IYGB

Special Paper V

Time: 3 hours 30 minutes

Candidates may use any calculator allowed by the Regulations of the Joint Council for Qualifications.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core Syllabus.
Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

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Question 1

The point P has position vector $2\mathbf{i}+2\mathbf{j}+21\mathbf{k}$.

a) Find the vector equation of the straight line l which passes through P and is parallel to the vector $\mathbf{i}-\mathbf{j}+5\mathbf{k}$. **(1)**

The points A and B have coordinates $(-1,2,3)$ and $(2,5,3)$, respectively.

The point C lies on l so that the triangle ABC is equilateral.

b) Find the two possible position vectors for C. **(8)**

Question 2

$$f(x) = b-(x-a)^2, \ x \in \mathbb{R}$$

$$g(x) = a+(x-b)^2, \ x \in \mathbb{R}.$$

The graph of $f(x)$ has a maximum at P and the graph of $g(x)$ has a minimum at Q, where P and Q are distinct points.

a) Given that $f(x)$ passes through Q, show that $g(x)$ passes through P. **(6)**

b) Given further that $f(x)$ touches the x axis, sketch both graphs in the same set of axes. **(3)**

Question 3

Find the real solutions for the following system of simultaneous equations

$$x^3+y^3=\frac{7}{2}$$
$$x^2y+xy^2=\frac{3}{2}.$$ **(9)**

Question 4

A surveyor views the top of a building, of height h, at an angle of elevation α.

The surveyor walks a distance a, directly towards the building.

From this new position he views the top of the building at an angle of elevation β.

Show that

$$h = \frac{a \sin\alpha \sin\beta}{\sin(\beta - \alpha)}.$$ **(8)**

Question 5

Solve the following simultaneous equations.

$$(2x)^{\ln 2} = (3y)^{\ln 3} \quad \text{and} \quad 3^{\ln x} = 2^{\ln y}.$$ **(8)**

Question 6

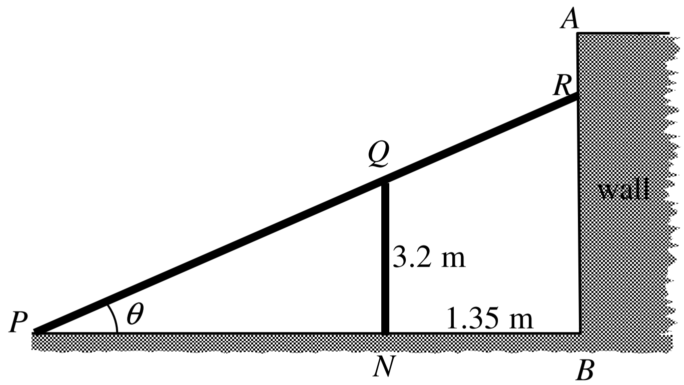
Sketch the graph of

$$f(x) = \arcsin(\cos x),$$

in the largest domain that the function is defined.

Indicate the coordinates of any intersections with the axes, and the coordinates of the cusps of the curve. **(6)**

Question 7



The figure above shows the wall AB of a certain structure, which is supported by a straight rigid beam PR, where P is on level ground and R is at some point on the wall.

In order to increase the rigidity of the support, the beam is rested on a steady pole NQ, of height 3.2 metres.

The pole is placed at a distance of 1.35 metres from the bottom of the wall B.

The beam PR is forming an acute angle θ with the horizontal ground PNB.

The angle θ is chosen so that the length of the beam PR, is least.

Determine the least value for the length of the beam PR, assuming that R lies on the wall, fully justifying that this is indeed the minimum value. **(12)**

Question 8

Use trigonometric identities to find a simplified expression for

$$\int \frac{\sin^8 x - \cos^8 x}{1 - \frac{1}{2}\sin^2 2x}\, dx .$$ **(10)**

Question 9

$$f(a) \equiv \frac{a}{a+1} + \sqrt{1+a^2+\frac{a^2}{a^2+2a+1}}\ ,\ a \in \mathbb{R}\ ,\ a \neq -1\ .$$

Show that $f(a)$ can be simplified to a linear polynomial in a. **(9)**

Question 10

On the 1st January 2000 a rare stamp was purchased at an auction for £16384 and by the 1st January 2010 its value was four times as large as its purchase price.

The future value of this stamp, £V, t years after the 1st January 2000 is modelled by the equation

$$V = Ap^t\ ,\ t \geq 0\ ,$$

where A and p are positive constants.

On the 1st January 1990 a different stamp was purchased for £2.

The future value of this stamp, £U, t years after the 1st January 1990 is modelled by the equation

$$U = Bq^t\ ,\ t \geq 0\ ,$$

where B and q are positive constants.

Given further that $q = p\sqrt{2}$, determine the year when the two stamps will achieve the same value according to their modelling equations. **(12)**

Question 11

A snowball is melting and its shape remains spherical at all times. The volume of the snowball, V cm^3, is decreasing at a rate proportional to its surface area.

Let t be the time in hours since the snowball's surface area was 4 m^2.

Sixteen hours later its surface area has reduced to 2.25 m^2.

By forming and solving a suitable differential equation, determine the value of t by which the snowball would have completely melted.

(11)

$\left[\text{volume of a sphere of radius } r \text{ is given by } \frac{4}{3}\pi r^3\right]$

$\left[\text{surface area of a sphere of radius } r \text{ is given by } 4\pi r^2\right]$

Question 12

A function is defined as

$$[x] \equiv \{\text{the greatest integer less or equal to } x\}.$$

The function f is defined as

$$f(n) = n\left[\frac{3}{5} + \frac{3n}{100}\right], \quad n \in \mathbb{N}.$$

Determine the value of

$$\sum_{n=1}^{82} f(n).$$

(9)

Question 13

The finite region R is bounded by the curve with equation

$$y = \sin x,\ 0 \le x \le \pi,$$

and the straight line with equation $y = \frac{1}{3}$.

The region R is rotated by 2π radians in the straight line with equation $y = \frac{1}{3}$ forming a solid of revolution.

Determine the exact volume of this solid. **(9)**

Question 14

Solve the following trigonometric equation

$$\arctan\left[x\cos\left(2\arcsin\frac{1}{x}\right)\right] = \frac{1}{4}\pi. \quad \textbf{(9)}$$

Question 15

The straight parallel lines l_1 and l_2 have respective equations

$$y = 3x + 5 \qquad \text{and} \qquad y = 3x + 2.$$

Two more straight lines, both passing through the origin O, intersect l_1 and l_2 forming a trapezium $ABCD$.

The trapezium $ABCD$ is isosceles with $|AB| = |CD| = 3\sqrt{5}$.

Determine the area of this trapezium. **(16)**

Question 16

$$0.6^x + 0.8^x = 1, \ x \in \mathbb{R}.$$

Find the only solution of the above equation, fully justifying the fact that it is the only solution.

(6)

Question 17 (*****)

The straight line L has equation

$$\frac{x}{p} + \frac{y}{q} = 1,$$

where p and q are non zero parameters, constrained by the equation

$$\frac{1}{p^2} + \frac{1}{q^2} = \frac{1}{2}.$$

The point P is the foot of the perpendicular from the origin O to L.

Show that for all values of p and q, P lies on a circle C, stating its radius. **(12)**

Question 18

By using an appropriate substitution or substitutions, followed by partial fractions show that

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x} \, dx = \frac{\ln 3}{20}.$$

(12)

Question 19

The n^{th} term of a series is given recursively by

$$A_n = \frac{a(2n+1)}{2n+4} A_{n-1}, \ n \in \mathbb{N}, \ n \geq 1,$$

where a is a positive constant.

Given further that $A_0 = 1$, show that

$$A_n = \left(\frac{a}{4}\right)^n \binom{2n+2}{n} \frac{1}{n+1}.$$

[*you may not use proof by induction*] **(14)**

Question 20

Given that n is an integer such that $n > 3$, use a detailed method to solve the following trigonometric equation.

$$\frac{1}{\sin\left[\frac{2\pi}{n}\right]} + \frac{1}{\sin\left[\frac{3\pi}{n}\right]} = \frac{1}{\sin\left[\frac{\pi}{n}\right]}.$$ **(10)**
