IYGB – MMS PAPER W – QUESTION 1

a)

X = NUMBER OF VOTERS IN FAVOUR

$X \sim B(40, 0.35)$

$H_0: p = 0.35$

$H_1: p > 0.35$, WHERE p IS THE PROPORTION OF "IN FAVOUR" VOTERS IN GENERAL

TESTING AT 5% SIGNIFICANCE ON THE BASIS THAT $x = 19$

$$P(X \geqslant 19) = 1 - P(X \leqslant 18)$$
$$= 1 - 0.93008\ldots$$
$$= 0.0699$$
$$= 6.99\%$$
$$> 5\%$$

THERE IS NO SIGNIFICANT EVIDENCE TO SUPPORT THE CAMPAIGN MANAGER'S CLAIM — NO SUFFICIENT EVIDENCE TO REJECT H_0

b)

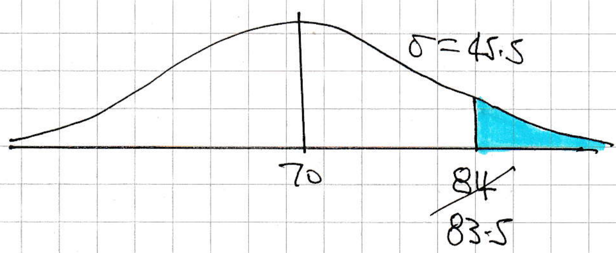
NOW SAMPLE $n = 200$, HYPOTHESES EXACTLY THE SAME

$X \sim B(200, 0.35)$

$P(X \geqslant 84) = \ldots$ NEED NOW TO BE FOUND BY APPROXIMATION

$Y \sim N(np, np(1-p))$

$Y \sim N(70, 45.5)$



IYGB - MMS PAPER W - QUESTION 1

$= P(Y > 83.5)$

$= 1 - P(Y < 83.5)$

$= 1 - P\left(Z < \dfrac{83.5 - 70}{\sqrt{45.5}}\right)$

$= 1 - \Phi(2.00137\ldots)$

$= 1 - 0.97732\ldots$

$= 0.02267$

$= 2.27\%$

$< 5\%$

THERE IS NOW SIGNIFICANT EVIDENCE TO SUPPORT THE CAMPAIGN MANAGER'S CLAIM - THERE IS SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - MMS PAPER W - QUESTION 2

PROCEED AS FOLLOWS

- $\bar{x} = 18.5$

$$\frac{\sum x}{n} = 18.5$$

$$\frac{\sum x}{20} = 18.5$$

$$\sum x = 370$$

- $\bar{y} = 25$

$$\frac{\sum y}{n} = 25$$

$$\frac{\sum y}{12} = 25$$

$$\sum y = 300$$

- $\sigma_x = 6.5$

$$\sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = 6.5$$

$$\sqrt{\frac{\sum x^2}{20} - 18.5^2} = 6.5$$

$$\frac{\sum x^2}{20} - 342.25 = 42.25$$

$$\frac{\sum x^2}{20} = 384.5$$

$$\sum x^2 = 7690$$

- $\sigma_y = 7.5$

$$\sqrt{\frac{\sum y^2}{n} - \bar{y}^2} = 7.5$$

$$\sqrt{\frac{\sum y^2}{12} - 25^2} = 7.5$$

$$\frac{\sum y^2}{12} - 625 = 56.25$$

$$\frac{\sum y^2}{12} = 681.25$$

$$\sum y^2 = 8175$$

COMBINING THE DATA INTO 32 OBSERVATIONS

- $$\text{MEAN}_{(32\text{ OBS})} = \frac{\sum x + \sum y}{20 + 12} = \frac{370 + 300}{32} = \frac{670}{32} = \frac{335}{16} \simeq 20.94$$

- $$\sigma_{(32\text{ OBS})} = \sqrt{\frac{\sum x^2 + \sum y^2}{32} - (20.94\ldots)^2} = \sqrt{\frac{7690 + 8175}{32} - (20.94\ldots)^2}$$

$$= 7.576433445\ldots \simeq 7.58$$

IYGB – MMS PAPER W – QUESTION 3

a) GENERATE THE NUMBERS ACCORDING TO THIS METHOD IGNORING REPEATS & NUMBERS OVER 750

270, 701, 016, 163, 635, 359, 597, ~~971~~, 716, ~~163~~, ~~635~~, 354, 548

b) NOT RANDOM AS THERE IS DEPENDENCE IN THE DIGITS

E.g. ONCE THE 270 HAS BEEN GENERATED, IF THE METHOD WAS RANDOM ANY NUMBER SHOULD BE POSSIBLE TO FOLLOW

INSTEAD ONLY THE NUMBERS 700, 701, 702, ..., 709 ARE NOW POSSIBLE

IYGB - MMS PAPER W - QUESTION 4

LET X = NUMBER OF STUDENTS WHICH ANSWERED CORRECTLY THE FIRST QUESTION.

$$\boxed{X \sim B(30, 0.2)}$$

a) $P(5 < X \leqslant 10) = P(6 \leqslant X \leqslant 10)$
$= P(X \leqslant 10) - P(X \leqslant 5)$
$= 0.97438\ldots - 0.42751\ldots$
$= 0.5469$ //

b) T = TOTAL MARKS OF THE 30 STUDENTS FROM THE FIRST QUESTION

$T = X \times 5 - (30 - X) \times 2$

$T = 5X + 2X - 60$

$T = 7X - 60$

$\Rightarrow P(T > 17) = P(7X - 60 > 17)$
$= P(7X > 77)$
$= P(X > 11)$
$= P(X \geqslant 12)$
$= 1 - P(X \leqslant 11)$
$= 1 - 0.9905$
$= 0.0095$ //

IYGB - MMS PAPER W - QUESTION 5

AS THE PROBABILITIES ARE GIVEN AS PERCENTAGES, PROCEED AS FOLLOWS

CONSIDER 100 JOURNEYS

		ARRIVALS (HOSTEAD)			
		EARLY	ON TIME	LATE	TOTAL
DEPARTURES	ON TIME	4	52	21	(77)
	LATE	(2)	(17)	(4)	(23)
	TOTAL	6	69	(25)	100

PUT THE INFO GIVEN IN "BLUE", & THEN FILL THE TABLE

a) FROM TABLE $= \frac{4}{6} = \frac{2}{3} = 0.667...$

b) FROM TABLE $= \frac{4}{25} = 0.16$

c) $P(\text{EARLY}) = 6\%$, $P(\text{ON TIME}) = 69\%$, $P(\text{LATE}) = 25\%$

$\therefore$ REQUIRED PROBABILITY $= [0.06 \times 0.69 \times 0.25] \times 6$ WAYS

$= 0.0621$

(6.21%)

IYGB - MMS PAPER W - QUESTION 6

$$\boxed{P(B) = 0.76 \qquad P(B|A) = 0.6 \qquad P(A' \cap B') = 0}$$

FORMING TWO EQUATIONS FROM THE EQUATIONS GIVEN

- IF $P(A' \cap B') = 0 \implies P(A \cup B) = 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$1 = P(A) + 0.76 - P(A \cap B)$$

$$\boxed{0.24 = P(A) - P(A \cap B)}$$

ALSO FROM THE CONDITIONAL PROBABILITY

- $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$

$$0.6 = \frac{P(B \cap A)}{P(A)}$$

$$P(B \cap A) = 0.6\,P(A)$$

$$\boxed{P(A \cap B) = 0.6\,P(A)}$$

COMBINING RESULTS

$$\implies 0.24 = P(A) - P(A \cap B)$$

$$\implies 0.24 = P(A) - 0.6\,P(A)$$

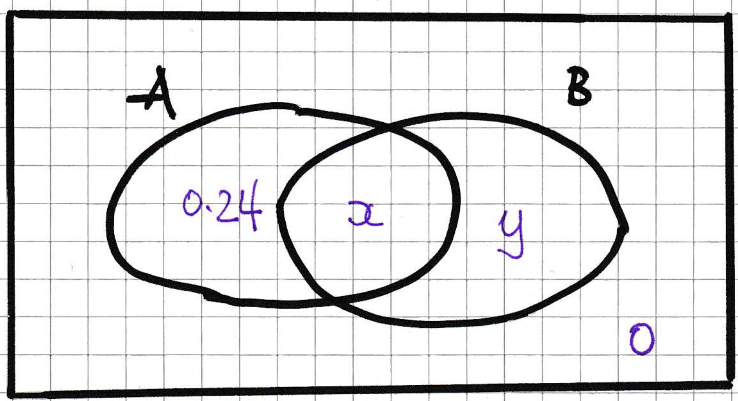
$$\implies 0.24 = 0.4\,P(A)$$

$$\implies P(A) = \frac{0.24}{0.4}$$

$$\implies P(A) = 0.6$$

IYGB - MMS PAPER W - QUESTION 6

ALTERNATIVE METHOD BY SETTING EQUATIONS DIRECTLY FROM A VENN DIAGRAM



$P(A' \cap B') = 0$
$P(B) = 0.76$
$P(B|A) = 0.6$

$x + y + 0.24 = 1$
$x + y = 0.76$

&

$$\frac{x}{x+0.24} = 0.6 \quad \leftarrow \text{CONDITIONAL}$$

$x = 0.6x + 0.144$

$0.4x = 0.144$

$x = 0.36$

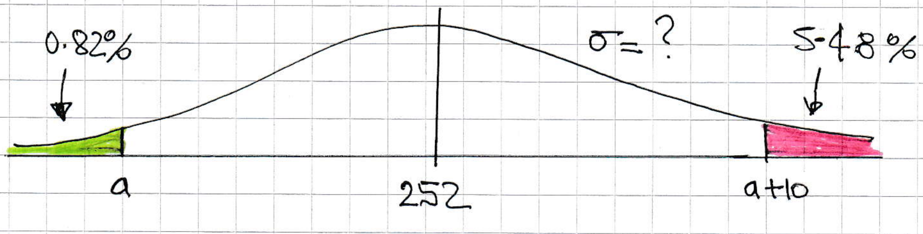
$\therefore \ P(A) = x + 0.36$

$P(A) = 0.6$

AS BEFORE

IYGB - MMS PAPER W - QUESTION 7

a) PUTTING INFORMATION INTO A DIAGRAM



X = VOLUME OF COFFEE DISPENSED (ml)
$X \sim N(252, \sigma^2)$

$\Rightarrow P(X<a) = 0.0082$
$\Rightarrow P(X>a) = 0.9918$
$\Rightarrow P\left(Z > \frac{a-252}{\sigma}\right) = 0.9918$
$\Rightarrow \Phi\left(\frac{a-252}{\sigma}\right) = 0.9918$
$\Rightarrow \frac{a-252}{\sigma} = -\Phi^{-1}(0.9918)$
$\Rightarrow \frac{a-252}{\sigma} = -2.40$
$\Rightarrow a - 252 = -2.40\sigma$
$\Rightarrow a = 252 - 2.40\sigma$

$\Rightarrow P(X>a+10) = 0.0548$
$\Rightarrow P(X<a+10) = 0.9452$
$\Rightarrow P\left(Z < \frac{a+10-252}{\sigma}\right) = 0.9452$
$\Rightarrow \Phi\left(\frac{a-242}{\sigma}\right) = 0.9452$
$\Rightarrow \frac{a-242}{\sigma} = \Phi^{-1}(0.9452)$
$\Rightarrow \frac{a-242}{\sigma} = 1.60$
$\Rightarrow a - 242 = 1.60\sigma$
$\Rightarrow a = 242 + 1.60\sigma$

$252 - 2.4\sigma = 242 + 1.6\sigma$
$10 = 4\sigma$
$\sigma = 2.5$

($a = 246$ FOR PART (b))

IYGB - MMS PAPER W - QUESTION 7

b) PROCEED AS FOLLOWS

$$P\left(X - 2a - 14 + \frac{64000}{X} < 0\right)$$

AS $X > 0$

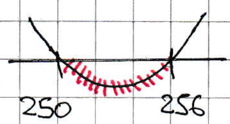
$$= P\left(X(X - 2a - 14) + 64000 < 0\right)$$

$$= P\left(X(X - 506) + 64000 < 0\right)$$

$$= P\left(X^2 - 506X + 64000 < 0\right)$$

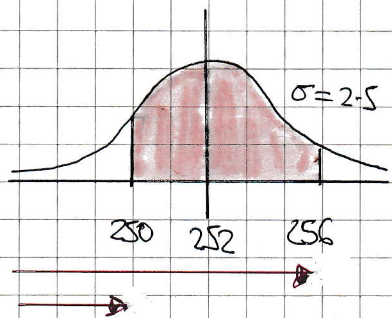
FACTORIZE BY THE QUADRATIC FORMULA

$$= P\left[(X - 250)(X - 256) < 0\right]$$



$$= P\left[250 < X < 256\right]$$

$$= P(X < 256) - P(X < 250)$$



$$= P(X < 256) - \left[1 - P(X > 250)\right]$$

$$= P(X < 256) + P(X > 250) - 1$$

$$= P\left(Z < \frac{256 - 252}{2.5}\right) + P\left(Z > \frac{250 - 252}{2.5}\right) - 1$$

$$= \Phi(1.6) + \Phi(-0.8) - 1$$

$$= 0.9452 + 0.7881 - 1$$

$$= 0.7333$$

IYGB - MMS PAPER W - QUESTION 7

c) SUMMARIZING ALL INFORMATION FOR THIS TEST

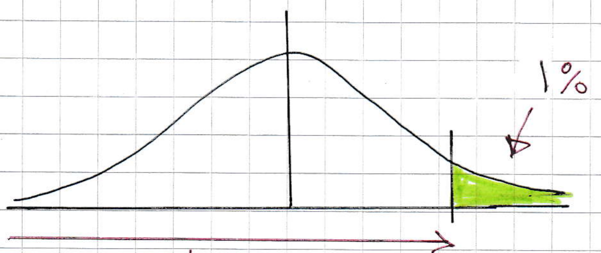
H_0 : $\mu = 252$

H_1 : $\mu > 252$, WHERE μ IS THE POPULATION MEAN

$n = 5$

$\sigma = 2.5$

$\bar{x} = 255$, 1% SIGNIFICANCE, ONE TAILED TEST



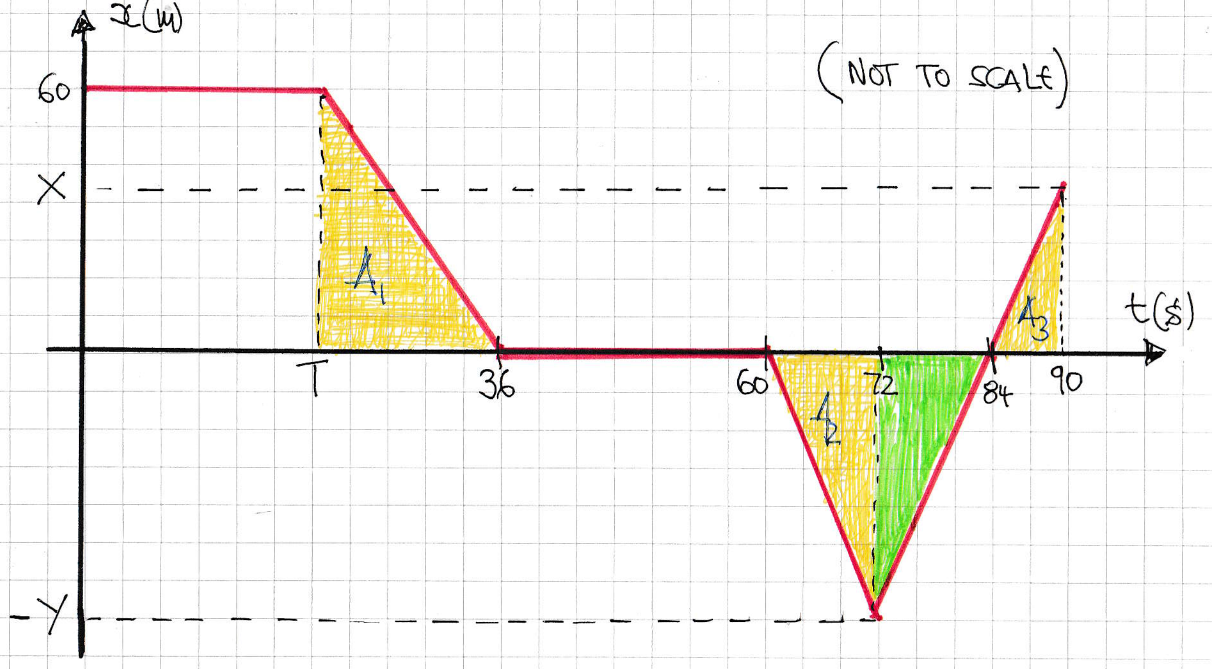
$\Phi^{-1}(0.99) = 2.3263$

$$Z\text{-STATISTIC} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{255 - 252}{2.5/\sqrt{5}} = 2.6833\ldots$$

AS $2.6833 > 2.3263$, THERE IS SIGNIFICANT EVIDENCE THAT μ IS GREATER THAN 252

THERE IS SUFFICIENT EVIDENCE TO REJECT H_0

IYGB - MMS PAPER W - QUESTION 8



SPEED IS EITHER 3 OR 0

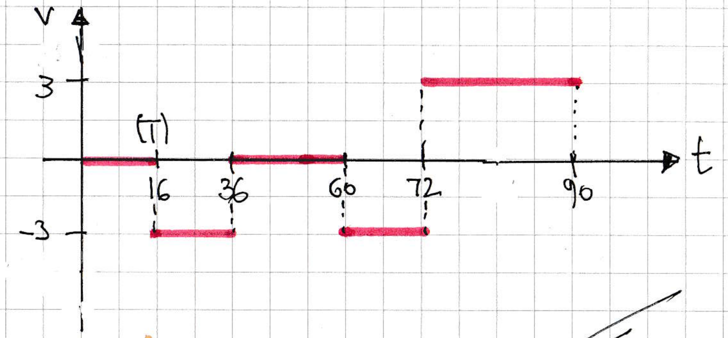
"DISTANCE = SPEED × TIME"

$$\left.\begin{aligned} 60 &= 3 \times (36 - T) \\ 20 &= 36 - T \\ T &= 16 \end{aligned}\right\} A_1$$

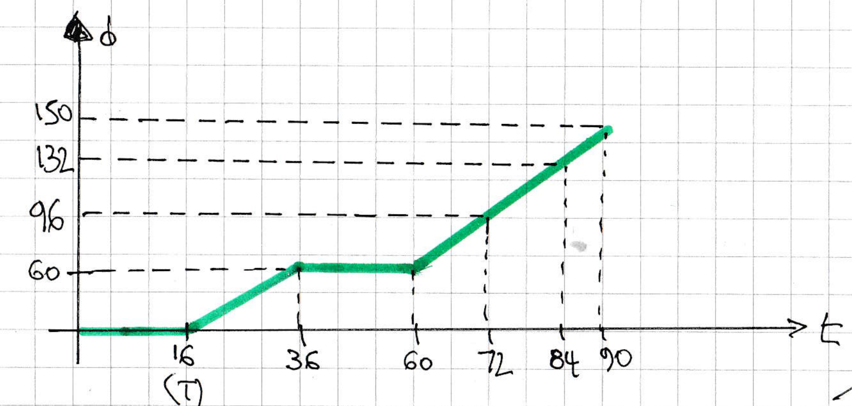
$$\left.\begin{aligned} Y &= 3 \times 12 \\ Y &= 36 \end{aligned}\right\} A_2$$

$$\left.\begin{aligned} X &= 3 \times 6 \\ X &= 18 \end{aligned}\right\} A_3$$

a) VELOCITY-TIME GRAPH

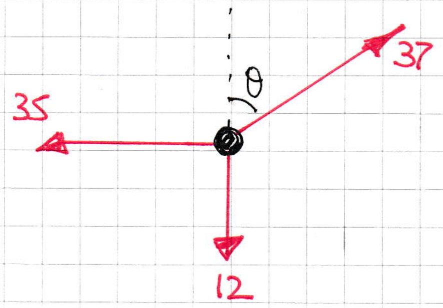


b) DISTANCE-TIME GRAPH



IYGB – MMS PAPER W – QUESTION 9

- LOOKING AT THE PARTICLE IN EQUILIBRIUM



- IF THE 35N FORCE GETS REMOVED, EVIDENTLY THERE WILL BE A RESULTANT OF 35N IN THE OPPOSITE DIRECTION

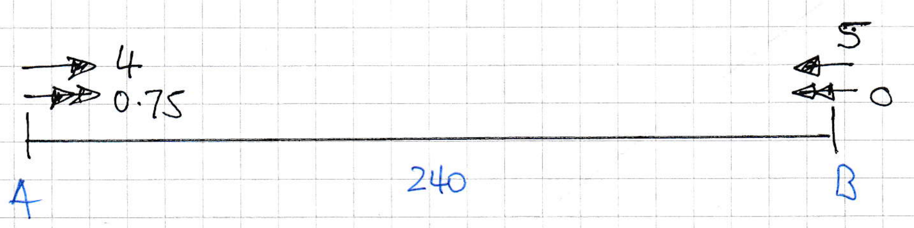
HENCE "$F = ma$" YIELDS

$$35 = m \times 14$$

$$m = 2.5 \text{ kg}$$

IYGB - MMS PAPER W - QUESTION 10

PUTTING THE INFORMATION INTO A DIAGRAM



TAKE "A" AS THE ORIGIN & USE $s = ut + \frac{1}{2}at^2$

$$s_A = 4t + \frac{1}{2}(0.75)t^2 \quad \leftarrow s = \cancel{s_0} + ut + \frac{1}{2}at^2$$

$$s_B = 240 - 5t + \frac{1}{2} \times 0 \times t^2 \quad \leftarrow s = s_0 + ut + \cancel{\frac{1}{2}at^2}$$

$$s_A = 4t + \frac{3}{8}t^2$$

$$s_B = 240 - 5t$$

MEETING IMPLIES $s_A = s_B$

$$\Rightarrow 4t + \frac{3}{8}t^2 = 240 - 5t$$

$$\Rightarrow 32t + 3t^2 = 1920 - 40t$$

$$\Rightarrow 3t^2 + 72t - 1920 = 0$$

$$\Rightarrow t^2 + 24t - 640 = 0$$

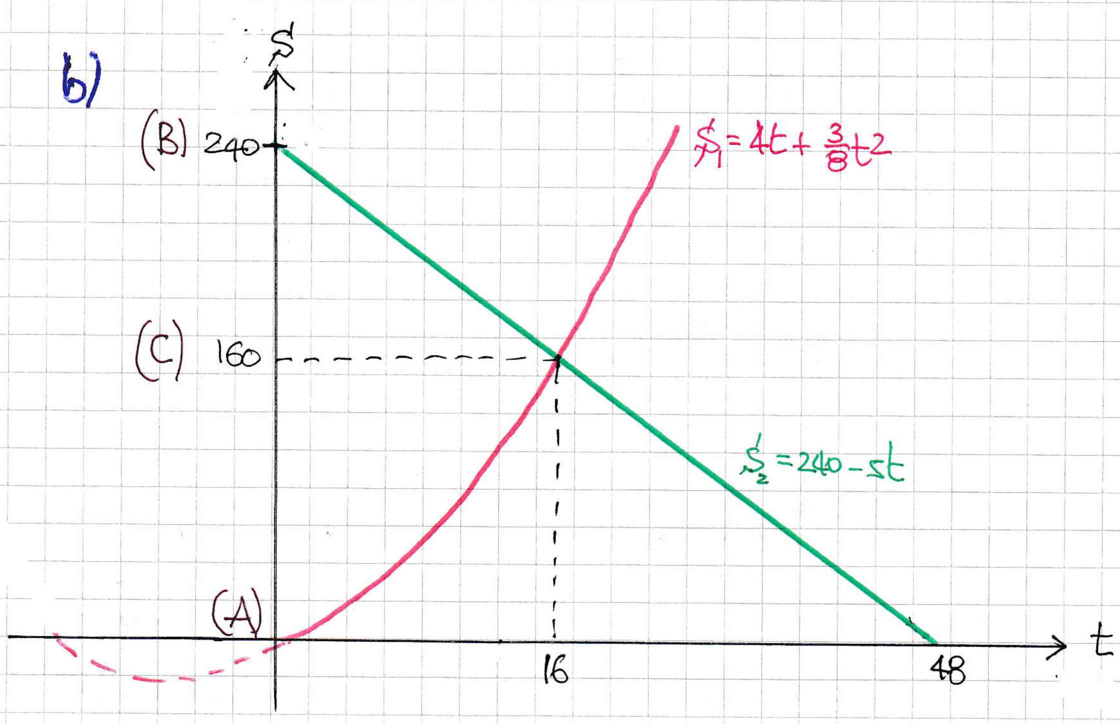
$$\Rightarrow (t - 16)(t + 40) = 0$$

$$\Rightarrow t = \begin{cases} 16 \\ \cancel{-40} \end{cases}$$

$$\therefore s_A = s_B = 240 - 5 \times 16 = 160\text{m} \qquad \text{i.e. } |AC| = 160\text{m}$$

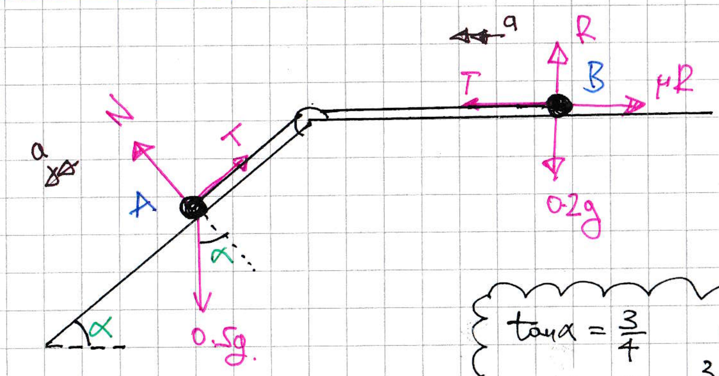
IYGB - MMS PAPER IV - QUESTION 10

b)



IYGB - MMS PAPER W - QUESTION 11

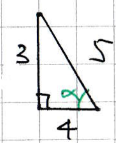
a) STARTING WITH A DIAGRAM



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$



USING STANDARD KINEMATICS

$u = 0 \text{ ms}^{-1}$
$a = ?$
$s = 2.25 \text{ m}$
$t = 1.5 \text{ s}$
$v =$

$s = \cancel{ut} + \frac{1}{2}at^2$

$2.25 = \frac{1}{2} \times a \times 1.5^2$

$a = 2 \text{ ms}^{-2}$

LOOKING AT THE EQUATION OF MOTION OF A, WITH $a = 2$

$\Rightarrow 0.5g\sin\alpha - T = 0.5a$ ← "F = ma"

$\Rightarrow 0.5g \times \frac{3}{5} - T = 0.5 \times 2$

$\Rightarrow 2.94 - T = 1$

$\Rightarrow T = 1.94 \text{ N}$

b) LOOKING AT THE EQUATION OF MOTION OF B

$\Rightarrow T - \mu R = 0.2a$ ← "F = ma"

$\Rightarrow 1.94 - \mu(0.2g) = 0.2 \times 2$

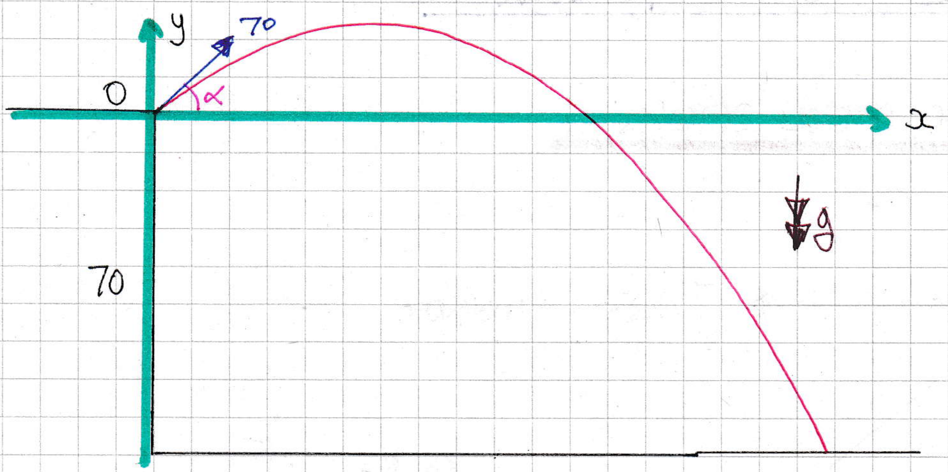
$\Rightarrow 1.94 - 1.96\mu = 0.4$

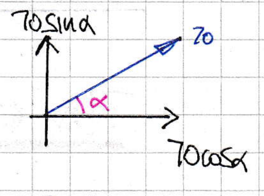
$\Rightarrow 1.54 = 1.96\mu$

$\Rightarrow \mu = \frac{1.54}{1.96}$

$\Rightarrow \mu = \frac{11}{14} \simeq 0.786$

IYGB - MMS PAPER W - QUESTION 12





$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

a) LOOKING AT THE VERTICAL MOTION

$u = 70\sin\alpha = 42$
$a = -9.8$
$s = -70$
$t = ?$
$v = ?$ (FOR PART b)

$\Rightarrow s = ut + \frac{1}{2}at^2$

$\Rightarrow -70 = 42t + \frac{1}{2}(-9.8)t^2$

$\Rightarrow -70 = 42t - 4.9t^2$

$\Rightarrow 4.9t^2 - 42t - 70 = 0$

$\Rightarrow 49t^2 - 420t - 700 = 0$

$\Rightarrow 7t^2 - 60t - 100 = 0$

$\Rightarrow (7t + 10)(t - 10) = 0$

$\Rightarrow t = \begin{cases} 10 \\ \cancel{-\frac{10}{7}} \end{cases}$ ← FLIGHT TIME

HORIZONTALLY NOW, DISTANCE = SPEED × TIME

$\Rightarrow x = 70\cos\alpha \times 10$

$\Rightarrow x = 70 \times \frac{4}{5} \times 10$

$\Rightarrow x = 560 \text{ m}$

b) LOOKING AT THE VERTICAL MOTION IN PART (a)

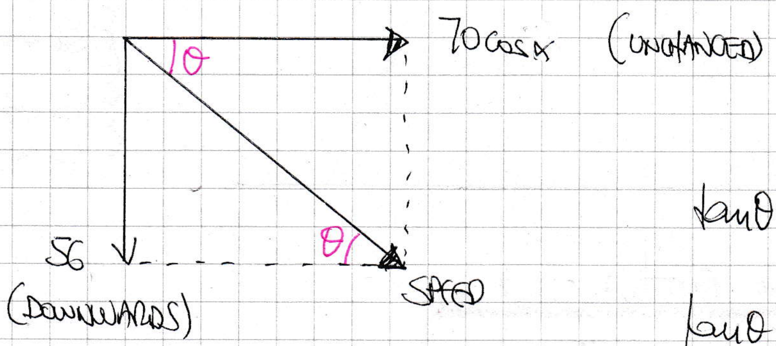
$\Rightarrow v = u + at$

$\Rightarrow v = 42 + (-9.8)10$

$\Rightarrow v = -56$

IYGB - MMS PAPER W - QUESTION 12

THUS AS THE PARTICLE REACHES A



$\tan\theta = \frac{56}{70\cos\alpha}$

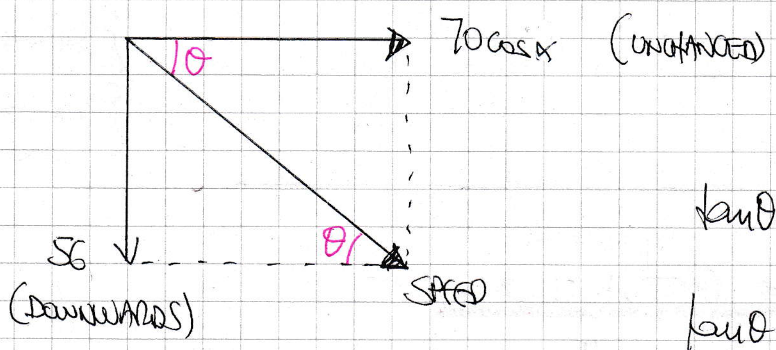
$\tan\theta = 1$

$\theta = 45°$

AS REQUIRED

IYGB - MMS PAPER W - QUESTION 12

THUS AS THE PARTICLE REACHES A



$\tan\theta = \dfrac{56}{70\cos\alpha}$

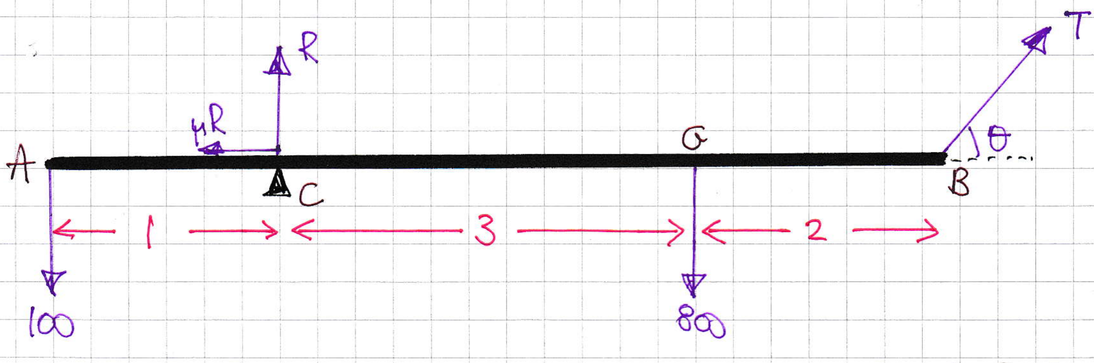
$\tan\theta = 1$

$\theta = 45°$

AS REQUIRED

IYGB - MMS PAPER W - QUESTION 13

STARTING WITH A DIAGRAM

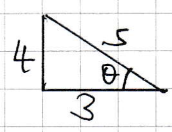


RESOLVING VERTICALLY & HORIZONTALLY

$(\uparrow)$ $R + T\sin\theta = 100 + 800$

$(\rightarrow)$ $\mu R = T\cos\theta$

$\sin\theta = 0.8 = \frac{4}{5}$



$\cos\theta = \frac{3}{5}$

TAKING MOMENTS ABOUT C

$100 \times 1 + T\sin\theta \times 5 = 800 \times 3$

$100 + 5T\sin\theta = 2400$

$5T\sin\theta = 2300$

$5T \times 0.8 = 2300$

$T = 575 \text{ N}$

THE OTHER TWO EQUATIONS NOW YIELD

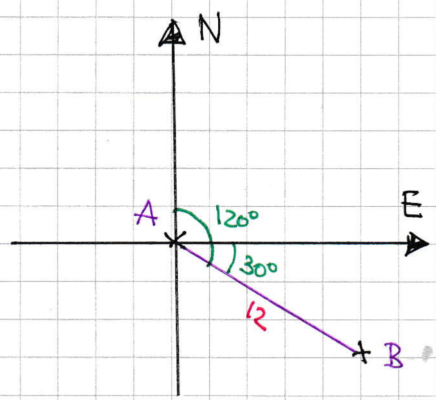
$$\left.\begin{array}{l} R + 575 \times 0.8 = 900 \\ \mu R = 575 \times 0.6 \end{array}\right\} \Longrightarrow \begin{array}{l} R = 440 \\ \mu R = 345 \end{array}$$

DIVIDING GIVES $\mu = \frac{345}{440} = \frac{69}{88}$

$\mu \simeq 0.784$

IYGB – MMS PAPER W – QUESTION 14

a)
- TAKE THE POSITION OF "A" AT NOON TO BE THE ORIGIN



- THEN AT NOON

$$\underline{r}_B = (12\cos 30)\underline{i} - (12\sin 30)\underline{j}$$

$$\underline{r}_B = 6\sqrt{3}\,\underline{i} - 6\underline{j}$$

- THEN THE POSITION VECTORS OF THE TWO SHIPS, t HOURS AFTER NOON, IS GIVEN BY

$$\underline{r}_A = (0\underline{i} + 0\underline{j}) + (7\underline{i} + 3\underline{j})t = 7t\underline{i} + 3t\underline{j}$$

$$\underline{r}_B = (6\sqrt{3}\underline{i} - 6\underline{j}) + (-3\underline{i} + 9\underline{j})t = (6\sqrt{3} - 3t)\underline{i} + (9t - 6)\underline{j}$$

- $$\underline{r}_B - \underline{r}_A = (6\sqrt{3} - 10t)\underline{i} + (6t - 6)\underline{j}$$

- WHEN B IS EAST OF A, $\underline{j}$ COMPONENT MUST BE ZERO

$$\Rightarrow 6t - 6 = 0$$

$$\Rightarrow 6t = 6$$

$$\Rightarrow t = 1$$

$$\therefore (i):\ 6\sqrt{3} - 10 \times 1 = -0.3923\ldots$$

$$\simeq \underline{392\text{ m} \ \text{EAST OF A}}$$

b)
- $|\underline{r}_B - \underline{r}_A|$ = DISTANCE BETWEEN THE SHIPS AT TIME t.

$$\Rightarrow |\underline{r}_B - \underline{r}_A| = \sqrt{(6\sqrt{3} - 10t)^2 + (6t - 6)^2}$$

IYGB – MMS PAPER W – QUESTION 14

$$\Rightarrow |\underline{r}_B - \underline{r}_A| = \sqrt{108 - 120\sqrt{3}t + 100t^2 + 36t^2 - 72t + 36}$$

$$\Rightarrow |\underline{r}_B - \underline{r}_A| = \sqrt{136t^2 - (72 + 120\sqrt{3})t + 144}$$

$$\Rightarrow |\underline{r}_B - \underline{r}_A|^2 = 136t^2 - (72 + 120\sqrt{3})t + 144$$

- LET $f(t) = 136t^2 - 24(3 + 5\sqrt{3})t + 144$

  BY COMPLETING THE SQUARE OR CALCULUS

$$\Rightarrow f'(t) = 272t - (72 + 120\sqrt{3})$$

- SOLVING FOR ZERO YIELDS

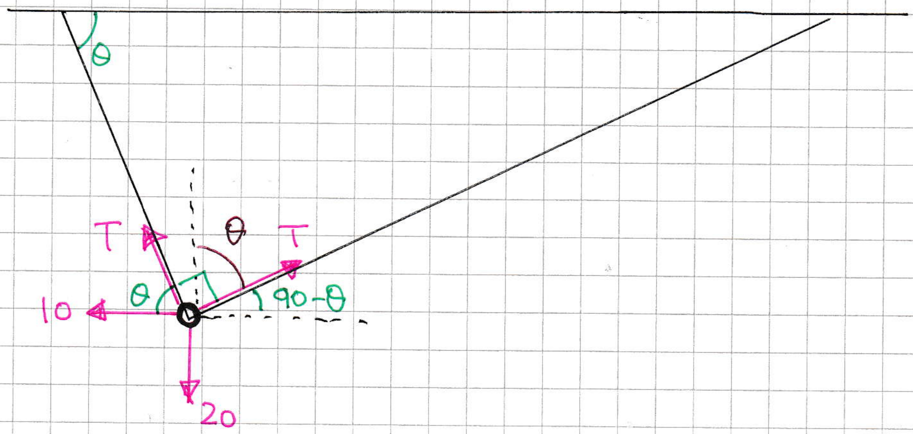
$$t = \frac{72 + 120\sqrt{3}}{272} \simeq 1.0288\ldots$$

$$\simeq 1 \text{ HOUR} - 2 \text{ MINUTES}$$

$$\simeq 13:02$$

IYGB – MMS PAPER W – QUESTION 15

STARTING WITH A DIAGRAM SHOWING THE "TRAPPED" RING IN EQUILIBRIUM



RESOLVING FORCES VERTICALLY AND HORIZONTALLY

(↕) $T\sin\theta + T\sin(90-\theta) = 20$ OR $T\sin\theta + T\cos\theta = 20$

(↔) $10 + T\cos\theta = T\cos(90-\theta)$ OR $10 + T\cos\theta = T\sin\theta$

SUBTRACTING EQUATIONS AS THEY ARE

$$T\sin\theta - 10 = 20 - T\sin\theta$$
$$2T\sin\theta = 30$$
$$\underline{T\sin\theta = 15}$$

& SIMILARLY $\underline{T\cos\theta = 5}$

HENCE WE NOW HAVE

$$T\sin\theta = 15$$
$$T\cos\theta = 5$$

DIVIDING GIVES $\tan\theta = 3$, SO $\theta \simeq 71.57°$

FINALLY WE HAVE

$$T = \frac{15}{\sin\theta} = \frac{15}{\sin(71.57..)} \simeq 15.81\text{ N}$$

$\therefore$ $T \simeq 15.8\text{N}$ & $\theta \simeq 71.6°$