IYGB - SYNOPTIC PAPER L - QUESTION 1

a) EXPANDING BINOMIALLY

$$(1+x)^{-1} = 1 + \frac{-1}{1}(x)^1 + \frac{-1(-2)}{1\times 2}(x)^2 + \frac{-1(-2)(-3)}{1\times 2\times 3}(x)^3 + O(x^4)$$

$$= 1 - x + x^2 - x^3 + O(x^3)$$

b) LET $g(x) = (1+x)^{-1}$

THEN $(1-2x)^{-1} = g(-2x)$

HENCE $g(-2x) = (1-2x)^{-1} = 1 - (-2x) + (-2x)^2 - (-2x)^3 + O(x^4)$

$$= 1 + 2x + 4x^2 + 8x^3 + O(x^4)$$

c) BY PARTIAL FRACTIONS OR DIRECT MULTIPLICATION

$$f(x) = (4x+1)(1+x)^{-1}(1-2x)^{-1}$$

$$f(x) = (1+4x)(1 - x + x^2 - x^3 + \dots)(1 + 2x + 4x^2 + 8x^3 + \dots)$$

$$f(x) = (1+4x)\left[\begin{array}{l} 1 + 2x + 4x^2 + 8x^3 + O(x^4) \\ \quad\; -x - 2x^2 - 4x^3 + O(x^4) \\ \qquad\qquad\; x^2 + 2x^3 + O(x^4) \\ \qquad\qquad\qquad -x^3 + O(x^4) \end{array}\right]$$

$$f(x) = (1+4x)(1 + x + 3x^2 + 5x^3 + O(x^4))$$

$$f(x) = \begin{array}{l} 1 + x + 3x^2 + 5x^3 + O(x^4) \\ \quad + 4x + 4x^2 + 12x^3 + O(x^4) \end{array}$$

$$\therefore f(x) = 1 + 5x + 7x^2 + 17x^3 + O(x^4)$$

- $(1+x)^{-1}$ IS VALID $|x|<1$, IE $-1<x<1$
- $(1-2x)^{-1}$ IS VALID $|2x|<1$, IE $-\frac{1}{2}<x<\frac{1}{2}$

$\therefore$ HERE $-\frac{1}{2}<x<\frac{1}{2}$

IYGB - SYNOPTIC PAPER L - QUESTION 2

a) WHEN t BECOMES VERY LARGE, IE $t \to \infty$

$$e^{-0.1t} \to 0$$
$$200e^{-0.1t} \to 0$$

$$\therefore \theta \to 225$$

$\therefore$ MAX TEMPERATURE IS $225°C$

b) WHEN $\theta = 125$

$$\Rightarrow 125 = 225 - 200e^{-0.1t}$$
$$\Rightarrow 200e^{-0.1t} = 100$$
$$\Rightarrow e^{-0.1t} = \frac{1}{2}$$
$$\Rightarrow e^{0.1t} = 2$$
$$\Rightarrow 0.1t = \ln 2$$
$$\Rightarrow t = 10\ln 2 \simeq 6.93147\ldots \simeq 7 \text{ min}$$

c) WORK AS FOLLOWS

$$\theta = 225 - 200e^{-0.1t}$$
$$\frac{d\theta}{dt} = 0 + 20e^{-0.1t}$$
$$\frac{d\theta}{dt} = 20e^{-0.1t}$$
$$10\frac{d\theta}{dt} = 200e^{-0.1t}$$
$$10\frac{d\theta}{dt} = 225 - \theta$$
$$\frac{d\theta}{dt} = \frac{1}{10}(225 - \theta)$$ // AS REQUIRED

FROM THE ORIGINAL EQUATION

$$\theta = 225 - 200e^{-0.1t}$$
$$200e^{-0.1t} = 225 - \theta$$

$$\left.\frac{d\theta}{dt}\right|_{\theta=125} = 10 \text{ °C/min}$$ //

IYGB - SYNOPTIC PAPER L - QUESTION 3

a) USING THE STANDARD EQUATION OF A CIRCLE

$$\Rightarrow (x-a)^2 + (y-b)^2 = r^2$$
$$\Rightarrow (x+3)^2 + (y-8)^2 = \left(\tfrac{1}{2}\sqrt{80}\right)^2$$
$$\Rightarrow (x+3)^2 + (y-8)^2 = 20$$

b) SOLVING SIMULTANEOUSLY WITH $y = 3x+7$

$$\Rightarrow (x+3)^2 + ((3x+7)-8)^2 = 20$$
$$\Rightarrow (x+3)^2 + (3x-1)^2 = 20$$
$$\Rightarrow x^2 + \cancel{6x} + 9 + 9x^2 - \cancel{6x} + 1 = 20$$
$$\Rightarrow 10x^2 = 10$$
$$\Rightarrow x^2 = 1$$
$$\Rightarrow x = \begin{cases} 1 \\ -1 \end{cases} \qquad y = \begin{cases} 3(1)+7 = 10 \\ 3(-1)+7 = 4 \end{cases}$$

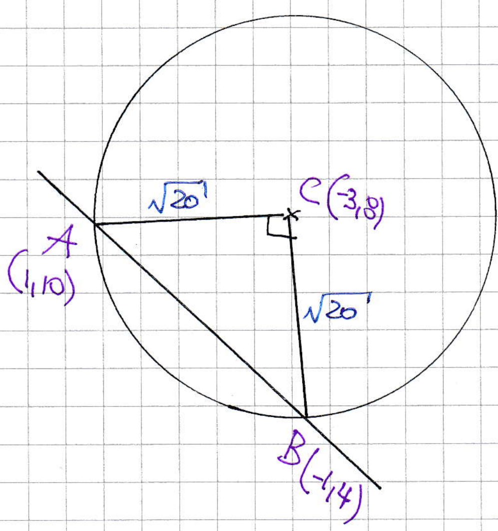
$$\therefore A(1,10) \;\&\; B(-1,4)$$

c) USING GRADIENTS

$$m_{AC} = \frac{8-10}{-3-1} = \frac{-2}{-4} = \frac{1}{2}$$

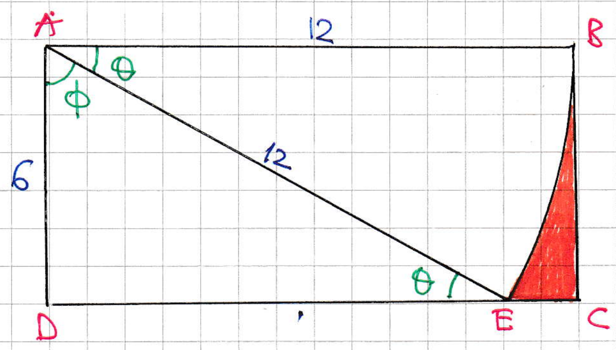
$$m_{BC} = \frac{8-4}{-3-(-1)} = \frac{4}{-2} = -2$$

NEGATIVE RECIPROCALS $\Rightarrow \hat{ACB} = 90°$



$$\& \text{ AREA} = \frac{1}{2}|AC||BC|$$
$$= \frac{1}{2}\sqrt{20}\sqrt{20}$$
$$= 10$$

IYGB - SYNOPTIC PAPER L - QUESTION 4



LOOKING AT THE DIAGRAM

$\sin\theta = \frac{6}{12}$ (TRIANGLE $\overset{\triangle}{ADE}$)

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

AREA OF RECTANGLE ABCD

$6 \times 12 = 72$

AREA OF TRIANGLE ADE

$= \frac{1}{2}|AD||AE|\sin\phi$

$= \frac{1}{2} \times 6 \times 12 \times \sin(90-30)$

$= 36\sin 60°$

$= 18\sqrt{3}$

AREA OF CIRCULAR SECTOR ABE

$= \pi r^2 \times \frac{\theta}{360}$

$= \pi \times 12^2 \times \frac{30}{360}$

$= 12\pi$

FINALLY WE OBTAIN THE AREA

REQUIRED AREA $= 72 - 18\sqrt{3} - 12\pi$

$\simeq 3.12$

IYGB - SYNOPTIC PAPER L - QUESTION 5

a) SOLVING SIMULTANEOUSLY TO FIND "A"

$$\left.\begin{array}{l} x^3+xy+y^3=10 \\ y=x+2 \end{array}\right\} \Rightarrow x^3+x(x+2)+(x+2)^3=10$$

$$\Rightarrow x^3+x^2+2x-10+(x+2)^3=0$$

NO NEED TO EXPAND

LET $f(x)=x^3+x^2+2x-10+(x+2)^3$

$f(0.1) = -0.528 < 0$

$f(0.2) = 1.096 > 0$

AS $f(x)$ IS CONTINUOUS AND CHANGES SIGN IN $(0.1, 0.2)$ THERE IS AT LEAST ONE ROOT IN THE INTERVAL

b) PREPARE THE "N-R ITEMS"

- $f(0.1) = -0.528$
- $f'(x) = 3x^2+2x+2+3(x+2)^2$

  $f'(0.1) = 15.46$

HENCE WE HAVE

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x = 0.1 - \frac{-0.528}{15.46}$$

$$x = 0.1341526\ldots$$

$\therefore$ APPROXIMATELY 0.134

IYGB - SYNOPTIC PAPER L - QUESTION 6

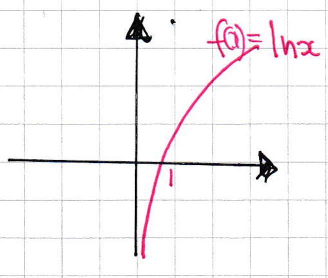
a) WITHOUT THE ORDER BEING IMPORTANT

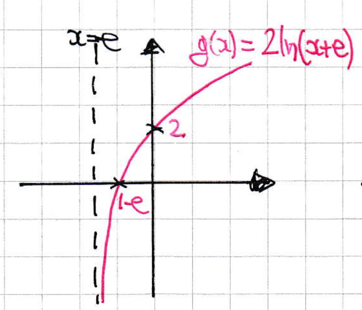
$$\ln(x) \longmapsto \ln(x+e) \longmapsto 2\ln(x+3)$$

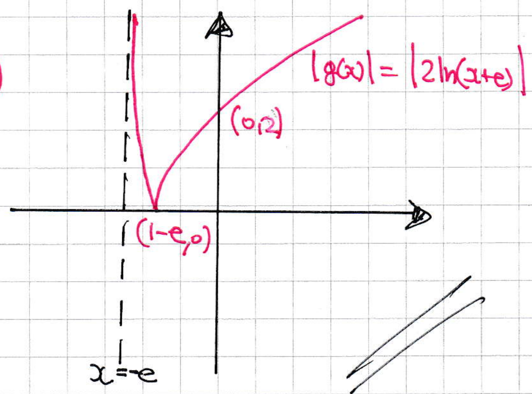
TRANSLATION BY $\begin{pmatrix} -e \\ 0 \end{pmatrix}$

VERTICAL STRETCH BY SCALE FACTOR 2

b) SKETCHING $g(x)$ AND $|g(x)|$







c) SOLVING $|g(x)| = 2$

$$|2\ln(x+e)| = 2 \implies 2\ln(x+e) = 2 \quad \text{OR} \quad 2\ln(x+e) = -2$$

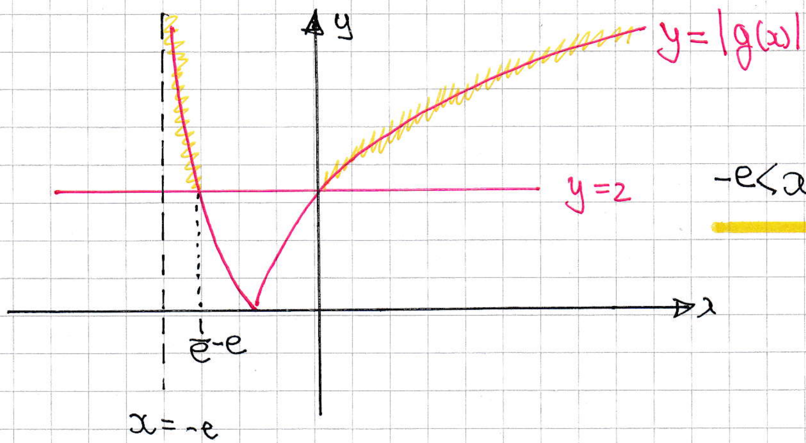
$$\ln(x+e) = 1 \qquad \ln(x+e) = -1$$

$$x+e = e \qquad x+e = e^{-1}$$

$$x = 0 \qquad x = e^{-1} - e$$

$$x = \frac{1}{e} - e$$

d) LOOKING AT THE GRAPH

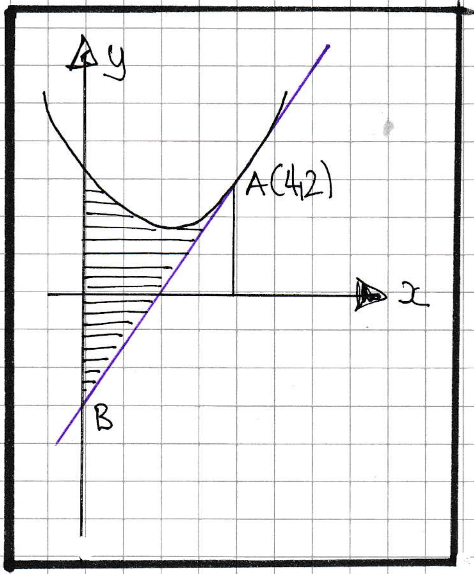


$$-e < x \leq \frac{1}{e} - e \quad \text{OR} \quad x \geq 0$$

IYGB - SYNOPTIC PAPER L - QUESTION 7

START BY FINDING THE EQUATION OF THE TANGENT.

- WHEN $x=4$, $y = 4^2 - 6\times4 + 10$
  $y = 2$ $\quad \therefore A(4,2)$



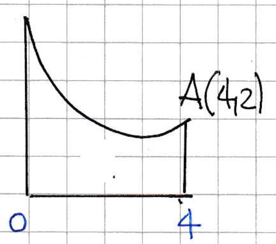
- $y = x^2 - 6x + 10$
  $\frac{dy}{dx} = 2x - 6$
  $\left.\frac{dy}{dx}\right|_{x=4} = 2$ ← TANGENT GRADIENT

- $y - y_0 = m(x - x_0)$
  $y - 2 = 2(x-4)$
  $y = 2x - 6$

FIND THE x & y INTERCEPT OF THE TANGENT.

$x=0$	$y=-6$	$(0,-6)$
$y=0$	$x=3$	$(3,0)$

LOOKING AT THE DIAGRAM BELOW



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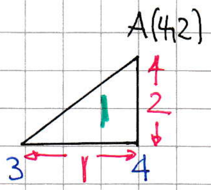

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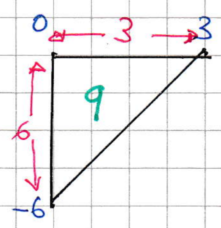


= REQUIRED AREA

$$\int_0^4 x^2 - 6x + 10 \, dx = \left[\tfrac{1}{3}x^3 - 3x^2 + 10x\right]_0^4$$

IYGB - SYNOPTIC PAPER L - QUESTION 7

$$= \left(\frac{64}{3} - 48 + 40\right) - (0)$$

$$= \frac{40}{3}$$

HENCE THE REQUIRED AREA IS $\frac{40}{3} - 1 + 9 = \frac{64}{3}$

IYGB - SYNOPTIC PAPER L - QUESTION 8

a) DIFFERENTIATE WITH RESPECT TO x

$$\Rightarrow \frac{d}{dx}(4xy) - \frac{d}{dx}\left((x+2)^2\right) = \frac{d}{dx}(y^2) - \frac{d}{dx}(5)$$

PRODUCT RULE — CHAIN RULE

$$\Rightarrow 4y + 4x\frac{dy}{dx} - 2(x+2) = 2y\frac{dy}{dx} - 0$$

$$\Rightarrow (4x - 2y)\frac{dy}{dx} = 2(x+2) - 4y$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x - 4y + 4}{4x - 2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x - 2y + 2}{2x - y}$$

b) "STATIONARY" $\Rightarrow \frac{dy}{dx} = 0$

$$\therefore \quad x - 2y + 2 = 0$$

$$x = 2y - 2$$

SUBSTITUTE INTO THE EQUATION OF THE CURVE

$$\Rightarrow 4y(2y-2) - (2y \cancel{-2} \cancel{+2})^2 = y^2 - 5$$

$$\Rightarrow 8y^2 - 8y - 4y^2 = y^2 - 5$$

$$\Rightarrow 3y^2 - 8y + 5 = 0$$

$$\Rightarrow (3y - 5)(y - 1) = 0$$

$$y = \begin{cases} 5/3 \\ 1 \end{cases} \qquad x = \begin{cases} 2 \times \frac{5}{3} - 2 = \frac{4}{3} \\ 2(1) - 2 = 0 \end{cases}$$

$$\therefore \left(\frac{4}{3}, \frac{5}{3}\right) \text{ \& } (0, 1)$$

IYGB - SYNOPTIC PAPER L - QUESTION 9

SWITCHING INTO INDICES FOR "COMFORT"

$$\frac{\sqrt[3]{16}-\sqrt[3]{2}}{\sqrt[3]{4}} = \frac{16^{\frac{1}{3}}-2^{\frac{1}{3}}}{4^{\frac{1}{3}}} = \frac{(2^4)^{\frac{1}{3}}-2^{\frac{1}{3}}}{(2^2)^{\frac{1}{3}}} = \frac{2^{\frac{4}{3}}-2^{\frac{1}{3}}}{2^{\frac{2}{3}}}$$

$$= \frac{2\times 2^{\frac{1}{3}}-2^{\frac{1}{3}}}{2^{\frac{2}{3}}} = \frac{2^{\frac{1}{3}}}{2^{\frac{2}{3}}}$$

NOW RATIONALIZING

$$= \frac{2^{\frac{1}{3}}\times 2^{\frac{1}{3}}}{2^{\frac{2}{3}}\times 2^{\frac{1}{3}}} = \frac{2^{\frac{2}{3}}}{2} = \frac{1}{2}\times 2^{\frac{2}{3}}$$

$$= \frac{1}{2}\times(2^2)^{\frac{1}{3}} = \frac{1}{2}\times 4^{\frac{1}{3}} = \frac{1}{2}\sqrt[3]{4}$$

IYGB - SYNOPTIC PAPER L - QUESTION 10

a) PROCEED AS FOLLOWS

$$\Longrightarrow \frac{dS}{dt} = 24\pi$$

$$\Longrightarrow \frac{dS}{dr} \times \frac{dr}{dt} = 24\pi$$

$$\Longrightarrow 8\cancel{\pi}r \times \frac{dr}{dt} = 24\cancel{\pi}$$

$$\Longrightarrow \frac{dr}{dt} = \frac{24}{8r}$$

$$\Longrightarrow \frac{dr}{dt} = \frac{3}{r}$$

SURFACE AREA OF SPHERE

$$S = 4\pi r^2$$

$$\frac{dS}{dr} = 8\pi r$$

b) RELATING THE VOLUME NEXT

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \times \frac{3}{r}$$

$$\frac{dV}{dt} = 12\pi r$$

$$\frac{dV}{dt} = \sqrt[3]{(12\pi r)^3}$$

$$\frac{dV}{dt} = \sqrt[3]{1728\pi^3 r^3}$$

$$\frac{dV}{dt} = \sqrt[3]{1728\pi^3 \times \frac{3V}{4\pi}}$$

$$\frac{dV}{dt} = \sqrt[3]{1296\pi^2 V}$$

AS REQUIRED

VOLUME OF A SPHERE

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$r^3 = \frac{3V}{4\pi}$$

c) SEPARATING VARIABLES & SOLVING THE O.D.E

$$\Longrightarrow \frac{dV}{dt} = (1296\pi^2)^{\frac{1}{3}} V^{\frac{1}{3}}$$

$$\Longrightarrow \frac{1}{V^{1/3}}\, dV = (1296\pi^2)^{\frac{1}{3}}\, dt$$

IYGB - SYNOPTIC PAPER L - QUESTION 10

$$\Rightarrow \int V^{-\frac{1}{3}}\,dV = \int (1296\pi^2)^{\frac{1}{3}}\,dt$$

$$\Rightarrow \frac{3}{2}V^{\frac{2}{3}} = (1296\pi^2)^{\frac{1}{3}}t + C$$

$$\Rightarrow V^{\frac{2}{3}} = \frac{2}{3}(1296\pi^2)^{\frac{1}{3}}t + C$$

AS REQUIRED

d) FINALLY WHEN $t = \sqrt[3]{36} = 36^{\frac{1}{3}}$, GIVEN THAT $t=0$ $V=64\pi$

$$(64\pi)^{\frac{2}{3}} = C$$

$$C = 16\pi^{\frac{2}{3}}$$

$$\therefore\ V^{\frac{2}{3}} = \frac{2}{3}(1296\pi^2)^{\frac{1}{3}}t + 16\pi^{\frac{2}{3}}$$

$$\Rightarrow V^{\frac{2}{3}} = \frac{2}{3}(1296\pi^2)^{\frac{1}{3}} \times 36^{\frac{1}{3}} + 16\pi^{\frac{2}{3}}$$

$$\Rightarrow V^{\frac{2}{3}} = \frac{2}{3}(46656\pi^2)^{\frac{1}{3}} + 16\pi^{\frac{2}{3}}$$

$$\Rightarrow V^{\frac{2}{3}} = \frac{2}{3} \times 36\pi^{\frac{2}{3}} + 16\pi^{\frac{2}{3}}$$

$$\Rightarrow V^{\frac{2}{3}} = 24\pi^{\frac{2}{3}} + 16\pi^{\frac{2}{3}}$$

$$\Rightarrow V^{\frac{2}{3}} = 40\pi^{\frac{2}{3}}$$

$$\Rightarrow V = \left(40\pi^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

$$\Rightarrow V = 40^{\frac{3}{2}}\pi$$

$$\Rightarrow V = 40\sqrt{40}\,\pi$$

IYGB - SYNOPTIC PAPER L - QUESTION 11

USING THE SUBSTITUTION GIVEN

$$\Longrightarrow u = 1 + x^2\operatorname{cosec}x$$

$$\Longrightarrow \frac{du}{dx} = 2x\operatorname{cosec}x - x^2\operatorname{cosec}x\cot x$$

$$\Longrightarrow \frac{du}{dx} = x\operatorname{cosec}x\,(2 - x\cot x)$$

$$\Longrightarrow dx = \frac{du}{(2 - x\cot x)(x\operatorname{cosec}x)}$$

SUBSTITUTE INTO THE INTEGRAL

$$\int \frac{2x - x^2\cot x}{x^2 + \sin x}\,dx = \int \frac{\cancel{x}\,\cancel{(2 - x\cot x)}}{x^2 + \sin x} \times \frac{du}{\cancel{(2 - x\cot x)}\,\cancel{x}\operatorname{cosec}x}$$

$$= \int \frac{1}{(x^2 + \sin x)\operatorname{cosec}x}\,du$$

$$= \int \frac{1}{x^2\operatorname{cosec}x + 1}\,du$$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \ln\left|1 + x^2\operatorname{cosec}x\right| + C$$

IYGB - SYNOPTIC PAPER L - QUESTION 12

a) AS THE TWO EXPRESSION OF $f(x)$ ARE IDENTICAL, WE MAY TRY DIFFERENT SENSIBLE VALUES OF x TO ELIMINATE

$$f(x) \equiv Ax^5+Bx^4+8x^2 \equiv (2x-1)(x-2)g(x)+169x-82$$

$$f(2) \equiv 32A+16B+32 = 0+338-82$$

$$f(\tfrac{1}{2}) \equiv \tfrac{1}{32}A+\tfrac{1}{16}B+2 = 0+\tfrac{169}{2}-82$$

TIDY THE EQUATIONS & SOLVE

$$\left.\begin{array}{l} 32A+16B=224 \\ \frac{1}{32}A+\frac{1}{16}B=\frac{1}{2} \end{array}\right\} \Longrightarrow \begin{array}{l} 2A+B=14 \\ A+2B=16 \end{array}$$

$$\boxed{B=14-2A}$$

$$\Rightarrow A+2(14-2A)=16$$

$$\Rightarrow A+28-4A=16$$

$$\Rightarrow 12=3A$$

$$\Rightarrow A=4 \qquad \& \quad B=6$$

b) USING THE ANSWERS FROM PART (a)

$$\Rightarrow 4x^5+6x^4+8x^2 \equiv (2x-1)(x-2)g(x)+169x-82$$

$$\Rightarrow 4x^5+6x^4+8x^2-169x+82 \equiv (2x-1)(x-2)g(x)$$

$$\Rightarrow 4x^5+6x^4+8x^2-169x+82 \equiv (2x^2-5x+2)g(x)$$

IYGB - SYNOPTIC PAPER L - QUESTION 12

BY LONG DIVISION

$$
\begin{array}{r|l}
 & 2x^3+8x^2+18x+41 \\
\hline
2x^2-5x+2 & 4x^5+6x^4+0x^3+8x^2-164x+82 \\
 & -4x^5+10x^4-4x^3 \\
 & 16x^4-4x^3+8x^2-164x+82 \\
 & -16x^4+40x^3-16x^2 \\
 & 36x^3-8x^2-164x+82 \\
 & -36x^3+90x^2-36x \\
 & 82x^2-205x+82 \\
 & -82x^2+205x-82 \\
 & 0
\end{array}
$$

$$\therefore \quad g(x) = 2x^3+8x^2+18x+41$$

c) **PROCEED AS FOLLOWS**

$$f(x) \equiv 4x^5+6x^4+8x^2 \equiv (x+2)^2 h(x) + Px + Q$$

$$f'(x) \equiv 20x^4+24x^3+16x \equiv 2(x+2)h(x) + (x+2)^2 h'(x) + P$$

$$f(-2) = -128+96+32 = 0 \quad -2P+Q$$

$$f'(-2) = 320-192-32 = P$$

$$\therefore \quad P = 96$$

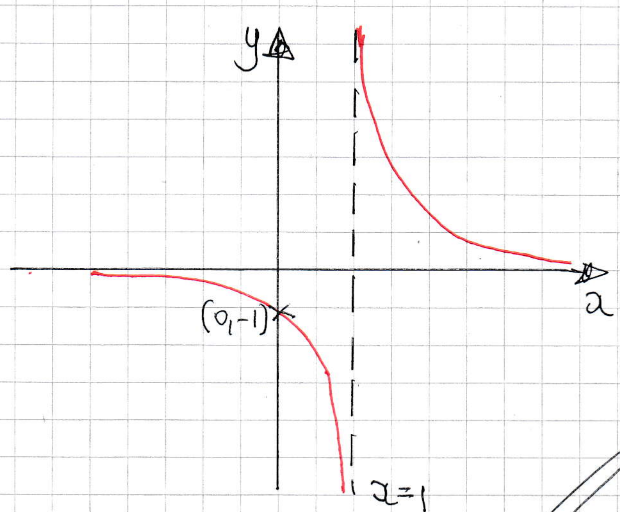
$$\& \quad Q-2P = 0$$

$$Q = 2P$$

$$Q = 192$$

IYGB – SYNOPTIC PAPER L – QUESTION 13

a) THIS IS A TRANSLATION OF $y = \frac{1}{x}$ BY 1 UNIT TO THE RIGHT



ASYMPTOTES
$y = 0$ & $x = 1$

b) WORKING FOR "INTERSECTIONS", ALTHOUGH WE ARE TOLD THERE ARE NOT ANY

$$\left.\begin{array}{l} y = \frac{1}{x-1} \\ y = a - 2x \end{array}\right\} \Rightarrow \frac{1}{x-1} = a - 2x$$

$$\Rightarrow 1 = (x-1)(a-2x)$$

$$\Rightarrow 1 = ax - 2x^2 - a + 2x$$

$$\Rightarrow 2x^2 - ax - 2x + 1 + a = 0$$

$$\Rightarrow 2x^2 - (a+2)x + (a+1) = 0$$

BUT THIS EQUATION HAS NO REAL ROOTS (NO INTERSECTIONS)

$$\Rightarrow B^2 - 4AC < 0$$

$$\Rightarrow [-(a+2)]^2 - 4 \times 2 \times (a+1) < 0$$

$$\Rightarrow (a+2)^2 - 8(a+1) < 0$$

IYGB – SYNOPTIC PAPER L – QUESTION 13

$$\Rightarrow \quad a^2 + 4a + 4 - 8a - 8 < 0$$

$$\Rightarrow \quad a^2 - 4a - 4 < 0$$

$$\Rightarrow \quad (a-2)^2 - 8 < 0$$

$$\Rightarrow \quad (a-2)^2 < 8$$

$$\Rightarrow \quad -\sqrt{8} < a - 2 < \sqrt{8}$$

$$\Rightarrow \quad 2 - \sqrt{8} < a < 2 + \sqrt{8}$$

$$\therefore \quad 2 - 2\sqrt{2} < a < 2 + 2\sqrt{2}$$

IYGB - SYNOPTIC PAPER L - QUESTION 14

a) REWRITE IN INDEX NOTATION & DIFFERENTIATE

$$y = \ln\left(x^{\frac{1}{2}}\right) + (\ln x)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{x^{\frac{1}{2}}} \times \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}(\ln x)^{-\frac{1}{2}} \times \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{2x} + \frac{1}{2x}(\ln x)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2x} + \frac{1}{2x\sqrt{\ln x}}$$

ALTERNATIVE FOR THE FIRST TERM

$$\frac{d}{dx}\left[\ln\sqrt{x}\right] = \frac{d}{dx}\left[\ln\left(x^{\frac{1}{2}}\right)\right] = \frac{d}{dx}\left[\frac{1}{2}\ln x\right] = \frac{1}{2x}$$

b) DIFFERENTIATING THE PRODUCT

$$y = (2x+1)^{\frac{1}{2}}(1-4x)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = (2x+1)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}} + (2x+1)^{\frac{1}{2}} \times \left(-\frac{1}{2}\right)(1-4x)^{-\frac{3}{2}}(-4x)$$

$$\frac{dy}{dx} = (2x+1)^{-\frac{1}{2}}(1-4x)^{-\frac{1}{2}} + 2(2x+1)^{\frac{1}{2}}(1-4x)^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = (2x+1)^{-\frac{1}{2}}(1-4x)^{-\frac{3}{2}}\left[(1-4x) + 2(2x+1)\right]$$

$$\frac{dy}{dx} = (2x+1)^{-\frac{1}{2}}(1-4x)^{-\frac{3}{2}}\left(1 - \cancel{4x} + \cancel{4x} + 2\right)$$

$$\frac{dy}{dx} = 3(2x+1)^{-\frac{1}{2}}(1-4x)^{-\frac{3}{2}}$$

IYGB - SYNOPTIC PAPER L - QUESTION 14

SOLVING FOR ZERO

$$\Longrightarrow \quad 3(1+x)^{-\frac{1}{2}}(1-4x)^{-\frac{3}{2}} = 0$$

$$\Longrightarrow \quad \frac{3}{2(1+x)^{\frac{1}{2}}(1-4x)^{3/2}} = 0$$

NO SOLUTIONS & HENCE NO TURNING POINTS //

c) BY THE QUOTIENT RULE

$$\frac{d}{dx}\left[\frac{2x-1}{(2x+1)^{\frac{1}{2}}}\right] = \frac{(2x+1)^{\frac{1}{2}}\times 2 - (2x-1)(2x+1)^{-\frac{1}{2}}\times\frac{1}{2}\times 2}{(2x+1)^1}$$

$$= \frac{2(2x+1)^{\frac{1}{2}} - (2x-1)(2x+1)^{-\frac{1}{2}}}{2x+1}$$

$$= \frac{(2x+1)^{-\frac{1}{2}}\left[2(2x+1) - (2x-1)\right]}{2x+1}$$

$$= \frac{(2x+1)^{-\frac{1}{2}}(4x+2-2x+1)}{2x+1} = \frac{2x+3}{(2x+1)^{\frac{1}{2}}(2x+1)}$$

$$= \frac{2x+3}{(2x+1)^{3/2}}$$

// AS REQUIRED

IYGB – SYNOPTIC PAPER L – QUESTION 15

START BY FINDING THE GRADIENT AT P

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{4\cos\theta}{-8\sin\theta} = -\frac{1}{2}\cot\theta$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = -\frac{1}{2}\cot\frac{\pi}{4} = -\frac{1}{2}$$

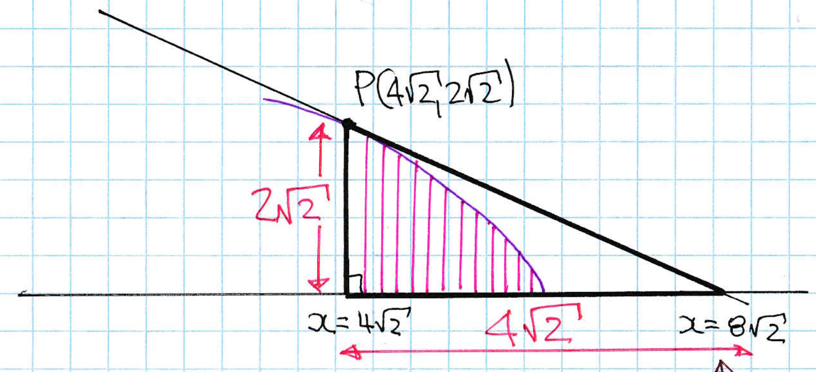
OBTAIN THE EQUATION OF THE TANGENT

- $x\Big|_{\frac{\pi}{4}} = 8\cos\frac{\pi}{4} = 8\times\frac{\sqrt{2}}{2} = 4\sqrt{2}$
- $y\Big|_{\frac{\pi}{4}} = 4\sin\frac{\pi}{4} = 4\times\frac{\sqrt{2}}{2} = 2\sqrt{2}$

I.E $P\left[4\sqrt{2}, 2\sqrt{2}\right]$

$$y - y_0 = m(x - x_0)$$
$$y - 2\sqrt{2} = -\frac{1}{2}(x - 4\sqrt{2})$$
$$2y - 4\sqrt{2} = -x + 4\sqrt{2}$$
$$2y + x = 8\sqrt{2}$$

NEXT WE FIND THE AREA OF THE TRIANGLE IN THE FOLLOWING DIAGRAM



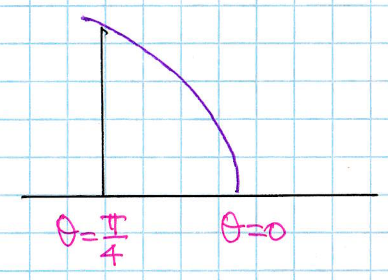
BY SETTING y=0 IN THE EQUATION OF THE TANGENT

$$\text{AREA}_{\triangle} = \frac{1}{2}\times 2\sqrt{2}\times 4\sqrt{2} = 8$$

FINALLY THE AREA SHOWN SHADED IN THE ABOVE DIAGRAM (AREA BETWEEN ELLIPSE & x AXIS)

FIRSTLY BY INSPECTION, THE ELLIPSE MEETS THE x AXIS AT $x = 8$, I.E $\theta = 0$

IYGB - QUESTION 15 - SYNOPTIC PAPER L



$$\text{AREA} = \int_{x_1}^{x_2} y(x)\,dx = \int_{\theta_1}^{\theta_2} y(\theta)\frac{dx}{d\theta}\,d\theta$$

$$\text{AREA} = \int_{\frac{\pi}{4}}^{0} 4\sin\theta\,(-8\sin\theta)\,d\theta$$

$$\text{AREA} = \int_{0}^{\frac{\pi}{4}} +32\sin^2\theta\,d\theta = \int_0^{\frac{\pi}{4}} 32\left(\tfrac{1}{2}-\tfrac{1}{2}\cos2\theta\right)d\theta$$

$$\text{AREA} = \int_0^{\frac{\pi}{4}} 16-16\cos2\theta\,d\theta = \left[16\theta - 8\sin2\theta\right]_0^{\frac{\pi}{4}}$$

$$\text{AREA} = (4\pi - 8) - (0)$$

$$\text{AREA} = 4\pi - 8$$

FINALLY THE REQUIRED AREA CAN BE FOUND

$$\Rightarrow \text{AREA OF TRIANGLE} - \text{AREA UNDER ELLIPSE} = 8 - (4\pi - 8)$$

$$= 16 - 4\pi$$

IYGB - SYNOPTIC PAPER L - QUESTION 16

- IF $f(x) = \sqrt{1+x^2}$ THEN $f(x+h) = \sqrt{1+(x+h)^2}$

- BY THE FORMAL DEFINITION OF THE DERIVATIVE

$$f'(x) = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim_{h \to 0}\left[\frac{\sqrt{1+(x+h)^2}-\sqrt{1+x^2}}{h}\right]$$

$$= \lim_{h \to 0}\left[\frac{\left[\sqrt{1+(x+h)^2}-\sqrt{1+x^2}\right]\left[\sqrt{1+(x+h)^2}+\sqrt{1+x^2}\right]}{h\left[\sqrt{1+(x+h)^2}+\sqrt{1+x^2}\right]}\right]$$

$$= \lim_{h \to 0}\left[\frac{\left[\cancel{1}+(x+h)^2\right]-\left[\cancel{1}+x^2\right]}{h\left[\sqrt{1+(x+h)^2}+\sqrt{1+x^2}\right]}\right]$$

$$= \lim_{h \to 0}\left[\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h\left[\sqrt{1+(x+h)^2}+\sqrt{1+x^2}\right]}\right]$$

$$= \lim_{h \to 0}\left[\frac{\cancel{h}(2x+h)}{\cancel{h}\left[\sqrt{1+(x+h)^2}+\sqrt{1+x^2}\right]}\right]$$

$$= \lim_{h \to 0}\left[\frac{2x+h}{\sqrt{1+(x+h)^2}+\sqrt{1+x^2}}\right]$$

$$= \frac{2x}{\sqrt{1+x^2}+\sqrt{1+x^2}}$$

$$= \frac{2x}{2\sqrt{1+x^2}}$$

$$= \frac{x}{\sqrt{1+x^2}}$$

//

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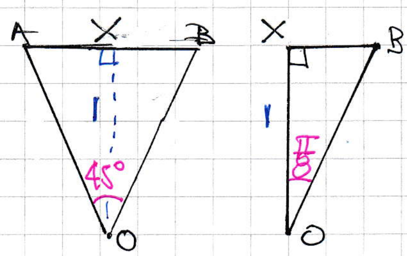
a) **LOOKING AT THE DIAGRAM**



$360 \div 8 = 45$ or $\frac{\pi}{4}$

$$\frac{|XB|}{|XO|} = \tan\frac{\pi}{8}$$

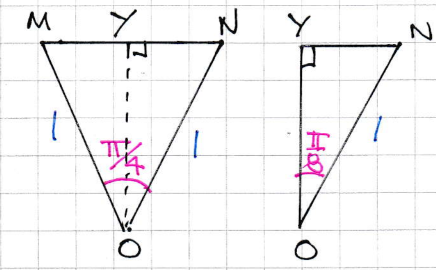
$$\frac{|XB|}{1} = \tan\frac{\pi}{8}$$

$$|XB| = \tan\frac{\pi}{8}$$

$$|AB| = 2\tan\frac{\pi}{8}$$

PERIMETER OF OCTAGON $= 8 \times 2\tan\frac{\pi}{8} = 16\tan\frac{\pi}{8}$ //

b) **LOOKING AT THE DIAGRAM AGAIN**



$$\frac{|YN|}{|ON|} = \sin\frac{\pi}{8}$$

$$\frac{|YN|}{1} = \sin\frac{\pi}{8}$$

$$|YN| = \sin\frac{\pi}{8}$$

$$|MN| = 2\sin\frac{\pi}{8}$$

PERIMETER OF OCTAGON $= 8 \times 2\sin\frac{\pi}{8} = 16\sin\frac{\pi}{8}$ //

c) **USING $\cos 2\theta \equiv 1 - 2\sin^2\theta$ WITH $\theta = \pi/8$**

$$\Rightarrow \cos\frac{\pi}{4} = 1 - 2\sin^2\frac{\pi}{8}$$

$$\Rightarrow 2\sin^2\frac{\pi}{8} = 1 - \cos\frac{\pi}{4}$$

$$\Rightarrow 2\sin^2\frac{\pi}{8} = 1 - \frac{\sqrt{2}}{2}$$

$$\Rightarrow \sin^2\frac{\pi}{8} = \frac{1}{2} - \frac{\sqrt{2}}{4}$$

$$\Rightarrow \sin^2\frac{\pi}{8} = \frac{2-\sqrt{2}}{4}$$

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$$\Rightarrow \sin\frac{\pi}{8} = +\sqrt{\frac{2-\sqrt{2}}{4}} \qquad \left(\frac{\pi}{8} \text{ IS ACUTE}\right)$$

$$\Rightarrow \sin\frac{\pi}{8} = \frac{1}{2}\sqrt{2-\sqrt{2}}$$

AS REQUIRED

d) USING THE DOUBLE ANGLE IDENTITY FOR $\tan 2\theta$ WITH $\theta = \pi/8$

$$\boxed{\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}}$$

$$\Rightarrow \tan\frac{\pi}{4} = \frac{2\tan \pi/8}{1-\tan^2\frac{\pi}{8}}$$

$$\Rightarrow 1 = \frac{2T}{1-T^2} \qquad \left\{T = \tan\frac{\pi}{8}\right\}$$

$$\Rightarrow 1 - T^2 = 2T$$

$$\Rightarrow 0 = T^2 + 2T - 1$$

$$\Rightarrow (T+1)^2 - 2 = 0$$

$$\Rightarrow (T+1)^2 = 2$$

$$\Rightarrow T + 1 = \pm\sqrt{2}$$

$$\Rightarrow T = -1 \pm \sqrt{2}$$

$$\Rightarrow \tan\frac{\pi}{8} = -1 \pm \sqrt{2}$$

$$\Rightarrow \tan\frac{\pi}{8} = -1 + \sqrt{2} \qquad \left(\text{AS } \frac{\pi}{8} \text{ IS ACUTE \& } -1-\sqrt{2} \text{ IS NEGATIVE}\right)$$

FINALLY WE HAVE

$$\text{PERIMETER } P_2 < \text{CIRCUMFERENCE } C < \text{PERIMETER } P_1$$

$$16\sin\frac{\pi}{8} < 2\pi \times 1 < 16\tan\frac{\pi}{8}$$

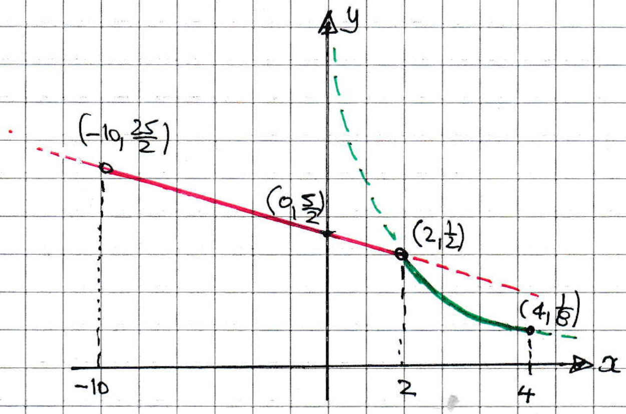
$$16\left(\frac{1}{2}\sqrt{2-\sqrt{2}}\right) < 2\pi < \left(-1+\sqrt{2}\right)\times 16$$

$$3.06 < \pi < 3.31$$

IYGB – SYNOPTIC PAPER L – QUESTION 18

START BY SKETCHING THE FUNCTION

$$f(x) = \begin{cases} \frac{5}{2} - x, & x \in \mathbb{R}, \ -10 < x < 2 \\ \frac{2}{x^2}, & x \in \mathbb{R} \ \ 2 \leq x \leq 4 \end{cases}$$



TREAT EACH SECTION SEPARATELY

$$\boxed{f_1(x) = \frac{5}{2} - x, \ -10 < x < 2}$$

$y = \frac{5}{2} - x$

$2y = 5 - 2x$

$2x = 5 - 2y$

$x = \frac{5}{2} - y$

$f_1^{-1}(x) = \frac{5}{2} - x$ (SELF INVERSE)

	$f_1(x)$	$f_1^{-1}(x)$
D	$-10 < x < 2$	$\frac{1}{2} < x < \frac{25}{2}$
R	$\frac{1}{2} < f_1(x) < \frac{25}{2}$	$-10 < f_1^{-1}(x) < 2$

$$\boxed{f_2(x) = \frac{2}{x^2}, \ 2 \leq x \leq 4}$$

$y = \frac{2}{x^2}$

$x^2 = \frac{2}{y}$

$x = \pm\sqrt{\frac{2}{y}}$

$x = +\sqrt{\frac{2}{y}}$

$f_2^{-1}(x) = \sqrt{\frac{2}{x}}$

	$f_2(x)$	$f_2^{-1}(x)$
D	$2 \leq x \leq 4$	$\frac{1}{8} \leq x \leq \frac{1}{2}$
R	$\frac{1}{8} \leq f_2(x) \leq \frac{1}{2}$	$2 \leq f_2^{-1}(x) \leq 4$

$$f^{-1}(x) = \begin{cases} \sqrt{\frac{2}{x}}, & x \in \mathbb{R}, \ \frac{1}{8} \leq x \leq \frac{1}{2} \\ \frac{5}{2} - x, & x \in \mathbb{R}, \ \frac{1}{2} < x < \frac{25}{2} \end{cases}$$

WITH RANGE $-10 < f^{-1}(x) \leq 4$

IYGB - SYNOPTIC PAPER L - QUESTION 19

$f(n) =$	$10, 13, 16, 19, 22, \ldots$	$3n+7$
$g(n) =$	$6, 12, 24, 48, 96, \ldots$	3×2^n

ADDING THE n^{th} TERMS

$$u_n = f(n) + g(n) = 3n + 7 + 3 \times 2^n$$

$$u_{n+1} = 3(n+1) + 7 + 3 \times 2^{n+1} = 3n + 10 + 3 \times 2^{n+1}$$

SUBTRACTING GIVES

$$u_{n+1} - u_n = \left(3n + 10 + 3 \times 2^{n+1}\right) - \left(3n + 7 + 3 \times 2^n\right)$$

$$= \cancel{3n} + 10 + 3 \times 2^{n+1} - \cancel{3n} - 7 - 3 \times 2^n$$

$$= 3 + 3 \times 2 \times 2^n - 3 \times 2^n$$

$$= 3 + 6 \times 2^n - 3 \times 2^n$$

$$= 3 + 3 \times 2^n$$

$$= 3(1 + 2^n)$$

FINALLY WE HAVE

$$u_{n+1} - u_n = 3(1 + 2^n)$$

$$u_{n+1} = u_n + 3(2^n + 1)$$

AS REQUIRED