IYGB – MMS PAPER E – QUESTION 1

a) USING A CALCULATOR IN STATISTICAL MODE

I) P.M.C.C = $r = 0.635$

II) REGRESSION LINE $\Rightarrow$ $L = a + bM$

$L = 80.3 + 3.94M$

b) USING THE REGRESSION LINE

I) IF $M = 9.8$

$L = 80.3 + 3.94 \times 9.8 = 119$

SHOULD BE RELIABLE (WITHIN REASON AS THE PMCC IS ONLY 0.635) AS THE VALUE OF M LIES WITHIN THE VALUES OF M THAT WAS USED TO CREATE THE REGRESSION LINE

II) IF $M = 20$

$L = 80.3 + 3.94 \times 20 = 159$

NOT LIKELY TO BE RELIABLE AS THIS VALUE OF M IS "WAY ABOVE" IS (EXTRAPOLATION)

c) a = "Y INTERCEPT"

a IS THE NUMBER OF LISTENERS IF NO MONEY WAS SPEND IN MARKETING/ADVERTISING

b = "GRADIENT"

b IS EXTRA LISTENERS PER 1000 SPENT ON MARKETING/ADVERTISING

d) "RESIDUAL = ACTUAL – PREDICTED"

(14.25)

125

$80.3 + 3.94 \times 14.25 = 136$

$\therefore$ RESIDUAL $= -11$

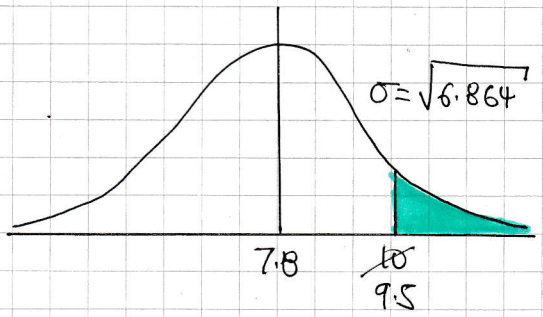
IYGB - MMS PAPER E - QUESTION 2

X = NUMBER OF PEOPLE WHO PREFER "READY SALTED" CRISPS

$X \sim B(65, 0.12)$

- MEAN $= E(X) = np = 65 \times 0.12 = 7.8$
- VARIANCE $= \text{Var}(X) = np(1-p) = 7.8 \times 0.88 = 6.864 > 5$

APPROXIMATE BY $Y \sim N(7.8, 6.864)$



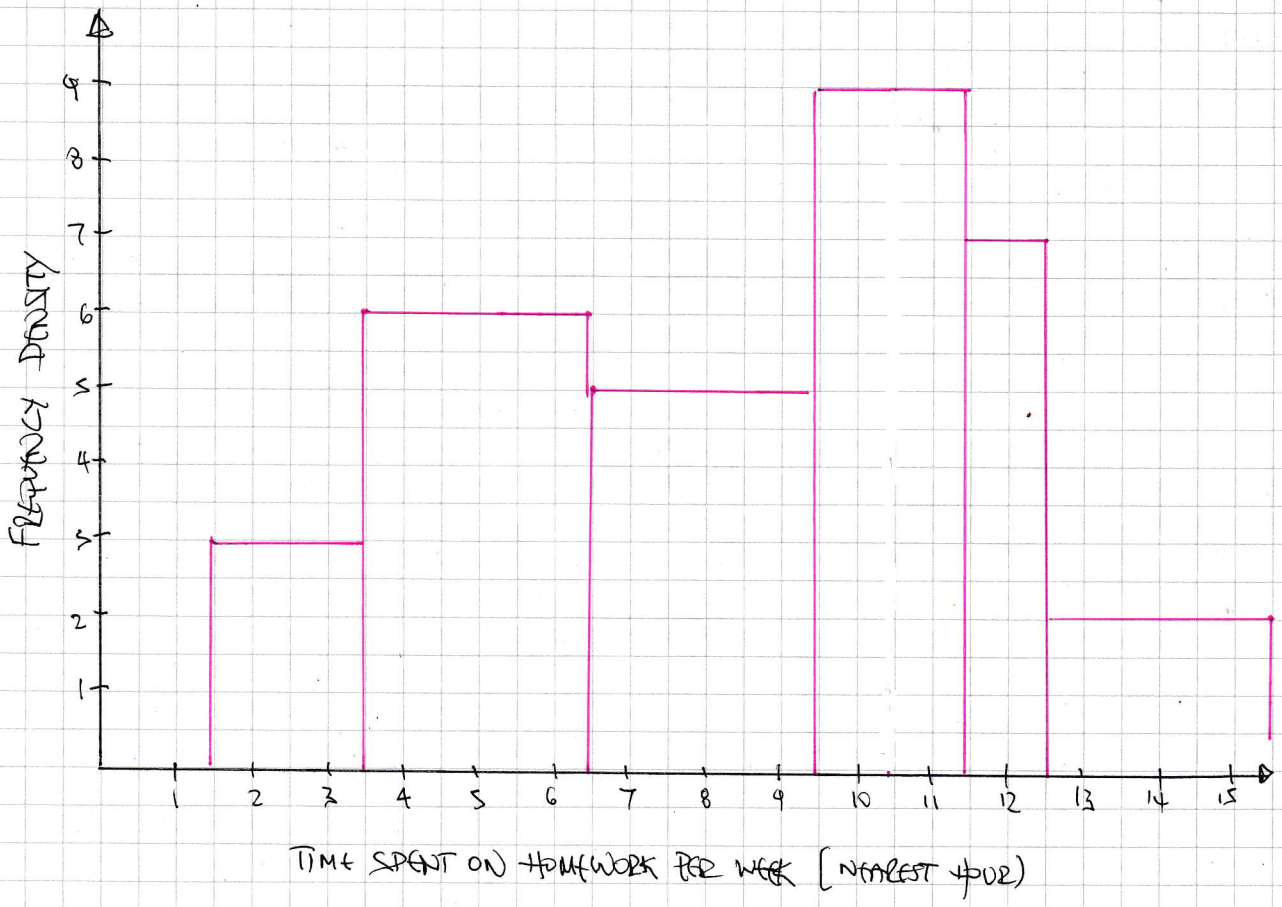
$$
\begin{aligned}
&P(X > 9) \\
&= P(X \geqslant 10) \\
&= P(Y > 9.5) \\
&= 1 - P(Y < 9.5) \\
&= 1 - P\left(Z < \frac{9.5 - 7.8}{\sqrt{6.864}}\right) \\
&= 1 - \Phi(0.6488738\ldots) \\
&= 1 - 0.74179\ldots \\
&= 0.2582
\end{aligned}
$$

IYGB - MMS PAPER E - QUESTION 3

a) STARTING WITH A TABLE WITH FREQUENCY DENSITY

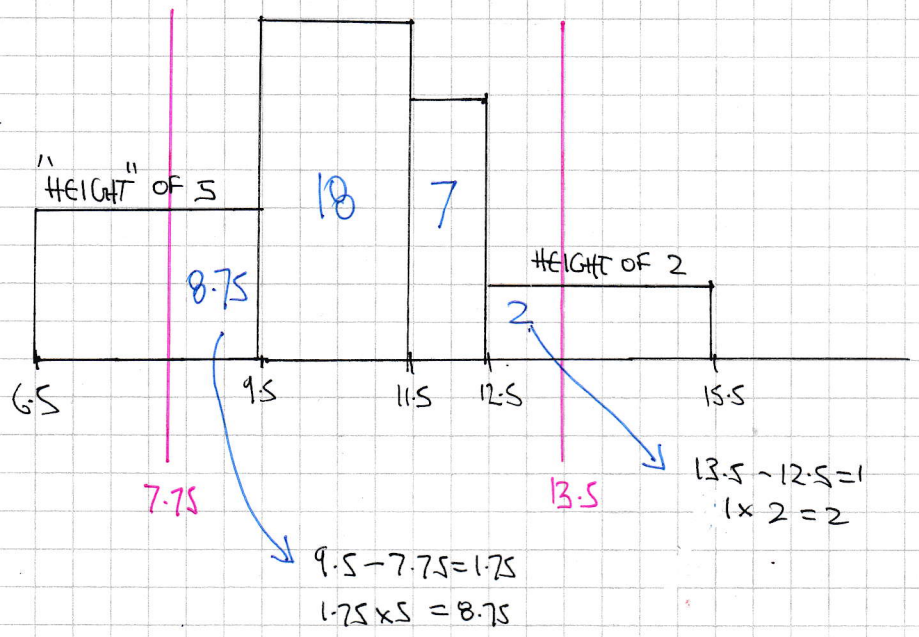
CLASS	CLASS WIDTH	FREQUENCY	FREQUENCY DENSITY
2–3	2	6	$6 \div 2 = 3$
4–6	3	18	$18 \div 3 = 6$
7–9	3	15	$15 \div 3 = 5$
10–11	2	18	$18 \div 2 = 9$
12	1	7	$7 \div 1 = 7$
13–15	3	6	$6 \div 3 = 2$

PLOTTING AS APPROPRIATE



IYGB – MMS PAPER E – QUESTION 3

b) USING THE HISTOGRAM (NOT TO SCALE THE DIAGRAM BELOW)



ESTIMATE $= 8.75 + 18 + 7 + 2 \simeq 36$ STUDENTS

c) EASIER TO USE INTERPOLATION (WHICH IS USING A HISTOGRAM WITHOUT DRAWING A DIAGRAM)

MEDIAN: $70 \div 2 = 35^{TH}$ OBS

MEDIAN LIES IN 7 – 9

$$Q_2 = 6.5 + \frac{35 - (6+18)}{18} \times 2$$

$$Q_2 = 6.5 + \frac{11}{9}$$

$$Q_2 = \frac{139}{18}$$

$\therefore$ MEDIAN $\simeq 7.7$

IYGB - MMS PAPER E - QUESTION 4

RECONSTRUCT THE TABLE

MILEAGES	MIDPOINTS (x)	$y = \frac{x-3325}{50}$	FREQUENCY (f)
$3250 \leqslant m < 3300$	3275	-1	19
$3300 \leqslant m < 3350$	3325	0	45
$3350 \leqslant m < 3400$	3375	1	16
$3400 \leqslant m < 3450$	3425	2	5
$3450 \leqslant m < 3500$	3475	3	2

CALCULATE SUMMARY STATISTICS IN y

$$\sum fy = 13 \qquad \sum fy^2 = 73 \qquad \sum f = 87$$

CALCULATE MEAN & STANDARD DEVIATION IN y

- $\bar{y} = \dfrac{\sum fy}{\sum f} = \dfrac{13}{87} \approx 0.1494\ldots$

- $\sigma_y = \sqrt{\dfrac{\sum fy^2}{\sum f} - \bar{y}^2} = \sqrt{\dfrac{73}{87} - \left(\dfrac{13}{87}\right)^2} \approx 0.90374\ldots$

UNCODING BACK INTO x

- $\bar{x} = \bar{y} \times 50 + 3325 \approx 3332$

- $\sigma_x = \sigma_y \times 50 \approx 45.187\ldots \approx 45.2$

IYGB – MMS PAPER E – QUESTION 5

X = NUMBER OF SIXES

$X \sim B\left(15, \frac{1}{6}\right)$

a) $P(X=0) = \binom{15}{0}\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{15} = 0.0649$

b) $P(X=3) = \binom{15}{3}\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{12} = 0.2363$

c) $P(X>3) = P(X \geqslant 4) = 1 - P(X \leqslant 3) =$ ...CALCULATOR...

$= 1 - 0.76848... = 0.2315$

d) REMODELLING WITH A NEW VARIABLE

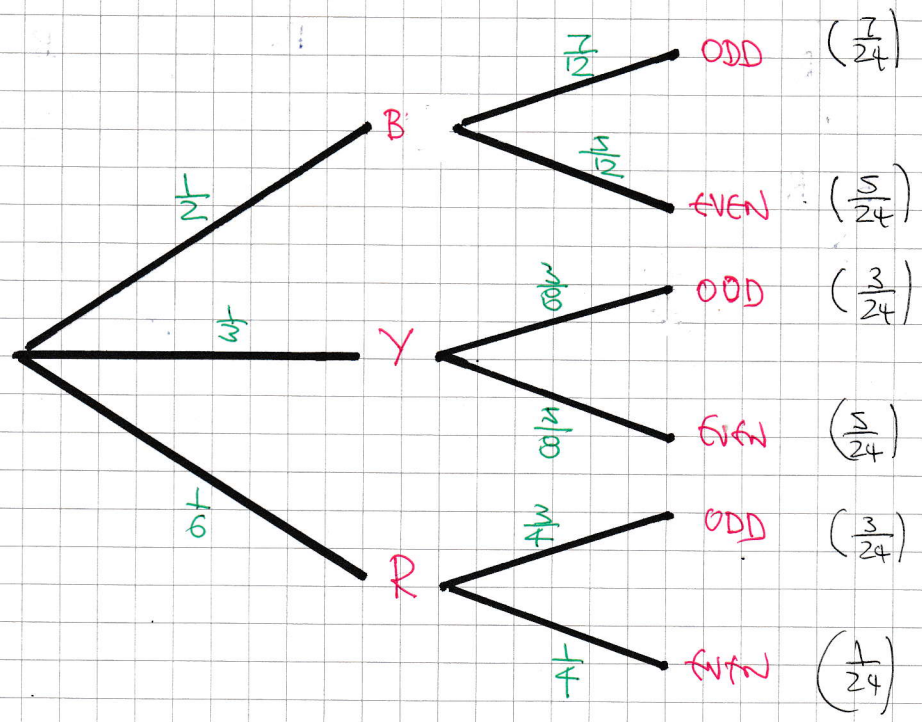
Y = NUMBER OF GRAMS WITH MORE THAN 3 SIXES

$Y \sim B(10, 0.2315)$

$P(Y=5) = \binom{10}{5}(0.2315)^{5}(1-0.2315)^{5} = 0.0449$

IYGB – MMS PAPER E – QUESTION 6

a) USING A TREE DIAGRAM FOR THIS PROBLEM



$$P(\text{EVEN}) = \left(\frac{1}{2}\times\frac{5}{12}\right)+\left(\frac{1}{3}\times\frac{5}{8}\right)+\left(\frac{1}{6}\times\frac{1}{4}\right) = \frac{5}{24}+\frac{5}{24}+\frac{1}{24} = \frac{11}{24}$$

b) $$P(\text{NOT RED}\,|\,\text{EVEN}) = \frac{P(\text{NOT RED}\cap\text{EVEN})}{P(\text{EVEN})}$$

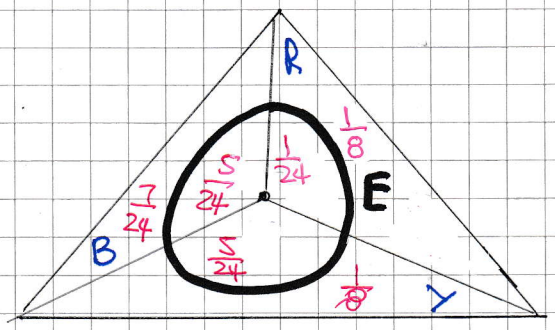
$$= \frac{P(B\cap E)+P(Y\cap E)}{\frac{11}{24}}$$

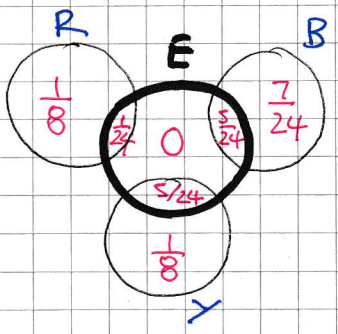
$$= \frac{\frac{5}{24}+\frac{5}{24}}{\frac{11}{24}}$$

$$= \frac{10}{11}$$

IYGB - MMS PAPER E - QUESTION 6

ALTERNATIVE BY VENN DIAGRAMS





IYGB - MMS PAPER E-QUESTION 7

$X \sim B(30, 0.4)$

$H_0 : p = 0.4$

$H_1 : p \neq 0.4$, p PROPORTION OF THE POPULATION

CRITICAL REGION REQUIRED AT 5% SIGNIFICANCE, TWO TAILED SO 2.5% IN EACH TAIL

$P(X \leq 6) = 0.0172 = 1.72\% < 2.5\%$

$P(X \leq 7) = 0.0435 = 4.35\% > 2.5\%$

$\vdots$

$P(X \geqslant 17) = 1 - P(X \leq 16) = 1 - 0.9519 = 0.0481 = 4.81\% > 2.5\%$

$P(X \geqslant 18) = 1 - P(X \leq 17) = 1 - 0.9788 = 0.0212 = 2.12\% < 2.5\%$

$\therefore$ CRITICAL REGION IS

$$\{0, 1, 2, \ldots, 6\} \cup \{18, 19, 20, \ldots, 30\}$$

IYGB - MMS PAPER E - QUESTION 8

$P(A) = 0.5 \quad P(B) = 0.2 \quad P(A|B) = 0.3$

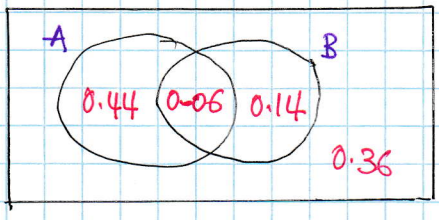
a) USING CONDITIONAL PROBABILITY FORMULA

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow 0.3 = \frac{P(A \cap B)}{0.2}$$

$$\Rightarrow P(A \cap B) = 0.06$$

FILL IN A VENN DIAGRAM

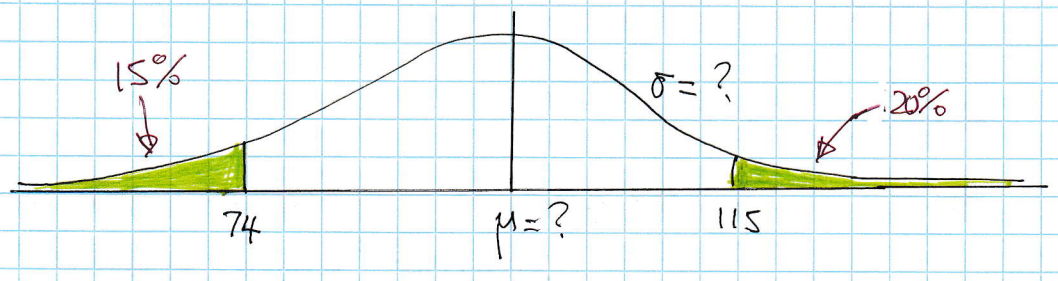


b) $P(A \cup B) = 1 - 0.36 = 0.64$ FROM DIAGRAM

c) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.06}{0.5} = 0.12$

IYGB - MMS PAPER E - QUESTION 9

PUTTING THE INFORMATION IN A DIAGRAM



T = time to complete exam

$T \sim N(\mu, \sigma^2)$

- $P(T<74) = 15\%$

$\Rightarrow P(T>74) = 85\%$

$\Rightarrow P\left(z > \frac{74-\mu}{\sigma}\right) = 0.85$

INVERSION

$\Rightarrow \frac{74-\mu}{\sigma} = -\Phi^{-1}(0.85)$

$\Rightarrow \frac{74-\mu}{\sigma} = -1.0364$

$\Rightarrow 74-\mu = -1.0364\sigma$

$\Rightarrow \boxed{74+1.0364\sigma = \mu}$

- $P(T>115) = 20\%$

$\Rightarrow P(T<115) = 80\%$

$\Rightarrow P\left(z > \frac{115-\mu}{\sigma}\right) = 0.8$

INVERSION

$\Rightarrow \frac{115-\mu}{\sigma} = +\Phi^{-1}(0.80)$

$\Rightarrow \frac{115-\mu}{\sigma} = 0.8416$

$\Rightarrow 115-\mu = 0.8416\sigma$

$\Rightarrow \boxed{115-0.8416\sigma = \mu}$

SOLVING SIMULTANEOUSLY

$\Rightarrow 74+1.0364\sigma = 115-0.8416\sigma$

$\Rightarrow 1.8780\sigma = 41$

$\Rightarrow \sigma = 21.83173589\ldots$

$\Rightarrow \sigma \approx 22$

& $\mu \approx 96.62641108\ldots$

$\mu \simeq 97$

IYGB – MMS PAPER E – QUESTION 9

b) SETTING UP HYPOTHESES

- H_0 : $\mu = 97$
- H_1 : $\mu > 97$, WHERE μ IS THE MEAN TIME FOR ALL STUDENTS (POPULATION MEAN)

$n = 10$, $\bar{T}_{10} = 108$, $\sigma = 22$, 5% SIGNIFICANCE

CRITICAL VALUE $\Phi^{-1}(0.95) = 1.6449$

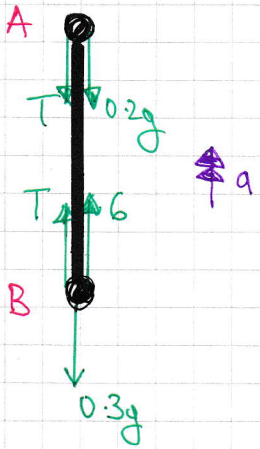
- Z STATISTIC $= \dfrac{\bar{T} - \mu}{\sigma/\sqrt{n}} = \dfrac{108 - 97}{22/\sqrt{10}} = 1.581138\ldots$

- AS $1.581138\ldots < 1.6449$, THERE IS NO SIGNIFICANT EVIDENCE AT 5%, TO SUPPORT THE EXAM SECRETARY'S BELIEF.

- THERE IS NO SUFFICIENT EVIDENCE TO REJECT H_0

IYGB – MMS PAPER E – QUESTION 10

STARTING WITH A DIAGRAM – MARK THE TRUSTS AS TENSIONS



"F = ma"

(A) $0 - T - 0.2g = 0.2a$

(B) $6 + T - 0.3g = 0.3a$

ELIMINATE THE TENSION/THRUST

$$6 - 0.5g = 0.5a$$

$$12 - g = a$$

$$a = 2.2 \text{ ms}^{-2}$$

FINALLY WE HAVE

$$-T - 0.2g = 0.2a$$

$$T + 0.2g = -0.2a$$

$$T = -0.2(g + a)$$

$$T = -0.2(9.8 + 2.2)$$

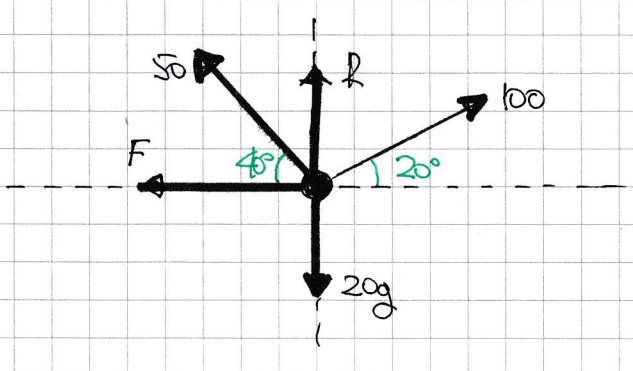
$$T = -0.2 \times 12$$

$$T = -2.4$$

$\therefore$ THRUST OF 2.4 N

IYGB - MMS PAPER E - QUESTION 11

STARTING WITH A DIAGRAM



RESOLVING FORCES IN EQUILIBRIUM

$(\updownarrow)$: $R + 100\sin 20^\circ + 50\sin 40^\circ = 20g$

$(\leftrightarrow)$: $F + 50\cos 40 = 100\cos 20^\circ$

SOLVING EACH EQUATION SEPARATELY

$R = 20g - 100\sin 20 - 50\sin 40$

$R = 129.658\ldots$

$R \approx 130\text{N}$

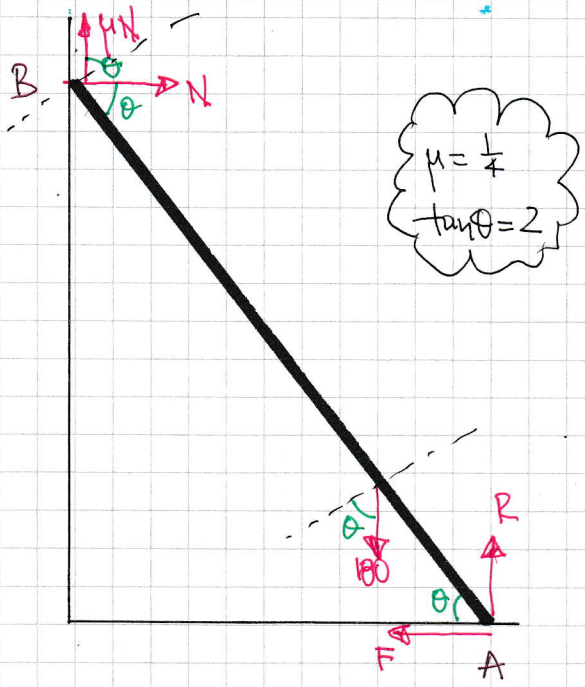
$F = 100\cos 20^\circ - 50\cos 40$

$F = 55.667\ldots$

$F \approx 55.7\text{ N}$

IYGB – MMS PAPER E – QUESTION 12

STARTING WITH A DETAILED DIAGRAM – LADDER IN LIMITING EQUILIBRIUM



$\mu = \frac{1}{4}$

$\tan\theta = 2$

(↑) $R + \mu N = 180$

(→) $N = F$

TAKING MOMENTS ABOUT A

$\curvearrowleft A$: $180\cos\theta \times 1.5 = N\sin\theta \times 6 + \mu N\cos\theta \times 6$

$270\cos\theta = 6N\sin\theta + 6\mu N\cos\theta$ $\quad \div \cos\theta$

$270 = 6N\tan\theta + 6\mu N$

$270 = 6F \times 2 + 6 \times \frac{1}{4} \times F$

$270 = 12F + \frac{3}{2}F$

$540 = 24F + 3F$

$27F = 540$

$F = 20$

IYGB - MMS PAPER E - QUESTION 13

a) SKETCHING A SPEED TIME GRAPH

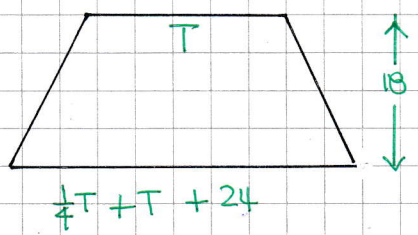
b) ACCELERATION = GRADIENT

$$\frac{\Delta v}{\Delta t} = -0.75 \implies \frac{0-18}{\Delta t} = -0.75$$

$$\implies \Delta t = 24$$

It DECELERATES FOR 24 s

c) i) TOTAL DISTANCE IS 1512



$$\implies \frac{(\frac{1}{4}T + T + 24) + T}{2} \times 18 = 1512$$

$$\implies (\tfrac{9}{4}T + 24) \times 9 = 1512$$

$$\implies \tfrac{9}{4}T + 24 = 168$$

$$\implies \tfrac{9}{4}T = 144$$

$$\implies 9T = 576$$

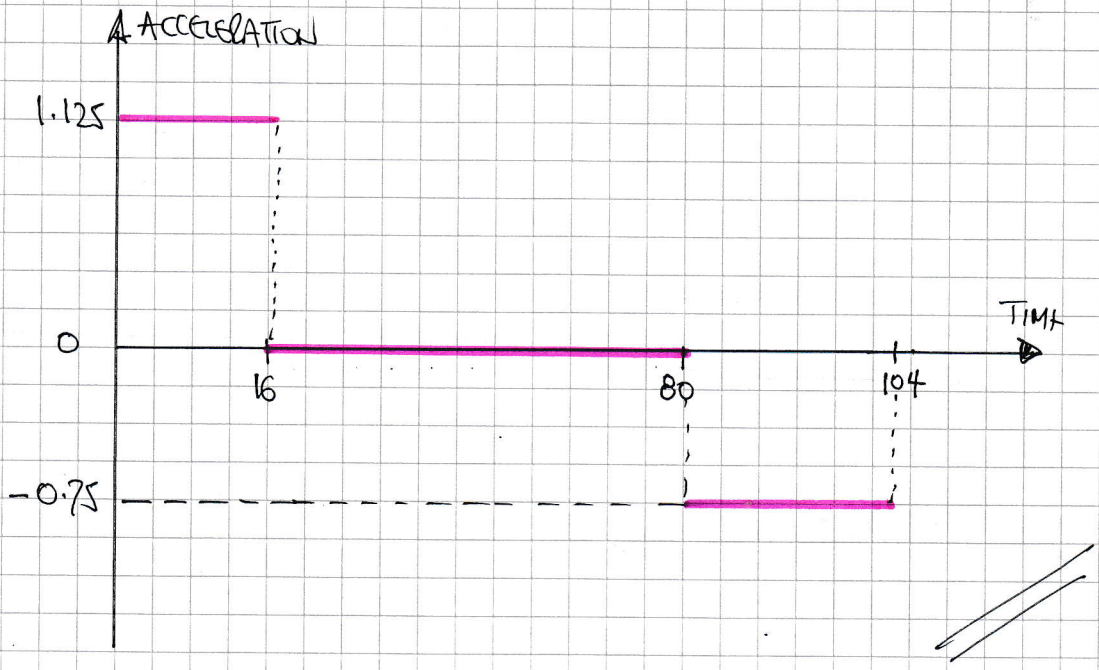
$$\implies T = 64$$

ii) ACCELERATION = GRADIENT $\implies a = \frac{18-0}{\frac{1}{4}T} = \frac{18}{\frac{1}{4}\times 64} = \frac{18}{16}$

$$\therefore a = \frac{9}{8} = 1.125 \text{ ms}^{-2}$$

IYGB - MMS PAPER E - QUESTION 13

c) FINALLY THE ACCELERATION - TIME GRAPH



IYGB - MMS PAPER E - QUESTION 14

a) DIFFERENTIATE W.R.T t, TO FIND AN EXPRESSION FOR THE ACCELERATION

$$v = t^2 - 4t - 12$$

$$a = \frac{dv}{dt} = 2t - 4$$

$$a\Big|_{t=3} = 2\times 3 - 4$$

$$a = 2 \text{ ms}^{-2}$$

b) SOLVE $v=0$

$$\Rightarrow t^2 - 4t - 12 = 0$$

$$\Rightarrow (t-6)(t+2) = 0$$

$$\Rightarrow t = 6, \cancel{-2}$$

$$\therefore a\Big|_{t=6} = 2\times 6 - 4$$

$$= 8 \text{ ms}^{-2}$$

c) INTEGRATE THE VELOCITY EXPRESSION, TO OBTAIN A DISPLACEMENT EXPRESSION

$$v = t^2 - 4t - 12$$

$$\Rightarrow x = \int v\,dt = \int t^2 - 4t - 12\,dt$$

$$\Rightarrow x = \frac{1}{3}t^3 - 2t^2 - 12t + C$$

$t=0 \quad x=20$

$$20 = 0 + C$$

$$\Rightarrow x = \frac{1}{3}t^3 - 2t^2 - 12t + 20$$

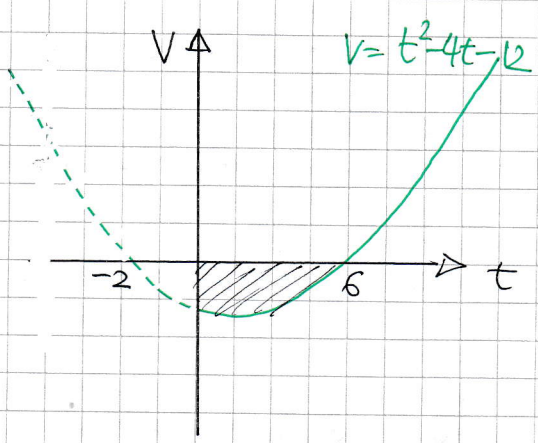
$$x\Big|_6 = \frac{1}{3}\times 6^3 - 2\times 6^2 - 12\times 6 + 20 = 72 - 72 - 72 + 20 = -52$$

$\therefore$ DISTANCE OF 52 m

IYGB - MMS PAPER E - QUESTION 14

c) ALTERNATIVE BY SPEED TIME GRAPH (FOR PART c)

- SKETCH THE SPEED TIME GRAPH.



- "SHADED AREA" $= \int_0^6 t^2-4t-12\,dt$

$$= \left[\frac{1}{3}t^3-2t^2-12t\right]_0^6$$

$$= (72-72-72)-0$$

$$= -72$$

- NOW THE PARTICLE WAS +20 (DISPLACEMENT) WHEN $t=0$
  HENCE THE DISPLACEMENT IS $-72+20=-52$

- THUS THE DISTANCE IS 52 m

1YGB – MMS PAPER E – QUESTION 15

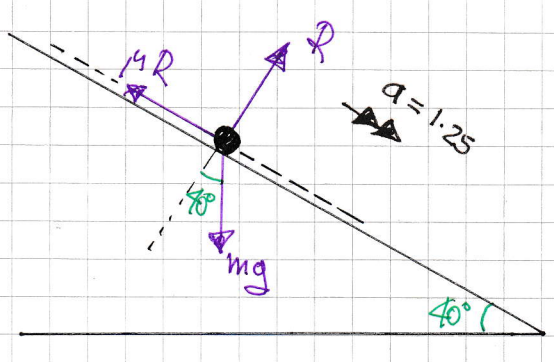
a) STARTING WITH KINEMATICS FOR THE JOURNEY A TO B

$u = 3\ ms^{-1}$
$a = ?$
$s = 22m$
$t = 4s$
$V = ?$

- $s = ut + \frac{1}{2}at^2$
  $22 = 3\times4 + \frac{1}{2}\times a \times 4^2$
  $22 = 12 + 8a$
  $8a = 10$
  $a = 1.25\ ms^{-2}$

- $V = u + at$
  $V = 3 + 1.25\times4$
  $V = 8\ ms^{-1}$

b) LOOKING AT THE DIAGRAM BELOW



RESOLVING PARALLEL (||) & PERPENDICULAR TO THE PLANE (⊥)

(⊥): $R = mg\cos40$ [EQUILIBRIUM]

(||): $mg\sin40 - \mu R = ma$ [$F = ma$]

SOLVING BY SUBSTITUTION

$\Rightarrow mg\sin40 - \mu(mg\cos40) = ma$

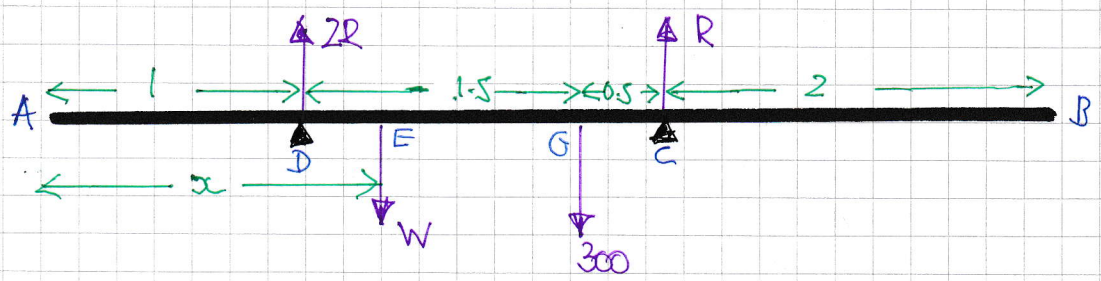
$\Rightarrow g\sin40 - \mu g\cos40 = a$

$\Rightarrow g\sin40 - a = \mu g\cos40$

$\Rightarrow \mu = \frac{g\sin40 - a}{g\cos40} = \frac{(9.8\times\sin40) - 1.25}{9.8\times\cos40} \simeq 0.673$

IYGB - MMS PAPER E - QUESTION 16

a) START WITH A DIAGRAM



RESOLVING VERTICALLY

$$2R + R = W + 300$$

$$3R = W + 300$$

$$R = \frac{1}{3}W + 100$$

TAKING MOMENTS ABOUT A

$$(2R \times 1) + (R \times 3) = Wx + 300 \times 2.5$$

$$2R + 3R = Wx + 750$$

$$5R = Wx + 750$$

$$5\left(\tfrac{1}{3}W + 100\right) = Wx + 750$$

$$\frac{5W}{3} + 500 = Wx + 750$$

$$5W + 1500 = 3Wx + 2250$$

$$5W - 3Wx = 750$$

$$W(5 - 3x) = 750$$

$$W = \frac{750}{5 - 3x}$$

∴ REQUIRED

b) TWO CONSTRAINTS TO BE SATISFIED

$0 < x < 5$ AND $W > 0$

$$\frac{750}{5 - 3x} > 0$$

$$5 - 3x > 0$$

$$-3x > -5$$

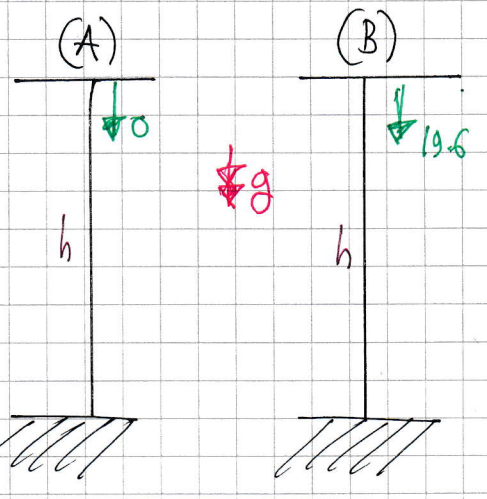
$$x < \frac{5}{3}$$

∴ COMBINING $0 < x < \frac{5}{3}$

IYGB – MMS PAPER E – QUESTION 17

START WITH A DETAILED DIAGRAM

FOR A	FOR B
$u = 0$	$u = 19.6$
$a = 9.8$	$a = 9.8$
$s = h$	$s = h$
$t = T$	$t = T-1$
$V = ?$	$V = ?$



USING $s = ut + \frac{1}{2}at^2$ FOR BOTH

A: $s = ut + \frac{1}{2}at^2$

$h = 0(T) + \frac{1}{2}(9.8)T^2$

$h = 4.9T^2$

B: $s = ut + \frac{1}{2}at^2$

$h = 19.6(T-1) + \frac{1}{2}(9.8)(T-1)^2$

$h = 19.6T - 19.6 + 4.9(T-1)^2$

BY SUBSTITUTION

$\Rightarrow 4.9T^2 = 19.6T - 19.6 + 4.9(T-1)^2$ $\quad \div 4.9$

$\Rightarrow T^2 = 4T - 4 + (T-1)^2$

$\Rightarrow T^2 = 4T - 4 + T^2 - 2T + 1$

$\Rightarrow 3 = 2T$

$\Rightarrow T = \frac{3}{2}$

FINALLY USING $h = 4.9T^2$

$h = 4.9 \times \left(\frac{3}{2}\right)^2$

$h = 11.025$

$h \simeq 11.0$ m.

IYGB – MMS PAPER E – QUESTION 18

a) FIRSTLY FIND THE VELOCITY OF THE BALL

- IF THE VELOCITY OF THE BALL WAS $4\underline{i}+3\underline{j}$, ITS SPEED WOULD HAVE BEEN $\sqrt{4^2+3^2} = 5\ ms^{-1}$
- AS THE SPEED IS $10\ ms^{-1}$, IE TWICE AS LARGE, THE VELOCITY OF THE BALL IS $2(4\underline{i}+3\underline{j}) = 8\underline{i}+6\underline{j}$

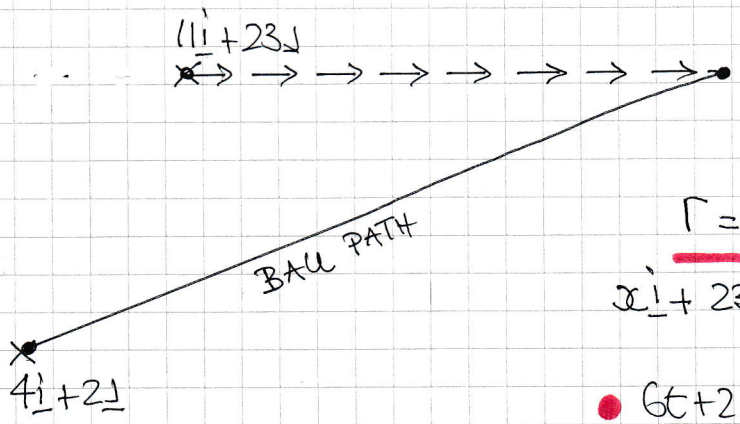
USING $\underline{r} = \underline{r}_0 + \underline{v}t$

$$\underline{r} = (4\underline{i}+2\underline{j}) + (8\underline{i}+6\underline{j})t$$

$$\underline{r} = (8t+4)\underline{i} + (6t+2)\underline{j}$$

AS REQUIRED

b) LOOKING AT A DIAGRAM – NOTE B IS RUNNING FAST



$$\underline{r} = (8t+4)\underline{i} + (6t+2)\underline{j}$$

$$x\underline{i} + 23\underline{j} = (8t+4)\underline{i} + (6t+2)\underline{j}$$

- $6t+2 = 23$
  $6t = 21$
  $t = 3.5$
- $x = 8\times 3.5 + 4$
  $x = 32$

$\therefore$ FROM $(11, 23)$ TO $(32, 23)$ IN 3.5 SECONDS

$$v = \frac{32-11}{3.5}$$

$$v = 6\ ms^{-1}$$