IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper D

Difficulty Rating: 3.755/1.2472

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 12 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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Question 1

$$f(x)=(1-x)^{\frac{1}{3}},\ -1<x<1.$$

a) Find the binomial expansion of $f(x)$ in ascending powers of x up and including the term in x^2.

$$g(x)=(8-3x)^{\frac{1}{3}},\ -\frac{8}{3}<x<\frac{8}{3}. \qquad (\mathbf{3})$$

b) Use the result of part **(a)** to find the binomial expansion of $g(x)$ in ascending powers of x up and including the term in x^2. $(\mathbf{3})$

c) Hence, show that

$$\sqrt[3]{7}\approx\frac{551}{288}. \qquad (\mathbf{3})$$

Question 2

Given that x is measured in radians, use small angle approximations to simplify the following expression.

$$\frac{\cos^2(3x)\ -\ 1}{2x\sin\left(\frac{3}{4}x\right)} \qquad (\mathbf{5})$$

Question 3

Show that

$$\sum_{r=1}^{12}\left[2r\ +\ 7+\ 2^r\right]\ =\ 8430. \qquad (\mathbf{6})$$

Detailed workings must be shown in this question.

Question 4

a) Use the trapezium rule with 5 equally spaced ordinates to estimate the value of the following integral.

$$\int_2^{18} \ln\left[\frac{2}{\sqrt{4+\sqrt{x}}}\right] dx. \qquad (\mathbf{4})$$

b) Use the answer of part **(a)** to estimate the value of

$$\int_2^{18} \ln\left(4+\sqrt{x}\right) dx. \qquad (\mathbf{4})$$

Question 5

Relative to a fixed origin O, the point A has coordinates $(6,-4,1)$.

The point B is such so that $\overrightarrow{BA} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

If the point M is the midpoint of OB, show that $\left|\overrightarrow{AM}\right| = k\sqrt{10}$, where k is a rational constant to be found. $(\mathbf{6})$

Question 6

The functions f and g are defined by

$$f(x) = 2x+3,\ x\in\mathbb{R},\ x\leq 4$$

$$g(x) = x^2-4,\ x\in\mathbb{R},\ x\geq 1.$$

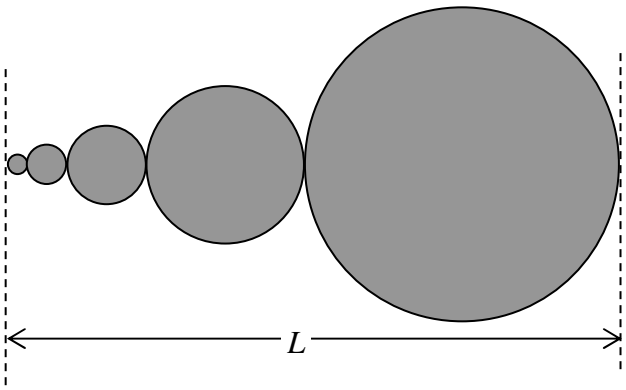
Find the domain and range of $gf(x)$. $(\mathbf{6})$

Question 7

If $\sin x = \frac{12}{13}$ and x is obtuse, show clearly that

$$\cot 2x = \frac{119}{120}. \qquad (\mathbf{5})$$

Question 8



The figure above shows a pattern of 5 circles, touching externally, whose centres lie on a straight line of length L units.

The radii of these circles form a geometric progression, where the radius of the smaller circle is 3 units and that of the fifth (larger) circle is 48 units.

a) Find the common ratio of the geometric progression. $(\mathbf{2})$

The pattern is extended by 5 more circles to 10 circles.

b) Determine the new value of L. $(\mathbf{3})$

c) Calculate, in terms of π, the total area of the 10 circles of the new pattern. $(\mathbf{5})$

Question 9

The curve C has equation

$$x = \sec^2 y + \tan y, \ 0 \leq y < \frac{\pi}{2}.$$

a) Show that

$$\frac{dy}{dx} = \frac{\cos^2 y}{2\tan y + 1}. \qquad (\mathbf{5})$$

b) Hence show that the equation of the normal to C at the point where $y = \frac{\pi}{4}$ is

$$4y + 24x = \pi + 72. \qquad (\mathbf{4})$$

Question 10

The value of a computer system V, in hundred of £, t years from when it was new, is depreciating at a rate proportional to its value cubed, at that time t.

The value of the computer system when it was new, was £1000.

a) By forming and solving a differential equation, show that

$$\frac{1}{V^2} = At + \frac{1}{100},$$

where A is a positive constant. $(\mathbf{7})$

b) Given that the value of the computer system halves after one year, find the value of t when the system is worth £250. $(\mathbf{5})$

Question 11

Use a suitable substitution to show that

$$\int_{\frac{1}{\ln 3}}^{\frac{1}{\ln 2}} \left(\frac{1}{x^2} + \frac{1}{x^3} \right) \mathrm{e}^{\frac{1}{x}} \, dx = \ln\left(\frac{27}{4} \right) \qquad \textbf{(10)}$$

Question 12

A curve lies entirely above the x axis and has parametric equations

$$x = \sin^2 t, \qquad y = 4\tan^3 t, \qquad 0 \leq t < \tfrac{1}{2}\pi .$$

The finite region R is bounded by the curve, the x axis and the straight line with equation $x = \frac{1}{2}$.

Use integration in parametric to find the exact area of R. **(14)**
