IYGB - MP2 PAPER D - QUESTION 1

a) $f(x) = (1-x)^{\frac{1}{3}}$

$$\Rightarrow f(x) = 1 + \frac{\frac{1}{3}}{1}(-x)^1 + \frac{\frac{1}{3}(-\frac{2}{3})}{1\times 2}(-x)^2 + O(x^3)$$

$$\Rightarrow f(x) = 1 - \frac{1}{3}x - \frac{1}{9}x^2 + O(x^3)$$

b) USING PART (a)

$$\Rightarrow g(x) = (8-3x)^{\frac{1}{3}} = 8^{\frac{1}{3}}\left(1-\frac{3}{8}x\right)^{\frac{1}{3}} = 2\left(1-\frac{3}{8}x\right)^{\frac{1}{3}}$$

$$\Rightarrow g(x) = 2f\left(\frac{3}{8}x\right)$$

$$\Rightarrow g(x) = 2\left[1 - \frac{1}{3}\left(\frac{3}{8}x\right)^1 - \frac{1}{9}\left(\frac{3}{8}x\right)^2 + O(x^3)\right]$$

$$\Rightarrow g(x) = 2\left[1 - \frac{1}{8}x - \frac{1}{64}x^2 + O(x^3)\right.$$

$$\Rightarrow g(x) = 2 - \frac{1}{4}x - \frac{1}{32}x^2 + O(x^3)$$

c) LET $x = \frac{1}{3}$ IN BOTH SIDES OF THE EXPANSION OF $g(x)$

$$\Rightarrow (8-3x)^{\frac{1}{3}} \simeq 2 - \frac{1}{4}x - \frac{1}{32}x^2$$

$$\Rightarrow \left(8 - 3\times\frac{1}{3}\right)^{\frac{1}{3}} \approx 2 - \frac{1}{4}\left(\frac{1}{3}\right) - \frac{1}{32}\left(\frac{1}{3}\right)^2$$

$$\Rightarrow 7^{\frac{1}{3}} \simeq 2 - \frac{1}{12} - \frac{1}{288}$$

$$\Rightarrow \sqrt[3]{7} \simeq \frac{551}{288}$$

AS REQUIRED

IYGB - MP2 PAPER D - QUESTION 2

USING THE APPROXIMATIONS FOR SMALL θ, IN RADIANS

- $\sin\theta \simeq \theta$
- $\cos\theta \simeq 1 - \frac{1}{2}\theta^2$

$$\frac{\cos^2 3x - 1}{2x\sin\left(\frac{3}{4}x\right)} \simeq \frac{\left[1 - \frac{1}{2}(3x)^2\right]^2 - 1}{2x\left(\frac{3}{4}x\right)}$$

$$\simeq \frac{\left(1 - \frac{9}{2}x^2\right)^2 - 1}{\frac{3}{2}x^2}$$

$$\simeq \frac{\left(1 - 9x^2 + \cancel{\frac{81}{4}x^4}\right) - 1}{\frac{3}{2}x^2}$$

$$\simeq \frac{-9x^2}{\frac{3}{2}x^2}$$

$$\simeq -6$$

ALTERNATIVE

$$\frac{\cos^2 3x - 1}{2x\sin\left(\frac{3}{4}x\right)} = -\frac{1-\cos^2 3x}{2x\sin\left(\frac{3}{4}x\right)} = -\frac{\sin^2 3x}{2x\sin\left(\frac{3}{4}x\right)}$$

$$\approx -\frac{(3x)^2}{2x\left(\frac{3}{4}x\right)}$$

$$\approx -\frac{9x^2}{\frac{3}{2}x^2}$$

$$\approx -6$$

IYGB - MP2 PAPER D - QUESTION 3

$$\sum_{r=1}^{12}(2r+7+2^r) = 8430$$

"USUALLY" WE GENERATE A FEW TERMS TO SEE A PATTERN

$$2\times1 + 7 + 2^1 = 11$$
$$2\times2 + 7 + 2^2 = 15$$
$$2\times3 + 7 + 2^3 = 21$$
$$2\times4 + 7 + 2^4 = 31$$
$$2\times5 + 7 + 2^5 = 49$$

NO OBVIOUS PATTERN !?

SPLIT THE SUM

$$\sum_{r=1}^{12}(2r+7) + \sum_{r=1}^{12}2^r$$

$$= \underbrace{(9+11+13+\dots+31)}_{\text{THIS IS AN A.P}} + \underbrace{(2+4+8+\dots+4096)}_{\text{G.P}}$$

THIS IS AN A.P: $a=9$, $d=2$, $L=31$, $n=12$

G.P: $a=2$, $r=2$, $n=12$

USING $S_n = \frac{n}{2}[a+L]$ & $S_n = \frac{a(1-r^n)}{1-r}$

$$= \frac{12}{2}[9+31] + \frac{2(1-2^{12})}{1-2}$$

$$= 240 + 8190$$

$$= 8430$$

IYGB – MP2 PAPER D – QUESTION 4

a) START BY FILLING A TABLE

x	2	6	10	14	18
$\ln\left[\frac{2}{\sqrt{4+\sqrt{x}}}\right]$	-0.1514	-0.2389	-0.2913	-0.3302	-0.3615
	FIRST	REST	REST	REST	LAST

BY THE TRAPEZIUM RULE

$$\int_2^{18} \ln\left[\frac{2}{\sqrt{4+\sqrt{x}}}\right] dx \simeq \frac{\text{THICKNESS}}{2}\left[\text{FIRST}+\text{LAST}+2\sum(\text{REST})\right]$$

$$\simeq \frac{4}{2}\left[-0.1514-0.3615+2(-0.2389-0.2913-0.3302)\right]$$

$$\simeq -4.467$$

b) PROCEED AS FOLLOWS

$$\Rightarrow \int_2^{18} \ln\left[\frac{2}{\sqrt{4+\sqrt{x}}}\right] dx = \int_2^{18} \ln 2 - \ln(4+\sqrt{x})^{\frac{1}{2}}\, dx$$

$$\Rightarrow -4.467 = (\ln 2)\int_2^{18} 1\, dx - \int_2^{18} \ln(4+\sqrt{x})^{\frac{1}{2}}\, dx$$

$$\Rightarrow -4.467 = (\ln 2)\left[x\right]_2^{18} - \frac{1}{2}\int_2^{18} \ln(4+\sqrt{x})\, dx$$

$$\Rightarrow -4.467 = 16\ln 2 - \frac{1}{2}\int_2^{18} \ln(4+\sqrt{x})\, dx$$

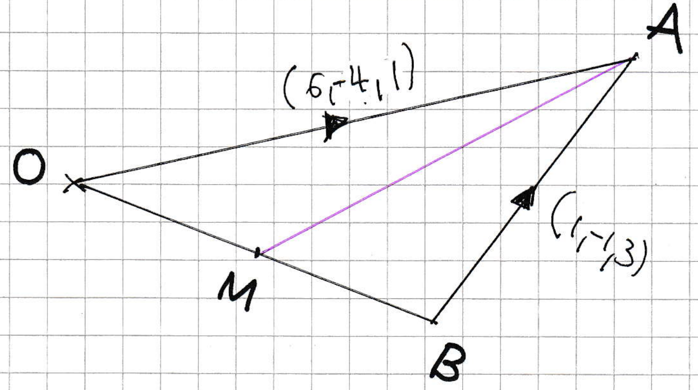
$$\Rightarrow \frac{1}{2}\int_2^{18} \ln(4+\sqrt{x})\, dx = 4.467 + 16\ln 2$$

$$\Rightarrow \int_2^{18} \ln(4+\sqrt{x})\, dx = 2(4.467+16\ln 2)$$

$$\Rightarrow \int_2^{18} \ln(4+\sqrt{x})\, dx \approx 31.1$$

IYGB - MP2 PAPER D - QUESTION 5

- PUT THE INFORMATION INTO A DIAGRAM



- FIND THE POSITION VECTOR (CO-ORDINATES) OF B

$$\Rightarrow \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$\Rightarrow \overrightarrow{OB} = (6,-4,1) - (1,-1,3)$$

$$\Rightarrow \overrightarrow{OB} = (5,-3,-2) \qquad \therefore B(5,-3,-2)$$

- NEXT THE COORDINATES OF M

$$\Rightarrow \overrightarrow{OM} = \frac{1}{2}\overrightarrow{OB} = \frac{1}{2}(5,-3,-2) = \left(\frac{5}{2},-\frac{3}{2},-1\right) \qquad \therefore M\left(\frac{5}{2},-\frac{3}{2},-1\right)$$

- THEN FIND THE VECTOR $\overrightarrow{AM}$

$$\Rightarrow \overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM} = -(6,-4,1) + \left(\frac{5}{2},-\frac{3}{2},-1\right)$$

$$\Rightarrow \overrightarrow{AM} = \left(-\frac{7}{2},\frac{5}{2},-2\right)$$

- FINALLY THE DISTANCE AM

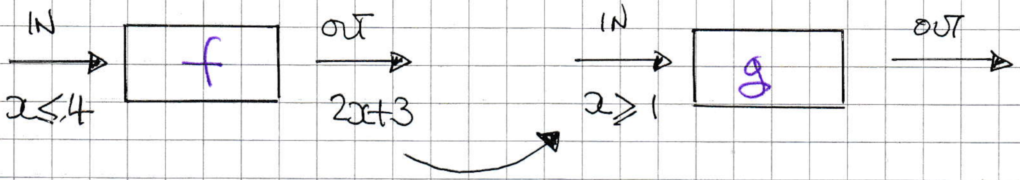
$$\Rightarrow |\overrightarrow{AM}| = \left|-\frac{7}{2},\frac{5}{2},-2\right| = \sqrt{\frac{49}{4}+\frac{25}{4}+4} = \sqrt{\frac{90}{4}} = \frac{3}{2}\sqrt{10}$$

$$\therefore k = \frac{3}{2}$$

IYGB – MP2 PAPER D – QUESTION 6

$$f(x) = 2x+3, \quad x \in \mathbb{R}, \quad x \leq 4$$
$$g(x) = x^2-4, \quad x \in \mathbb{R}, \quad x \geq 1$$

LOOKING AT THE DIAGRAM BELOW



THE DOMAIN MUST SATISFY

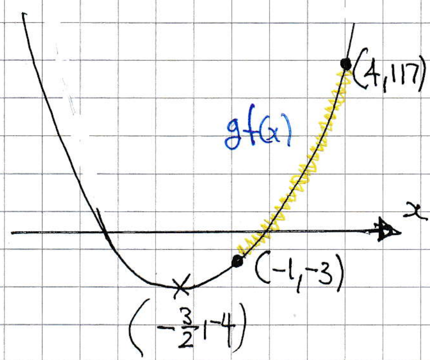
- $x \leq 4$ AND
- $2x+3 \geq 1$
  $2x \geq -2$
  $x \geq -1$

$$\therefore \quad -1 \leq x \leq 4$$

TO FIND THE RANGE IT BEST TO FIND AN EXPRESSION FOR THE COMPOSITION

$$g(f(x)) = g(2x+3) = (2x+3)^2 - 4$$

LOOKING AT THE GRAPH WITH THE DOMAIN ABOVE

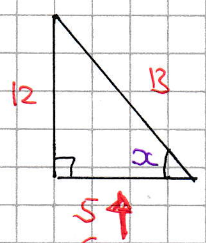


$$-3 \leq gf(x) \leq 117$$

IYGB – MP2 PAPER D – QUESTION 7

COLLECT ALL THE INFORMATION FIRST

$$\sin x = \frac{12}{13} \begin{matrix} \leftarrow \text{OPP} \\ \leftarrow \text{HYP} \end{matrix}$$



(PYTHAGORAS $5^2+12^2=13^2$)

OBTUSE ANGLES HAVE NEGATIVE COSINE

$$\cos x = -\frac{5}{13}$$

USING THE DOUBLE ANGLE IDENTITIES

$$\cot 2x = \frac{\cos 2x}{\sin 2x} = \frac{1-2\sin^2 x}{2\sin x\cos x} = \frac{1-2\left(\frac{12}{13}\right)^2}{2\times\frac{12}{13}\times\left(-\frac{5}{13}\right)} = \frac{1-\frac{288}{169}}{-\frac{120}{169}}$$

$$= \frac{169-288}{-120} = \frac{288-169}{120} = \frac{119}{120}$$

IYGB – MP2 PAPER D – QUESTION 8

a) USING $u_n = ar^{n-1}$

$\Rightarrow 48 = 3 \times r^4$

$\Rightarrow 16 = r^4$

$\Rightarrow r = +2$ (ALL TERMS ARE POSITIVE)

b) USING THE SUM FORMULA FOR $n=10$ & SUM, NOTING THAT IT NEEDS TO BE DOUBLED (DIAMETER SUM IS NEEDED)

$$L_{NEW} = 2 \times \frac{a(r^n - 1)}{r-1} \qquad r=2,\ a=3,\ n=10$$

$$L_{NEW} = 2 \times \frac{3(2^{10}-1)}{2-1}$$

$$L_{NEW} = 6138$$

c) FORM AN EXPRESSION TO SEE THE PATTERN

$$\Rightarrow \text{AREA} = \underset{\substack{\uparrow \\ 1ST}}{\pi \times 3^2} + \underset{\substack{\uparrow \\ 2ND}}{\pi \times (2\times3)^2} + \underset{\substack{\uparrow \\ 3RD}}{\pi \times (2^2\times 3)^2} + \underset{\substack{\uparrow \\ 4th}}{\pi(2^3\times3)^2} + \dots + \pi(2^9\times3)^2$$

$$\Rightarrow \text{AREA} = \pi \times 3^2\left[\underbrace{1 + 2^2 + 2^4 + 2^6 + \dots + 2^{18}}_{10 \text{ terms}}\right]$$

$$\Rightarrow \text{AREA} = 9\pi \times \left[1 + 4 + 16 + 64 + \dots + 262144\right]$$

$$\Rightarrow \text{AREA} = 9\pi \times \frac{1(4^{10}-1)}{4-1}$$

$$\Rightarrow \text{AREA} = 3145725\pi$$

IYGB - MP2 PAPER D - QUESTION 9

a) DIFFERENTIATE USING THE INVERSE RULE

$$x = \sec^2 y + \tan y$$

$$\frac{dx}{dy} = 2\sec y(\sec y \tan y) + \sec^2 y$$

$$\frac{dx}{dy} = 2\sec^2 y \tan y + \sec^2 y$$

$$\frac{dx}{dy} = \sec^2 y(2\tan y + 1)$$

$$\frac{dx}{dy} = \frac{2\tan y + 1}{\cos^2 y}$$

$$\frac{dy}{dx} = \frac{\cos^2 y}{2\tan y + 1} \quad // \quad \text{AS REQUIRED}$$

b) OBTAIN x CO-ORDINATE AND GRADIENT

$$x\Big|_{y=\frac{\pi}{4}} = \sec^2\frac{\pi}{4} + \tan\frac{\pi}{4} = 2 + 1 = 3$$

$$\frac{dy}{dx}\Big|_{y=\frac{\pi}{4}} = \frac{\cos^2\frac{\pi}{4}}{2\tan\frac{\pi}{4} + 1} = \frac{\frac{1}{2}}{2+1} = \frac{\frac{1}{2}}{3} = \frac{1}{6}$$

USING GRADIENT -6 & $(3, \frac{\pi}{4})$

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - \frac{\pi}{4} = -6(x - 3)$$

$$\Rightarrow y - \frac{\pi}{4} = -6x + 18$$

$$\Rightarrow 4y - \pi = -24x + 72$$

$$\Rightarrow 4y + 24x = \pi + 72 \quad //$$

AS REQUIRED

IYGB — MP2 PAPER D — QUESTION 10

a) FORM A DIFFERENTIAL EQUATION FROM THE GIVEN INFORMATION

$$\frac{dV}{dt} = -kV^3$$

RATE DECREASES PROPOR VALUE CUBED

V = VALUE, IN £100
t = TIME, IN YEARS

t=0 , V=10

SOLVING BY SEPARATIONS OF VARIABLES

$$\Rightarrow dV = -kV^3\,dt$$

$$\Rightarrow -\frac{1}{V^3}\,dV = k\,dt$$

$$\Rightarrow \int -V^{-3}\,dV = \int k\,dt$$

$$\Rightarrow \frac{1}{2}V^{-2} = kt + c$$

$$\Rightarrow \frac{1}{2V^2} = kt + C$$

APPLY THE GIVEN CONDITION t=0 V=10 (£1000)

$$\Rightarrow \frac{1}{2\times10^2} = C$$

$$\Rightarrow C = \frac{1}{200}$$

$$\Rightarrow \frac{1}{2V^2} = kt + \frac{1}{200}$$

$$\Rightarrow \frac{1}{V^2} = 2kt + \frac{1}{100} \qquad (\text{LET } A = 2k)$$

$$\Rightarrow \frac{1}{V^2} = At + \frac{1}{100}$$

AS REQUIRED

IYGB – MP2 PAPER D – QUESTION 10

b) USING THE FACT THAT WHEN $t=1$ $V=5$ (£500/HALF)

$$\Rightarrow \frac{1}{5^2} = A \times 1 + \frac{1}{100}$$

$$\Rightarrow \frac{1}{25} = A + \frac{1}{100}$$

$$\Rightarrow A = \frac{3}{100}$$

$$\Rightarrow \frac{1}{V^2} = \frac{3}{100}t + \frac{1}{100}$$

FINALLY WHEN $V=2.5$ (VAL = £250)

$$\Rightarrow \frac{1}{2.5^2} = \frac{3}{100}t + \frac{1}{100}$$

$$\Rightarrow \frac{1}{6.25} = \frac{3}{100}t + \frac{1}{100}$$

$\times 100$

$$\Rightarrow 16 = 3t + 1$$

$$\Rightarrow 3t = 15$$

$$\Rightarrow t = 5$$

IYGB - MP2 PAPER D - QUESTION 11

USING THE SUBSTITUTION $u = \frac{1}{x} \iff x = \frac{1}{u}$

$$\frac{du}{dx} = -\frac{1}{x^2}$$

$$dx = -x^2\,du$$

$$dx = -\frac{1}{u^2}\,du$$

- $x = \frac{1}{\ln 3} \longmapsto u = \ln 3$
- $x = \frac{1}{\ln 2} \longmapsto u = \ln 2$

TRANSFORMING THE INTEGRAL

$$\int_{\frac{1}{\ln 3}}^{\frac{1}{\ln 2}} \left(\frac{1}{x^2} + \frac{1}{x^3}\right) e^{\frac{1}{x}}\,dx = \int_{\ln 3}^{\ln 2} (u^2 + u^3)e^u \left(-\frac{1}{u^2}\,du\right)$$

$$= \int_{\ln 2}^{\ln 3} (1+u)e^u\,du$$

CONTINUE BY INTEGRATION BY PARTS

$$= \Big[(u+1)e^u\Big]_{\ln 2}^{\ln 3} - \int_{\ln 2}^{\ln 3} e^u\,du$$

$u+1$	1
e^u	e^u

$$= \Big[(u+1)e^u - e^u\Big]_{\ln 2}^{\ln 3} = \Big[ue^u\Big]_{\ln 2}^{\ln 3}$$

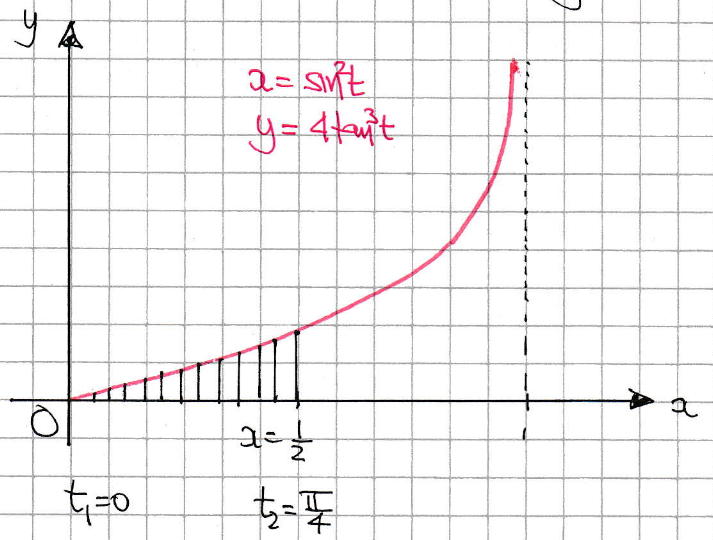
$$= 3\ln 3 - 2\ln 2 = \ln 27 - \ln 4 = \ln\left(\frac{27}{4}\right)$$

IYGB - MP2 PAPER D - QUESTION 12

START WITH A DIAGRAM — BY INSPECTION $t=0 \Longrightarrow (0,0)$

— $0 \leqslant x < 1$

— $0 \leqslant y < \infty$



- $x=0 \longmapsto t=0$
- $x=\frac{1}{2}$ $\quad \frac{1}{2} = \sin^2 t$

  $\sin t = \pm\frac{1}{\sqrt{2}}$

  $t = +\frac{\pi}{4}$

SETTING UP A PARAMETRIC INTEGRAL

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\,\frac{dx}{dt}\,dt = \int_0^{\frac{\pi}{4}} 4\tan^3 t\,(2\sin t\cos t\,dt)$$

$$= \int_0^{\frac{\pi}{4}} 8\tan^3 t\sin t\cos t\,dt = 8\int_0^{\frac{\pi}{4}} \frac{\sin^3 t}{\cos^3 t}\times\sin t\cos t\,dt$$

$$= 8\int_0^{\frac{\pi}{4}} \frac{\sin^4 t}{\cos^2 t}\,dt = 8\int_0^{\frac{\pi}{4}} \frac{(1-\cos^2 t)^2}{\cos^2 t}\,dt$$

EXPAND AND SPLIT THE FRACTION

$$\text{AREA} = 8\int_0^{\frac{\pi}{4}} \frac{1-2\cos^2 t+\cos^4 t}{\cos^2 t}\,dt = 8\int_0^{\frac{\pi}{4}} \sec^2 t - 2 + \cos^2 t\;\,dt$$

$$= 8\int_0^{\frac{\pi}{4}} \sec^2 t - 2 + \left(\frac{1}{2}+\frac{1}{2}\cos 2t\right)\,dt$$

IYGB - MP2 PAPER D - QUESTION 12

FINALLY TIDY & EVALUATE

$$\text{AREA} = 8\int_0^{\frac{\pi}{4}} \sec^2 t - \frac{3}{2} + \frac{1}{2}\cos 2t \ dt$$

$$= 8\left[\tan t - \frac{3}{2}t + \frac{1}{4}\sin 2t\right]_0^{\frac{\pi}{4}}$$

$$= 8\left[\left(1 - \frac{3}{2}\times\frac{\pi}{4} + \frac{1}{4}\right) - 0\right]$$

$$= 8\left[\frac{5}{4} - \frac{3\pi}{8}\right]$$

$$= 10 - 3\pi$$