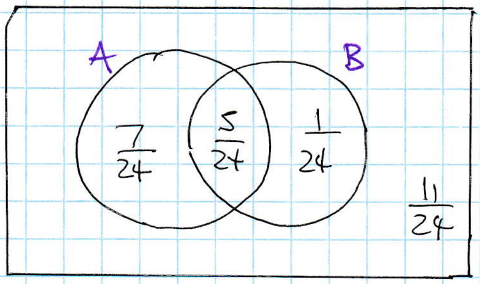
IYGB - MMS PAPER C - QUESTION 1



a) $P(A \cup B) = \frac{7}{24} + \frac{5}{24} + \frac{1}{24} = \frac{13}{24}$

b) $P[(A \cap B') \cup (B \cap A') \cup (A' \cap B')] = \frac{7}{24} + \frac{1}{24} + \frac{11}{24} = \frac{19}{24}$

$P(A' \cup B')$

c) $P(B \cap A') = \frac{1}{24}$

d) $P[(A \cap B') \cup (A' \cap B)] = \frac{7}{24} + \frac{1}{24} = \frac{8}{24}$

e) $P(A' \cap B') = \frac{11}{24}$

f) $P[(A \cap B) \cup (A' \cap B')] = \frac{5}{24} + \frac{11}{24} = \frac{16}{24}$

g) $P(A \cup (A' \cap B')) = \frac{7}{24} + \frac{5}{24} + \frac{11}{24} = \frac{23}{24}$

$P(A \cup B')$

IYGB - MMS PAPER C - QUESTION 2

a) EXPLANATORY (INDEPENDENT) IS THE "N" AS IT IS "N" THAT AFFECTS THE TEMPERATURE AND NOT THE OTHER WAY ROUND

RESPONSE VARIABLE (DEPENDENT) IS THE "T"

b) I) FROM A STATISTICAL CALCULATOR

$$\text{P.M.C.C} = r = 0.845$$

II) ALSO USING A CALCULATOR

$$T = 43.7 + 0.0076N$$

c) a – "y INTERCEPT"

a IS THE TEMPERATURE OF THE DRILL BIT BEFORE IT IS REVOLVED

b – "GRADIENT"

b IS THE EXTRA TEMPERATURE RISE PER REVOLUTION OF THE DRILL BIT

a IS UNLIKELY TO BE 43.7°C AS THIS WOULD REPRESENT THE "ROOM TEMPERATURE"

d) USING THE REGRESSION LINE

$$T = 43.7 + 0.0076N$$

- $T_{1600} = 43.7 + 0.0076 \times 1600 \simeq 55.9°$

UNRELIABLE AS "N" IS WAY ABOVE THE LARGEST VALUE OF N THAT WAS USED TO CREATE THE EQUATION OF THE REGRESSION LINE

- $T_{825} = 43.7 + 0.0076 \times 825 \simeq 50.0$

POSSIBLY RELIABLE AS IT LIES WITHIN THE VALUES OF "N" WHICH WAS USED TO CREATE THE REGRESSION LINE

IYGB - MMS PAPER C - QUESTION 3

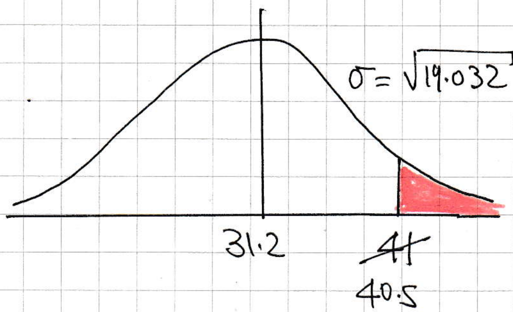
X = NUMBER OF FEMALE STUDENTS

$X \sim B(80, 0.39)$

- MEAN $= E(X) = np = 80 \times 0.39 = 31.2$
- VARIANCE $= \mathrm{Var}(X) = np(1-p) = 31.2 \times 0.61 = 19.032 > 5$

APPROXIMATE BY $Y \sim N(31.2, 19.032)$

$$P(X > 40)$$
$$= P(X \geq 41)$$
$$= P(Y > 40.5)$$
$$= 1 - P(Y < 40.5)$$
$$= 1 - P\left(Z < \frac{40.5 - 31.2}{\sqrt{19.032}}\right)$$
$$= 1 - \Phi(2.13177\ldots)$$
$$= 1 - 0.983487\ldots$$
$$= 0.0165$$



IYGB – MMS PAPER C – QUESTION 4

PREPARING FOR ALL THE ITEMS REQUIRED IN THE QUESTION, BY ADDING MORE COLUMNS

TIMES (nearest hour)	MIDPOINT	FREQUENCY		CLASS WIDTH	FREQUENCY DENSITY
11 – 14	12.5	24	24	4	$24 \div 4 = 6$
15 – 17	16	24	48	3	$24 \div 3 = 8$
18 – 19	18.5	19	67	2	$19 \div 2 = 9.5$
20	20	11		1	$11 \div 1 = 11$
21 – 23	22	21		3	$21 \div 3 = 7$
24 – 28	26	15		5	$15 \div 5 = 3$

a) USING CALCULATOR TO OBTAIN SUMS

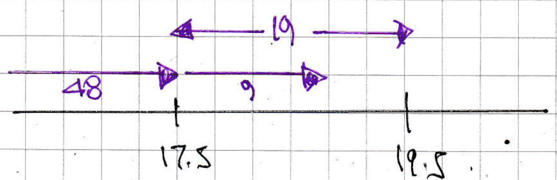
$$\sum x = 2107.5 \qquad \sum x^2 = 41100.75 \qquad n = 114$$

- MEAN $= \bar{x} = \frac{\sum x}{n} = \frac{2107.5}{114} \simeq 18.5$
- S.D $= \sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{41100.75}{114} - 18.4868...^2} \simeq 4.33$

b) AS THE NUMBER OF DATA IS LARGE

$Q_2 = \frac{1}{2} \times 114 = 57^{TH}$ OBS WHICH LIE IN 18 – 19

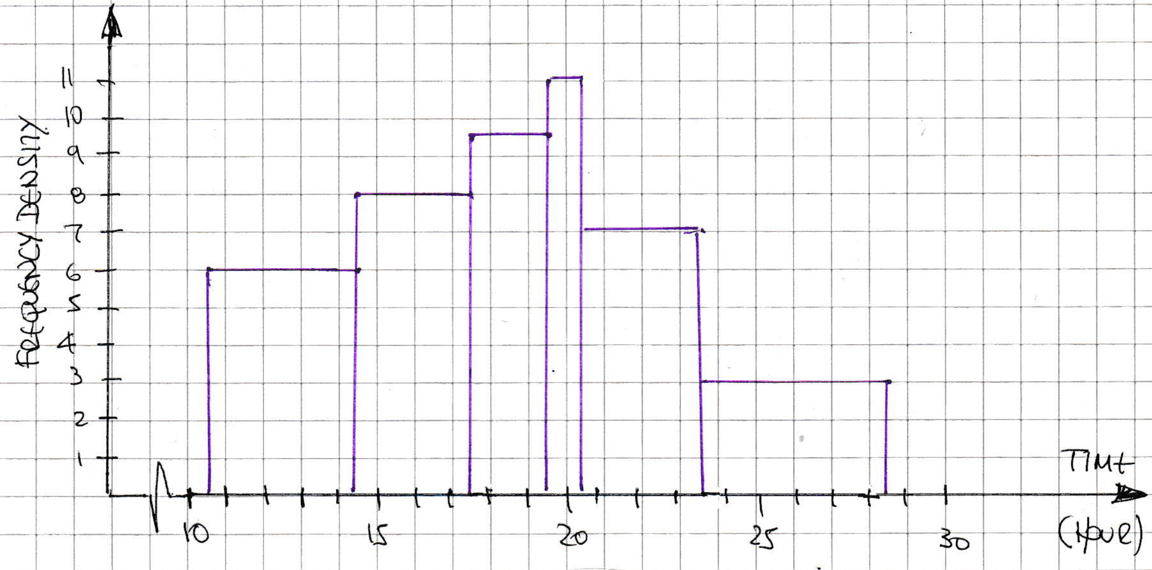
BY LINEAR INTERPOLATION



$$Q_2 \simeq 17.5 + \frac{9}{19} \times 2 \simeq 18.4$$

YGB - MMS PAPER C - QUESTION 4

c) USING THE FREQUENCY DENSITY FROM THE TABLE



d) CALCULATE BOUNDS

$\bar{x} + 3\sigma = 18.5 + 3 \times 4.33 = 31.49$

$\bar{x} - 3\sigma = 18.5 - 3 \times 4.33 = 5.51$

ALL THE DATA (100 %) LIES WITHIN 3 STANDARD DEVIATIONS OF THE MEAN

e) IN SUPPORT OF A NORMAL

- MEDIAN ≈ MEAN
- ALL DATA IS CONTAINED WITHIN 3σ

AGAINST A NORMAL

- HISTOGRAM SHOWS POSITIVE SKEW

POSSIBLY NOT APPROPRIATE DUE TO THE SKEW

IYGB - MMS PAPER C - QUESTION 5

$X \sim B(40, 0.2)$

- H_0 : $p = 0.2$
- H_1 : $p \neq 0.2$, p IS THE PROPORTION IN THE ENTIRE POPULATION

CRITICAL REGION REQUIRED, AT 5%, TWO TAILED, IE 2.5% IN EACH TAIL

$P(X \leq 2) = 0.0079 = 0.79\% < 2.5\%$

$P(X \leq 3) = 0.0285 = 2.85\% > 2.5\%$

$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$

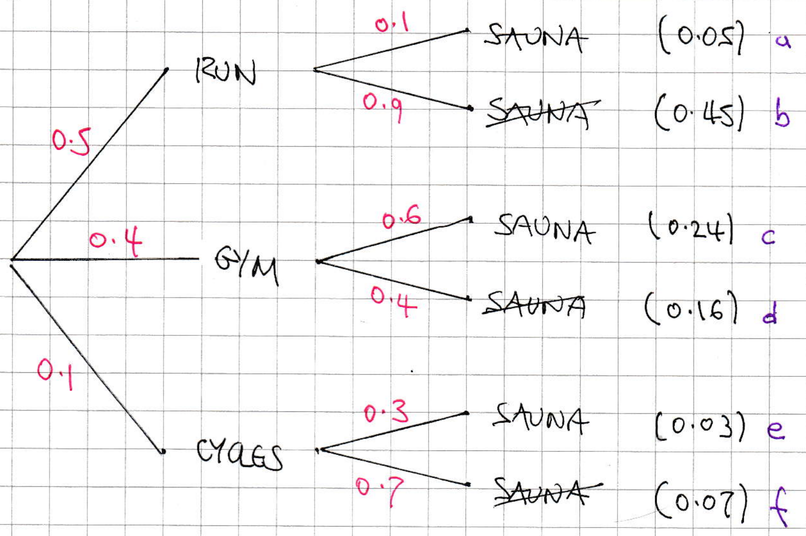
$P(X \geq 13) = 1 - P(X \leq 12) = 1 - 0.9568 = 0.0432 = 4.32\% > 2.5\%$

$P(X \geq 14) = 1 - P(X \leq 13) = 1 - 0.9806 = 0.0194 = 1.94\% < 2.5\%$

$\therefore$ CRITICAL REGION IS $\{0, 1, 2\} \cup \{14, 15, 16, \ldots 40\}$

IYGB - MMS PAPER C - QUESTION 6

a) USING A TREE DIAGRAM



$$P(\text{SAUNA}) = \text{"}a+c+e\text{"} = 0.05 + 0.24 + 0.03 = 0.32$$

b) $$P(\text{GYM} \mid \text{SAUNA}) = \frac{P(\text{GYM} \cap \text{SAUNA})}{P(\text{SAUNA})} = \frac{c}{a+c+e} = \frac{0.24}{0.32} = 0.75$$

c) $$P(\text{NO RUN} \mid \text{NO SAUNA}) = \frac{P(\text{NO RUN} \cap \text{NO SAUNA})}{P(\text{NO SAUNA})}$$

$$= \frac{d+f}{b+d+f}$$

$$= \frac{0.16 + 0.07}{0.16 + 0.07 + 0.45} \quad \leftarrow \text{OR } 1 - 0.32$$

$$= \frac{23}{68} \approx 0.3382$$

IYGB – MMS PAPER C – QUESTION 7

X = VOLUME OF OIL (ml).

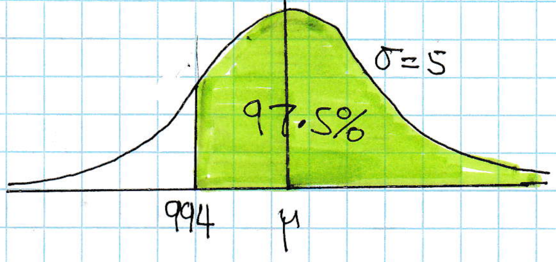
$X \sim N(\mu, 5^2)$

a) LOOKING AT THE DIAGRAM BELOW

$P(X > 994) = 97.5\%$

$P\left(Z > \frac{994-\mu}{5}\right) = 0.975$

$\Big\downarrow$ INVERSION



$\frac{994-\mu}{5} = -\Phi^{-1}(0.975)$

$\frac{994-\mu}{5} = -1.96$

$994 - \mu = 9.8$

$\mu = 1003.8$

b) $P(X < 992)$

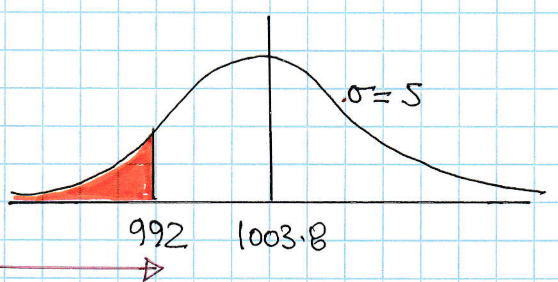
$= 1 - P(X > 992)$

$= 1 - P\left(Z > \frac{992-1003.8}{5}\right)$

$= 1 - \Phi(-2.36)$

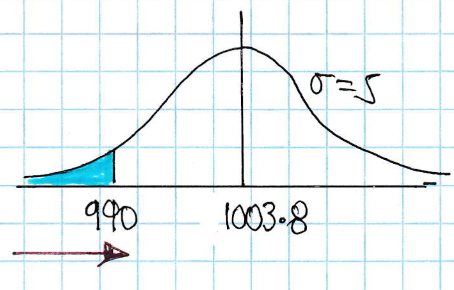
$= 1 - 0.9909$

$= 0.0091$



IYGB – MMS PAPER C – QUESTION 7

c) FIRST CALCULATE THE PROBABILITY OF A BOTTLE HAVING LESS THAN 990 ml



$$
\begin{aligned}
&P(X<990) \\
&= 1 - P(X>990) \\
&= 1 - P\left(Z > \frac{990-1003.8}{5}\right) \\
&= 1 - \Phi(-2.76) \\
&= 1 - 0.99711 \quad \text{(CALCULATOR)} \\
&= 0.00289
\end{aligned}
$$

THE REQUIRED PROBABILITY CAN NOW BE FOUND

$$
\begin{aligned}
P(X<990 \mid \text{FINE}) &= P(X<990 \mid X<992) \\
&= \frac{P(X<990 \cap X<992)}{P(X<992)} \\
&= \frac{P(X<990)}{P(X<992)} \\
&= \frac{0.00289}{0.0091} \\
&= 0.31758\ldots \\
&\approx 0.318
\end{aligned}
$$

IYGB - MMS PAPER C - QUESTION 8

a) ORGANISE OUTCOMES

0,0	0,1 / 1,0	0,3 / 3,0	1,1	1,3 / 3,1	3,3
$\frac{1}{6}\times\frac{1}{6}$	$\frac{1}{6}\times\frac{1}{3}\times2$	$\frac{1}{6}\times\frac{1}{2}\times2$	$\frac{1}{3}\times\frac{1}{3}$	$\frac{1}{3}\times\frac{1}{2}\times2$	$\frac{1}{2}\times\frac{1}{2}$

LET $Y = X_1 + X_2$

y	0	1	2	3	4	6
$P(Y=y)$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$

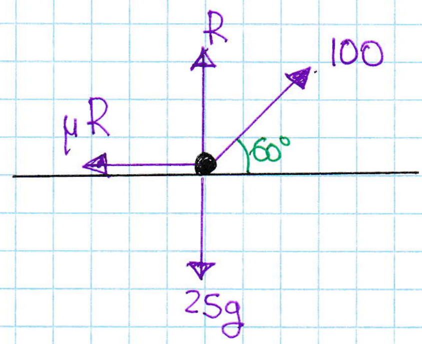
b) ORGANIZE "NEW" OUTCOMES WITH $X_1 > X_2$

1,0   3,0   3,1

$$\left(\frac{1}{3}\times\frac{1}{6}\right)+\left(\frac{1}{2}\times\frac{1}{6}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right) = \frac{1}{18}+\frac{1}{12}+\frac{1}{6} = \frac{11}{36}$$

IYGB – MMS PAPER C – QUESTION 9

FORMING TWO EQUATIONS, BASED ON THE DIAGRAM OPPOSITE



$(\updownarrow)$: $R + 100\sin 60 = 25g$

$(\leftrightarrow)$: $\mu R = 100\cos 60$

FROM THE FIRST EQUATION

$$R = 25g - 100\sin 60$$

SUBSTITUTE INTO THE OTHER EQUATION

$$\mu(25g - 100\sin 60) = 100\cos 60$$

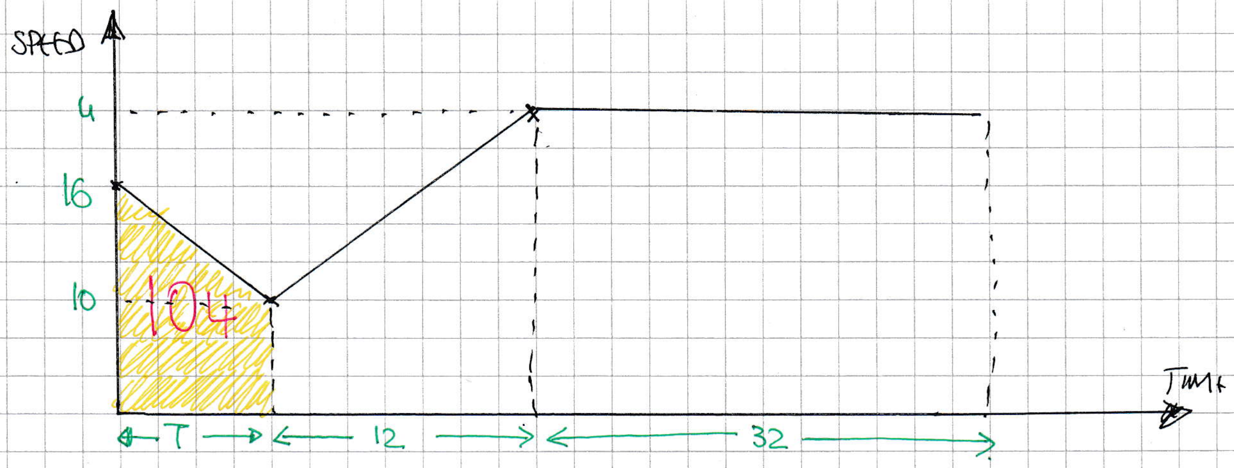
$$\mu = \frac{100\cos 60}{25g - 100\sin 60}$$

$$\mu = 0.3156616\ldots$$

$$\mu \simeq 0.316$$

IYGB - MMS PAPER C - QUESTION 10

a) DRAWING A SPEED TIME GRAPH



b) AREA OF YELLOW TRAPEZIUM IS 104

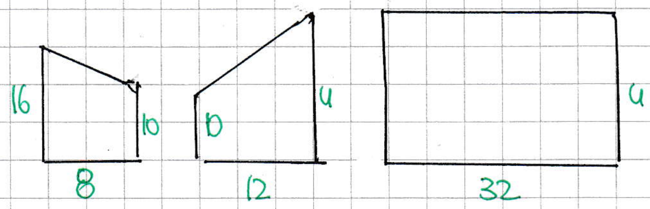
$$\frac{16+10}{2} \times T = 104$$

$$13T = 104$$

$$T = 8$$

$\therefore$ TOTAL TIME $= 8 + 12 + 32 = 52\text{ s}$

c) TOTAL AREA (DISTANCE) IS 1000 m



$= 1000$

$$104 + \frac{10+u}{2} \times 12 + 32u = 1000$$

$$104 + (10+u) \times 6 + 32u = 1000$$

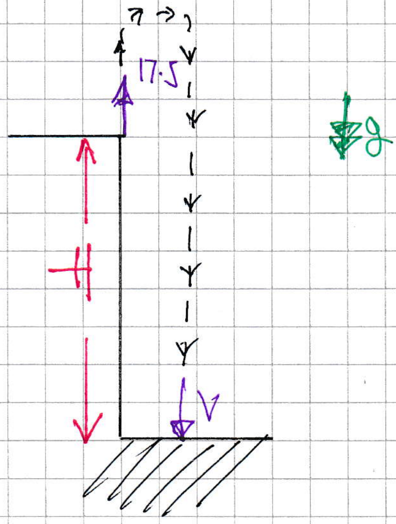
$$104 + 60 + 6u + 32u = 1000$$

$$38u = 836$$

$$u = 22$$

IYGB - MMS PAPER C - QUESTION 11

a) LOOKING AT THE DIAGRAM, USING DISPLACEMENTS & CONSIDERING THE ENTIRE JOURNEY



$$\left|\begin{array}{l} u = +17.5 \\ a = -9.8 \\ s = -H \\ t = 5 \\ v = \text{(ALSO NEEDED FOR PART (b))} \end{array}\right.$$

USING $\Rightarrow$ "$s = ut + \frac{1}{2}at^2$"

$$-H = 17.5 \times 5 + \tfrac{1}{2}(-9.8) \times 5^2$$
$$-H = 87.5 - 122.5$$
$$-H = -35$$
$$H = 35$$

b) V CAN FIRSTLY BE FOUND FROM PART (a)

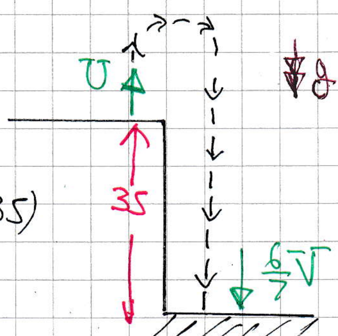
$$\left|\begin{array}{l} u = 17.5 \\ a = -9.8 \\ s = -35 \\ t = 5 \\ v = ? \end{array}\right|$$

"$v = u + at$" $\Rightarrow$ $v = 17.5 - 9.8 \times 5$

$v = -31.5$

$\therefore$ SPEED $V = 31.5$

NEW DIAGRAM, AND CONSIDERING DISPLACEMENTS FOR THE ENTIRE JOURNEY



$$\left|\begin{array}{l} u = U \\ a = -9.8 \\ s = -35 \\ t = \text{—} \\ v = -\frac{6}{7}V \end{array}\right|$$

$\Rightarrow$ "$v^2 = u^2 + 2as$"

$\Rightarrow$ $\left(-\frac{6}{7}V\right)^2 = U^2 + 2(-9.8)(-35)$

$\Rightarrow$ $\frac{36}{49}V^2 = U^2 + 686$

IYGB - MMS PAPER C - QUESTION 11

$$\Longrightarrow \frac{36}{49}(31.5)^2 = v^2 + 686$$

FAILURE

$$\Longrightarrow 729 = v^2 + 686$$

$$\Longrightarrow 43 = v^2$$

$$\Longrightarrow v = +\sqrt{43}$$

$$\Longrightarrow v \simeq 6.56 \text{ ms}^{-1}$$

IYGB – MMS PAPER C – QUESTION 12

a) USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$\Rightarrow 8\underline{i} + 20\underline{j} = 12\underline{i} + 12\underline{j} + \underline{v} \times 2$

$\Rightarrow -4\underline{i} + 8\underline{j} = 2\underline{v}$

$\Rightarrow \underline{v} = -2\underline{i} + 4\underline{j}$

b) USING $\underline{r} = \underline{r}_0 + \underline{v}t$ AGAIN WITH VELOCITY FOUND IN (a)

$\Rightarrow \underline{r} = 12\underline{i} + 12\underline{j} + (-2\underline{i} + 4\underline{j})t$

$\Rightarrow \underline{r} = (12 - 2t)\underline{i} + (12 + 4t)\underline{j}$

c) INTERCEPTION IMPLIES SAME POSITION VECTOR AT THE SAME TIME

$\underline{r}_A = \underline{r}_B \Rightarrow (12 - 2t)\underline{i} + (12 + 4t)\underline{j} = (2t + k)\underline{i} + (2t + 33)\underline{j}$

$$\Rightarrow \begin{cases} 12 - 2t = 2t + k \\ 12 + 4t = 2t + 33 \end{cases}$$

$$\Rightarrow \begin{cases} k = 12 - 4t \\ 2t = 21 \end{cases}$$

$$\Rightarrow \begin{cases} k = 12 - 4t \\ 4t = 42 \end{cases}$$

$\Rightarrow k = -30$

IYGB - MMS PAPER C - QUESTION 13

a) FIND AN EXPRESSION FOR THE VELOCITY

$$\Rightarrow a = 6t - 18$$

$$\Rightarrow v = \int 6t - 18 \, dt$$

$$\Rightarrow v = 3t^2 - 18t + C$$

APPLY CONDITION $t=0 \quad v=+15$

$15 = 0 - 0 + C$

$C = 15$

USE THE VELOCITY EXPRESSION WITH $v=0$

$$\Rightarrow v = 3t^2 - 18t + 15$$

$$\Rightarrow 0 = 3t^2 - 18t + 15$$

$$\Rightarrow 0 = t^2 - 6t + 5$$

$$\Rightarrow 0 = (t-1)(t-5)$$

$$\Rightarrow t = \begin{cases} 1 \\ 5 \end{cases}$$

b) INTEGRATE THE VELOCITY EXPRESSION, TO OBTAIN A DISPLACEMENT

$$\Rightarrow v = 3t^2 - 18t + 15$$

$$\Rightarrow x = \int 3t^2 - 18t + 15 \, dt$$

$$\Rightarrow x = t^3 - 9t^2 + 15t + D$$

WHEN $t=0 \quad x=0$ (ORIGIN)

$\therefore D = 0$

$$\Rightarrow \boxed{x = t^3 - 9t^2 + 15t}$$

IYGB - MMS PAPER G - QUESTION 13

$$x_1 = 1^3 - 9\times1^2 + 15\times1 = 1-9+15 = 7$$

$$x_5 = 5^3 - 9\times5^2 + 15\times5 = 125-225+75 = -25$$

$\therefore$ TOTAL DISTANCE IS $7+25 = 32\text{ m}$

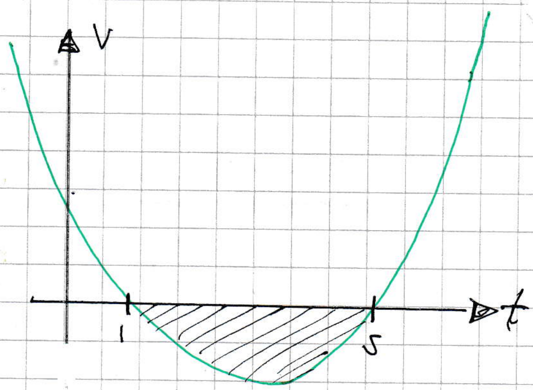
ALTERNATIVE FOR PART (b) BY SPEED TIME GRAPH

SKETCHING

$$V = 3t^2 - 18t + 15$$

$$V = 3\left[t^2 - 6t + 5\right]$$

$$V = 3(t-1)(t-5)$$



$$\therefore \text{ DISTANCE} = \left|\int_1^5 3t^2 - 18t + 15 \, dt\right|$$

$$= \left|\left[t^3 - 9t^2 + 15t\right]_1^5\right|$$

$$= \left|(125 - 225 + 75) - (1 - 9 + 15)\right|$$

$$= \left|-25 - 7\right|$$

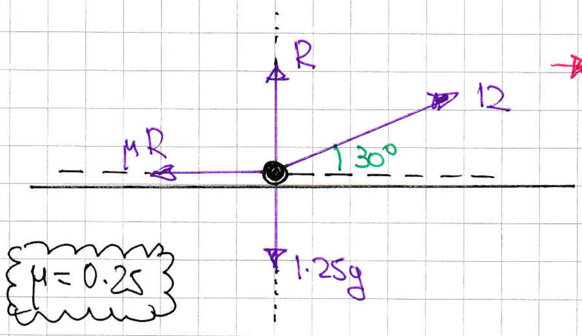
$$= \left|-32\right|$$

$$= 32\text{ m}$$

AS ABOVE

IYGB - MMS PAPER C - QUESTION 14

a) STARTING WITH A DETAILED DIAGRAM



RESOLVING VERTICALLY (EQUILIBRIUM) & HORIZONTALLY ("F = ma")

(↑): $R + 12\sin 30 = 1.25g$

$R + 6 = 12.25$

$\underline{R = 6.25\text{ N}}$

(→): $12\cos 30 - \mu R = ma$

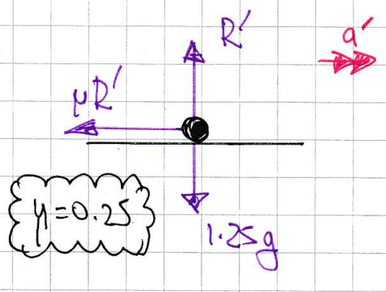
$12 \times \frac{\sqrt{3}}{2} - 0.25(6.25) = 1.25a$

$6\sqrt{3} - 1.5625 = 1.25a$

$a = 7.063843876...$

$a \simeq 7.06\text{ ms}^{-2}$

b) RECALCULATE THE ACCELERATION IN THE ABSENCE OF PULL



(↑): $R' = 1.25g$ [EQUILIBRIUM]

(→): $-\mu R' = ma'$ ["F = ma"]

$\Rightarrow -\mu(\cancel{1.25}g) = \cancel{1.25}a'$

$\Rightarrow \underline{a' = -2.45\text{ ms}^{-2}}$

FINALLY KINEMATICS

$u = 7.35\text{ ms}^{-1}$

$a = -2.45\text{ ms}^{-2}$

$s =$

$t = ?$

$v = 0$

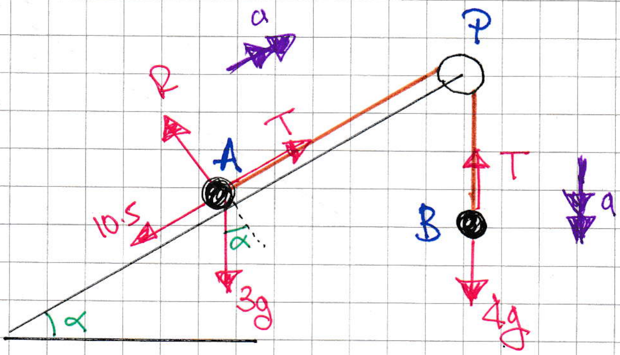
$v = u + at \Rightarrow 0 = 7.35 + (-2.45)t$

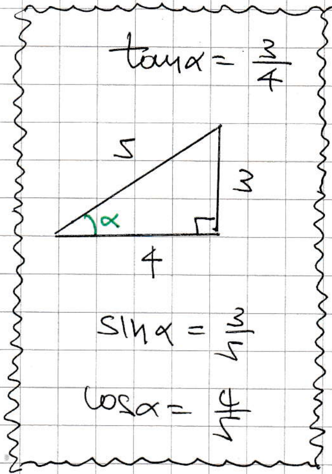
$\Rightarrow 2.45t = 7.35$

$\Rightarrow t = 3\text{ s}$

IYGB - MMS PAPER C - QUESTION 15

a) START WITH A DETAILED DIAGRAM





LOOKING AT THE EQUATION OF MOTION FOR EACH PARTICLE

(A): $T - 10.5 - 3g\sin\alpha = 3a$ ← "$F = ma$" FOR EACH

(B): $4g - T = 4a$

$\Rightarrow 4g - 10.5 - 3g\sin\alpha = 7a$ ← ADDING THE EQUATIONS

$\Rightarrow 4g - 10.5 - 3g\left(\frac{3}{5}\right) = 7a$

$\Rightarrow 11.06 = 7a$

$\Rightarrow a = 1.58 \text{ ms}^{-2}$

b) FIRST WE NEED TO FIND THE TENSION IN THE STRING

$\Rightarrow 4g - T = 4a$

$\Rightarrow 4g - 4a = T$

$\Rightarrow T = 4 \times 9.8 - 4 \times 1.58$

$\Rightarrow T = 32.88 \text{ N}$

IYGB – MMS PAPER C – QUESTION 15

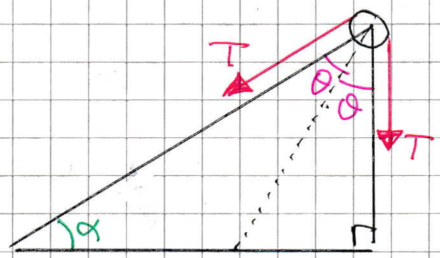
LOOKING AT THE DIAGRAM BELOW

- $\theta = \dfrac{90-\alpha}{2} = \dfrac{90 - \arctan\frac{3}{4}}{2}$

  $\theta = 26.6°$ TO THE VERTICAL

  $\uparrow$

  $\left(\arctan\frac{1}{2}\right)$



- MAGNITUDE OF FORCE ON P

  $F = 2T\cos\theta = 2 \times 32.88 \times \cos(26.6\ldots) = 58.8175\ldots$

  $F \simeq 58.8 \text{ N}$ //

c) USE KINEMATICS FOR CONSTANT ACCELERATION

$u = 0 \text{ ms}^{-1}$
$a = 1.58 \text{ ms}^{-2}$
$s = ?$
$t = 2 \text{ s}$
$v = ?$

TWO SECONDS INTO THE MOTION

- $v = u + at$

  $v = 0 + 1.58 \times 2$

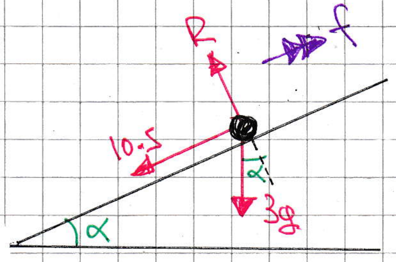
  $v = 3.16 \text{ ms}^{-1}$

- $s = \left(\dfrac{u+v}{2}\right)t$

  $s = \left(\dfrac{0+3.16}{2}\right) \times 2$

  $s = 3.16 \text{ m}$

NEXT RECALCULATE THE ACCELERATION (DECELERATION)



(NOTE THAT THERE IS NO TENSION AFTER 2 s)

"$F = ma$" IN THE DIRECTION OF MOTION

$-10.5 - 3g\sin\alpha = 3f$

$-10.5 - 17.64 = 3f$

$f = -9.38 \text{ ms}^{-2}$

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FINALLY KINEMATICS UNDER THE NEW DECELERATION

$u = 3.16 \text{ ms}^{-1}$

$a = -9.38 \text{ ms}^{-2}$

$s = ?$

$t =$

$v = 0$

"$v^2 = u^2 + 2as$"

$0 = 3.16^2 + 2(-9.38)s$

$18.76s = 9.9856$

$s = 0.53228\ldots$

$\therefore$ TOTAL DISTANCE UP THE PLANE IS

$= 3.16 + 0.53228\ldots$

$\approx 3.69 \text{ m}$ // 3sf