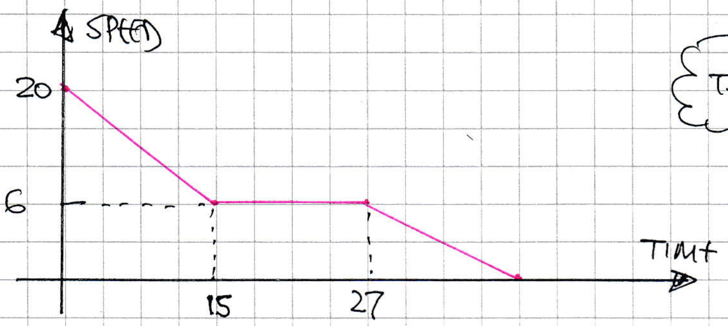
$$t = \frac{28 - 7\sqrt{10}}{9.8}$$

$$t = \frac{20 - 5\sqrt{10}}{7}$$

$$t = 0.598$$

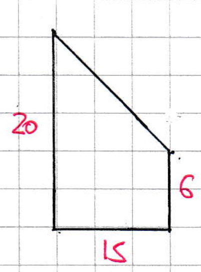
IYGB - MMS PAPER A - QUESTION 14

a) SKETCHING THE SPEED TIME GRAPH FROM THE INFORMATION GIVEN



TOTAL DISTANCE = 315

b) FIND THE AREA COVERED IN THE FIRST 15 SECONDS



$$\text{DISTANCE} = \text{AREA} = \frac{20+6}{2} \times 15$$

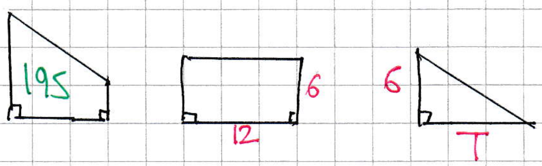
$$= 13 \times 15$$

$$= 195 \text{ m}$$

c) LOOKING AT THE TOTAL DISTANCE FOR THE JOURNEY

$\Rightarrow$ TOTAL DISTANCE = 315

$\Rightarrow$ $195 + (12 \times 6) + \frac{1}{2} \times 6 \times T = 315$



$\Rightarrow$ $195 + 72 + 3T = 315$

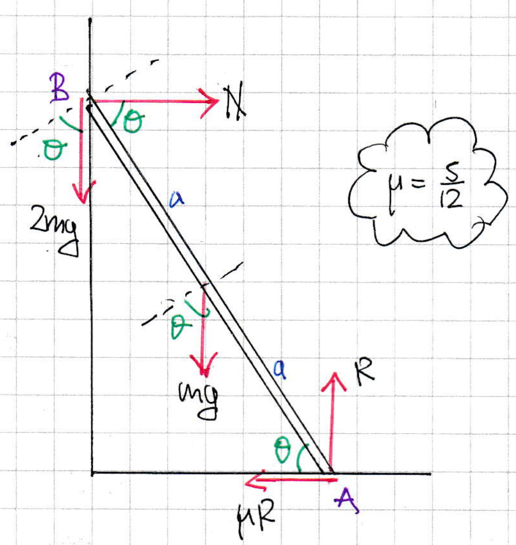
$\Rightarrow$ $3T = 48$

$\Rightarrow$ $T = 16$

$\therefore$ TOTAL TIME $= 15 + 12 + 16 = 43 \text{ s}$

IYGB - MMS PAPER A - QUESTION 15

LOOKING AT THE DIAGRAM BELOW



$(\uparrow): R = 3mg$

$(\rightarrow): N = \mu R$

TAKING MOMENTS ABOUT A

$$\Rightarrow mg\cos\theta \times a + 2mg\cos\theta \times 2a = N\sin\theta \times 2a$$

$$\Rightarrow 2N\cancel{a}\sin\theta = 5mg\cancel{a}\cos\theta$$

$$\Rightarrow 2(\mu R)\sin\theta = 5mg\cos\theta$$

$$\Rightarrow 2 \times \frac{5}{12} \times 3mg \times \sin\theta = 5mg\cos\theta$$

$$\Rightarrow \frac{5}{2}\cancel{mg}\sin\theta = 5\cancel{mg}\cos\theta$$

$$\Rightarrow \sin\theta = 2\cos\theta$$

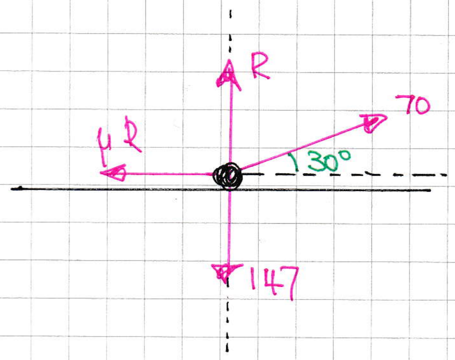
$$\Rightarrow \frac{\sin\theta}{\cos\theta} = 2$$

$$\Rightarrow \tan\theta = 2$$

$$\Rightarrow \theta \simeq 63.4°$$

IYGB - MMS PAPER A - QUESTION 16

a) STARTING WITH A DIAGRAM & NOTING THAT CONSTANT SPEED IMPLIES EQUILIBRIUM



$(\updownarrow)$: $R + 70\sin 30 = 147$

$R + 35 = 147$

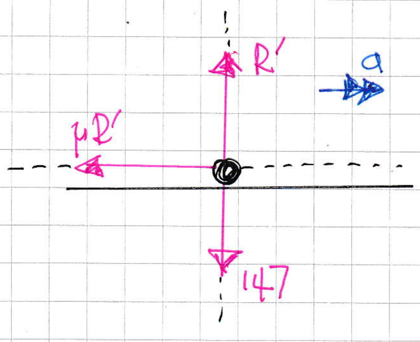
$\underline{R = 112}$

$(\leftrightarrow)$: $70\cos 30 = \mu R$

$70 \times \frac{\sqrt{3}}{2} = \mu \times 112$

$\mu = \frac{5}{16}\sqrt{3} \simeq 0.541$

b) CALCULATE THE ACCELERATION (DECELERATION) ONCE "PULL" IS REMOVED



$(\updownarrow)$: $R' = 147$ [EQUILIBRIUM]

$(\leftrightarrow)$: $-\mu R' = ma$ ["F = ma"]

$\Rightarrow -\left(\frac{5}{16}\sqrt{3}\right)(147) = \frac{147}{g}a$

$\Rightarrow -\frac{5}{16}\sqrt{3} = \frac{a}{g}$

$\Rightarrow \underline{a = -\frac{5}{16}\sqrt{3}g} \simeq -5.3044...$

!! $mg = 147$, $m = \frac{147}{g}$!!

FINALLY KINEMATICS

$u = ?$
$a = -5.3044 \text{ ms}^{-2}$
$s = 12.25 \text{ m}$
$t = -$
$v = 0$

$v^2 = u^2 + 2as \Rightarrow 0 = u^2 + 2(-5.3044...)(12.25)$

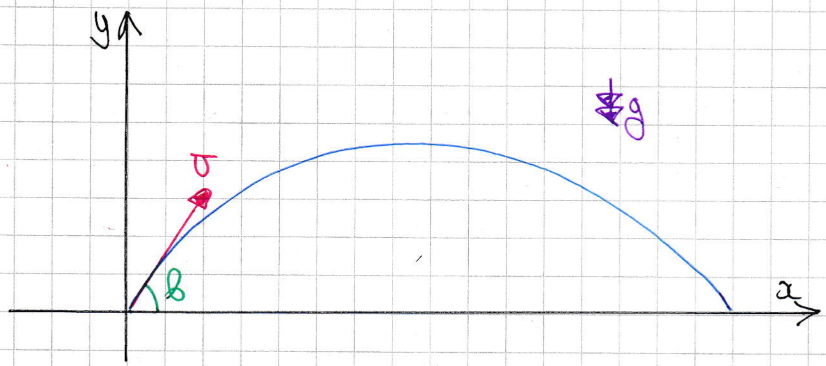
$\Rightarrow u^2 = 129.957...$

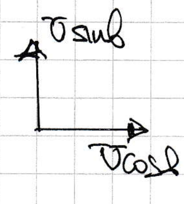
$\Rightarrow |u| = 11.3999...$

$\Rightarrow u \simeq 11.4 \text{ ms}^{-1}$

IYGB - MMS PAPER A - QUESTION 17

START WITH A DIAGRAM, LETTING THE PROJECTION SPEED BE U





HORIZONTALLY AT TIME "T"

$x = (U\cos\theta)T$

$x = UT\cos\theta$

$T = \frac{x}{U\cos\theta}$ (SUBSTITUTE TO y)

VERTICALLY AT TIME "T" $(s = ut + \frac{1}{2}at^2)$

$y = UT\sin\theta + \frac{1}{2}(-g)T^2$

$y = \cancel{U}\left(\frac{x}{\cancel{U}\cos\theta}\right)\sin\theta - \frac{1}{2}g\left(\frac{x}{U\cos\theta}\right)^2$

$$\boxed{y = x\tan\theta - \frac{gx^2}{2U^2\cos^2\theta}}$$

USING (4,2) & (36,12) WITH THIS EQUATION

$(4,2) \Rightarrow 2 = 4\tan\theta - \frac{16g}{2U^2\cos^2\theta}$

$\Rightarrow \frac{8g}{U^2\cos^2\theta} = 4\tan\theta - 2$

$\Rightarrow \frac{4g}{U^2\cos^2\theta} = 2\tan\theta - 1 \quad \times 27$

$\Rightarrow \frac{108g}{U^2\cos^2\theta} = 54\tan\theta - 27$

$(36,12) \Rightarrow 12 = 36\tan\theta - \frac{1296g}{2U^2\cos^2\theta}$

$\Rightarrow \frac{648g}{U^2\cos^2\theta} = 36\tan\theta - 12$

$\Rightarrow \frac{54g}{U^2\cos^2\theta} = 3\tan\theta - 1 \quad \times 2$

$\Rightarrow \frac{108g}{U^2\cos^2\theta} = 6\tan\theta - 2$

COMBINE AND SOLVE

$\Rightarrow 54\tan\theta - 27 = 6\tan\theta - 2$

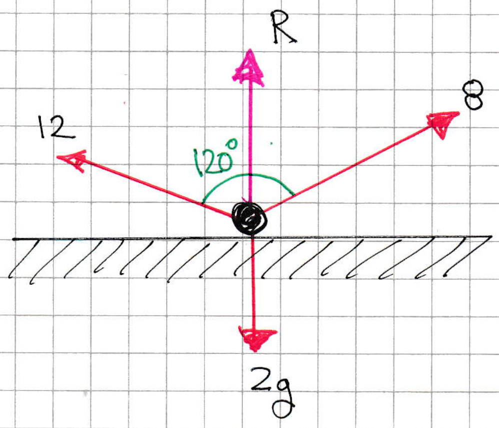
$\Rightarrow 48\tan\theta = 25$

$\Rightarrow \tan\theta = \frac{25}{48}$

$\theta \simeq 27.5°$

IYGB - MMS PAPER A - QUESTION 18

LOOKING AT THE DIAGRAMS & NOTING THAT THE ONLY REACTION IS NORMAL AS THE SURFACE IS SMOOTH



- BY THE COSINE RULE

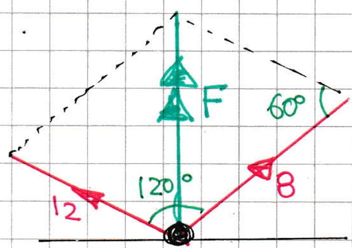
$|F|^2 = 8^2 + 12^2 - 2 \times 8 \times 6 \times \cos 60$

$|F|^2 = 64 + 144 - 48$

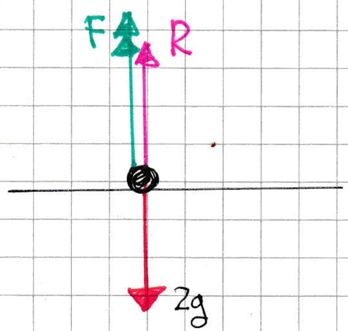
$|F| = \sqrt{160}$

$\uparrow$

(RESULTANT OF THE 8N & 12N)



- THIS RESULTANT MUST ACT IN A VERTICAL DIRECTION OTHERWISE THERE WOULD BE A "SIDEWISE" FORCE (NOTE NO FRICTION)

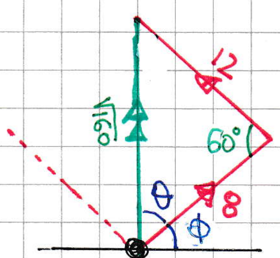


$\therefore \quad R + F = 2g$

$R = 2g - \sqrt{160}$

$R \approx 6.95 \text{ N}$

- BY THE SINE RULE (OPPOSITE)



$$\frac{\sin 60}{\sqrt{160}} = \frac{\sin \theta}{12}$$

IYGB - MMS PAPER A - QUESTION 18

$$\Rightarrow \quad \sin\theta = \frac{12\sin 60}{\sqrt{160}}$$

$$\Rightarrow \quad \sin\theta = 0.8215\ldots$$

$$\Rightarrow \quad \theta = 55.24^\circ \qquad (\theta \neq 124.76^\circ)$$

$$\Rightarrow \quad \phi = 34.76^\circ$$