IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper F

Difficulty Rating: 3.4100/1.0811

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 13 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

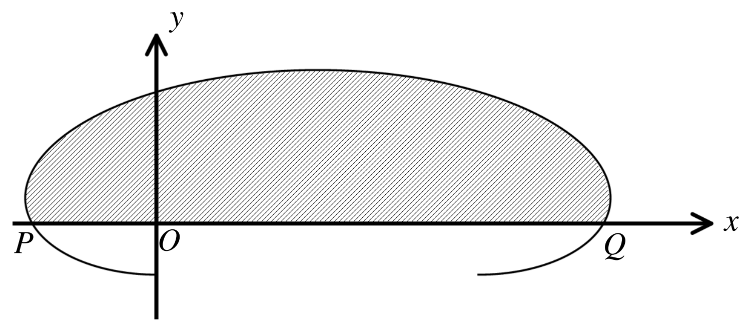
Question 1

$$x^3 + 10x - 4 = 0.$$

a) Show that the above equation has a root α, which lies between 0 and 1. **(2)**

b) Use the Newton-Raphson method twice, starting with $x_1 = 0.5$ to find, correct to 4 decimal places, an approximation for α. **(6)**

Question 2



The figure above shows a curve known as a re-entrant cycloid, with parametric equations

$$x = \theta - 4\sin\theta, \quad y = 1 - 2\cos\theta, \quad 0 \le \theta \le 2\pi.$$

The curve crosses the x axis at the points P and Q.

a) Find the value of θ at the points P and Q. **(3)**

b) Show that the area of the finite region bounded by the curve and the x axis, shown shaded in the figure above, is given by the integral

$$\int_{\theta_1}^{\theta_2} 1 - 6\cos\theta + 8\cos^2\theta \; d\theta,$$

where θ_1 and θ_2 must be stated. **(4)**

c) Find an exact value for the above integral. **(5)**

Question 3

Solve the trigonometric equation

$$\sin\theta\cos\frac{\pi}{5} = \frac{1}{2} - \cos\theta\sin\frac{\pi}{5}, \quad 2\pi < \theta < 4\pi,$$

giving the answers in terms of π. **(5)**

Question 4

It is given that $|y| = 2$, $y \in \mathbb{R}$.

a) Find the possible values of $|3y-1|$. **(2)**

It is next given that $5 \le t \le 13$, $t \in \mathbb{R}$.

b) Express the above inequality in the form $|t-a| \le b$, where a and b are positive integers to be stated. **(2)**

It is finally given that

$$\left|x-\sqrt{2}\right| = \left|x+5\sqrt{2}\right|, \; x \in \mathbb{R}.$$

c) Determine the value of x. **(3)**

Question 5

The points $A(5,-1,0)$, $B(3,5,-4)$, $C(12,2,8)$ are referred relative to a fixed origin O.

The point D is such so that $\overrightarrow{AD} = 2\overrightarrow{BC}$.

Determine the distance CD. **(6)**

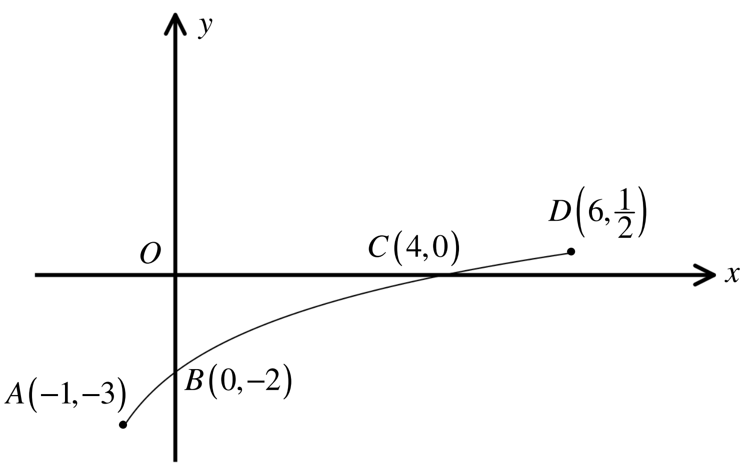
Question 6

$$y = \arctan\left(\tfrac{1}{2}x\right),\ x \in \mathbb{R}.$$

By writing $y = \arctan\left(\tfrac{1}{2}x\right)$ as $x = f(y)$, show that

$$\frac{dy}{dx} = \frac{2}{x^2+4}.$$ **(5)**

Question 7



The figure above shows the graph of the function $f(x)$, defined for $-1 \le x \le 6$.

Sketch the graph of $f^{-1}(x)$, marking clearly the end points of the graph and any points where it crosses the coordinate axes. **(3)**

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Question 8

A curve is given parametrically by

$$x = 3 + 2\cos\theta, \quad y = -3 + 2\sin\theta, \quad 0 \le \theta < 2\pi .$$

Show clearly that

$$\frac{dy}{dx} = \frac{3-x}{3+y}. \qquad \textbf{(5)}$$

Question 9

The sum to infinity of a geometric progression is four times as large as its second term.

a) Show that the common ratio of the series is $\frac{1}{2}$. **(4)**

It is further given that the sum of the first four terms of the progression is 5760.

b) Find the first term of the progression. **(3)**

The sum of the first k terms of the progression is **three less** than its sum to infinity.

c) Use algebra to determine the value of k. **(4)**

Question 10

a) Find an estimate for the following integral, by using the trapezium rule with 5 equally spaced **ordinates.** to for

$$\int_1^2 e^{\frac{1}{10}x^2} \; dx . \qquad \textbf{(4)}$$

b) Use the answer of part **(a)** to find estimates for

$$\int_1^2 e^{1+\frac{1}{10}x^2} \; dx . \qquad \textbf{(3)}$$

Question 11

Use the substitution $u = 1 + x^2 e^{-3x}$ to find an expression for

$$\int \frac{x(2-3x)}{e^{3x}+x^2}\,dx\,.$$ **(8)**

Question 12

$$f(x) = \sqrt{1-x},\ -1 < x < 1,$$

a) Expand $f(x)$ in ascending powers of x, up and including the term in x^2. **(3)**

b) Use the expansion of part **(a)** to show that if y is numerically small

$$\sqrt{1-4y+y^2} \approx 1-2y-\frac{3}{2}y^2\,.$$ **(5)**

Question 13

A container is the shape of a hollow **inverted** right circular cone has base radius 20 cm and height 80 cm. The container is filled with water and is supported in an upright position.

Water is leaking out of a hole at the vertex of the cone.

Let h cm be the height of the water in the container, where h is measured from the vertex of the cone, and t minutes be the time from the instant since $h = 80$.

The rate at which the volume of the water is decreasing is directly proportional to the height of the water in the container.

a) By relating the measurements of the container to that of the volume of the water in the container, show that

$$\frac{dh}{dt} = -\frac{A}{h},$$

where A is a positive constant.

$\left[\text{volume of a cone of radius } r \text{ and height } h \text{ is given by } \frac{1}{3}\pi r^2 h\right]$ **(7)**

When $t = 1$, $h = 78$.

b) Determine the value of t by which all the water would have leaked out of the container. **(8)**

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