IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper B

Difficulty Rating: 3.815/1.2814

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 13 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

If x is in radians

$$\frac{d}{dx}(\sin x) = \cos x .$$

Prove the validity of the above result from first principles.

You may assume that if h is small and measured in radians, then as $h \to \infty$

$$\frac{\cos(h)-1}{h} \to 0 \quad \text{and} \quad \frac{\sin(h)}{h} \to 1 .$$

(5)

Question 2

A sequence of numbers, u_1, u_2, u_3, u_4, ... , is defined by

$$u_n = \frac{1}{1-u_{n-1}} , \quad u_1 = 2 .$$

Determine the value of

$$\sum_{n=1}^{20} u_n .$$

(5)

Question 3

The point $P\left(\frac{2}{5}, -\frac{2}{3}\right)$ lies on the curve C with parametric equations

$$x = \frac{1}{t+a} , \quad y = \frac{1}{t-a} , \quad t \in \mathbb{R} , \quad t \neq \pm a ,$$

where a is a non zero constant.

Show that the gradient at P is $\frac{25}{9}$.

(7)

Question 4

The points $A(1,1,2)$, $B(2,1,5)$, $C(4,0,1)$ and D form the parallelogram $ABCD$, where the above coordinates are measured relative to a fixed origin.

a) Find the coordinates of D. **(2)**

The points E, B and D are collinear, so that B is the midpoint of ED.

b) Determine the coordinates of E. **(2)**

The point F is such so that $ABEF$ is also a parallelogram.

c) Find the coordinates of F. **(2)**

d) Show that B is the midpoint of FC. **(2)**

e) Prove that $ADBF$ is another parallelogram. **(2)**

Question 5

A curve C has implicit equation

$$\frac{(x+2y)^2}{4x-y}+y=3x+2.$$

a) Show clearly that

$$\frac{dy}{dx}=\frac{2kx-ky+8}{6y+kx+2},$$ **(6)**

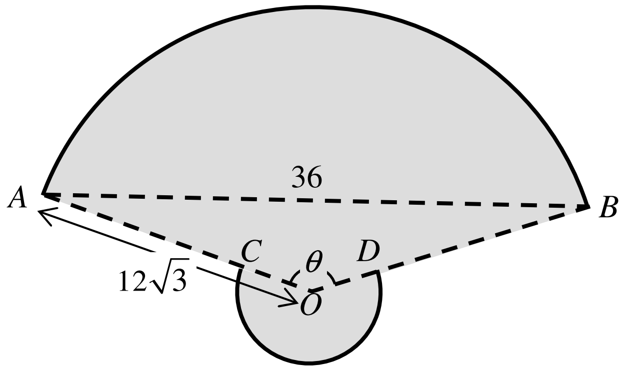
where k is a constant to be found.

b) Find the gradient at each of the points on C, where $x=2$. **(6)**

Question 6

Show by using algebra, that the sum of the integers between 1 and 600 inclusive, that are **not** divisible by 3, is 120000. **(5)**

Question 7



The figure above shows a model of the region used by shot putters in to throw the shot. The throwing region consists of a **minor** circular sector OAB of radius $12\sqrt{3}$ metres subtending an angle θ radians at O. The chord AB is 36 metres.

The shot putter's region COD is a **major** circular sector of radius $3\sqrt{3}$ metres, where C and D lie on OA and OB, respectively.

a) Show that $\theta = \frac{2}{3}\pi$. **(3)**

b) Find, in terms of π, the total area of throwing region and shot putter's region. **(4)**

c) Show further that the total perimeter of the throwing region and the shot putter's region, shown shaded in the figure above, is

$$6(2\pi + 3)\sqrt{3}.$$

(3)

Question 8

$$\frac{\cos\theta\cos 2\theta}{\cos\theta + \sin\theta} = \frac{1}{2}, \quad 0 \le x < 2\pi.$$

Given that $\cos\theta + \sin\theta \ne 0$, find the solutions of the above trigonometric equation, giving the answers in radians in terms of π. **(8)**

Question 9

The function f is defined by

$$f(x) = 2 + \sqrt{x}\ ,\ x \in \mathbb{R},\ x \geq 0\ .$$

a) Evaluate $ff(49)$. **(1)**

b) Find an expression for the inverse function, $f^{-1}(x)$. **(2)**

c) Sketch in the same set of axes the graph of $f(x)$ and the graph of $f^{-1}(x)$, clearly marking the line of reflection between the two graphs. **(3)**

d) Show that $x = 4$ is the only solution of the equation $f(x) = f^{-1}(x)$. **(3)**

Question 10

$$e^{x}\frac{dy}{dx} + y^{2} = xy^{2}\ ,\ x > 0\ ,\ y > 0$$

Show that the solution of the above differential equation subject to $y = e$ at $x = 1$, is

$$y = \frac{1}{x}e^{x}\ .$$

(8)

Question 11

After a road accident, fuel is leaking from a tanker onto a flat section of the motorway forming a circle of thickness 3 mm.

Petrol is leaking at a steady rate of $0.0008\ \text{m}^3\text{s}^{-1}$

Find, in terms of π, the rate at which the radius of the circle of petrol is increasing at the instant when the radius has reached 6 m. **(5)**

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Question 12

$$y = \arcsin x\,,\ -\frac{1}{2} \le x \le \frac{1}{2}\,,\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}\,.$$

a) By finding $\frac{dx}{dy}$ and using an appropriate trigonometric identity show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}\,.$$ **(4)**

A curve C has equation

$$y = x\arcsin 2x\,,\ -\frac{1}{2} \le x \le \frac{1}{2}\,,\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}\,.$$

b) Find the exact value of $\frac{dy}{dx}$ at the point on C where $x = \frac{1}{4}$. **(5)**

Question 13

Determine, in terms of a, the value of the following integral.

$$\int_{\frac{2}{a}}^{\frac{17}{a}} \frac{2ax}{\sqrt{ax-1}}\,dx\,,\ a \ne 0\,.$$

You may find the substitution $u^2 = ax - 1$ useful in this question. **(7)**
