IYGB

Special Paper F

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

Question 1

Solve the following trigonometric equation.

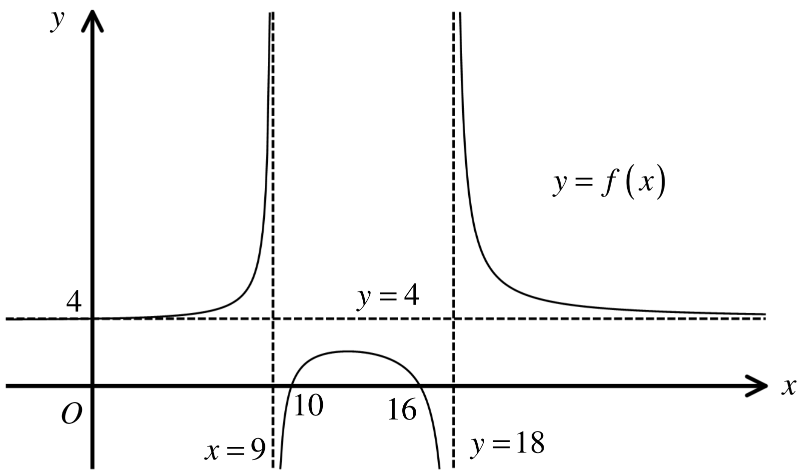
$$\arctan 2x + \arctan x = \arctan 3, \quad x \in \mathbb{R}. \qquad (7)$$

Question 2

Determine, in exact form where appropriate, the solutions of the following equation.

$$x^4 + 2(x+2)^2 = 3x^3 + 6x^2. \qquad (7)$$

Question 3

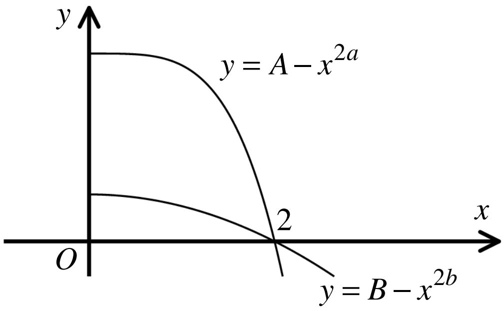


The figure above shows the curve with equation $y = f(x)$.

The equations of the three asymptotes to the curve, and the three intercepts of the curve with the coordinate axes are marked in the figure.

Sketch a detailed graph of $y^2 = \left|f(|x|)\right|$. **(6)**

Question 4



The figure above shows the curves with equations

$$y = A - x^{2a} \quad \text{and} \quad y = B - x^{2b} \ , \quad x \geq 0 \, ,$$

where A and B are positive constants with $A > B$, and a and b are positive integers with $a + b = 4$.

Both curves meet the x axis at the point $(2,0)$.

Find the exact value of x, for which both curves have the same gradient. **(8)**

Question 5

A cyclist travelling at **constant** speed V km/h covers a distance of 125 km.

If he was to decrease his speed by 5 km/h it would have taken him an extra $1\frac{1}{4}$ hours to cover the same distance.

Find the value of V. **(7)**

Question 6

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times \ldots \times u_{k-1} \times u_k .$$

Use a clear method to determine the value of k given that

$$\prod_{m=1}^{4}\prod_{n=1}^{3}[kmn] = 4 \times 96^7 . \qquad \textbf{(7)}$$

Question 7

The following quadratic in x is given below

$$x^2 + 3kx + k^2 = 7x + 3k ,$$

where k is a constant.

Show that the above quadratic has real solutions whose difference is at least 2. **(7)**

Question 8

A curve has equation

$$9yx^2 - 6x(y+1) + y + 1 = 0 , \ x \in \mathbb{R} , \ x \neq \tfrac{1}{3} .$$

Find, in exact form where appropriate, the three solutions of the equation

$$2\frac{d^2y}{dx^2} = 6x + 1$$

where $\frac{d^2y}{dx^2}$ represents the second derivative of the above equation. **(10)**

Question 9

The straight line L_1 has equation

$$y = 6 - 2x,$$

The straight line L_2 passes through the point $A(2,7)$ and meets L_1 at the point B.

Given that L_1 and L_2 intersect each other at $45°$, determine the coordinates of B. **(10)**

Question 10

$$f(x) \equiv \frac{2-3x}{(1-x)(1-2x)}, \ -\tfrac{1}{2} < x < \tfrac{1}{2}.$$

Show that $f(x)$ can be written in the form

$$f(x) = \sum_{r=0}^{\infty} \left[x^r \, g(r) \right],$$

where $g(r)$ is a simplified function to be found. **(11)**

Question 11

$$I = \int_{-2}^{2} \frac{1}{\sqrt{1 - ax + a^2}} \, dx, \ a > 0, \ a \neq 0.$$

Find the two possible values of I, giving the answer in terms of a where appropriate.

(10)

Question 12

Solve the following logarithmic equation.

$$3+8\log_{\frac{1}{k}}\left[\sqrt{8+4\sqrt{3}}-\sqrt{8-4\sqrt{3}}\right]=0, \quad k>0, \ k\neq 1. \qquad \textbf{(11)}$$

Question 13

OAB is a triangle and $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}$.

- The point C lies on OB so that $OC:CB=3:1$.
- The point P lies on AC so that $AP:PC=2:1$.
- The point Q lies on AB so that O, P and Q are collinear.

Determine the ratio $AQ:QB$. **(11)**

Question 14

Use differentiation to find a simplified general solution for the following differential equation.

$$\left(x^2-1\right)\left(\frac{dy}{dx}\right)^2-2xy\left(\frac{dy}{dx}\right)+y^2=1. \qquad \textbf{(12)}$$

Question 15

Use appropriate integration techniques to show that

$$\int_0^1 4x\arctan x \ dx = \pi-2. \qquad \textbf{(11)}$$

Question 16

A function is defined as

$$f(x) \equiv x^3 - \tfrac{1}{2}\lambda x + \lambda - 8, \quad x \in \mathbb{R},$$

where λ is a non zero constant.

The equation $f(x) = 0$ has exactly two real distinct roots.

The equation $f(x) = k$, where k is a constant, has three distinct real roots.

By considering $f(2)$, or otherwise, determine the range of values of k. **(14)**

Question 17

The function f is defined as

$$f(x) \equiv kx - \frac{x^3}{x^2+1}, \quad x \in \mathbb{R},$$

where k is a positive constant.

Given that f is increasing for $x \in \mathbb{R}$, show that $k > \frac{9}{8}$ and hence sketch the graph of f, showing clearly the behaviour of f at $\pm\sqrt{2}$. **(15)**

Question 18

By considering the trigonometric identity for $\tan(A-B)$, with $A = \arctan(n+1)$ and $B = \arctan(n)$, sum the following series

$$\sum_{n=1}^{\infty} \arctan\left(\frac{1}{n^2+n+1}\right).$$

You may assume the series converges. **(10)**

Question 19

A family of straight lines passes through the point with coordinates $(4,2)$.

The variable point M denotes the midpoint of the x and y intercepts of this family of straight lines.

Sketch a detailed graph of the curve that M traces, for this family of straight lines. **(12)**

Question 20

The triangle ABC is isosceles with $|AB| = |AC|$ and $\measuredangle BAC = 36°$.

The angle bisector of $\measuredangle ABC$ meets AC at the point D.

By using trigonometry in the above construction, or otherwise, show that

$$\cos 36° = \frac{1}{2}\left(1+\sqrt{5}\right).$$ **(14)**
