IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper H

Difficulty Rating: 3.8450/1.2993

Time: 2 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 11 questions in this question paper.
The total mark for this paper is 100.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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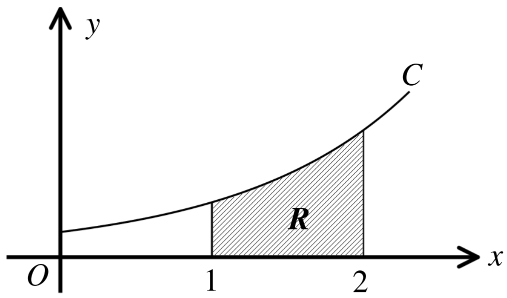
Question 1

The points $A(-2,-10,-17)$ and $B(25,-1,19)$ are referred relative to a fixed origin O.

The point C is such so that ACB forms a straight line.

Given further that $\frac{|\overrightarrow{AC}|}{|\overrightarrow{CB}|}=\frac{2}{7}$, determine the coordinates of C. **(4)**

Question 2



The figure above shows the curve C, given parametrically by

$$x=\ln t, \quad y=t+\sqrt{t}\ , \quad 1\le t\le 10 .$$

The finite region R is bounded by C, the straight lines with equations $x=1$ and $x=2$, and the x axis.

a) Show that the area of R is given by

$$\int_{T_1}^{T_2} 1+t^{-\frac{1}{2}}\ dt ,$$ **(4)**

stating the values of T_1 and T_2.

b) Hence find an exact value for the area of R. **(2)**

Question 3

It is given that θ and φ are such so that

$$\tan\theta = t \quad \text{and} \quad \tan\varphi = t-1,$$

where t is a constant.

It is further given that

$$\frac{1}{\cos^2\theta} - \frac{1}{\cos^2\varphi} = 3.$$

a) Show clearly that $t = 2$. **(5)**

b) Determine the exact value of $\tan(\theta+\varphi)$, showing clearly all the steps in the workings. **(3)**

Question 4

The terms of a geometric progression are $u_1, u_2, u_3, u_4, u_5, \ldots$

a) Given that $u_4 = 6$ and $u_3 + u_5 = 20$, show that

$$3r^2 - 10r + 3 = 0,$$

where r is the common ratio of the progression. **(5)**

b) Given further that the progression has a sum to infinity determine its value. **(4)**

Question 5

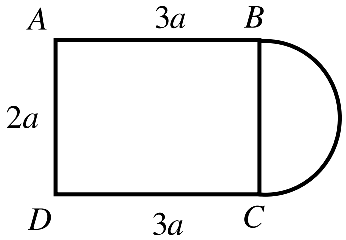


figure 1

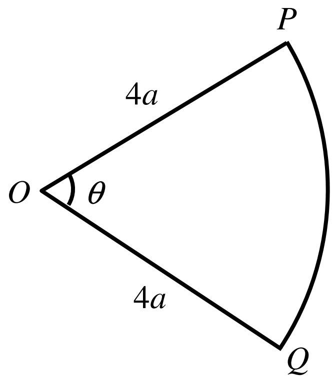


figure 2

Figure 1, shows a rectangle $ABCD$ where $|AB| = |DC| = 3a$ and $|AD| = |BC| = 2a$.

A semicircle with diameter BC is attached to the rectangle. The rectangle and the semicircle are to be considered as a single composite shape X.

Figure 2, shows a circular sector OPQ where $|OP| = |OQ| = 4a$. The sector has its centre at O, and $\measuredangle POQ = \theta$ radians. The sector is denoted as shape Y.

a) Given that the area of X is equal to the area of Y, express θ in terms of π. **(4)**

b) Given further that the perimeter Y is greater than the perimeter of X, show that the difference between the perimeter of X and Y is

$$\frac{3}{4}a(4-\pi).$$

(4)

Question 6

By using the substitution $u = \sqrt[3]{x}$, or otherwise, show that

$$\int_0^{\sqrt{27}} \frac{2}{x + \sqrt[3]{x}} \, dx = 6\ln 2.$$

(7)

Question 7

A curve C has implicit equation

$$2xy = 2^x + y^2.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{y - 2^{x-1}\ln 2}{y - x}. \qquad (5)$$

The point P lies on C, where $x = 2$.

b) Find an equation of the tangent to C at P. **(5)**

Question 8

The function $f(x)$ is defined by

$$f(x) \equiv 3 - 2x^2,\ x \in \mathbb{R},\ x \le 0.$$

a) State the range of $f(x)$. **(1)**

b) Show that

$$ff(x) = -8x^4 + 24x^2 - 15$$

and hence solve the equation

$$ff(x) = -47. \qquad (5)$$

c) Find an expression for the inverse function, $f^{-1}(x)$. **(3)**

d) Solve the equation

$$f(x) = f^{-1}(x). \qquad (4)$$

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Question 9

Show, with detailed workings, that

a) $\frac{d}{dx}(\cos 2x \tan 2x) = 2\cos 2x$. **(3)**

b) $\frac{d}{dx}\left(\frac{x^2}{(3x-1)^2}\right) = -\frac{2x}{(3x-1)^3}$. **(5)**

Question 10

At the point P, which lies on the curve with equation

$$x = \ln\left(y^3 - y\right),$$

the gradient is 4 .

The point P is close to the point with coordinates $(7.5\ , 12)$.

a) Show that the y coordinate of P is a solution of the equation

$$y^3 - 12y^2 - y + 4 = 0 .$$ **(5)**

b) Use the Newton Raphson method once on the equation of part **(a)**, in order to determine the coordinates of P, correct to two decimal places. **(6)**

Question 11

Water is drained from a large hole at the bottom of a tank of height 4 m.

Let $V \text{ m}^3$ and $x \text{ m}$ be the volume and the height of the water in the tank, respectively, at time t minutes since the water started draining out.

Suppose further that the shape of the tank is such so that V and x are related by

$$V = \frac{5}{3}x^3 .$$

The rate at which the volume of the water is drained is proportional to the square root of its height, so that it can be modelled by the differential equation

$$\frac{dV}{dt} = -kx^{\frac{1}{2}},$$

where k is a positive constant.

a) Given that it takes 32 minutes to empty the full tank, show that …

i. … $5x^{\frac{3}{2}}\frac{dx}{dt} = -k$. **(4)**

ii. … $t = 32 - x^{\frac{5}{2}}$. **(7)**

When the tank is completely empty, water begins to pour in at the constant rate of 0.5 m^3 per minute and continues to drain out at the same rate as before.

b) Show further that …

i. … $\frac{dx}{dt} = \frac{1-4\sqrt{x}}{10x^2}$. **(3)**

ii. … the height of the water cannot exceed $\frac{1}{16}$ of a metre. **(2)**

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