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IYGB

Special Extension Paper A

Time: 3 hours 30 minutes

Candidates may NOT use any calculator.

Information for Candidates

This practice paper follows the Advanced Level Mathematics Core and the Advanced Level Further Pure Mathematics Syllabi of recent years.

Booklets of *Mathematical formulae and statistical tables* may NOT be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Scoring

Total Score $= T$, Number of non attempted questions $= N$, Percentage score $= P$.

$P = \frac{1}{2}T + N$ (rounded up to the nearest integer)

Distinction $P \geq 70$, Merit $55 \leq P \leq 69$, Pass $40 \leq P \leq 54$

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Question 1

Show clearly that

$$1^3 - 2^3 + 3^3 - 4^3 + \ldots - 40^3 = -33200 .$$ **(4)**

Question 2

The straight line L_1 has vector equation

$$\mathbf{r} = 4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k} + \lambda(3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}),$$

where λ is a scalar parameter.

The plane Π has vector equation

$$\mathbf{r} \cdot (4\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) = 17 .$$

The point P is the intersection of L_1 and Π.

The acute angle θ is formed between L_1 and Π.

The straight line L_2 lies on Π, passes through P so that the acute angle between L_1 and L_2 is also θ.

Find a vector equation for L_2. **(10)**

Question 3

Find the solutions of the equation

$$w^4 = 16(1 - w)^4 ,$$

giving the answers in the form $x + \mathrm{i}y$, where $x \in \mathbb{R}$, $y \in \mathbb{R}$. **(8)**

Question 4

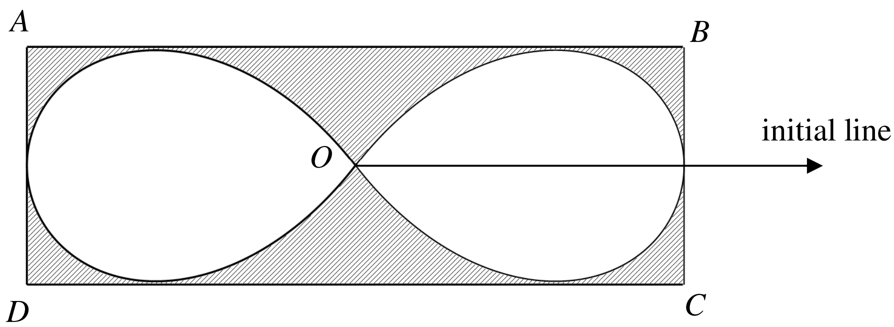
The straight line l_1 passes though the points $A(30,0)$ and $B(0,10)$.

The straight line l_2 is perpendicular to l_1.

The point P is the intersection between l_1 and l_2, and the point Q is the point where l_2 meets the y axis.

Given further that the area of the triangle OQP is 2.4, where O is the origin, find the possible area of the quadrilateral $OQPA$. **(10)**

Question 5



The figure above shows the rectangle $ABCD$ enclosing the curve with polar equation

$$r^2 = \cos 2\theta, \quad \theta \in \left[0, \tfrac{1}{4}\pi\right] \cup \left[\tfrac{3}{4}\pi, \tfrac{5}{4}\pi\right] \cup \left[\tfrac{7}{4}\pi, 2\pi\right).$$

Each of the straight line segments AB and CD is a tangent to the curve parallel to the initial line, while each of the straight line segments AD and BC is a tangent to the curve perpendicular to the initial line.

Show with detailed calculations that the total area enclosed between the curve and the rectangle $ABCD$ is $\sqrt{2}-1$. **(12)**

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Question 6

Investigate the convergence or divergence of each of the following two series using standard tests and justifying every step in the workings.

a) $\displaystyle\sum_{n=1}^{\infty}\left[\frac{1}{n(n+3)}\right].$ **(1)**

b) $\displaystyle\sum_{n=4}^{\infty}\left[\frac{1}{n(n-3)}\right].$ **(7)**

You may not conclude simply by summing each the series.

Question 7

By using a suitable substitution, solve the differential equation

$$\frac{dy}{dx}=\frac{x-y}{x+y},$$

subject to the condition $y=1$ at $x=1$. **(10)**

Question 8

The quadratic equation

$$ax^2+bx+1=0, \quad a\neq 0,$$

where a and b are constants, has roots α and β.

Find, in terms of α and β, the roots of the equation

$$x^2+\left(b^3-3ab\right)x+a^3=0.$$ **(8)**

Question 9

$$y = \ln\left(2 - e^{x}\right), \; x < \ln 2 .$$

Show clearly that

$$e^{y}\left[\frac{d^{3}y}{dx^{3}} + 3\frac{dy}{dx}\frac{d^{2}y}{dx^{2}} + \left(\frac{dy}{dx}\right)^{3}\right] + e^{x} = 0 ,$$

and hence find the first 3 non zero terms in the Maclaurin expansion of

$$y = \ln\left(2 - e^{x}\right), \quad x < \ln 2 \; .$$ **(10)**

Question 10

A curve C has intrinsic equation

$$s = \ln\left(\tan\psi + \sec\psi\right) + \tan\psi\sec\psi , \; 0 \le \psi < \frac{\pi}{2},$$

where s is the arc length is measured from the point with Cartesian coordinates $(0,1)$, and ψ is the angle the tangent to C makes with the positive x axis.

It is further given that the gradient at $(0,1)$ is zero.

Show that the Cartesian equation of C is

$$y = \frac{1}{4}x^{2} + 1 .$$ **(12)**

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Question 11

A sequence of numbers is given by the recurrence relation

$$u_{n+1} = \frac{5u_n - 1}{4u_n + 1}, \ u_1 = 1, \ n \in \mathbb{N}, \ n \geq 1.$$

Prove by induction that the n^{th} term of the sequence is given by

$$u_n = \frac{n+2}{2n+1}.$$

(7)

Question 12

The point P lies on an ellipse whose foci are on the x axis at the points S and T.

Given further that the triangle STP is right angled at T, show that

$$e = \frac{1 - \tan\frac{1}{2}\theta}{1 + \tan\frac{1}{2}\theta},$$

where e is the eccentricity of the ellipse, and θ is the angle PST. **(10)**

Question 13

Find the value of

$$\frac{1}{\pi}\int_{-\pi}^{\pi} \frac{\sin\frac{9}{2}x}{\sin\frac{1}{2}x} \, dx.$$

You may assume that the integrand is continuous at $x = 0$. **(14)**

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Question 14

Solve the equation

$$7x + 12y = 220,$$

given further that $x \in \mathbb{N}$, $y \in \mathbb{N}$. **(10)**

Question 15

A curve has equation

$$f(x) \equiv \frac{x-2}{3-x}, \qquad x \in \mathbb{R},\ x \neq 3.$$

Sketch in separate set of axes the graph of

- $y = f(x)$
- $y^2 = -f(x)$
- $y = f'(x)$

You must show in each case the equations of any asymptotes and the coordinates of any intersections with the coordinate axes.

(10)

Question 16

Use De Moivre's theorem to find a multiple angle cosine expression and use this expression to show that

$$\cos 36° = \frac{1}{4}\left(1 + \sqrt{5}\right).$$ **(12)**

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Question 17

The product operator $\prod$, is defined as

$$\prod_{i=1}^{k}[u_i] = u_1 \times u_2 \times u_3 \times u_4 \times ... \times u_{k-1} \times u_k .$$

By writing $\sin x$ as an infinite factorized polynomial, where each of the factors represents a zero of $\sin x$, derive Wallis's formula

$$\frac{\pi}{2} = \prod_{r=1}^{\infty} \frac{4r^2}{4r^2 - 1} .$$

You may assume without proof that

$$\lim_{x \to 0}\left[\frac{\sin x}{x}\right] = 1 . \qquad \textbf{(10)}$$

Question 18

Find a general solution of the following differential equation.

$$y = x\frac{dy}{dx} + e^{\frac{dy}{dx}} . \qquad \textbf{(9)}$$

Question 19

Use appropriate integration techniques to show that

$$\int_{\operatorname{arsinh}\frac{1}{\sqrt{3}}}^{\operatorname{arsinh}\sqrt{3}} \operatorname{sech} x(1 - \operatorname{sech} x)\ dx = \frac{\pi}{12} . \qquad \textbf{(15)}$$

Question 20

Use an appropriate method to sum the following series

$$\sum_{r=1}^{\infty} \frac{r \times 2^r}{(r+2)!}.$$

You may assume the series converges. **(11)**

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