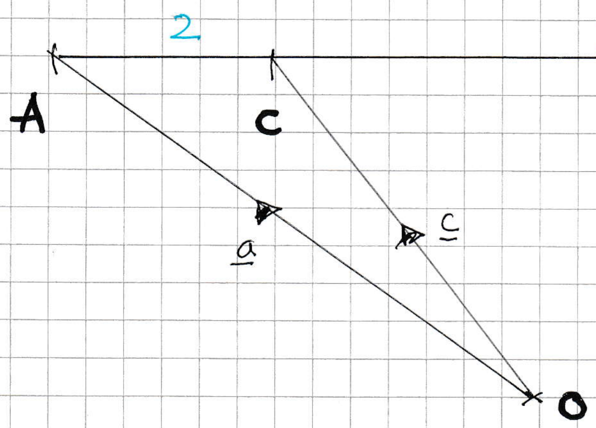
IYGB – MP2 PAPER H – QUESTION 1

$A(-2,-10,-17)$ $\quad B(25,-1,19)$ $\quad |\overrightarrow{AC}| : |\overrightarrow{CB}|$
$2 : 7$



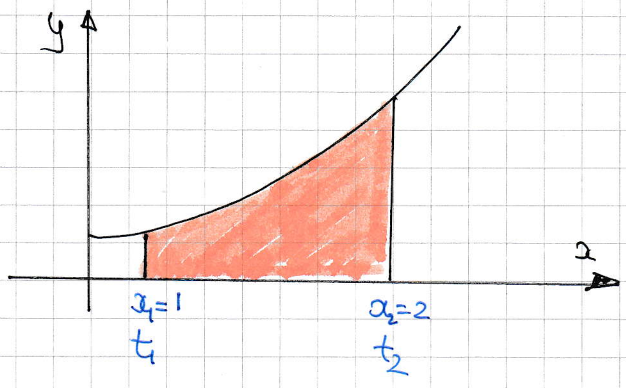
LOOKING AT THE DIAGRAM

$$\Rightarrow \overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$$
$$\Rightarrow \overrightarrow{OC} = \overrightarrow{OA} + \frac{2}{9}\overrightarrow{AB}$$
$$\Rightarrow \underline{c} = \underline{a} + \frac{2}{9}(\underline{b} - \underline{a})$$
$$\Rightarrow \underline{c} = \underline{a} + \frac{2}{9}\underline{b} - \frac{2}{9}\underline{a}$$
$$\Rightarrow \underline{c} = \frac{7}{9}\underline{a} + \frac{2}{9}\underline{b}$$
$$\Rightarrow \underline{c} = \frac{1}{9}\left[7\underline{a} + 2\underline{b}\right]$$
$$\Rightarrow \underline{c} = \frac{1}{9}\left[7(-2,-10,-17) + 2(25,-1,19)\right]$$
$$\Rightarrow \underline{c} = \frac{1}{9}(36,-72,-81)$$
$$\Rightarrow \underline{c} = (4,-8,-9)$$

$\therefore C(4,-8,-9)$

IYGB - MP2 PAPER H - QUESTION 2

a) LOOKING AT THE DIAGRAM TO CONVERT LIMITS



- $x=1$, $\ln t=1$, $t=e \leftarrow t_1$
- $x=2$, $\ln t=2$, $t=e^2 \leftarrow t_2$

$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{t_1}^{t_2} y(t)\,\frac{dx}{dt}\,dt = \int_{e}^{e^2} \left(t+t^{\frac{1}{2}}\right)\underbrace{\left(\frac{1}{t}\right)}_{\frac{dx}{dt}}\,dt$$

$$= \int_{e}^{e^2} t\times\frac{1}{t} + t^{\frac{1}{2}}\times\frac{1}{t}\,dt = \int_{e}^{e^2} 1+t^{-\frac{1}{2}}\,dt$$

AS REQUIRED

b) INTEGRATING PART (a)

$$\dots = \left[t+2t^{\frac{1}{2}}\right]_{e}^{e^2} = \left[e^2+2(e^2)^{\frac{1}{2}}\right]-\left[e+2e^{\frac{1}{2}}\right]$$

$$= e^2+2e-e-2e^{\frac{1}{2}}$$

$$= e^2+e-2\sqrt{e}$$

IYGB - MP2 PAPER H - QUESTION 3

a) WORKING AS FOLLOWS

$$\Rightarrow \frac{1}{\cos^2\theta} - \frac{1}{\cos^2\phi} = 3$$

$$\Rightarrow \sec^2\theta - \sec^2\phi = 3$$

$$\Rightarrow (1+\tan^2\theta) - (1+\tan^2\phi) = 3$$

$1 + \tan^2 A \equiv \sec^2 A$

$$\Rightarrow \tan^2\theta - \tan^2\phi = 3$$

$$\Rightarrow t^2 - (t-1)^2 = 3$$

$$\Rightarrow t^2 - (t^2 - 2t + 1) = 3$$

$$\Rightarrow \cancel{t^2} - \cancel{t^2} + 2t - 1 = 3$$

$$\Rightarrow 2t = 4$$

$$\Rightarrow t = 2$$

b) USING THE TANGENT COMPOUND IDENTITY

$$\Rightarrow \tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{t + (t-1)}{1 - t(t-1)}$$

$$= \frac{2t-1}{1-t^2+t} = \frac{2t-1}{1-2^2+2} = \frac{2\times2-1}{3-2^2}$$

$$= \frac{3}{-1} = -3$$

IYGB - MP2 PAPER H - QUESTION 4

a) USING THE FORMULA $u_n = ar^{n-1}$

$u_4 = 6$

$ar^3 = 6$

$u_3 + u_5 = 20$

$ar^2 + ar^4 = 20$

$ar^2(1+r^2) = 20$

DIVIDING THE EQUATIONS

$$\frac{\cancel{a}r^{\cancel{2}}(1+r^2)}{\cancel{a}r^{\cancel{3}}} = \frac{20}{6} \implies \frac{1+r^2}{r} = \frac{10}{3}$$

$$\implies 3(1+r^2) = 10r$$

$$\implies 3r^2+3 = 10r$$

$$\implies 3r^2 - 10r + 3 = 0$$

AS REQUIRED

b) SOLVING THE QUADRATIC

$$\implies (3r-1)(r-3) = 0$$

$$\implies r = \frac{1}{3} \quad \text{or} \quad \cancel{3}$$

NO SUM TO INFINITY FOR THIS VALUE

USING $ar^3 = 6$

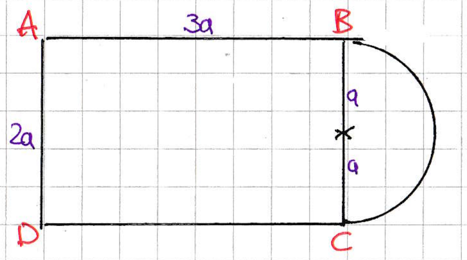
$$\implies a \times \left(\frac{1}{3}\right)^3 = 6$$

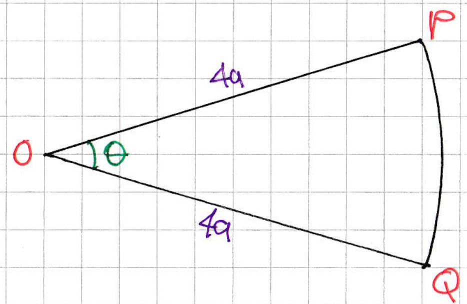
$$\implies a = 162$$

FINALLY $S_\infty = \frac{a}{1-r} = \frac{162}{1-\frac{1}{3}} = \frac{162}{\frac{2}{3}} = 243$

IYGB - MP2 PAPER H - QUESTION 5

a) LOOKING AT THE DIAGRAMS





"AREA OF X" = "AREA OF Y"

$$(2a)(3a) + \frac{1}{2}\pi a^2 = \underbrace{\frac{1}{2}(4a)^2\theta}_{\text{"}\frac{1}{2}r^2\theta^c\text{"}}$$

$$\Rightarrow 6a^2 + \frac{1}{2}\pi a^2 = 8a^2\theta$$

$$\Rightarrow 6 + \frac{1}{2}\pi = 8\theta$$

$$\Rightarrow 12 + \pi = 16\theta$$

$$\Rightarrow \theta = \frac{1}{16}(\pi + 12)$$

b) PERIMETER OF Y - PERIMETER X

$$= \Big[4a + 4a + \underbrace{(4a)\theta}_{\text{"}L = r\theta\text{"}}\Big] - \Big[2a + 3a + 3a + \underbrace{\tfrac{1}{2}(2\pi a)}_{\text{HALF CIRCUMFERENCE}}\Big]$$

$$= 8a + 4a\theta - [8a + \pi a]$$

$$= \cancel{8a} + 4a\theta - \cancel{8a} - \pi a$$

$$= 4a\theta - \pi a$$

$$= 4a \times \frac{1}{16}(\pi + 12) - \pi a$$

$$= \frac{1}{4}a(\pi + 12) - \pi a$$

$$= \frac{1}{4}\pi a + 3a - \pi a$$

$$= 3a - \frac{3}{4}\pi a$$

$$= \frac{3}{4}a(4 - \pi)$$

AS REQUIRED

IYGB – MP2 PAPER T – QUESTION 6

USING THE SUBSTITUTION GIVEN

$$\int_0^{\sqrt{27}} \frac{2}{\sqrt[3]{x}+x}\,dx = \int_0^{\sqrt{3}} \frac{2}{u+u^3}\,(3u^2\,du)$$

$$= \int_0^{\sqrt{3}} \frac{6u^2}{u(1+u^2)}\,du = \int_0^{\sqrt{3}} \frac{6u}{u^2+1}\,du$$

$$= 3\int_0^{\sqrt{3}} \frac{2u}{u^2+1}\,du$$

$$\boxed{\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|+C}$$

- $u=\sqrt[3]{x}$

$u^3=x$

$x=u^3$

$\frac{dx}{du}=3u^2$

$dx=3u^2\,du$

- $x=\sqrt{27}$

$27^{\frac{1}{2}}=u^3$

$3^{\frac{3}{2}}=u^3$

$3^{\frac{1}{2}}=u$

- $x=0,\ u=0$

USING THE ABOVE RESULT (OR ANOTHER SUBSTITUTION)

$$\dots = 3\Big[\ln|u^2+1|\Big]_0^{\sqrt{3}} = 3\left[\ln 4 - \cancel{\ln 1}\right]$$

$$= 3\times 2\ln 2$$

$$= \underline{\underline{6\ln 2}}$$

AS REQUIRED

IYGB - MP2 PAPER H - QUESTION 7

a) DIFFERENTIATE W.R.T x

$$\Rightarrow \frac{d}{dx}(2xy) = \frac{d}{dx}(2^x + y^2)$$

$$\Rightarrow 2y + 2x\frac{dy}{dx} = 2^x \ln 2 + 2y\frac{dy}{dx}$$

$$\boxed{\frac{d}{dx}(a^x) = a^x \ln a}$$

$$\Rightarrow 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2^x \ln 2 - 2y$$

$$\Rightarrow (2x - 2y)\frac{dy}{dx} = -2y + 2^x \ln 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2y + 2^x \ln 2}{2x - 2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2(y - 2^{x-1}\ln 2)}{-2(y - x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 2^{x-1}\ln 2}{y - x}$$

// AS REQUIRED

b) FIRST FIND THE FULL COORDINATES OF P

WHEN $x = 2$ $\Rightarrow 2xy = 2^x + y^2$

$$\Rightarrow 4y = 4 + y^2$$

$$\Rightarrow 0 = y^2 - 4y + 4$$

$$\Rightarrow 0 = (y-2)^2$$

$$\Rightarrow y = 2$$

$$\therefore P(2,2)$$

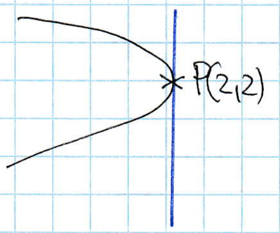
NEXT FIND THE GRADIENT AT P

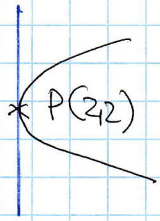
$$\left.\frac{dy}{dx}\right|_{(2,2)} = \frac{2 - 2\ln 2}{2 - 2} = \frac{2 - 2\ln 2}{0} = \infty$$ ← INFINITE GRADIENT

IYGB - MP2 PAPER H - QUESTION 7

HENCE THE TANGENT AT $P(2,2)$ IS "VERTICAL"

i.e. EITHER OR

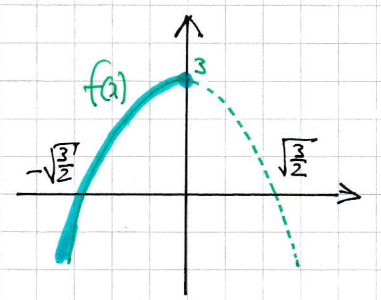




$\therefore$ EQUATION OF TANGENT IS $x=2$

IYGB - MP2 PAPER H - QUESTION 8

a) SKETCHING THE FUNCTION



RANGE

$f(x) \leq 3$

b) FINDING THE COMPOSITION $ff(x)$

$$f(f(x)) = f(3-2x^2) = 3-2(3-2x^2)^2 = 3-2(9-12x^2+4x^4)$$

$$= 3-18+24x^2-8x^4 = -8x^4+24x^2-15$$

AS REQUIRED

SOLVING THE REQUIRED EQUATION

$$\Rightarrow -8x^4+24x^2-15 = -47$$

$$\Rightarrow -8x^4+24x^2+32 = 0$$

$\div(-8)$

$$\Rightarrow x^4-3x^2-4 = 0$$

THIS IS A QUADRATIC IN x^2

$$\Rightarrow (x^2+1)(x^2-4) = 0$$

$$\Rightarrow (x^2+1)(x-2)(x+2) = 0$$

IRREDUCIBLE AS $x^2+1 \neq 0$

$$\Rightarrow x = \begin{cases} \cancel{2} \\ -2 \end{cases}$$

NOT IN THE DOMAIN

$$\Rightarrow x = -2$$

IYGB - MP2 PAPER H - QUESTION 8

c) USING STANDARD METHODOLOGY

$\Rightarrow y = 3 - 2x^2$

$\Rightarrow 2x^2 = 3 - y$

$\Rightarrow x^2 = \frac{3-y}{2}$

$\Rightarrow x = \pm\sqrt{\frac{3-y}{2}}$ BUT $x \leq 0$

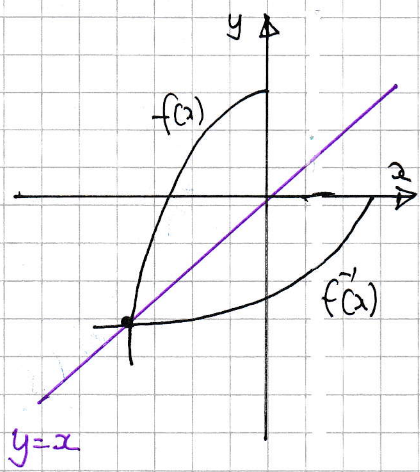
$\Rightarrow x = -\sqrt{\frac{3-y}{2}}$

$\therefore f^{-1}(x) = -\sqrt{\frac{3-x}{2}}$

d) SOLVING $f = f^{-1}$ I.E $3 - 2x^2 = -\sqrt{\frac{3-x}{2}}$ IS UNPLEASANT

WE SOLVE INSTEAD EITHER $f(x) = x$

OR $f^{-1}(x) = x$ (DIAGRAM)



$\Rightarrow 3 - 2x^2 = x$

$\Rightarrow 0 = 2x^2 + x - 3$

$\Rightarrow 0 = (2x+3)(x-1)$

$\Rightarrow x = \cancel{1}$ NOT IN THE DOMAIN, $-\frac{3}{2}$

IYGB - MP2 PAPER H - QUESTION 9

a) MANIPULATE BEFORE DIFFERENTIATING!!

$$\frac{d}{dx}\left[\cos 2x \tan 2x\right] = \frac{d}{dx}\left[\cancel{\cos 2x} \times \frac{\sin 2x}{\cancel{\cos 2x}}\right] = \frac{d}{dx}\left[\sin 2x\right]$$

$$= 2 \times \cos 2x = 2\cos 2x$$

AS REQUIRED

OR BY THE PRODUCT RULE

$$\frac{d}{dx}\left[\cos 2x \tan 2x\right] = -2\sin 2x \tan 2x + \cos 2x \sec^2 2x \times 2$$

$$= 2\cos 2x \sec^2 2x - 2\sin 2x \tan 2x$$

$$= 2\left[\cos 2x \times \frac{1}{\cos^2 2x} - \sin 2x \times \frac{\sin 2x}{\cos 2x}\right]$$

$$= 2\left[\frac{1}{\cos 2x} - \frac{\sin^2 2x}{\cos 2x}\right]$$

$\cos^2 2x + \sin^2 2x \equiv 1$

$$= 2\left[\frac{1 - \sin^2 2x}{\cos 2x}\right] = 2 \times \frac{\cos^{\cancel{2}} 2x}{\cancel{\cos 2x}}$$

$$= 2\cos 2x$$

AS REQUIRED

b) BY THE QUOTIENT RULE

$$\frac{d}{dx}\left[\frac{x^2}{(3x-1)^2}\right] = \frac{(3x-1)^2 \times 2x - x^2 \times 2(3x-1) \times 3}{\left[(3x-1)^2\right]^2}$$

$$= \frac{2x(3x-1)^2 - 6x^2(3x-1)}{(3x-1)^4} = \frac{\cancel{(3x-1)}\left[2x(3x-1) - 6x^2\right]}{(3x-1)^{\cancel{4}\,3}}$$

$$= \frac{6x^2 - 2x - 6x^2}{(3x-1)^3} = \frac{-2x}{(3x-1)^3}$$

$$= -\frac{2x}{(3x-1)^3}$$

AS REQUIRED

IYGB - MP2 PAPER H - QUESTION 10

a) START BY DIFFERENTIATION

$$\Rightarrow x = \ln(y^3 - y)$$

$$\Rightarrow \frac{dx}{dy} = \frac{3y^2 - 1}{y^3 - y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^3 - y}{3y^2 - 1}$$

SETTING $\frac{dy}{dx} = 4$

$$\Rightarrow \frac{y^3 - y}{3y^2 - 1} = 4$$

$$\Rightarrow y^3 - y = 12y^2 - 4$$

$$\Rightarrow y^3 - 12y^2 - y + 4 = 0$$

AS REQUIRED

b) LET $f(y) = y^3 - 12y^2 - y + 4$

- $f'(y) = 3y^2 - 24y - 1$
- $f(12) = -8$
- $f'(12) = 143$

BY THE NEWTON RAPHSON METHOD

$$\Rightarrow y_{r+1} = y_r - \frac{f(y_r)}{f'(y_r)}$$

IYGB - MP2 PAPER H - QUESTION 10

$$\Rightarrow \quad y = 12 - \frac{f(12)}{f'(12)}$$

$$\Rightarrow \quad y = 12 - \frac{-8}{143}$$

$$\Rightarrow \quad y = \frac{1724}{143}$$

$$\Rightarrow \quad y \simeq 12.05594\ldots$$

& hence $x = 7.46176\ldots$

$\therefore \; P(7.46, 12.06)$ // AS REQUIRED

IYGB - MP2 PAPER H - QUESTION 11

a) USING THE O.D.E GIVEN

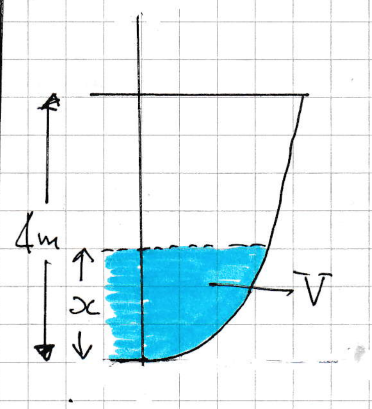
(I) $\frac{dV}{dt} = -kx^{\frac{1}{2}}$

$\frac{dV}{dx} \times \frac{dx}{dt} = -kx^{\frac{1}{2}}$

$5x^2 \frac{dx}{dt} = -kx^{\frac{1}{2}}$

$5x^{\frac{3}{2}} \frac{dx}{dt} = -k$

AS REQUIRED



$V = \frac{5}{3}x^3$

$\frac{dV}{dx} = 5x^2$

(II) SOLVING BY SEPARATION OF VARIABLES

$\Rightarrow 5x^{\frac{3}{2}}\,dx = -k\,dt$

$\Rightarrow \int 5x^{\frac{3}{2}}\,dx = \int -k\,dt$

$\Rightarrow \boxed{2x^{\frac{5}{2}} = -kt + C}$

WHEN $t=0$, $x=4$ (GIVEN)

$\Rightarrow 2\times4^{\frac{5}{2}} = -k\times0 + C$

$\Rightarrow C = 64$

$\Rightarrow \boxed{2x^{\frac{5}{2}} = 64 - kt}$

WHEN $t=32$, $x=0$ (TANK IS EMPTY IN 32 MINUTES)

$\Rightarrow 2\times0^{\frac{5}{2}} = 64 - k\times32$

$\Rightarrow 0 = 64 - 32k$

$\Rightarrow k = 2$

$\Rightarrow 2x^{\frac{5}{2}} = 64 - 2t$

IYGB - MP2 PAPER H - QUESTION 11

$$\Rightarrow \quad 2t = 64 - 2x^{\frac{5}{2}}$$

$$\Rightarrow \quad t = 32 - x^{\frac{5}{2}}$$

AS REQUIRED

b) REMODELLING THE O.D.E BY LOOKING AT THE ORIGINAL O.D.E

$$\frac{dV}{dt} = \underbrace{-kx^{\frac{1}{2}}}_{\text{LEAKAGE ONLY}} \qquad \text{NOW} \qquad \frac{dV}{dt} = \underbrace{-kx^{\frac{1}{2}}}_{\text{LEAKAGE AS BEFORE}} + \underbrace{\frac{1}{2}}_{\text{WATER IN AT CONSTANT RATE}}$$

(I) RELATING THE RATES AS BEFORE

$$\Rightarrow \quad \frac{dV}{dt} = \frac{1}{2} - kx^{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{dV}{dx} \times \frac{dx}{dt} = \frac{1}{2} - 2x^{\frac{1}{2}}$$

$$\Rightarrow \quad 5x^2 \frac{dx}{dt} = \frac{1}{2} - 2x^{\frac{1}{2}}$$

$$\Rightarrow \quad 10x^2 \frac{dx}{dt} = 1 - 4x^{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{1 - 4x^{\frac{1}{2}}}{10x^2}$$

AS REQUIRED

(II) FINALLY THE HEIGHT WILL NO LONGER CHANGE WHEN $\frac{dx}{dt} = 0$

$$\frac{1 - 4x^{\frac{1}{2}}}{10x^2} = 0 \quad \Rightarrow \quad 1 - 4x^{\frac{1}{2}} = 0$$

$$\Rightarrow \quad 4x^{\frac{1}{2}} = 1$$

$$\Rightarrow \quad x^{\frac{1}{2}} = \frac{1}{4}$$

$$\Rightarrow \quad x = \frac{1}{16}$$

← LIMITING VALUE FOR x

AS THE HEIGHT STARTS FROM $x = 0$ & ITS LIMITING VALUE IS $\frac{1}{16}$, IT CANNOT EXCEED $\frac{1}{16}$