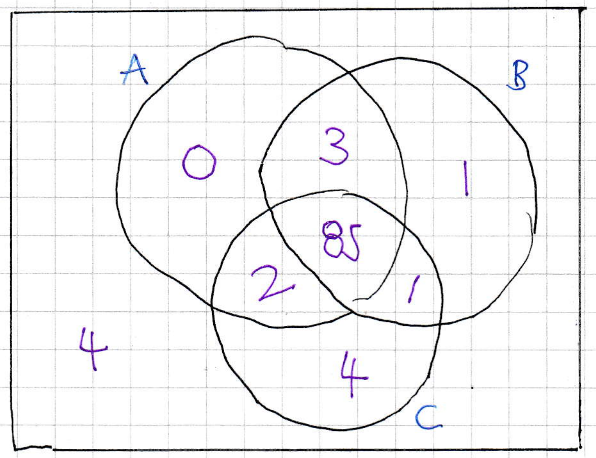
IYGB - MMS PAPER B - QUESTION 1

a) STARTING WITH A "TRIPLE" VENN DIAGRAM



LOOKING AT THE VENN DIAGRAM

b) $P(\text{TWO OUT OF THE THREE}) = \frac{3+2+1}{100} = \frac{6}{100}$

c) $P(\text{B BUT NOT C}) = \frac{3+1}{100} = \frac{4}{100}$

d) $P(B') = \frac{2+4+4}{100} = \frac{10}{100}$

e) FINALLY THE CONDITIONAL PROBABILITY

$$P(A \cap C \mid \text{LIKES TWO}) = \frac{\frac{2}{100}+\frac{85}{100}}{\frac{3}{100}+\frac{2}{100}+\frac{1}{100}+\frac{85}{100}} = \frac{87}{91} \simeq 0.956$$

IYGB – MMS PAPER B – QUESTION 2

LOOKING AT THE SUMMARY STATISTICS

$$\sum_{i=1}^{5}\left(\frac{x_i-255}{2}\right)=50 \qquad \sum_{i=1}^{5}\left(\frac{x_i-255}{2}\right)^2=1650$$

LET $y=\dfrac{x-255}{2}$

$$\sum y = 50 \qquad \sum y^2 = 1650 \qquad n=5$$

CALCULATE THE MEAN & STANDARD DEVIATION IN y

$$\bar{y}=\frac{\sum y}{n}=\frac{50}{5}=10$$

$$\sigma_y=\sqrt{\frac{\sum y^2}{n}-\bar{y}^2}=\sqrt{\frac{1650}{5}-10^2}=\sqrt{230}$$

UNCODE THE MEAN & STANDARD DEVIATION BACK INTO x.

- $\bar{x}=\bar{y}\times 2+255$

  $\bar{x}=10\times 2+255$

  $\bar{x}=275$

- $\sigma_x=\sigma_y\times 2$

  $\sigma_x=2\sqrt{230}$

  $\sigma_x\simeq 30.3$

IYGB – MMS PAPER B – QUESTION 3

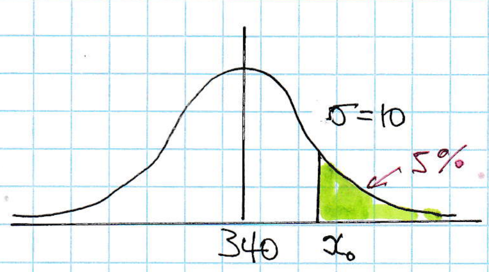
a) I) PUTTING THE INFORMATION INTO A DIAGRAM

X = weight of pop-corn bags
$X \sim N(340, 10^2)$

$\Rightarrow P(X > x_0) = 5\%$

$\Rightarrow P(X < x_0) = 95\%$

$\Rightarrow P\left(Z < \frac{x_0 - 340}{10}\right) = 0.95$



↓ INVERSION

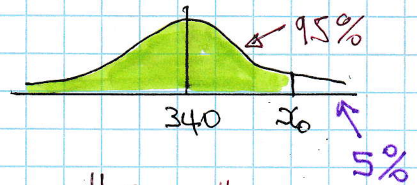
$$\frac{x_0 - 340}{10} = +\Phi^{-1}(0.95)$$

$$\frac{x_0 - 340}{10} = 1.6449$$

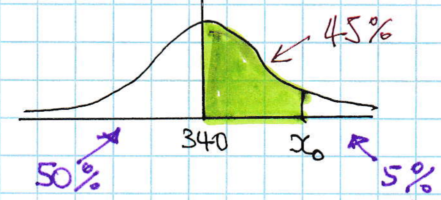
$$x_0 = 356.449$$

$\therefore$ x_0 IS 356 grams

II) LOOKING AT THE DIAGRAMS



"GIVEN"



"REQUIRED"

$$P(X > \mu \mid X < x_0) = \frac{\text{"}45\%\text{"}}{\text{"}95\%\text{"}} = \frac{45}{95} = \frac{9}{19}$$

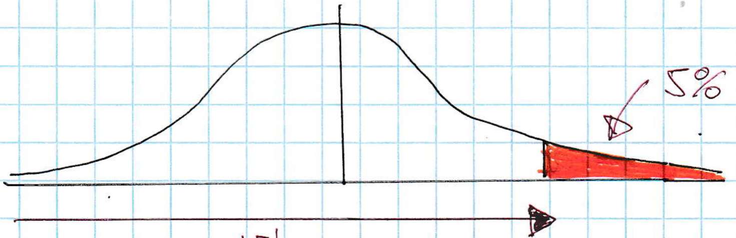
$\simeq 0.4737$

IYGB – MMS PAPER B – QUESTION 3

b) SETTING UP THE HYPOTHESES

- $H_0 : \mu = 320$
- $H_1 : \mu > 320$, WHERE μ IS THE MEAN WEIGHT OF ALL BAGS OF POPCORNS

$n = 5$, $\bar{x}_5 = 327$, $\sigma = 10$, 5% SIGNIFICANCE



$\Phi^{-1}(0.95) = 1.6449$ ← CRITICAL VALUE

- Z STATISTIC $= \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{327 - 320}{10/\sqrt{5}} = 1.5652\ldots$

- AS $1.5652 < 1.6449$ THERE IS NO SIGNIFICANT EVIDENCE (AT 5%) THAT THE MEAN WEIGHT OF BAGS IS OVER 320

- THERE IS NO SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - MMS PAPER B - QUESTION 4

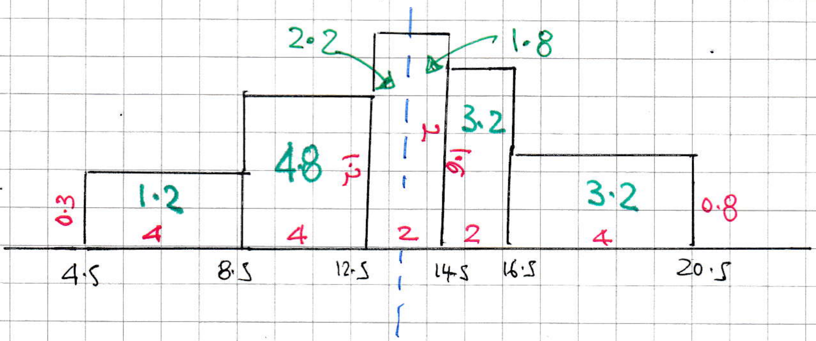
"HEIGHT BETWEEN 5 cm AND 8 cm" $\Rightarrow$ $4.5 \leq h < 8.5$

AREA OF 4×0.3 CORRESPONDS 18 PLANTS

AREA	:	PLANTS
1.2	:	18
1	:	15

(THIS IS NEEDED FOR THE MEAN & STANDARD DEVIATION)

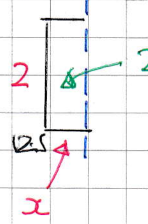
a) FIND THE MEDIAN, WITHOUT INVOLVING FREQUENCIES.



TOTAL AREA

$0.3 \times 4 = 1.2$
$1.2 \times 4 = 4.8$
$2 \times 2 = 4$
$1.6 \times 2 = 3.2$
$0.8 \times 4 = 3.2$
$ 16.4$

$\frac{1}{2}$ THE AREA IS 8.2



$\therefore Q_2 = 12.5 + 1.1$

$Q_2 = 13.6$

$2x = 2.2$

$x = 1.1$

b) RECONSTRUCT THE "TABLE OF DATA"

PLANT HEIGHT (NEAREST cm)	MIDPOINTS	AREA	FREQUENCY (AREA × 15)
5 – 8	6.5	1.2	18
9 – 12	10.5	4.8	72
13 – 14	13.5	4	60
15 – 16	15.5	3.2	48
17 – 20	18.5	3.2	48
			246

IYGB -

OBTAINING THE SUMS BY CALCULATOR

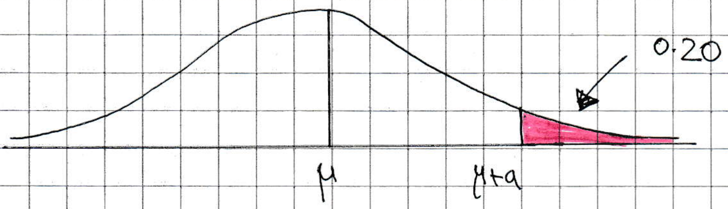
$$\boxed{\begin{aligned}\Sigma fx &= 3315\\ \Sigma fx^2 &= 47593.5\end{aligned}}$$

- $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{3315}{246} = 13.48$ //

- $S.D = \sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}} = \sqrt{\frac{47593.5}{246} - (13.48...)^2} \simeq 3.45$ //

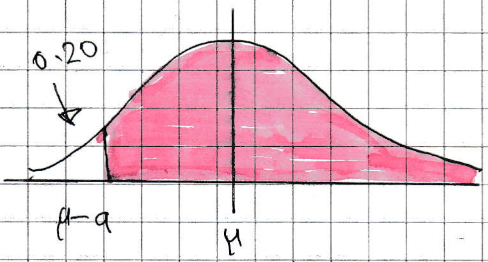
IYGB – MMS PAPER B – QUESTION 5

LOOKING AT THE INFORMATION GIVEN FOR $X \sim N(\mu, \sigma^2)$



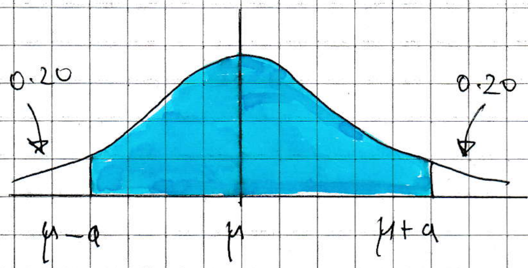
$$P(X > a) = 0.20$$

a)



$$P(X > \mu - a) = 1 - 0.2$$
$$= \underline{0.8}$$

b)



$$P(|X - \mu| < a)$$
$$= P(\mu - a < X < \mu + a)$$
$$= 1 - 2 \times 0.2$$
$$= \underline{0.6}$$

c)

$$P(X < \mu + a \mid X > \mu - a) = \frac{P(X < \mu + a) \cap B(X > \mu - a)}{P(X > \mu - a)}$$

$$= \frac{P(\mu - a < X < \mu + a)}{P(X > \mu - a)}$$

$$= \frac{0.6}{0.8}$$

$$= \underline{0.75}$$

IYGB – MMS PAPER B – QUESTION 6

a)
- FIXED TRIALS
- TWO OUTCOMES
- CONSTANT PROBABILITY OF SUCCESS
- INDEPENDENCE OF EVENTS

b) X = NUMBER OF SALES (ROY)

I) $X \sim B(15, 0.05)$

$$P(X=0) = \binom{15}{0}(0.05)^0(0.95)^{15}$$
$$= 0.4633$$

II) $X \sim B(20, 0.05)$

$$P(X>3) = P(X \geqslant 4)$$
$$= 1 - P(X \leqslant 3)$$
$$= 1 - 0.9841$$
$$= 0.0159$$

c) WE REQUIRE THAT

$$\Rightarrow P(X \geqslant 1) > 0.99$$
$$\Rightarrow 1 - P(X=0) > 0.99$$
$$\Rightarrow -P(X=0) > -0.01$$
$$\Rightarrow P(X=0) < 0.01$$
$$\Rightarrow \binom{n}{0}(0.05)^0(0.95)^n < 0.01$$
$$\Rightarrow 0.95^n < 0.01$$
$$\Rightarrow \log(0.95^n) < \log(0.01)$$
$$\Rightarrow n\log(0.95) < \log(0.01)$$
$$\Rightarrow n > \frac{\log(0.01)}{\log(0.95)}$$ (DIVIDED BY NEGATIVE)
$$\Rightarrow n > 89.78\ldots$$

$$\therefore n = 90$$

IYGB - MMS PAPER B - QUESTION 6

d) ORGANIZING THE OUTCOMES USING Y = SALES MADE BY ROX

$X \sim B(20, 0.05)$ & $Y \sim B(20, 0.15)$

I) $P(X=2) \times P(Y=2) = \binom{20}{2}(0.05)^2(0.95)^{18} \times \binom{20}{2}(0.15)^2(0.85)^{18} = 0.0433$

II) $P(X=0) = \binom{20}{0}(0.05)(0.95)^5 = 0.3585$

$P(X=1) = \binom{20}{1}(0.05)^1(0.95)^4 = 0.3774$

$P(X=2) = \binom{20}{2}(0.05)^2(0.95)^3 = 0.1887$

$P(X=3) = \binom{20}{3}(0.05)^3(0.95)^2 = 0.0596$

$P(X=4) = \binom{20}{4}(0.05)^4(0.95)^1 = 0.0133$

$P(X=5) = \binom{20}{5}(0.05)^5(0.95)^0 = 0.0022$

$P(Y=0) = \binom{20}{0}(0.15)^0(0.85)^5 = 0.0388$

$P(Y=1) = \binom{20}{1}(0.15)^1(0.85)^4 = 0.1368$

$P(Y=2) = \binom{20}{2}(0.15)^2(0.85)^3 = 0.2293$

$P(Y=3) = \binom{20}{3}(0.15)^3(0.85)^2 = 0.2428$

$P(Y=4) = \binom{20}{4}(0.15)^4(0.85)^1 = 0.1821$

$P(Y=5) = \binom{20}{5}(0.15)^5(0.85)^0 = 0.1028$

SO WE HAVE

$(0,5)$	$(5,0)$	$(1,4)$	$(4,1)$	$(2,3)$	$(3,2)$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
0.0369	0.0001	0.0687	0.0018	0.0458	0.0137

ADDING TO OBTAIN THE DESIRED RESULT OF 0.167

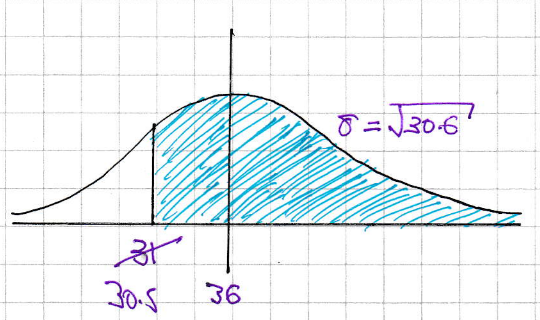
IYGB – MMS PAPER B – QUESTION 6

e) FINALLY APPROXIMATE BY A NORMAL DISTRIBUTION

$Y \sim B(240, 0.15)$

MEAN $= 240 \times 0.15 = 36$

VARIANCE $= 36 \times 0.85 = 30.6$

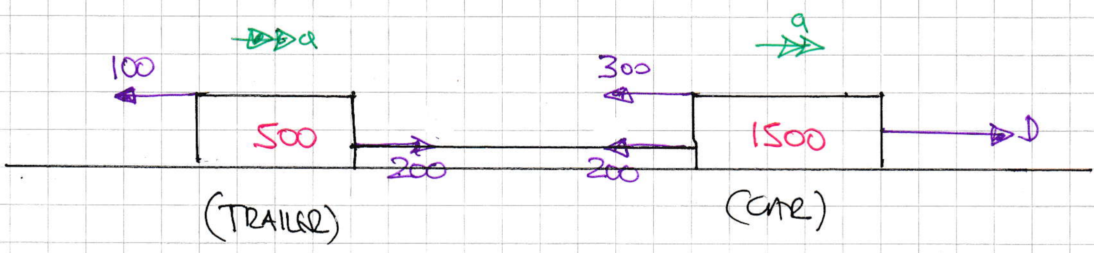


$W \sim N(30, 30.6)$

$$
\begin{aligned}
P(Y > 30) &= P(Y \geqslant 31) \\
&= P(W > 30.5) \\
&= P\left(Z > \frac{30.5 - 36}{\sqrt{30.6}}\right) \\
&= \Phi(-0.9943) \\
&= 0.8394
\end{aligned}
$$

IYGB - MMS PAPER B - QUESTION 7

a) STARTING WITH A DIAGRAM, IGNORING "VERTICAL" FORCES



TRAILER

$200 - 100 = 500a$

$100 = 500a$

$a = 0.2 \text{ ms}^{-2}$ //

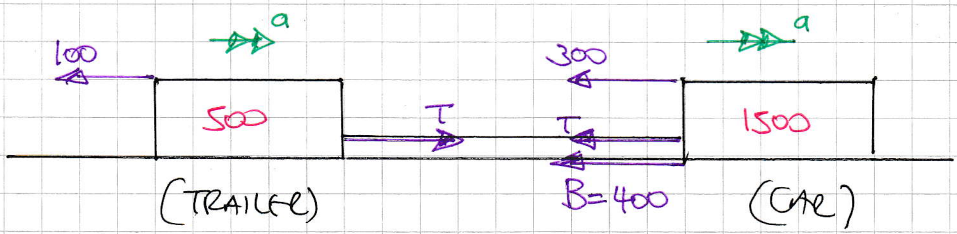
CAR

$D - 300 - 200 = 1500a$

$D - 500 = 1500 \times 0.2$

$D = 800 \text{N}$ //

b) REMODELLING THE DIAGRAM - LEAVE THE TENSION, WHICH IS NOW THRUST, MARKED THE "WRONG" WAY



(TRAILER) : $T - 100 = 500a$

(CAR) : $-300 - T - 400 = 1500a$

$\Rightarrow$ ADDING TO GET

$-800 = 2000a$

$a = -0.4 \text{ ms}^{-2}$

AND $T - 100 = 500(-0.4)$

$T - 100 = -200$

$T = -100 \text{ N}$

$\therefore$ DECELERATION OF 0.4 ms^{-2} & THRUST OF 100 N

IYGB – MMS PAPER B – QUESTION 8

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$ WE OBTAIN

$$\Rightarrow 65\underline{i} + 59\underline{j} = 20\underline{i} + 35\underline{j} + \frac{1}{2}\underline{v}$$

$$\Rightarrow 45\underline{i} + 24\underline{j} = \frac{1}{2}\underline{v}$$

$$\Rightarrow \underline{v} = 90\underline{i} + 48\underline{j}$$

b) USING $\underline{p} = \underline{p}_0 + \underline{v}t$

$$\Rightarrow \underline{p} = (20\underline{i} + 35\underline{j}) + (90\underline{i} + 48\underline{j})t$$

$$\Rightarrow \underline{p} = (90t + 20)\underline{i} + (48t + 35)\underline{j}$$

c) PROCEED TO FIND VELOCITY VECTOR FOR Q

- IF THE VELOCITY WAS $24\underline{i} - 7\underline{j}$, THEN ITS SPEED WOULD HAVE BEEN

$$\sqrt{24^2 + (-7)^2} = \sqrt{576 + 49} = 25 \text{ km h}^{-1}$$

- BUT THE SPEED IS IN FACT 125 km h^{-1}, IE 5 TIMES LARGER
- $\underline{V}_Q = 5(24\underline{i} - 7\underline{j}) = 120\underline{i} - 35\underline{j}$

FINALLY WE CAN FIND AN EXPRESSION FOR $\underline{q}$

$$\Rightarrow \underline{q} = \underline{q}_0 + \underline{V}_Q t$$

$$\Rightarrow \underline{q} = 200\underline{j} + (120\underline{i} - 35\underline{j})t$$

$$\Rightarrow \underline{q} = 120t\,\underline{i} + (200 - 35t)\underline{j}$$

IYGB - MMS PAPER B - QUESTION 8

WHEN $t=2$

$$\underline{p} = (90\times2+20)\underline{i} + (48\times2+35)\underline{j} = 200\underline{i} + 131\underline{j}$$

$$\underline{q} = (120\times2)\underline{i} + (200-35\times2)\underline{j} = 240\underline{i} + 130\underline{j}$$

$\therefore$ $P(200, 131)$ & $Q(240, 130)$

$$|PQ| = \sqrt{(200-240)^2 + (131-130)^2}$$

$$= \sqrt{1600+1}$$

$$= \sqrt{1601}$$

$$\simeq 40.0 \text{ km}$$

IYGB - MMS PAPER B - QUESTION 9

a) LOOKING AT THE JOURNEY A TO C

$u = 5\ ms^{-1}$
$a = ?$
$s = 500\ m$
$t = 20\ s$
$v = -$

$$s = ut + \tfrac{1}{2}at^2$$
$$500 = 5 \times 20 + \tfrac{1}{2} \times a \times 20^2$$
$$500 = 100 + 200a$$
$$400 = 200a$$
$$a = 2\ ms^{-2}$$

b) LOOKING AT THE JOURNEY FROM A TO B, WITH $a=2$

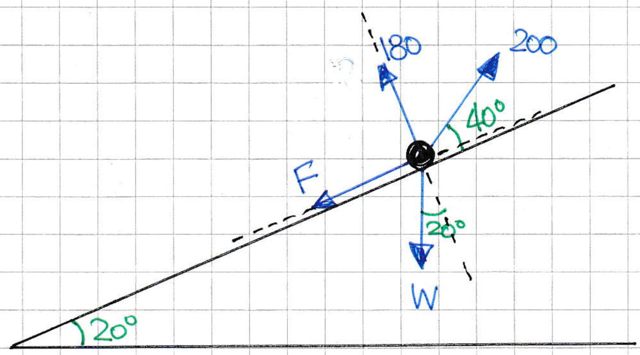
$u = 5\ ms^{-1}$
$a = 2\ ms^{-2}$
$s = 300\ m$
$t = ?$
$v = ?$

I) $v^2 = u^2 + 2as$
$$v^2 = 5^2 + 2 \times 2 \times 300$$
$$v^2 = 25 + 1200$$
$$v^2 = 1225$$
$$v = 35\ ms^{-1}$$

II) $v = u + at$
$$35 = 5 + 2t$$
$$30 = 2t$$
$$t = 15\ s$$

IYGB - MMS PAPER B - QUESTION 10

START WITH A DETAILED DIAGRAM



RESOLVING PARALLEL & PERPENDICULAR TO THE INCLINED PLANE

$(\parallel): \quad F + W\sin 20 = 200\cos 40°$

$(\perp): \quad 180 + 200\sin 40° = W\cos 20°$

SOLVING TO FIND W FROM THE SECOND EQUATION

$$W = \frac{180 + 200\sin 40}{\cos 20°}$$

$$W = 328.3600\ldots$$

$$W \simeq 328\text{N}$$

FINALLY WE CAN OBTAIN F

$$F = 200\cos 40 - W\sin 20$$

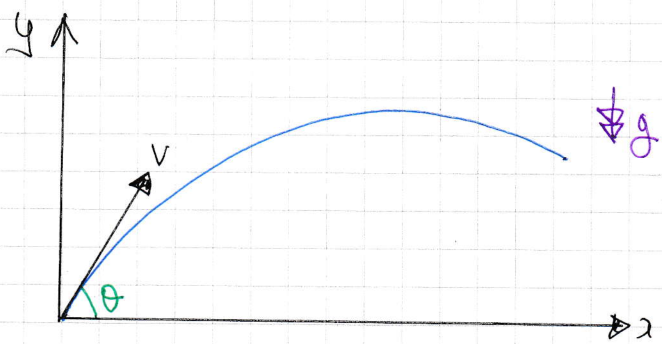
$$F = 200\cos 40 - (328.36\ldots)\sin 20$$

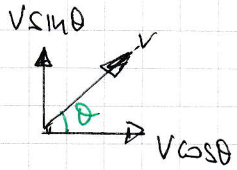
$$F = 40.9031\ldots$$

$$F \simeq 40.9\text{ N}$$

IYGB – MMS PAPER B – QUESTION 11

STARTING WITH A GENERAL DIAGRAM





FOR THE PARTICLE P,

$$\left.\begin{array}{l} x = V\cos\theta \times t \\ y = V\sin\theta \times t + \frac{1}{2}(-g)t^2 \end{array}\right\} \Rightarrow \begin{array}{l} x = Vt\cos\theta \\ y = Vt\sin\theta - 4.9t^2 \end{array}$$

COLLISION TAKES PLACE WHEN $t=5$ AND NOTE $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $V = 42$, $g = 9.8$

$$x = 42 \times 5 \times 0.6$$
$$y = 42 \times 5 \times 0.8 - 4.9 \times 5^2 \qquad \therefore (x,y) = (126, 45.5)$$

NOW PARTICLE Q IS AT $(126, 45.5)$ WHEN $t=2$ WITH $\theta = \psi$ AND $v = U$

$$\left.\begin{array}{l} x = U\cos\psi \times t \\ y = U\sin\psi \times t + \frac{1}{2}(-g)t^2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 126 = U\cos\psi \times 2 \\ 45.5 = 2U\sin\psi - 4.9 \times 2^2 \end{array}\right\} \Rightarrow$$

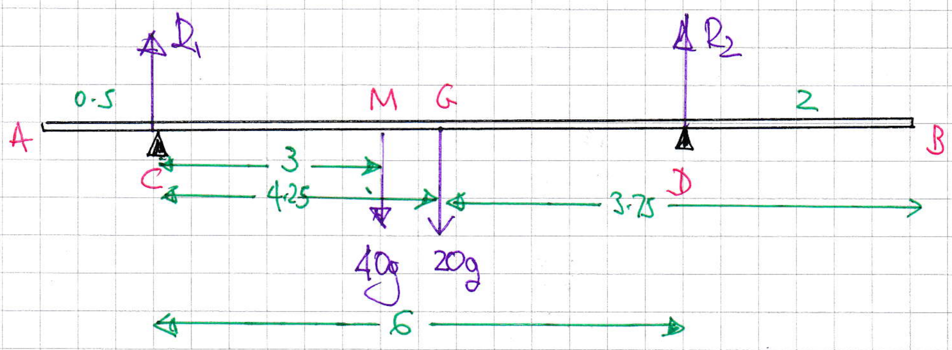
$$\boxed{\begin{array}{l} U\cos\psi = 63 \\ U\sin\psi = 32.55 \end{array}}$$

- DIVIDING $\tan\psi = \frac{31}{60}$
- SQUARING $U = \sqrt{63^2 + 32.55^2}$

$\therefore \psi \approx 27.3°$ & $U \approx 70.9 \text{ ms}^{-1}$

IYGB - MMS PAPER B - QUESTION 12

a) LOOKING AT A DIAGRAM



TAKING MOMENTS ABOUT C

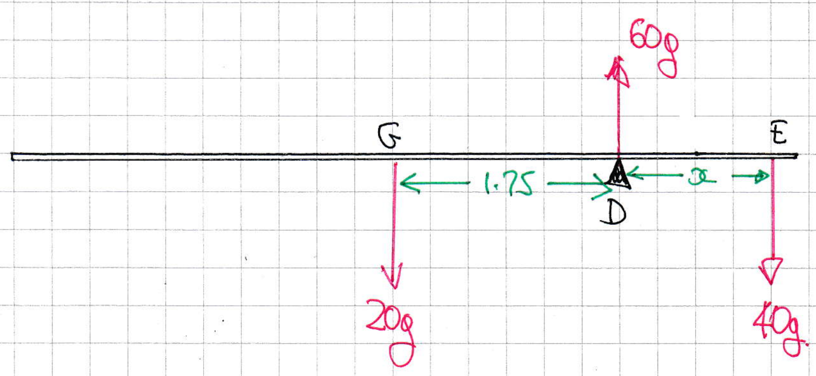
$$(40g \times 3) + (20g \times 4.25) = R_2 \times 6$$
$$120g + 85g = 6R_2$$
$$6R_2 = 205g$$
$$R_2 = 334.8333\ldots$$

RESOLVING VERTICALLY

$$R_1 + R_2 = 40g + 20g$$
$$R_1 = 60g - 334.8333\ldots$$
$$R_1 = 253.1666\ldots$$

∴ REACTIONS AT C & D ARE 253N & 335N RESPECTIVELY

b) "TILTING ABOUT D" $\Longrightarrow$ $R_1 = 0$ & $R_2 = 60g$

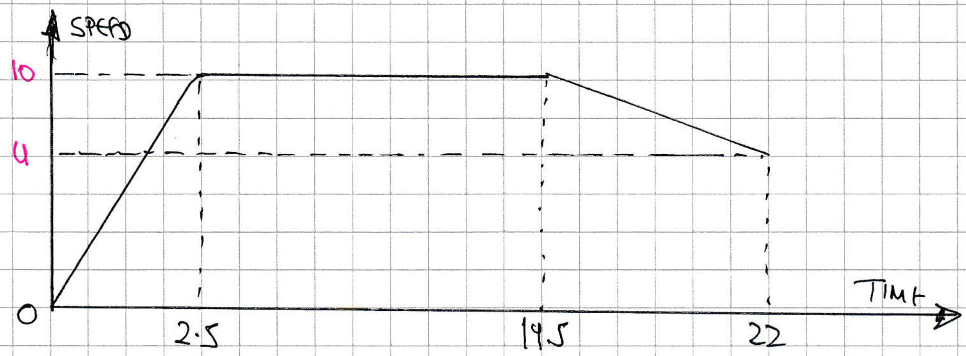


D: $20g \times 1.75 = 40g\,x$

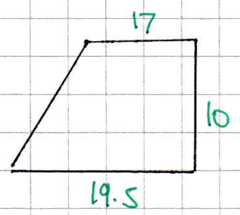
$$35g = 40g\,x$$
$$40x = 35$$
$$x = 0.875 \text{ m}$$

IYGB – MMS PAPER B – QUESTION 13

a) LOOKING AT THE SPEED TIME GRAPH



AREA OF TRAPEZIUM



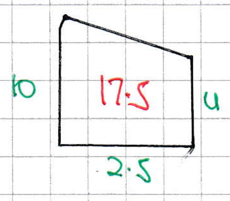
$$\text{DISTANCE} = \frac{17 + 19.5}{2} \times 10$$

$$= 18.25 \times 10$$

$$= 182.5 \text{ m}$$

b) AS THIS IS A 200 METRE RACE...

- $200 - 182.5 = 17.5$



$$\frac{10 + u}{2} \times 2.5 = 17.5$$

$$(10 + u) \times 2.5 = 35$$

$$10 + u = 14$$

$$u = 4 \text{ ms}^{-1}$$

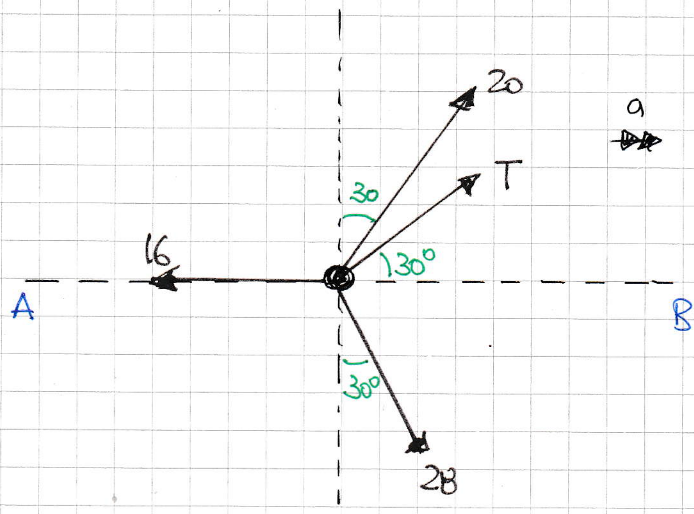
c) USING FACT THAT ACCELERATION = GRADIENT

$$a = \frac{u - 10}{22 - 19.5} = \frac{4 - 10}{2.5} = \frac{-6}{2.5} = -2.4$$

$\therefore$ DECELERATION OF 2.4 ms^{-2}

IYGB – MMS PAPER B – QUESTION 14

LOOKING AT THE DIAGRAM – IF THE PARTICLE IS ACCELERATING "ALONG AB" IT MUST BE IN EQUILIBRIUM "PERPENDICULAR" TO AB



$(\updownarrow)$: $20\cos 30 + T\sin 30 = 28\cos 30$ (EQUILIBRIUM)

$\Rightarrow 20 + T\tan 30 = 28$ DIVIDE THROUGH BY $\cos 30°$, TO CREATE $\tan 30°$

$\Rightarrow T\tan 30 = 8$

$\Rightarrow T = 8\sqrt{3}$ N

NOW IN THE DIRECTION AB THERE IS ACCELERATION, THEREFORE RESULTANT

$(\rightarrow)$: $20\sin 30 + T\cos 30 + 28\sin 30 - 16 = 80a$ $(F = ma)$

$\Rightarrow 20 \times \frac{1}{2} + 8\sqrt{3} \times \frac{\sqrt{3}}{2} + 28 \times \frac{1}{2} - 16 = 80a$

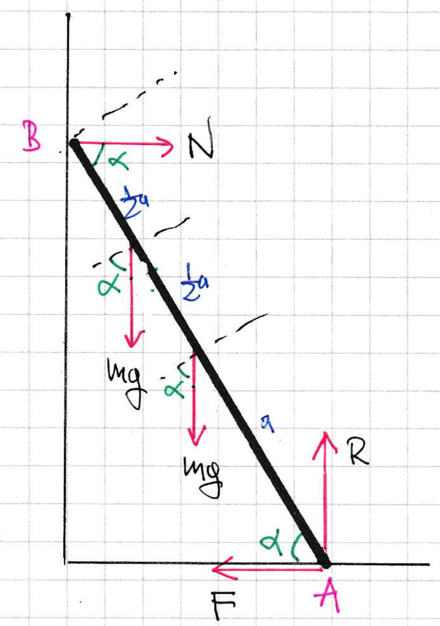
$\Rightarrow 10 + 12 + 14 - 16 = 80a$

$\Rightarrow 80a = 20$

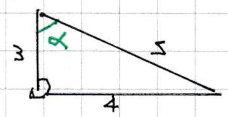
$\Rightarrow a = 0.25 \text{ ms}^{-2}$

IYGB - MMS PAPER B - QUESTION 15

STARTING WITH A DIAGRAM



$\tan\alpha = \frac{4}{3}$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

$(\uparrow)$: $R = 2mg$

$(\rightarrow)$: $N = F$

TAKING MOMENTS ABOUT A

$$\overset{\curvearrowleft}{A}: \quad mg\cos\alpha \times \cancel{a} + mg\cos\alpha \times \frac{3}{2}\cancel{a} = N\sin\alpha \times 2\cancel{a}$$

$$\frac{5}{2}mg\cos\alpha = 2N\sin\alpha$$

$$\frac{5}{2}mg = 2N\tan\alpha$$

$$\frac{5}{2}mg = \frac{8}{3}N$$

$$\frac{5}{2}mg = \frac{8}{3}F$$

$$F = \frac{15}{16}mg$$

BUT $F \leqslant \mu R$

$$\Rightarrow \frac{15}{16}mg \leqslant \mu(2mg)$$

$$\Rightarrow \frac{15}{16} \leqslant 2\mu$$

$$\Rightarrow \mu \geqslant \frac{15}{32}$$