IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper Z

Difficulty Rating: 4.4280/1.5267

Time: 2 hours 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 13 questions in this question paper.
The total mark for this paper is 125.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

An arithmetic series has common difference 2.

The 3^{rd}, 6^{th} and 10^{th} terms of the arithmetic series are the respective first three terms of a geometric series.

Determine in any order the first term of the arithmetic series and the common ratio of the geometric series. **(6)**

Question 2

An arithmetic series has first term 2 and common difference X.

A geometric series has first term 2 and common ratio X.

The sum of the 11^{th} term of the arithmetic series and the 11^{th} term of the geometric series is 900.

a) Show that X is a solution of the equation

$$X^{10} + 5X = 449.$$ **(3)**

b) Show further that

$$1.8 < X < 1.9.$$ **(2)**

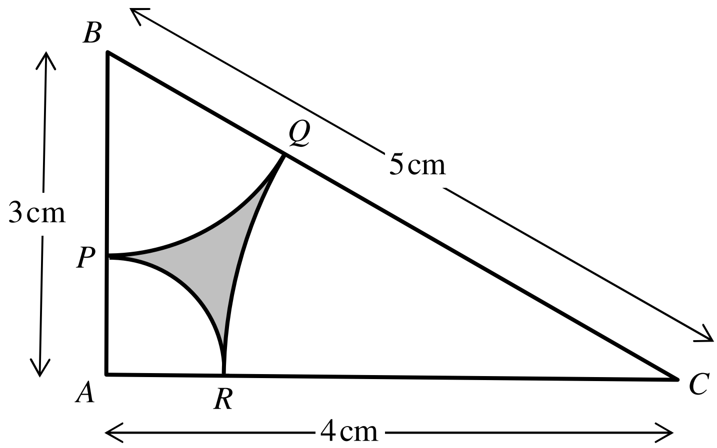
c) Use the Newton Raphson approximation method **twice**, with a starting value of 1.8, to find an approximate value for X, giving the answer correct to 3 decimal places. **(6)**

Question 3

Show, with a detailed method, that

$$\frac{d}{dx}\left[\ln\left(\frac{1}{\sqrt{x^2+1}-x}\right)\right] = \frac{1}{\sqrt{x^2+1}}.$$ **(7)**

Question 4



The figure above shows a triangle ABC where $\measuredangle BAC = 90°$.

The lengths of AB, AC and BC are 3 cm, 4 cm and 5 cm, respectively.

Three arcs are drawn inside the triangle with centres the three vertices of the triangle.

The arcs so that they touch each other in pairs at the points P, Q and R.

Find the area of the shaded region, correct to three significant figures. **(8)**

Question 5

$$I = \int \frac{\cos^3 x}{\left(1+\sin^2 x\right)\sin x}\, dx .$$

By using the substitution $u = \sin x + \operatorname{cosec} x$, or otherwise, show that

$$I = \ln\left|\frac{\sin x}{1+\sin^2 x}\right| + \text{constant}$$ **(10)**

Question 6

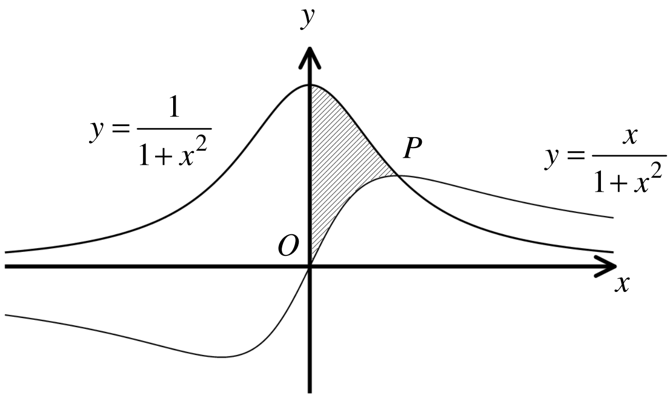
$$y = \arctan x \,, \; x \in \mathbb{R} \,.$$

a) By rewriting the above equation in the form $x = f(y)$, show clearly that

$$\frac{dy}{dx} = \frac{1}{1+x^2} \,.$$ **(4)**

The figure below shows the graphs of the curves with equations

$$y = \frac{1}{1+x^2} \qquad \text{and} \qquad y = \frac{x}{1+x^2} \,.$$



The two graphs intersect at the point $P\left(1, \frac{1}{2}\right)$.

b) Find the exact area of the finite region bounded by the two curves and the y axis, shown shaded in the figure. **(6)**

Question 7

A curve C is given by the parametric equations

$$x = \tan\theta - \sec\theta, \quad y = \cot\theta - \operatorname{cosec}\theta, \quad 0 < \theta < \frac{\pi}{2}.$$

Show clearly that …

a) … a Cartesian equation of C is

$$(x^2 - 1)(y^2 - 1) = 4xy. \qquad \textbf{(7)}$$

b) … $\dfrac{dy}{dx} = \dfrac{1 - y^2}{2x}$. **(7)**

Question 8

The function f is defined as

$$f(x) = \ln(4 - 2x), \ x \in \mathbb{R}, \ x < 2.$$

a) Find in exact form the coordinates of the points where the graph of $f(x)$ crosses the coordinate axes. **(2)**

Consider the following sequence of transformations T_1, T_2 and T_3.

$$\ln x \xrightarrow{T_1} \ln(x + 4) \xrightarrow{T_2} \ln(2x + 4) \xrightarrow{T_3} \ln(-2x + 4)$$

b) Describe geometrically the transformations T_1, T_2 and T_3, and hence sketch the graph of $f(x)$.

Indicate clearly any asymptotes and coordinates of intersections with the axes. **(5)**

c) Find, an expression for $f^{-1}(x)$, the inverse function of $f(x)$. **(3)**

d) State the domain and range of $f^{-1}(x)$. **(2)**

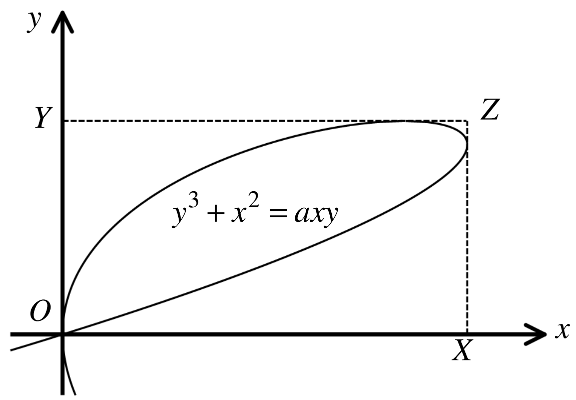
Question 9

$$(1+x)\frac{dy}{dx} = y(1-x), \ \ y>0, \ \ x>-1.$$

Solve the above given differential equation, subject to the boundary condition $y=1$ at $x=0$, to show that

$$y=(x+1)^2 e^{-x}.$$ **(10)**

Question 10



The figure above shows the curve with equation

$$y^3+x^2=axy,$$

where a is a positive constant.

The point Y lies on the y axis so that the straight line segment YZ is a tangent to the curve parallel to the x axis. Similarly the point X lies on the x axis so that the straight line segment XZ is a tangent to the curve parallel to the y axis.

The area of the rectangle $OYZX$, where O is the origin, is 288 square units.

Determine the value of a. **(12)**

Question 11

The variables x, y, z and t are related by the equations

$$z = \sqrt{t^3 + 8t^{\frac{1}{2}} + 1} \qquad y = \frac{1}{(x+3)^2} \qquad \ln(x+3)^3 = \frac{1}{3}z,$$

where $x > -3$ and $x \geq 0$.

Find the value of z, when $t = 4$ and hence determine the value of $\frac{dy}{dt}$, when $y = e^{-2}$.

(10)

Question 12

The distinct acute angles θ and φ, $\theta > \varphi$ satisfy the equation

$$f(\theta,\varphi) = g(\theta,\varphi)\tan\varphi,$$

where the functions f and g are defined as

$$f(\theta,\varphi) \equiv \sin(\theta-\varphi) \quad \text{and} \quad g(\theta,\varphi) \equiv \cos(\theta-\varphi) - 2\tan\varphi\sin(\theta-\varphi).$$

Use trigonometric identities to show that

$$\tan\theta = 2\tan\varphi.$$

(10)

Question 13

Without the use of any calculating aid and by showing full workings, show that

$$(0.9)^{0.9} \approx 0.91.$$

(5)
