IYGB GCE

Mathematics MMS

Advanced Level

Practice Paper T

Difficulty Rating: 4.7600/1.6129

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 14 questions in this question paper.
The total mark for this paper is 150.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

SECTION 1 – STATISTICS

Question 1

A certain type of drug takes on average 8 minutes to act.

a) Assuming this time is Normally distributed with a standard deviation of 1.5, find the time under which the drug begins to act in 5% of the patients. **(4)**

A patient is given this drug.

b) Given that the drug took less than 8 minutes to act, determine the probability that it actually took more than 5 minutes to act. **(4)**

c) Given instead that the drug took more than 5 minutes to act, determine the probability that it actually took less than 8 minutes to act. **(3)**

Question 2

A discrete random variable X has distribution

$$X \sim \mathrm{B}\left(8, \tfrac{1}{4}\right).$$

Two independent observations of X, denoted by X_1 and X_2 are considered.

a) Determine $\mathrm{P}(X_1 + X_2 \le 3)$. **(5)**

Ten independent observations of X are selected at random.

b) Determine the probability that half of these observations will be a 2. **(4)**

Question 3

A box contains 5 balls of which 2 are white.

Balls are drawn from the box one after the other, without being replaced, until both the white balls are picked.

If the second ball picked is white, determine the probability that exactly 4 balls were picked out of the box.

(9)

Question 4

A multiple choice paper has n questions, where $n > 20$.

Each question has 5 options of which only 1 is correct.

A pass is obtained if at least 20 questions are answered correctly. The probability of obtaining a pass by randomly guessing the answers is less than 2.5%.

By using a distributional approximation, calculate the least value of n. **(12)**

Question 5

The probability distribution of a discrete random variable X is given by

$$P(X = x) = \begin{cases} k & x = 1 \\ \frac{1}{2}P(X = x-1) & x = 2,3,4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

Three independent observations of X are made, denoted by X_1, X_2 and X_3, and the variable Y is defined as $Y = X_1 + X_2 + X_3$.

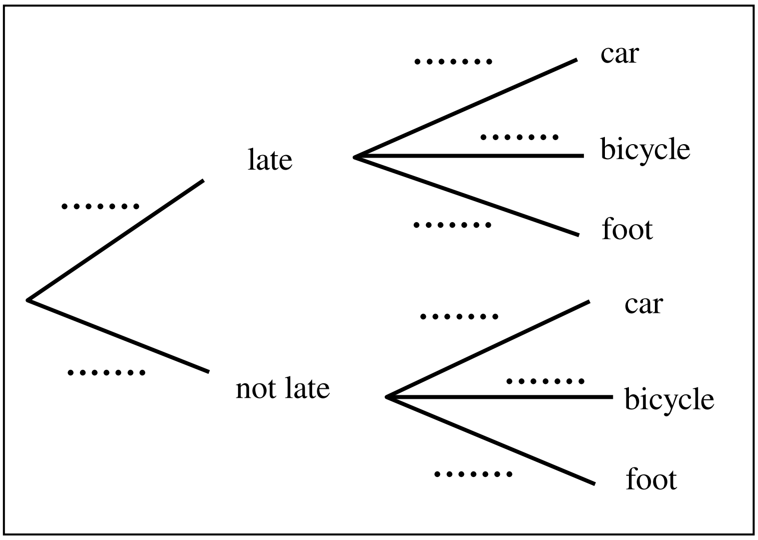
If Y is an even number, determine the probability that Y is greater than 9. **(10)**

Question 6

The probability that Phil goes to work by car, by bicycle or on foot are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively.

The respective probabilities of Phil being **late** when using these 3 forms of transport are $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$.

Complete the following tree diagram to illustrate the above information.



(10)

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Question 7

The events C and D are such so that

$$\mathrm{P}(C)=\tfrac{1}{3}, \qquad \mathrm{P}(D)=\tfrac{7}{36}, \qquad \mathrm{P}\big[(C\cap D')\cup(C'\cap D)\big]=\tfrac{13}{36}.$$

a) Find, showing a full clear method, the value of $\mathrm{P}(C\cap D)$. **(8)**

If **instead** the events C and D satisfy

$$\mathrm{P}(C)=\frac{k}{k+2}, \quad \mathrm{P}(D)=\frac{2}{k} ,$$

where k is a positive constant such that $\mathrm{P}(C)<1$, $\mathrm{P}(D)<1$.

b) Show that C and D cannot be mutually exclusive. **(6)**

SECTION 2 - MECHANICS

Question 8

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

Two particles are moving with constant acceleration on a horizontal surface where O is contained.

At time $t = 0$ s, one of the particles is at the point with position vector $(7\mathbf{i} + 2\mathbf{j})$ m, moving with velocity $(\mathbf{i} + 2\mathbf{j})$ ms^{-1} and constant acceleration $\left(\frac{1}{4}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$ ms^{-2}.

At time $t = 0$ s, the other particle is at the point with position vector $(\mathbf{i} - \mathbf{j})$ m, moving with velocity $(2\mathbf{i} - 2\mathbf{j})$ ms^{-1} and constant acceleration $\left(\frac{1}{2}\mathbf{i} - \frac{1}{4}\mathbf{j}\right)$ ms^{-2}.

Calculate the distance between the two particles, at the instant when they are moving in parallel directions to one another.

[You may ignore any motion taking place prior to time $t = 0$ s] **(11)**

Question 9

Two forces, both of magnitude 5 N each, have a resultant of magnitude 8 N.

These two forces act on a particle, of mass m kg, which remains at rest on a smooth horizontal surface. The surface makes an acute angle θ with one of the 5 N forces.

Given that the surface exerts a force of 4 N to the particle determine the exact value of m and the exact value of $\cos\theta$.

*You **may not** use any calculating aid in this question.* **(10)**

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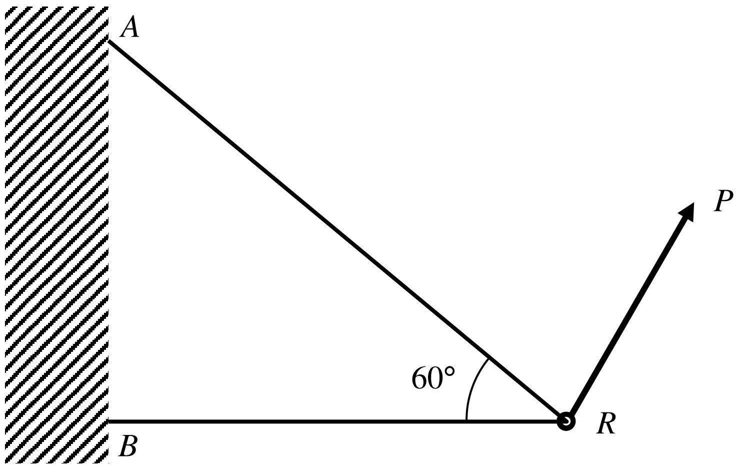
Question 10



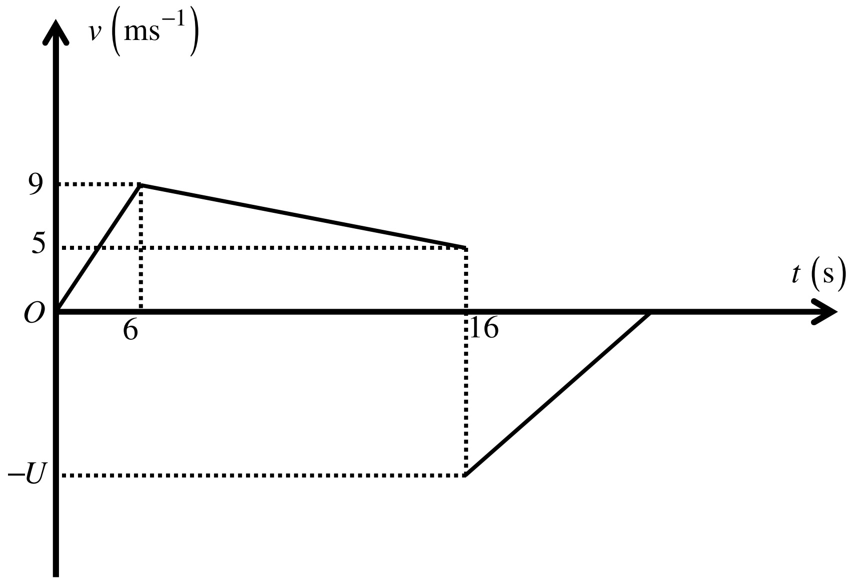
A light inextensible string is threaded through a ring R, of weight W. The two ends of the string, A and B, are attached on a wall with A vertically above B.

The ring is in equilibrium by a force P acting on R, so that BR is horizontal with $\measuredangle BRA = 60°$, as shown in the figure above.

Determine, in terms of W, the magnitude of P, when the tension in the string is least.

(8)

Question 11



A particle of mass 2 kg is released from rest from a point A on an incline plane and begins to move down a line of greatest slope of the plane.

The plane has a different coefficient of friction at different sections so the resistance to the motion of the particle has different values at different sections of the plane, as the particle slides down.

The particle accelerates uniformly to a speed of $9\ \text{ms}^{-1}$ in 6 s as it reaches point B.

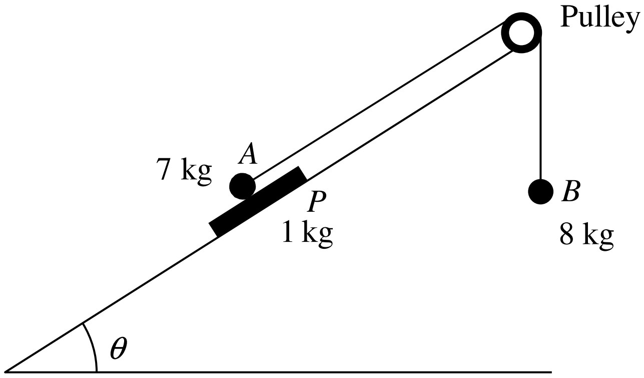
The coefficient of friction increases at B so the particle continues to slide down with constant deceleration for 10 s achieving a speed of $5\ \text{ms}^{-1}$ as it reaches point C.

At C the particle is instantaneously projected with speed $U\ \text{ms}^{-1}$, **up** a line of greatest slope of the plane, coming to rest at B.

If the **normal** reaction between the plane and the particle has a magnitude of 15.68 N, determine the value of U, correct to 2 decimal places.

(13)

Question 12



A rough plate P, of mass 1 kg, is placed on a fixed rough plane, inclined at an angle α to the horizontal, where $\tan\theta = 0.75$.

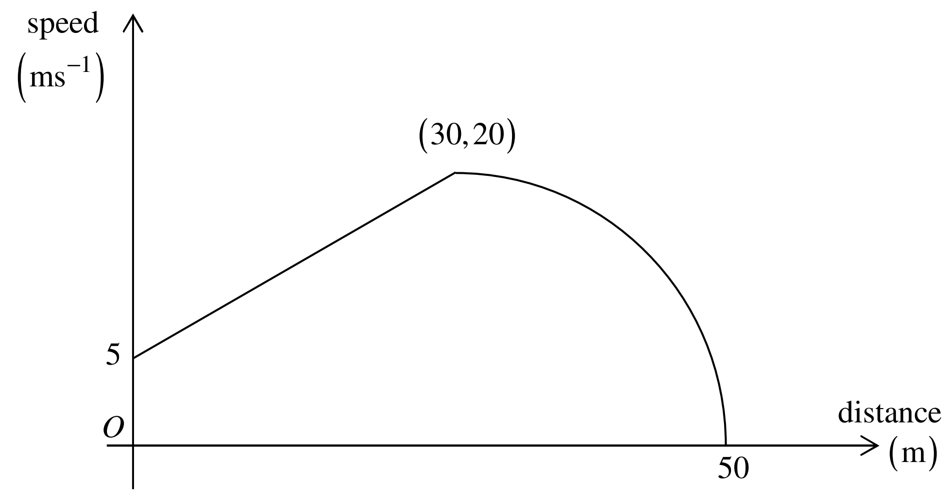
A particle A, of mass 7 kg, is placed on the top surface of P and is connected to another particle B, of mass 8 kg, by a light inextensible string, which passes over a smooth pulley that is located at the top the plane.

B is hanging freely at the end of the incline plane vertically below the pulley, as shown in the figure above. The two particles, the plate, the pulley and the string lie in a vertical plane parallel to the line of greatest slope of the incline plane.

When the system is released from rest with the string taut, B begins accelerate downwards at 2 ms^{-2}.

Given that P is in equilibrium, while A is accelerating on its top surface, determine the range of possible values of the coefficient of friction between P and the plane. **(12)**

Question 13



The speed distance graph of the journey of a particle is shown above.

It consists of a straight line segment joining the point $(0,5)$ to $(30,20)$, joined to a quarter circle of radius 20. The total distance covered by the particle is 50 m.

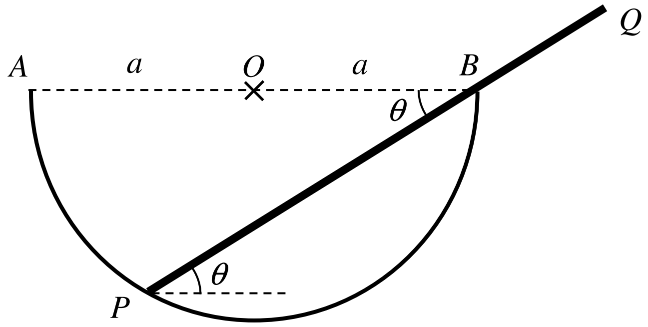
Determine in exact form the total journey time of the particle.

You may assume without proof that

$$\int \frac{1}{\sqrt{a^2-(u-b)^2}}\,du = \arcsin\left(\frac{u-b}{a}\right)+\text{constant}$$

(11)

Question 14



A smooth hollow hemispherical bowl of radius a and centre at O, is fixed so that its circular rim lies in a horizontal plane.

A smooth uniform rod PQ, of length L, rests in equilibrium with its end P at some point inside the bowl, as shown in the figure above.

The rod is in contact with the rim of the hemisphere at some point B, so that $|PB| < L$, and is inclined at an angle θ to the horizontal.

Show that

$$L = \frac{4a\cos 2\theta}{\cos\theta}.$$

(10)