IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper W

Difficulty Rating: 4.4080/1.5075

Time: 2 hours 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 17 questions in this question paper.
The total mark for this paper is 125.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

Prove by first principles, and by using the small angle approximations for $\sin x$ and $\cos x$, that

$$\frac{d}{dx}(\sec x) = \sec x \tan x .$$ **(7)**

Question 2

By considering $\left(\sqrt{2}\right)^{\sqrt{2}}$, or otherwise, prove that an irrational number raised to the power of an irrational number **can be** a rational number.

(4)

Question 3

A curve has implicit equation

$$2x\sin y + 2\cos 2y = 1, \quad 0 \le y \le 2\pi .$$

Determine the equations of the two straight lines, which are parallel to the y axis, and are tangents to the above curve.

(9)

Question 4

The radius r of a circle is changing so that

$$\frac{dr}{dt} = \frac{1}{r^2} .$$

Show that the rate at which the area of the circle A changes satisfies the equation

$$\frac{dA}{dt} = \sqrt{\frac{4\pi^3}{A}} .$$ **(6)**

Question 5

$$f(x) \equiv (1-8x)^{\frac{1}{4}}, \ |x| < \frac{1}{8}.$$

a) Find the first four terms in the binomial series expansion of $f(x)$. **(3)**

The term of lowest degree in the series expansion of

$$(1+ax)\left(1+bx^2\right)^5 - f(x),$$

is the term in x^3.

b) Determine the value of each of the constants a and b, and hence state the coefficient of x^3. **(7)**

Question 6

The points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are referred relative to a fixed origin O.

If the point P is such so that $\overrightarrow{AP} : \overrightarrow{PB} = \lambda : \mu$, use vector algebra to show that

$$\overrightarrow{OP} = \frac{(\mu x_1 + \lambda x_2)\mathbf{i} + (\mu y_1 + \lambda y_2)\mathbf{j} + (\mu z_1 + \lambda z_2)\mathbf{k}}{\lambda + \mu}.$$ **(6)**

Question 7

The function $f(x)$ has domain $x \in \mathbb{R}$, $-1 \le x \le 5$.

It is further given that $f'(x) > 0$ and $f''(x) < 0$

Find a possible equation of $f(x)$, which ***does not*** contain exponentials. **(3)**

Question 8

A sequence u_1, u_2, u_3, u_4, … is given by

$$u_{n+1} = f\left(n, u_n\right).$$

The first few terms of the sequence are

$$2,\ -1,\ 5,\ -4,\ 8,\ -7,...$$

Find an expression for u_{n+1}, in the form $u_{n+1} = f\left(n, u_n\right)$. **(4)**

Question 9

The functions f and g are defined as

$$f(x) \equiv |3x + a| + b,\ x \in \mathbb{R}$$

$$g(x) \equiv 2x + 5,\ x \in \mathbb{R},$$

where a and b are positive constants.

The graph of f meets the graph of g at the points P and Q.

Given that the coordinates of P are $(0,5)$, find the coordinates of Q in terms of a. **(7)**

Question 10

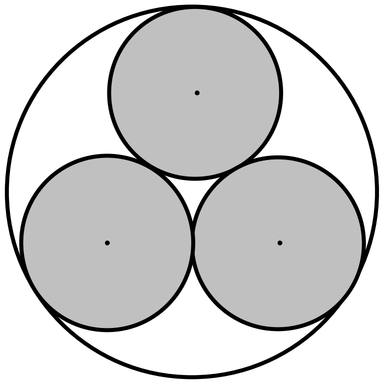
Liquid is kept in containers, which due to evaporation and ongoing chemical reactions, at the end of each month the volume of the liquid in these containers reduces by 4% compared with the volume at the start of the same month.

At the start of each month a new container is filled up with 200 litres of liquid, so that at the end of thirty months there are 30 containers with liquid.

Calculate the total amount of liquid in the 30 containers at the end of 30 months. **(7)**

Question 11

The figure below shows the plan of three identical circular cylinders of radius 6 cm, that fit snugly inside a larger cylinder.



Show that the radius of the larger cylinder is $6+4\sqrt{3} \text{ cm}$. **(6)**

Question 12

The curve with equation $xy = 3$ is traced by the following parametric equations

$$x = \frac{4tp}{t+p}, \quad y = \frac{4}{t+p}, \quad t, p \in \mathbb{R}, \; t \neq p$$

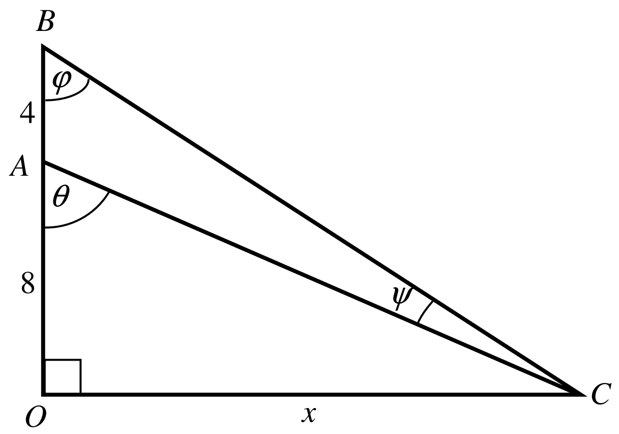
where t and p are parameters.

Find the relationship between t and p, giving the answer in the form $p = f(t)$. **(6)**

Question 13

The diagram below shows a right angled triangle OBC where $|OC| = x$ and the point A on OB so that $|OA| = 8$, $|AB| = 4$.

The angles OAC, OBC and ACB are denoted by θ, φ and ψ respectively.



By considering a relationship between the angles θ, φ and ψ, show that

$$\tan\psi = \frac{4x}{96 + x^2} .$$ **(7)**

Question 14

The curve C has equation

$$y = 4 \times 8^{x+1} - 2^{x+1} .$$

Show that an equation of the tangent to the curve, at the point where C crosses the x axis is given by

$$y = (x+2)\ln 2 .$$ **(7)**

Question 15

It is given that

$$u^2 = \frac{1-x^2}{(1-x)^2}, \ x \neq \pm 1.$$

a) Show clearly that …

i. … $x = \dfrac{u^2-1}{u^2+1}$. **(3)**

ii. … $1-x^2 = \dfrac{4u^2}{\left(u^2+1\right)^2}$ **(3)**

iii. … $\dfrac{dx}{du} = \dfrac{4u}{\left(u^2+1\right)^2}$. **(3)**

b) Hence show further that

$$\int \frac{3}{(4x+5)\sqrt{1-x^2} - 3\left(1-x^2\right)}\, dx = \frac{2\sqrt{1-x}}{\sqrt{1-x} - 3\sqrt{1+x}} + \text{constant}.$$ **(9)**

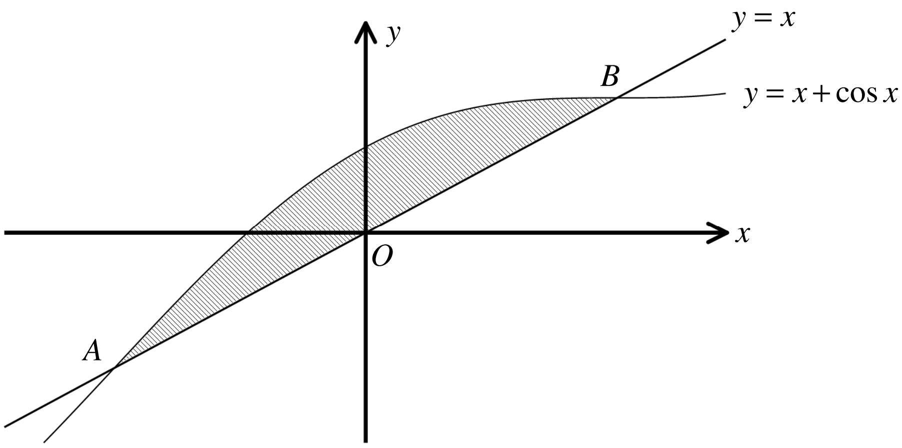
Question 16

Solve the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 1,$$

given that $y = -\frac{1}{4}$ and $\dfrac{dy}{dx} = 1$ at $x = 0$, giving the answer in the form $y = f(x)$. **(10)**

Question 17



The figure above shows the graph of the curve C with equation

$$y = x + \cos x,\ x \in \mathbb{R}$$

and the straight line L with equation

$$y = x,\ x \in \mathbb{R}.$$

Show that the area of the finite region bounded by C and L is 2 square units. **(8)**